Nowcasting Recessions using the SVM Machine Learning Algorithm*

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Abstract

We introduce a novel application of Support Vector Machines (SVM), an important Machine Learning algorithm, to determine the beginning and end of recessions in real time. Nowcasting, "forecasting" a condition about the present time because the full information about it is not available until later, is key for recessions, which are only determined months after the fact. We show that SVM has excellent predictive performance for this task, and we provide implementation details to facilitate its use in similar problems in economics and finance.

1. Introduction

Real-time business cycle dating is of central importance in modern macroeconomics. Recessions reflect great dislocation in the economy and are often the source of societal anxiety. During a recession, unemployment is usually higher, and output is lower. Accurately identifying turning points from expansions to recessions has broad use for policymakers, business executives, academics, and individuals. Additionally, investors with enough resources to use this information in their investment process may change their portfolios as the economy turns from growth to contraction.

There have been several attempts in the literature to accurately predict the timing of recessions. This paper applies an important Machine Learning algorithm, Support Vector Machines (SVM), to this task and compares its performance to other approaches. SVM is particularly useful when the amount of data available is limited, since it has a self-regularizing feature that reduces overfitting. It also has a built-in, in-sample metric that can be used to characterize the out-of-sample performance, a useful feature that most other methods do not have.

The National Bureau of Economic Research (NBER) provides the official dating of expansions and recessions. The NBER's Business Cycle Dating Committee periodically assesses the prevailing conditions in the macroeconomy and determines if the economy is in an expansion or a recession[1,2]. The committee releases announcements about the dates of the turning points. Due to the reliance on macroeconomic data, which may be revised or released with a lag, and the committee's preference for accuracy over timeliness, the NBER has historically announced turning points with a delay between four and 21 months (Giusto and Piger, 2017).

Is it possible to identify business cycle turning points in a more timely manner? This is the focus of a large body of literature going back to Burns and Mitchell (1946) and greatly expanded in a series of papers by Stock and Watson (1989, 2002). This question is also the

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[1] NBER uses "expansion" and "contraction" to describe business cycles. In this paper, we use recession and contraction interchangeably.

[2] NBER defines a recession as a "significant decline in economic activity spread across the economy, lasting more than a few months." The committee does not follow a fixed rule of labeling a recession as at least two consecutive quarters of negative GDP growth. (https://www.nber.org/cycles/jan08bcdc_memo.html)s

central focus of our paper. We use SVM, a simple but powerful machine learning algorithm, to identify turning points in the macroeconomy.

Machine learning algorithms have been shown to be useful tools in many settings outside of the social sciences, and only more recently have been adopted more extensively in economics. It is straightforward to set up SVM for classification, and computation is relatively cheap compared to prevailing models in the macroeconomic modeling literature. Dynamic factor Markov-switching model (Diebold and Rudebusch, 1996; Chauvet, 1998; Chauvet and Piger, 2008) and dynamic probit (Fossati, 2016) models are significantly more computationally intensive to estimate.

Traditional business cycle analysis treats recessions and expansions as intrinsic shifts in the macroeconomic process (Stock and Watson, 1989). These intrinsic shifts are often characterized by high non-linearities reflected by key state variables. SVM has shown to be a versatile and useful tool to capture non-linearities, hence it is a good candidate tool to apply in a recession-prediction exercise.

We start the paper with a brief description of SVM and a discussion of its advantages and disadvantages. We then provide details of our implementation of this algorithm. There are two main empirical choices: the kernel, which models the nonlinearity implemented by the algorithm, and the soft margin cost, which controls the tradeoff between model stability and penalty for misclassified data. We use a radial basis kernel (Scholkopf et al, 1997) and we select the soft margin cost parameter using 10-fold cross validation.

We include four variables as inputs to our SVM model: the monthly log difference in nonfarm payrolls, the log difference in the average monthly S&P 500 price level, the level of the production index from the Manufacturing ISM Report on Business, and the 10-year U.S. Treasury yield minus the federal funds rate. These four variables represent information from four important and distinct markets: the labor market, the stock market, the goods market, and the bond market.

We fit the model once a month to form nowcasts and forecasts. Model estimation occurs once all data referring to the previous calendar month become available. Our financial variables (the S&P 500 price and the slope of the yield curve) are available following the last trading session of each month. The Institute for Supply Management (ISM) report is released on the

first business day of each month. The Employment Situation Report (containing nonfarm payrolls) typically comes out on the first Friday of each month. Given this release schedule, model estimation, as well as the formulation of nowcasts and forecasts, usually occurs within the first seven days of the calendar month. *We are careful to not use variables that may not be available in real time or values that are later revised.* Only information available at the time of each nowcast or forecast is included in the model.

Our sample is from 1959 to 2018. We use the first 14 years to train the model before making the first nowcast in 1973. From 1959 to 1973, there are two recessions identified by the NBER in 1960 to 1961 and 1969 to 1970. These episodes provide targets to train the model.

There are six NBER-defined recessions from 1973 to 2018. The SVM model successfully captures all six events. The model identifies the transition from expansion to recession (and back) typically within one to three months of the NBER definition. We use classification error to formally evaluate the model performance. The SVM model misclassified 5.3% of the months from 1973 to 2018. To put this figure in perspective, recessions occurred 13% of the time. A trivial classifier that always predicts expansion would have a classification error of 13%. Not surprisingly, SVM greatly improves on this benchmark.

The SVM model is competitive with other recession-prediction models proposed in the literature. Diebold and Rudebusch (1996) propose the dynamic-factor Markov-switching (DFMS) model, Chauvet (1998) uses the DFMS model to forecast recessions, and Chauvet and Hamilton (2006) propose the GDP-based model; both are published by the Federal Reserve Bank of St. Louis. We transform the smoothed recession probabilities from the DFMS into a recession indicator variable in two ways. First, we classify all months with probability greater than 50% to be recessions. Second, we use the methodology put forward in Chauvet and Piger (2008). The DFMS model has a classification error of 4.7% for the cutoff rule and 9.3% for the Chauvet and Piger method. The GDP-based model has a classification error of 7.3%. Although the SVM model slightly underperforms the DFMS model with the cutoff rule, its results are available with minimal delay and it is easier to estimate than the DFMS model. The GDP-based model only uses one variable and makes quarterly predictions. Both elements may contribute to its higher classification error compared to SVM.

