In[1]:=
```
Clear@"`*"
res = Assuming[beta >= 0 && t > 0 && alpha > 0 && a > 0,
  Integrate[Exp[-2 a (q - q0)^2] Exp[-2 ((1 + q^2 - b q) alpha + beta b q) t],
   {q, -Infinity, Infinity}]]
```

Out[2]=
$$\frac{e^{\frac{t\left(-4\,a\left(b\,\text{beta}\,q0+\text{alpha}\left(1-b\,q0+q0^2\right)\right)+\left(\text{alpha}^2\left(-4+b^2\right)-2\,\text{alpha}\,b^2\,\text{beta}+b^2\,\text{beta}^2\right)t\right)}{2\left(a+\text{alpha}\,t\right)}}\sqrt{\frac{\pi}{2}}}{\sqrt{a+\text{alpha}\,t}}$$

In[3]:=
```
res2 = res /. b → 0
```

Out[3]=
$$\frac{e^{\frac{t\left(-4\,a\,\text{alpha}\left(1+q0^2\right)-4\,\text{alpha}^2\,t\right)}{2\left(a+\text{alpha}\,t\right)}}\sqrt{\frac{\pi}{2}}}{\sqrt{a+\text{alpha}\,t}}$$

In[4]:=
```
alpha^2 (-4 + b^2) - 2 alpha b^2 beta + b^2 beta^2 // FullSimplify
```

Out[4]=
$$(\text{alpha}\,(-2+b)-b\,\text{beta})\,(\text{alpha}\,(2+b)-b\,\text{beta})$$

In[5]:=
```
FullSimplify[1/(2 (a + alpha t)) t (-4 a (b beta q0 + alpha (1 - b q0 + q0^2)) +
    (alpha^2 (-4 + b^2) - 2 alpha b^2 beta + b^2 beta^2) t) /. beta → alpha]
```

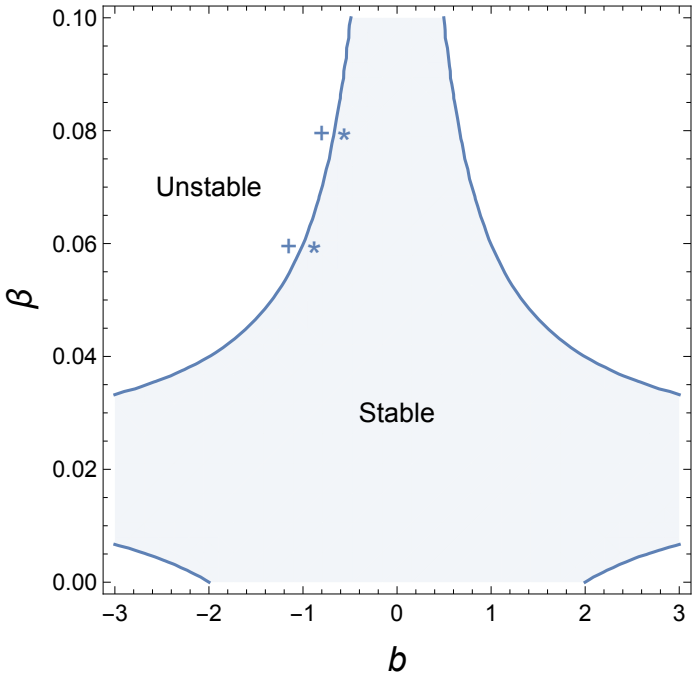
Out[5]=
$$-\frac{2\,\text{alpha}\,t\left(a+a\,q0^2+\text{alpha}\,t\right)}{a+\text{alpha}\,t}$$

In[6]:=
```
res5 = (alpha (-2 + b) - b beta) (alpha (2 + b) - b beta) /. alpha → 0.02
```

Out[6]=
$$(0.02\,(-2+b)-b\,\text{beta})\,(0.02\,(2+b)-b\,\text{beta})$$

In[7]:=
```
fig = Show[RegionPlot[res5 < 0, {b, -3, 3},
    {beta, 0.0, 0.1}, BoundaryStyle → None, PlotStyle → Opacity[0.08],
    FrameTicksStyle -> 12, FrameLabel → {b, β}, LabelStyle → 18],
  ContourPlot[res5 == 0, {b, -3, 3}, {beta, 0.0, 0.1}, PlotLegends → Automatic],
  ListPlot[{{-0.8, 0.08}, {-1.15, 0.06}}, PlotMarkers → {"+", 16}],
  ListPlot[{{-0.56, 0.08}, {-0.88, 0.06}}, PlotMarkers → {"*", 20}],
  Graphics[{Text[Style["Stable", FontSize → 14], {0, 0.03}],
    Text[Style["Unstable", FontSize → 14], {-2, 0.07}]}]]
```

Out[7]=



In[8]:=
```
filename = FileNameJoin[{NotebookDirectory[], "phase.pdf"}];
Export[filename, fig];
```