Vanilla and Barrier Options under Stochastic Volatility

TP 2 – Derivatives (60206A) – HEC Montréal – Pascal François

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The two parts of this assignment are independent.

PART ONE

1] Working under the Heston (1993) stochastic volatility model with EMM dynamics:

$$dS_t = (r - y)S_t dt + \sqrt{v_t} S_t dW_t,$$
$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t} dZ_t,$$
$$dW_t dZ_t = \rho dt,$$

use the closed-form formula to evaluate the three European call options with the following parameters: $S_0 = 100$, $T = 21/252$, $r = 0.05$, $y = 0.02$, $v_0 = 0.25$, $\kappa = 0.2$, $\theta = 0.2$, $\sigma = 0.3$, $\rho = -0.2$, and $K \in \{95, 100, 105\}$.

2] By means of Monte Carlo simulations with 100,000 paths and daily time step ($\Delta t = 1/252$), evaluate the three down-and-out European calls with the same parameters and daily monitored barrier at level $H = 90$.

3] Refine your answers to question 2] by using the European call premiums as control variates. To this end, split the 100,000 simulations into 40 batches of 2,500 paths, and regress, for each strike, the down-and-out call simulated value over the vanilla call simulated value. For each strike, plot the data points and their associated regression line. Report the slope coefficients and corresponding R-squared.

PART TWO

1] Download the Excel data file containing the time series of the adjusted closing price for the S&P TSX Composite index over the November 6, 2020 - November 11, 2024, period. Plot the evolution of the index price and that of the index returns. Compute the sample return volatility assuming 252 trading days within a year.

2] Estimate the NGARCH model from maximum likelihood. Initialize the variance with the square of the first sample return. Use a daily risk-free rate of 2.75%/365 (average 3-month Canadian T-

Bill yield over the period). Verify that the unconditional volatility of your estimation is comparable to the level of the sample volatility.

3] Simulate 100,000 paths of daily index returns for the next 63 days. Evaluate the 51 European 3-month (63-day) calls with strikes ranging from 23,000 to 28,000 by increments of 100.

4] Plot the smile corresponding to the 51 simulated call prices.