Giusto and Piger (2017) use learning vector quantization (LVQ) to identify business cycle turning points. Their model has a very low classification error (1.8%) from 1977 to 2013. In the same period, SVM has a classification error of 4.8%. While the LVQ model identifies recessions more effectively than SVM, it requires significant delays to do so: an average lag of 134 days for NBER peaks and an average lag of 234 days for NBER troughs. In comparison, the SVM model identifies these turning points with a delay less than 30 days. If the econometrician's objective is to minimize classification error, the LVQ model may be preferred over SVM. If real-time recession identification is the goal, SVM would be preferred.

We also investigate the ability of SVM to form forecasts of recessions. We form one, two, and three-month ahead forecasts using the SVM model and evaluate them using the same methodology as for the nowcasts. Classification error increases with forecast horizon, but the changes are modest. The one-month ahead forecasts have a classification error of 6.7%, whereas the two and three-month ahead forecasts have classification errors of 6.8% and 8.0%. Expansions and recessions are somewhat persistent phases, such that even without the most recent data, the SVM model can provide useful identification of business cycle turning points.

Another advantage of the SVM algorithm over others is that it automatically identifies critical points for the classification task within the training set. These points can help with the interpretability of the solution, often a difficult task for Machine Learning algorithms. We compare critical points of the SVM algorithm against the NBER chronology and offer some interpretation for how the SVM algorithm performs classification.

Our central contribution to is demonstrate the merit of a new methodology applied to a large existing macroeconomics literature on classifying recessions. SVM is a novel technique to be used for time series analysis in macroeconomics. We show it works well in classifying NBER-defined recessions. In an attempt to provide a link from the statistical and machine-learning methodology to the macroeconomic forecasting literature, we hope to expand the macroeconomist's toolkit to form superior nowcasts and forecasts.

Our paper fits into the large literature forecasting and nowcasting recessions. Existing approaches include the probit model, dynamic probit, and other linear probability models. Stock and Watson (1989) proposed a "recession index", a time series of probability that the economy will be in a recession in six months. Estrella and Mishkin (1998) show financial variables

contain information about future recessions using a static probit model. Wright (2006) also uses a probit model in investigating the usefulness of the Treasury yield curve in recession prediction. Kauppi and Saikkonen (2005) argue dynamic probit models are superior to static probit models in forecasting recessions. Compared to these studies, our work allows for more complex non-linearities, which expands the richness in dynamics that may be captured by our model.

There is also an emerging literature in using statistical and machine learning methods in macroeconomic time series analysis. Qi (2001) applies neural networks to forecasting recessions and identifies several useful variables. Berge (2015) examines using boosting in recession prediction and finds improvement to a Bayesian model averaging benchmark. Giusto and Piger (2017) use learning vector quantization to identify recessions, which results in shorter time lag compared to the NBER announcements. Our paper complements these papers in demonstrating the efficacy of machine learning algorithms for macroeconomic applications.

The paper is laid out as follows. Section 2 starts with an overview of support vector machines and a discussion of our implementation of SVM. Section 3 presents the data, our empirical modeling choices, and the nowcasting and forecasting results. We summarize our conclusions in Section 4. Technical details about the use of SVM can be found in the Appendix.

2. Support Vector Machine

SVM is a popular and flexible method for classification. It is widely used in many areas, including text and hypertext categorization, hand-written character recognition, image classification, and numerous others. Its theoretical development was mainly due to Vapnik and can be found in Vapnik (1996). We provide an overview of SVM in the following sections. For a more in-depth introduction, please see Friedman et al. (2009).

2.1 Linear SVM

We start with a discussion of the linear SVM, the building block of more sophisticated SVM algorithms. Consider a classification problem with two classes, labeled as $y_i \in \{-1,1\}$. We have predictor variables x_i paired with y_i. Suppose the positive and negative classes are linearly separable, then we can construct a hyperplane to completely separate the two classes:

$$\{x: x^T \beta + \beta_0 = 0\}$$

The normal vector β and constant β_0 could be rescaled by a constant and have no effect on the hyperplane (2β, $2\beta_0$ give the same hyperplane). To uniquely identify the parameters, let us impose the unit norm on β, $\|\beta\| = 1$, where $\|.\|$ is the Euclidean norm. We may classify the data points according to the following rule:

$$y_i = 1 \ if \ x_i^T \beta + \beta_0 > 0$$

$$y_i = -1 \ if \ x_i^T \beta + \beta_0 < 0$$

This setup gives us a hyperplane that separates the two classes such that $y_i(x_i^T\beta + \beta_0) > 0$ for all i. To induce the largest separation between the two classes, we want to maximize the margin $y_i(x_i^T\beta + \beta_0)$. This leads to the formal optimization problem:

$$\underset{\beta,\beta_0}{\text{Maximize}} \ M \quad s.t. \quad y_i(x_i^T\beta + \beta_0) \geq M, \forall i \quad (1)$$

$$\|\beta\| = 1$$

where M is the margin, or the closest distance between each class and the separating hyperplane. The two classes are separated by a distance of $2M$. This setup is known as a "maximum margin classifier" because it picks the hyperplane with the largest separation between the positive and negative classes. The above problem cannot be solved using standard optimization methods because it is not convex. Friedman et al. (2009) demonstrate that the above optimization problem can be transformed into the following convex problem:

$$\underset{\beta,\beta_0}{\text{Minimize}} \ \|\beta\| \quad s.t. \quad y_i(x_i^T\beta + \beta_0) \geq 1, \forall i \quad (2)$$

This optimization problem is convex and can be solved using standard optimization packages.

Now suppose the two classes are not perfectly separable by a hyperplane. We still want to provide the largest separation between the two classes, but now possibly allowing for some data points to be misclassified. We can introduce slack variables ξ_i to allow for misclassification:

$$\underset{\beta,\beta_0}{\text{Minimize}} \ \|\beta\| \quad s.t. \quad y_i(x_i^T\beta + \beta_0) \geq 1 - \xi_i, \forall i \quad (3)$$

$$\xi_i \geq 0$$

$$\sum_i \xi_i \leq C$$

Note that (3) is just (2) with slack variables to allow for potential misclassification. If $\xi_i < 1$, i is still classified correctly, although it violates the margin. Misclassification occurs if $\xi_i > 1$. The last constraint limits the total number of misclassifications by restricting the sum of all ξ_i to be smaller than some constant C, called the soft margin cost.

2.2 Nonlinear SVMs

Nonlinear SVM (referred to simply as SVM) extends the idea of the linear version. If the original data can be easily separated by a hyperplane, linear SVM performs well. However, in many applications, the original data may not be linearly separable even if we allow for misclassification. SVM solves this problem by mapping the original feature space into a higher-dimensional space, then looks for a separating hyperplane in the higher-dimensional space. If a separating hyperplane is found, it can be mapped back into the original feature space. SVM performs this mapping using a kernel function. The kernel function is used to compute inner products in the transformed, higher-dimensional space and can be viewed as a measure of similarity.

