Calculation of truncated Demagnetisation Field - Direct Approach

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February 14, 2012

Abstract

In this document the mathematics of the program truncdemag1.py are explained. The demagnetisation field is found using an approach that truncates an infinite domain and uses one scalar potential.

The Demagnetisation Field inside a Magnetic Body

The demagnetisation field $\overrightarrow{H}_{demag}$ related to a magnetic body is generated by its magnetisation field $\overrightarrow{M}$, and tends to oppose it.In this context the region in space occupied by the magnetic object of interest (usually a ferromagnetic material) is denoted by Ω_M. The complement of this region Ω_V is considered to be a vacuum, which extends to infinity and over which the demagnetisation field $\overrightarrow{H}_{demag}$ is also defined. According to basic electrodynamics the two fields are related by

$$\nabla\cdot \overrightarrow{H}_{demag} = -\nabla\cdot \overrightarrow{M} \text{ in } \Omega_M \tag{1}$$

The assumption is also made that there are no free currents and that the electric displacement field does not change over time so that

$$\nabla \times \overrightarrow{H}_{demag} = 0 \tag{2}$$

This means that we can introduce a magnetic potential function ϕ, such that

$$\overrightarrow{H}_{demag} = -\nabla\phi$$

which means that

$$-\nabla^2\phi = \nabla\cdot \overrightarrow{M}$$

so that our demagnetisation field can be obtained by solving a Poisson problem.

Poisson Problem for the Scalar Potential

The poisson problem for the scalar potential ϕ reads

$$\begin{cases} -\nabla^2\phi = \nabla\cdot \overrightarrow{M} & \text{in } \Omega_M \\ -\nabla^2\phi = 0 & \text{in } \Omega_V \\ & \\ \text{jump}(\frac{\partial\phi}{\partial n}) = -n\cdot \overrightarrow{M} & \text{on } \partial\Omega_M \\ \lim_{\overrightarrow{r}\in\Omega_M\to\partial\Omega_M} \phi = \lim_{\overrightarrow{r}\in\Omega_V\to\partial\Omega_M} \phi & \text{on } \partial\Omega_M \\ & \\ \phi \to 0 & \text{as } \overrightarrow{r} \in \Omega_V \to \infty \end{cases}$$

Where n denotes the unit outward normal. The prescribed jump in the normal derivative is a consequence of the divergence theorem applied to the definition of $\overrightarrow{H}_{demag}$, equation 1. Using equation 2 and stokes theorem we see that the tangential component of $\nabla\phi$ or $\overrightarrow{H}_{demag}$ is continuous over $\partial\Omega_M$. Which means that the scalar potential only differs by a constant as it crosses $\partial\Omega_M$. This constant is set to 0, which means that ϕ is continuous over $\partial\Omega_M$. Finally we expect the demag fiel $\overrightarrow{H}_{demag}$ to decay to 0 far away from the magnetic body, which results in the open boundary condition $|\overrightarrow{H}_{demag}| \to 0$ as $\overrightarrow{r} \in \Omega_V \to \infty$ for the potential ϕ.

Truncation of the Domain

Since a computer cannot simulate an infinite domain there needs to be some way of dealing with the open boundary condition

$$\phi \to 0 \text{ as } \overrightarrow{r} \in \Omega_V \to \infty$$

Here the domain is truncated, using the rule of thumb that it should be about 5 times as large as in order to get decent results.