SVM has several advantages compared to other classification methods. Different choices of kernel functions make this method versatile and effective in high-dimensional space. It is even effective when the number of dimensions is greater than the number of samples because of its self-regularizing nature (see Appendix for more details). SVM can also be used for unsupervised learning (Ben-Hur et al., 2001) and regressions (Drucker et al., 1997).

There are also some disadvantages in using SVM. The choice of kernel function can be a double-edged sword, and a poor choice could lead to undesirable results. SVM does not directly produce probability estimates, but rather just the final classification outcome. Inference for SVM can be a challenge, since the parameters may be difficult to interpret. For our purposes, prediction is the ultimate goal and we relegate inference to a lower priority.

2.3 SVM Implementation

We implement a standard SVM as outlined above, using a radial basis kernel. The radial basis kernel is a common choice for SVM implementation. It can be represented as follows:

$$K(u, v) = \exp(-\gamma \, \|u - v\|^2)$$

where $\|u - v\|^2$ is the squared Euclidean distance between the two vectors u and v. γ controls the size of the kernel. Small γ is associated with large variance for the Gaussian kernels, so if v is a support vector, it will influence the classification of u even if the distance between them is large. Large γ is associated with small variance for the Gaussian kernels, so the support vectors have short influence on deciding the class. The size of γ also relates to how much the Gaussian kernels will overlap with one another. As such, the choice of γ is related to the dimensionality of the input space, which depends on the number of input variables. The ideal choice of γ scales each of the Gaussian distributions to have partial overlaps with others, but not too much.

If in each coordinate the variable is Gaussian with the same variance, the total variance of the resulting high-dimensional Gaussian would be proportional to the number of dimensions. Since γ is inversely proportional to the variance, it would be inversely proportional to the number of dimensions of the input space. This consideration leads to a heuristic choice of γ that equals the inverse of the number of predictors.

The other important parameter choice is the soft margin cost function C, the cost of misclassification. This parameter controls the tradeoff between model stability and penalizing for misclassified data points. We select C with the following procedure. We start with a set of candidate values for C. On each of the first 100 backtest dates, we run each candidate model (with a different C) 100 times using 10-fold cross validation. We compute a regret measure for each date by taking the difference between each cross-validation error and the lowest cross-validation error of that date. This measure allows us to compare models across different samples. We then select the value of C that minimizes the total regret over the 100 backtest dates.

3. Classifying Recessions

We use SVM to classify NBER-defined recessions. We first discuss the target variable, followed by our choice of predictors, then provide a discussion of our results.

3.1 Data

The National Bureau of Economic Research (NBER) announces a set of dates for the U.S. business cycle expansions and contractions[3]. The NBER's Business Cycle Dating Committee uses data from the National Income and Product Accounts (NIPA) to determine the dates for economic expansions and recessions. Expansions are defined as the previous trough to the current peak in economic activity; recessions are defined as previous peak to the current trough. Additionally, economic cycles defined as trough to trough and peak to peak are available.

Our model is monthly, the same frequency as the NBER recession indicator variable. NBER-defined recessions are marked as the positive class and expansions are marked as the negative class. Our goal is to perform classification on the positive and negative classes using SVM.

We use four monthly series that contain information about the prevailing state of the macroeconomy. These four variables broadly capture information from four distinct markets:

1. Monthly log difference in nonfarm payrolls from the Employment Situation Report published by the Bureau of Labor Statistics (BLS). We use the same transformation as the datasets compiled by the Federal Reserve Bank of St. Louis. This variable has been used in multiple past studies (see Camacho et al., 2012) and captures the conditions in the labor market.

2. Log difference in average monthly price of the S&P 500 (Estrella and Mishkin, 1998; Qi, 2001). This series captures information from the stock market.

3. Production index from the Manufacturing ISM Report on Business published by the Institute for Supply Management. Lahiri and Monokroussos (2013) show that this variable contains useful information for forecasting GDP. Bok et al. (2018) uses this

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[3] https://www.nber.org/cycles.html.

variable for macroeconomic nowcasting. We extend this idea to nowcasting recessions. This varaible provides information about the goods market.

4. 10-year Treasury yield minus the federal funds rate. Numerous academic papers have shown the usefulness of this variable in forecasting recessions (Stock and Watson, 1989; Wright, 2006). Also known as the term spread, the slope of the Treasury yield curve reflects conditions in the bond market.

Numerous papers have investigated predictive variables for recessions. We use four variables that have been proposed in the literature. There may be a potential selection bias among these variables stemming from a multiple comparisons problem, as failed predictors are rarely published. To the extent there is a selection bias among these variables, our model will inherit that bias.

We restrict the number of variables to keep the model parsimonious. The small number of variables is comparable to other studies using machine learning techniques to produce reduced-form forecasts in macroeconomics. For example, Davig and Hall (2017) use the same four variables in a Naïve Bayes classifier to forecast recessions. Giusto and Piger (2017) also use four variables, albeit somewhat different from our choices, as inputs for real-time identification of recessions using an algorithm called learning vector quantization.

In modern macroeconomics, the economy is commonly thought to be driven by a small set of unobserved state variables. Observed macroeconomic variables are combinations of these latent state variables plus some measurement error. The latent state variables are slow-moving and contain some lead-lag structure. Given this consideration, we not only include the current value of the four variables above, but also 11 lags to allow for any persistent effects in the underlying macroeconomy. In effect, we are using one year of information of each of the four variables as input to our SVM model, for a total of 48 input variables.

Macroeconomic data are often revised. At a given point in time, the data available in real-time up to that point is known as the vintage series. If there are revisions to the historical data in the next period, those vintage data are overwritten with the revised series. Using the revised series may build in a look-ahead bias in forecasting, as we are using information we would not have had in real-time. To alleviate this problem, we are careful in considering vintage variables in our forecasts. At each point in time, we only use real-time information that the

econometrician could have had access to. We do not use revised data until the time they are known.

3.2 Modeling Choices

We estimate our model monthly when all the data become available for that month. S&P 500 and Treasury yields are available daily. Manufacturing ISM Report on Business typically comes out on the first business day of each month at 10 am Eastern time[4]. The Bureau of Labor Statistics releases the Employment Situation Report typically on the first Friday of each month[5]. This report contains nonfarm payroll data, as well as other macroeconomic data including the unemployment rate, average hourly earnings, etc. We run our model after nonfarm payroll is released by the BLS. The SVM nowcasts produces at the beginning of month t are evaluated against the NBER definition in month t-1. Therefore, the SVM model produces real-time forecasts with a delay equal to the time between the end of one month and the first Friday of the following month, typically fewer than 10 days. The SVM forecasts are compared to the NBER definition in month t, t+1, or t+2 depending on the forecast horizon.

The NBER's Business Cycle Dating Committee uses NIPA data to determine whether the economy is in an expansion or a recession. NIPA data are often released with a lag and are sometimes revised. As a result, the NBER business cycle dates have historically been reported with a delay between four and 21 months (Giusto and Piger, 2017). We are mindful of this lag in reporting, and we are careful in our modeling choices to account for it. When we fit the model, we stop the training period 12 months before the date of estimation.

We use an expanding window when estimating our model. The start date of our estimation is January 1959, comparable to other studies (for example, Davig and Hall, 2017 also starts in January 1959). We evaluate the model nowcasts and forecasts starting in 1973, to allow for a 14-year training period from 1959 to 1973. In this period, there were two recessions, one from April 1960 to February 1961, and another from December 1969 to November 1970. These events provide the positive examples that our model is initially trained on.

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[4] https://www.instituteforsupplymanagement.org/ISMReport/content.cfm?ItemNumber=10745&SSO=1
[5] https://www.bls.gov/news.release/empsit.nr0.htm

Our model produces nowcasts and forecasts of the macroeconomy and either classifies the prevailing conditions as expansion or recession. We compare our model predictions with the NBER business-cycle chronology to evaluate model performance.

3.3 SVM Nowcasting

We evaluate the usefulness of the SVM model by comparing the predicted expansions and recessions to those from the official NBER business cycle chronology. Figure 1 displays recession predictions from the SVM model. The official NBER recession dates are shown in grey bars. There were six NBER recessions in our sample from 1973 to 2018, and the SVM model was able to identify all six events. However, SVM model does not always identify the exact month of the recession according to the NBER definition. To understand the differences between NBER recessions and the SVM nowcasts, we compile the dates of peaks and troughs of each business cycle in Table 1.

SVM Recession Predictions
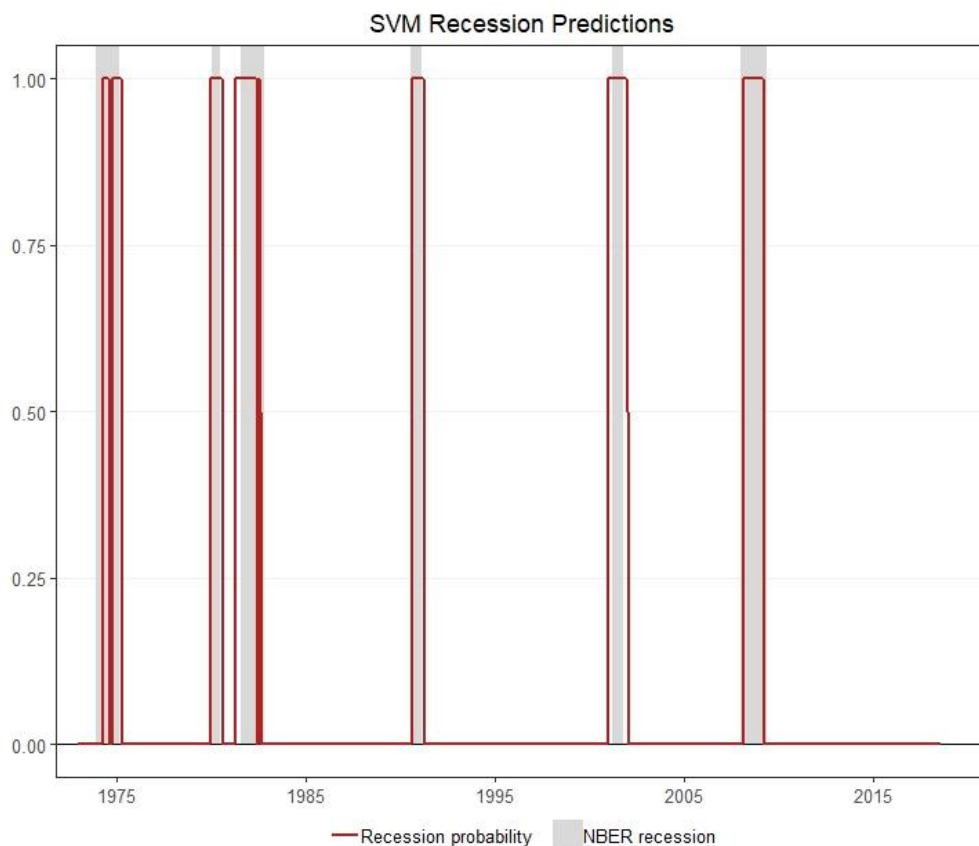
—Recession probability   NBER recession

Figure 1: SVM Recession Predictions. The vertical axis shows the recession probability. The red line represents the recession predictions from the SVM model. The official NBER recessions are shaded in grey. The sample is from 1973 to 2018.

Table 1 illustrates that the SVM model identifies NBER peaks and troughs within one to three months of the NBER's own definitions. Using four variables and their lags, the SVM model provides real-time identification of business cycle turning points whereas the NBER announces their business cycle dating with a four to 21-month lag. While SVM does not identify business cycle turning points to the exact month, its simplicity and timeliness are clear advantages over the NBER methodology.

How does the SVM model compare to other recession-prediction models? We compare the SVM model to two models published by the Federal Reserve Bank of St. Louis: the dynamic-factor Markov-switching model and the GDP-based recession prediction model.

NBER Peak	SVM Peak	NBER Trough	SVM Trough
Nov 1973	Apr 1974	Mar 1975	Apr 1975
Jan 1980	Dec 1979	Jul 1980	Aug 1980
Jul 1981	Apr 1981	Nov 1982	Aug 1982
Jul 1990	Aug 1990	Mar 1991	Apr 1991
Mar 2001	Dec 2000	Nov 2001	Jan 2002
Dec 2007	Feb 2008	Jun 2009	Mar 2009

Table 1: SVM Predictions of NBER Turning Points. We compare the NBER defined peaks and troughs with those using SVM.

The original work on DFMS model is due to Diebold and Rudebusch (1996), and Chauvet (1998) uses four monthly variables to identify business cycle turning points. Chauvet and Piger (2008) expand Chauvet (1998) to analyze the performance of the DFMS model. The St. Louis Fed releases the smoothed U.S. recession probabilities from the DFMS model. We take these probabilities and impose a threshold of 50% for classification. If the probability of recession is greater than or equal to 50%, we will classify that month as being in a recession. If the probability is lower than 50%, it is classified as an expansionary month.

Chauvet and Hamilton (2006) propose a GDP-based recession prediction model. The model identifies recessions at the quarterly frequency for the quarter just preceding the most recently available GDP numbers. The St. Louis Fed publishes both the probability of recession and the time series of GDP-based recession indicator. We compare the SVM model to the latter, transforming the quarterly series into by assuming the three months within the quarter are all in the same class, either recession or expansion. Admittedly, comparing a quarterly series of recession predictions to monthly models may put the quarterly model at a disadvantage due to coarser granularity.

We compare SVM, DFMS, and GDP-based models on the metric of classification error. Classification error is a simple measure of how classification models perform when compared to the actual outcomes. It is defined as follows:

$$Error = \frac{\#\ Incorrect\ observations}{\#\ Total\ observations}$$

where the classification $Error$ for a model is the ratio of the number of incorrectly classified data points to the total number of observations. Incorrectly classified data are a combination of Type I and Type II errors. In our setting, an incorrect observation corresponds to if the model classified a month as a recession but NBER classified it as an expansion, or if the model classified a month as an expansion but the NBER classified it as a recession. We present the classification errors for the three models in Table 2.

The Federal Reserve Bank of St. Louis publishes the smoothed recession probabilities from the DFMS model. We transform the probabilities into recession predictions in two ways. First, we use a simple cutoff rule: if the probability exceeds 50% in a month, that month is classified as a recession. Second, we use the methodology outlined in Chauvet and Piger (2008). Three consecutive months of probabilities greater than 80% indicates the beginning of a new recession; three consecutive months of probabilities lower than 20% indicates the beginning of a new expansion. These are marked as "DFMS (50% cutoff)" and "DFMS (CP)" in Table 2.

From 1973 to 2018, recessions occurred 13% of the time. If a model always predicts expansion, it would have a classification error of 13%. All models in Table 2 have errors much lower than this naïve model. The SVM model has a classification error of 5.3%. The DFMS model with a 50% cutoff rule has a slightly lower rate of 4.7%, indicating the more sophisticated

structure of DFMS may capture some nuances beyond the SVM model. Using the three-month rule put forward by Chauvet and Piger (2008) results in a larger classification error of 9.3% for the DFMS. Compared to SVM, DFMS is much more computationally intensive to estimate. SVM provides a simpler alternative which achieves a similar classification error.

Another attractive aspect of SVM is its built-in, in-sample metric that can be used to characterize out-of-sample performance. This generalization result can be stated as:

$$E(e_{out}) \leq \frac{E[\# \ of \ support \ vectors]}{N}$$

where e_{out} is the out-of-sample classification error estimated by cross validation, and the expected value is taken with respect to different data sets. Intuitively, support vectors correspond to the effective parameters used in an SVM model, so this generalization result relates the expected out-of-sample error to the ratio of number of parameters to the number of observations. See the Appendix for more details. This result allows us to place at least a somewhat loose upper bound on the out-of-sample performance of the model. In our SVM model, the ratio of support vectors to observations declines steadily over the sample period – starting from a high of 43% to a low of 26% on the most recent backtest date. Although the bound does not pin down e_{out} exactly, its value is consistent with the fact that the out-of-sample performance has improved over time as the expanding training window includes more recession examples from which to learn.

Classification error 1973-2018			
SVM	DFMS (50% cutoff)	DFMS (CP)	GDP-Based
5.3%	4.7%	9.3%	7.3%

Table 2: Classification Errors for SVM, DFMS, and GDP-based Models. Classification error is calculated as $Error = \frac{\# \ Incorrect \ observations}{\# \ Total \ observations}$, where $\# \ Incorrect \ observations$ is the total number of observations that are not classified correctly. $\# \ Total \ observations$ is the total number of data points for which the models are evaluated. We transform the DFMS smoothed recession probabilities from the St. Louis Fed into recession predictions in two ways. First, we

use a 50% cutoff; any month with probability greater than 50% is classified as a recession. Second, we use the methodology from Chauvet and Piger (2008): three consecutive months of probabilities greater than 80% indicates the start of a new recession; three consecutive months of probabilities below 20% indicates the start of a new expansion.

The DFMS model presents a smooth recession probability, which uses the most recent data available, and is potentially influenced by data that was not available the first time a recession probability for a given month was calculated. In contract, the SVM model produces nowcasts in real-time, using the most recent data and never revising the predictions in the past. The DFMS model also has a two-month lag in reporting the recession probabilities because one of the inputs is real manufacturing and trade sales produced by the U.S. Census Bureau, which is only available after a two-month lag. The SVM model remains competitive to the DFMS model even without any revisions or a two-month lag.

The GDP-based model has a higher classification error of 7.3% in this period. We conjecture this result is due to the combination of only use GDP as the input and coarser output frequency. The SVM model uses a broader data set and produces a monthly series, allowing greater flexibility compared to the GDP-based model.

Giusto and Piger (2017) propose using learning vector quantization model to identify business cycle turning points. Their sample is from 1976 through 2013. We compare our results to theirs in this sample period. The classification errors of these two models are reported in Table 3.

The LVQ model has a lower classification error compared to SVM. As indicated in their paper, Giusto and Piger (2017) demonstrate that the LVQ model typically identifies business cycle peaks and troughs within one month of the NBER definitions. While the LVQ identifies recessions more effectively than SVM, it requires significant delay to do so. Giusto and Piger (2017) are on average 134 days late in identifying NBER peaks (compared to 224-day delay from the NBER) and 234 days late in identifying NBER troughs (446-day delay from the NBER). In contrast, SVM can identify recessions with a much shorter delay of under 10 days. If the econometrician's objective is to minimize classification error, then LVQ may be preferred over SVM. If the econometrician values real-time identification, SVM is the preferred model.

Classification Error 1976-2013	
SVM	LVQ
4.8%	1.8%

Table 3: Classification Errors for SVM and LVQ Models, 1977-2013. Classification error is calculated as $Error = \frac{\#\ Incorrect\ observations}{\#\ Total\ observations}$, where $\#\ Incorrect\ observations$ is the total number of observations that are not classified correctly. $\#\ Total\ observations$ is the total number of data points for which the models are evaluated.

3.4 SVM Forecasting

We investigate the efficacy of SVM as a forecasting model, using the information set available in real time to predict whether the economy will be in a recession in the future. We use the same set of four variables and their lags in our model and maintain our model parameter choices from the previous section. The classification error from SVM forecasts are shown in Table 4.

SVM Forecast Classification Error		
1-Month	2-Month	3-Month
6.7%	6.8%	8.0%

Table 4: Classification Errors for SVM Forecasts. We compute classification error for forecasts made using the SVM model. Forecast horizons include one, two, and three months. The sample is 1973 to 2018.

From 1973 to 2018, the SVM model produced nowcasts with a classification error of 5.3%. At the one-month forecasting horizon, this figure increase to 6.7%. Perhaps not surprisingly, as the forecast horizon increases, the classification error also increases. At the two-month horizon, the classification error is 6.8%. At the three-month horizon, it is 8.0%. Forecasts made at longer

horizons do not use the most recent information, which results in noisier identification of the macroeconomic conditions.

3.5 SVM Critical Points

Another advantage of the SVM algorithm over others is that it automatically identifies critical points for the classification task within the training set. These points can help with the interpretability of the solution, often a difficult task for Machine Learning algorithms that many applications require for practical purposes.

The critical points are the support vectors that define the SVM classification boundary. Technically, support vectors are training points that have a positive Lagrange multiplier $\alpha > 0$ in the SVM solution (please see the Appendix). The SVM algorithm produces two types of support vectors, which play different roles:

1. Margin support vectors are critical points that are reliably classified into one of the categories. They achieve the margin with no error ($\xi_i = 0$).
2. Non-margin support vectors are critical points that are less reliably classified. They violate the margin ($\xi_i > 0$). The violation can be slight so as not to alter their correct classification, or severe enough to alter that classification and result in an in-sample error.

In Figure 2, we plot margin support vectors and non-margin support vectors separately. The y-coordinate indicates the value of α for each support vector. Notice that the plot shows the month of each training point including those that end up being support vectors, but it does not show the input variables the algorithm used to decide the support vectors. A support vector at the transition between recession and expansion only depends on the input variables without knowing the actual timing of recessions and expansions.

We train the SVM model using the full sample and compare α against the NBER chronology. α corresponding to expansions are colored red; α corresponding to recessions are colored blue. The left panel shows the case for margin support vectors (with $\xi_i = 0$) and the right panel show non-margin support vectors (with $\xi_i > 0$).
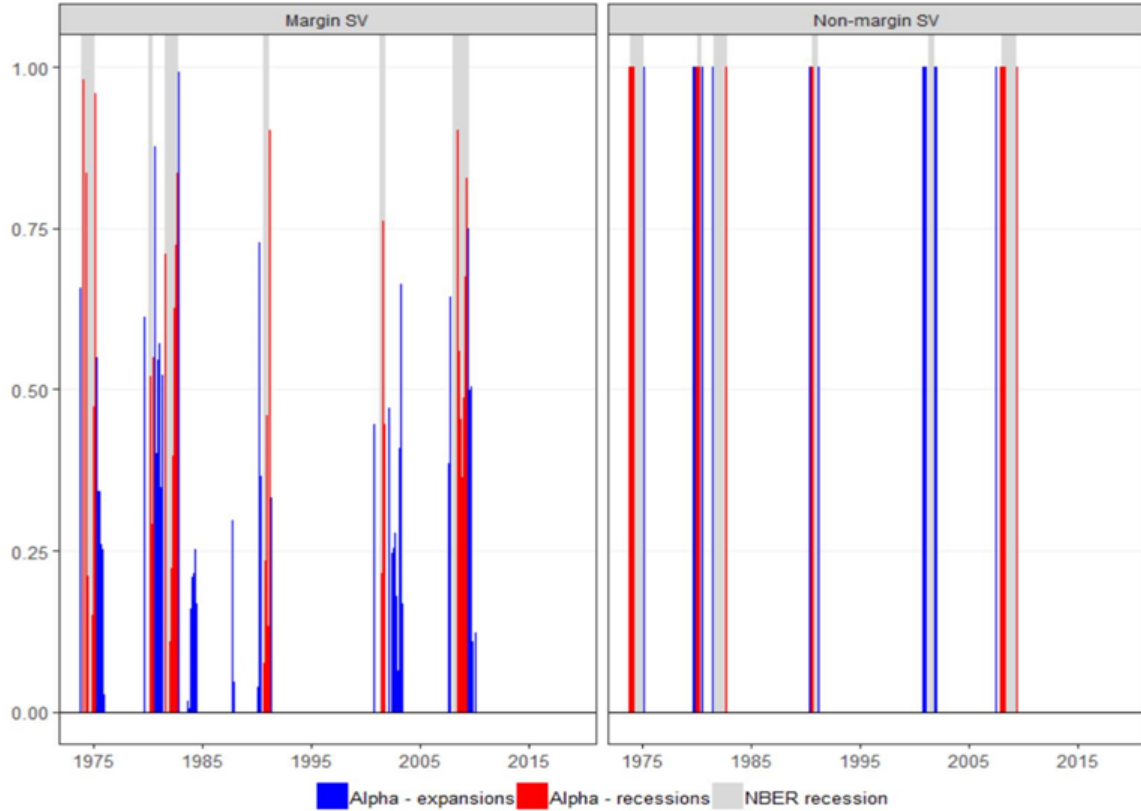
Figure 2:SVM Dual Parameter and NBER Recessions. We plot the dual parameter *α* of SVM. NBER recessions are shaded in gray.

The non-margin support vectors are all at the boundaries between expansions and recessions, where classification is the most difficult, leading to margins being violated ($\xi_i > 0$). In contrast, the margin support vectors are inside recession or expansion cycles without crossing over to the other class, which leads to $\xi_i = 0$. Furthermore, many non-margin support vectors associated with the recession class correspond to the beginning months of some of the worst recessions in our sample: the 1973-1975 downturn, the first of the double-dip recessions in the early 1980s, and the Financial Crisis.

4. Conclusion

Many problems in economics and finance can be viewed as prediction exercises. Statistical and machine learning techniques, with their impressive predictive power, are

becoming increasingly more common in economics and finance applications. In this paper, we address a classic issue in macroeconomics, recession identification and prediction, using support vector machines. We find SVM is a useful tool for this application.

We consider four variables as inputs to SVM: the monthly log difference in nonfarm payrolls, the log difference in the average monthly price of the S&P 500 price level, the level of the production index from the Manufacturing ISM Report on Business, and the 10-year U.S. Treasury yield minus the federal funds rate. These variables reflect prevailing conditions in the labor market, stock market, goods market, and bond market. For SVM, we use a radial basis kernel, and the soft margin cost is chosen using10-fold cross validation. We train our model from 1959 to 1973 and compare model predictions to NBER chronology from 1973 to 2018.

The SVM model achieves a classification error of 5.3% from 1973 to 2018. In comparison, the dynamic factor Markov-switching model (Diebold and Rudebusch, 1996; Chauvet, 1998) has a classification error of 4.7% or 9.3%, depending on how probabilities are transformed into a recession indicator variable, and the GDP-based model of Chauvet and Hamilton (2006) has a classification error of 7.3%. While the DFMS slightly outperforms the SVM model, it is more difficult to set up and computationally intensive to estimate. The SVM model could also be used to forecast recessions. The one, two, and three-month forecasts result in classification errors of 6.7%, 6.8%, and 8.0%.

Predicting turning points is inherently interesting, but a focus on recession prediction classifies the economy into just two phases. Within a recession or expansion, the economy does not behave in a uniform way. As such, other models favor alternative approaches which may help uncover intra-phase behavior. For example, Aruoba et al. (2009) produces cardinal measurements for the prevailing macroeconomic conditions. Researchers may also consider combining distinct models for forecasting recessions and for the "level" of the economy. Each model has its own value for investors and policymakers.

One interesting future research direction would be to expand the data to include other countries. An out-of-sample test provides the best evaluation of statistical models. In particular, it would be interesting to see if SVM can help identify Euro area expansions and recessions, using as training data the chronology maintained by the Business Cycle Dating Committee at the Centre for Economic Policy Research (CEPR). Moving beyond OECD countries, China and

India could provide relevant test cases – although data limitations may hamper model performance. Another potentially interesting research direction is to apply SVM to forecast continuous target variables (support vector regression) in macroeconomics. SVM may be used to forecast GDP growth, unemployment, industrial production, or any other key macroeconomic variables.

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Appendix

We summarize some technical details of SVM – highlighting the self-regularizing feature of the method as well as the built-in, in-sample metric that can be used to characterize out-of-sample performance.

To illustrate these points, we consider a classification problem with two classes, labeled as $y_i \in \{-1,1\}$. Additionally, we have d predictor variables contained in the vector x_i which are paired with each y_i. Let N denote the number of data points in our sample. If we assume that the two classes are linearly separable, then we can construct a hyperplane such that the in-sample classification error is zero. We describe this hyperplane as

$$\{x : x^T\beta + \beta_0 = 0\}$$

There are an infinite number of such separating hyperplanes, but SVM seeks the one with the largest "margin," which is simply the distance between the separating hyperplane and the nearest data point. In Section 2.1, we stated – without proof – that the problem of finding this maximum margin hyperplane could be expressed as the following optimization problem:

$$\underset{\beta,\beta_0}{\text{Minimize}} \ \|\beta\| \quad s.t. \quad y_i(x_i^T\beta + \beta_0) \geq 1, \forall i$$

Our goal now is to demonstrate that the solution to this formulation is indeed the separating hyperplane with the maximum margin. To this aim, let x_n be the nearest data point to the hyperplane $x^T\beta + \beta_0 = 0$. It will be convenient to normalize (β, β_0) such that $|x_n^T\beta + \beta_0| = 1$. Note that this normalization is without loss of generality, since any rescaling of (β, β_0) does not change the hyperplane itself.

To find the distance between x_n and the hyperplane, note that β is orthogonal to the hyperplane. This is straightforward to show. Consider points x_p and $x_{p'}$ on the hyperplane. Since the points lie on the hyperplane, it must be the case that $x_p^T\beta + \beta_0 = 0$ and $x_{p'}^T\beta + \beta_0 = 0$. Subtracting these two equations, we get

$$\left(x_p^T\beta + \beta_0\right) - \left(x_{p'}^T\beta + \beta_0\right) = 0$$
$$(x_p - x_{p'})^T\beta = 0$$

Since x_p and $x_{p'}$ are arbitrary points on the hyperplane, β must be perpendicular to the hyperplane. Geometrically, this means that the distance between x_n and the hyperplane is equal to the absolute value of the projection of $x_n - x$ onto β, where x is any point on the hyperplane. Recall that the projection is the inner product between $x_n - x$ and $\hat{\beta}$, where $\hat{\beta} = \frac{\beta}{\|\beta\|}$ is of unit length. So, we have:

$$distance = \left|\hat{\beta}^T(x_n - x)\right|$$

$$= \frac{1}{\|\beta\|}\left|\beta^T x_n - \beta^T x\right|$$

$$= \frac{1}{\|\beta\|}\left|(\beta^T x_n + \beta_0) - (\beta^T x + \beta_0)\right|$$

$$= \frac{1}{\|\beta\|}\left|(\beta^T x_n + \beta_0) - (0)\right|$$

$$= \frac{1}{\|\beta\|}\left|(1) - (0)\right|$$

$$= \frac{1}{\|\beta\|}$$

where the third equality comes from adding and subtracting β_0, the fourth equality comes from the fact that x lies on the hyperplane, and the fifth equality comes from the normalization of β.

At this point, we have shown that the problem of finding the hyperplane with the maximum margin can be expressed as:

$$\underset{\beta, \beta_0}{\text{Maximize}} \ \frac{1}{\|\beta\|} \quad s.t. \quad \min_{n=1,2,\dots,N} \left|x_n^T \beta + \beta_0\right| = 1 \quad (1a)$$

where the constraint reflects our normalization of (β, β_0). To make this problem tractable, let's express it in an equivalent form but without the minimum function in the constraint. To do this, observe that

$$\left|x_n^T \beta + \beta_0\right| = y_n(x_n^T \beta + \beta_0) \ \forall n = 1, 2, \dots, N \quad (2a)$$

This follows because the hyperplane is assumed to separate the data correctly, so the class label ($\{-1, 1\}$) must agree with the signal $(x_n^T \beta + \beta_0)$ – meaning that the product will always be positive. With Equation (2a) in mind, let's consider the following optimization problem:

$$\underset{\beta, \beta_0}{\text{Minimize}} \ \|\beta\| \quad s.t. \quad y_n(x_n^T\beta + \beta_0) \geq 1 \ \forall n = 1,2,\dots,N \quad (3a)$$

We now argue that Problem (3a) is equivalent to Problem (1a). It is straightforward to see that minimizing the objective is equivalent to maximizing the reciprocal of the same objective. Moving to the constraint, note that Equation (2a) allows us to rewrite the original constraint as:

$$\underset{n=1,2,\dots,N}{\min} \ y_n(x_n^T\beta + \beta_0) = 1$$

While it immediately follows that $y_n(x_n^T\beta + \beta_0) \geq 1 \ \forall n = 1,2,\dots,N$, replacing the original constraint with these inequality constraints raises the possibility that the optimal value of (β, β_0) could be achieved with all of the constraints nonbinding – which would violate the requirement that the minimum value of $y_n(x_n^T\beta + \beta_0)$ equals 1. However, a simple argument shows that this outcome is not possible. For any candidate solution with all nonbinding constraints, a better outcome can always be achieved by using the slack in the inequality constraints to reduce the magnitude of the vector β. So, we have demonstrated that solving Problem (3a) yields the separating hyperplane with the maximum margin. For convenience, let's trivially rewrite this problem as

$$\underset{\beta, \beta_0}{\text{Minimize}} \ \frac{1}{2}\beta^T\beta \quad s.t. \quad y_n(x_n^T\beta + \beta_0) \geq 1 \ \forall n = 1,2,\dots,N \quad (4a)$$

where the objective function is now the norm squared which removes the square root – yielding a quadratic program (QP). Additionally, the objective is scaled by ½ in order to simplify the resulting calculations.

Interestingly, notice that we have shown that maximizing the margin corresponds mathematically to minimizing the norm of the weights, β, which is similar to weight decay or regularization used in ridge regression. Recall that ridge regression augments the typical mean-squared error criterion of a linear model with a regularization term that penalizes the size of the weights:

$$\underset{w}{\text{Minimize}} \ \frac{1}{N}\sum_{n=1}^{N}(y_n - x_n^Tw)^2 \quad s.t. \quad w^Tw \leq C$$

where w is a vector of weights (or regression coefficients) and C is a constraint on the norm squared of w. In ridge regression, the aim is to optimize the in-sample error subject to a

constraint on the norm of the weights. In SVM, the aim is to minimize the norm of the weights subject to a constraint on the in-sample error. This correspondence between SVM and regularization is the justification for SVM being "self-regularizing."

To gain some additional insight about SVM, let's look at the Lagrange formulation of Problem (4a). Starting with the Lagrangian, we have:

$$L(\beta, \beta_0, \alpha) = \frac{1}{2}\beta^T\beta - \sum_{n=1}^{N} \alpha_n(y_n(x_n^T\beta + \beta_0) - 1)$$

which is simply a weighted sum of the objective and the constraint functions. The coefficients, α_n, are the Lagrange multipliers. The Lagrange dual function, $g(\alpha)$, is given by

$$g(\alpha) = \inf_{\beta, \beta_0} L(\beta, \beta_0, \alpha)$$

To minimize L over (β, β_0), let's set the gradient equal to zero:

$$\nabla_\beta L = \beta - \sum_{n=1}^{N} \alpha_n y_n x_n = 0$$

$$\frac{\partial L}{\partial \beta_0} = -\sum_{n=1}^{N} \alpha_n y_n = 0$$

Substituting into L, we obtain the Lagrange dual function:

$$g(\alpha) = \sum_{n=1}^{N} \alpha_n - \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} y_n y_m \alpha_n \alpha_m x_n^T x_m$$

Now, we can write the Lagrange dual problem as:

$$\underset{\alpha}{\text{Maximize}} \, g(\alpha) \quad s.t. \quad \alpha_n \geq 0 \, \forall n = 1, 2, \ldots, N$$

$$and \sum_{n=1}^{N} \alpha_n y_n = 0$$

where $\alpha_n \geq 0$ are often referred to as the dual constraints and are part of the Karush-Kuhn-Tucker (KKT) conditions. Note that the dual problem is a QP, so we can easily obtain the solution $\alpha^* = (\alpha_1^*, \ldots, \alpha_N^*)$. We can plug this solution into the expression for $\nabla_\beta L$ to obtain

$$\beta = \sum_{n=1}^{N} \alpha_n^* y_n x_n$$

From complementary slackness – another KKT condition – we know that:

$$\alpha_n^* (y_n (\beta^T x_n + \beta_0) - 1) = 0 \quad \forall n = 1, 2, \ldots, N$$

So, if $\alpha_n^* > 0$, then $y_n(\beta^T x_n + \beta_0) = 1$ which means that x_n achieves the margin. These points are known as "support vectors" (SV's). Data points that are not support vectors must have $\alpha_n^* = 0$. So, the optimal weight vector, β, is completely determined by the support vectors:

$$\beta = \sum_{x_n \text{ is } SV} \alpha_n^* y_n x_n$$

Additionally, we can solve for β_0 using any SV:

$$y_n (\beta^T x_n + \beta_0) = 1$$

Collectively, this implies that the separating hyperplane with the maximum margin will not change if non-SV data points are removed from the training sample.

This leads to another feature of SVM worth emphasizing: its built-in, in-sample metric that can be used to characterize out-of-sample performance. This generalization result can be stated as:

$$E(e_{out}) \leq \frac{E[\# \text{ of } SV's]}{N}$$

where e_{out} is the out-of-sample classification error estimated by cross validation, and the expected value is taken with respect to different data sets. A simple proof of this result follows from the mechanics of leave-one-out cross validation. Leave-one-out cross validation works by removing a single data point, training on the remaining $N - 1$ points, and using the left-out point for validation. This process is then repeated for all points in the sample. The cross-validation error is defined as the average of all N validation errors:

$$e_{CV} = \frac{1}{N} \sum_{n=1}^{N} e_n$$

Since the cross-validation error approximates the true out-of-sample error, it suffices to show that:

$$E(e_{CV}) \leq \frac{E[\# \ of \ SV's]}{N}$$

To compute e_{CV}, we need to know each cross-validation error e_n. First, suppose that the left-out point is not a support vector, then we know that $e_n = 0$. This follows from the earlier observation that the optimal separating hyperplane will not change if non-SV data points are removed. So, we will still have zero in-sample error. On the other hand, suppose that the left-out point is a support vector, then we know that the separating hyperplane may change, and so the left-out point may not be correctly classified. But, since the error is binary, we know that $e_n \leq 1$. Therefore,

$$e_{CV} = \frac{1}{N} \sum_{n=1}^{N} e_n \leq \frac{\# \ of \ SV's}{N}$$

Taking expectations gives the result.