



Portland State University

W'23 CS 485/585

Intro to Cryptography

3000 BC

Concise Oxford English Dictionary

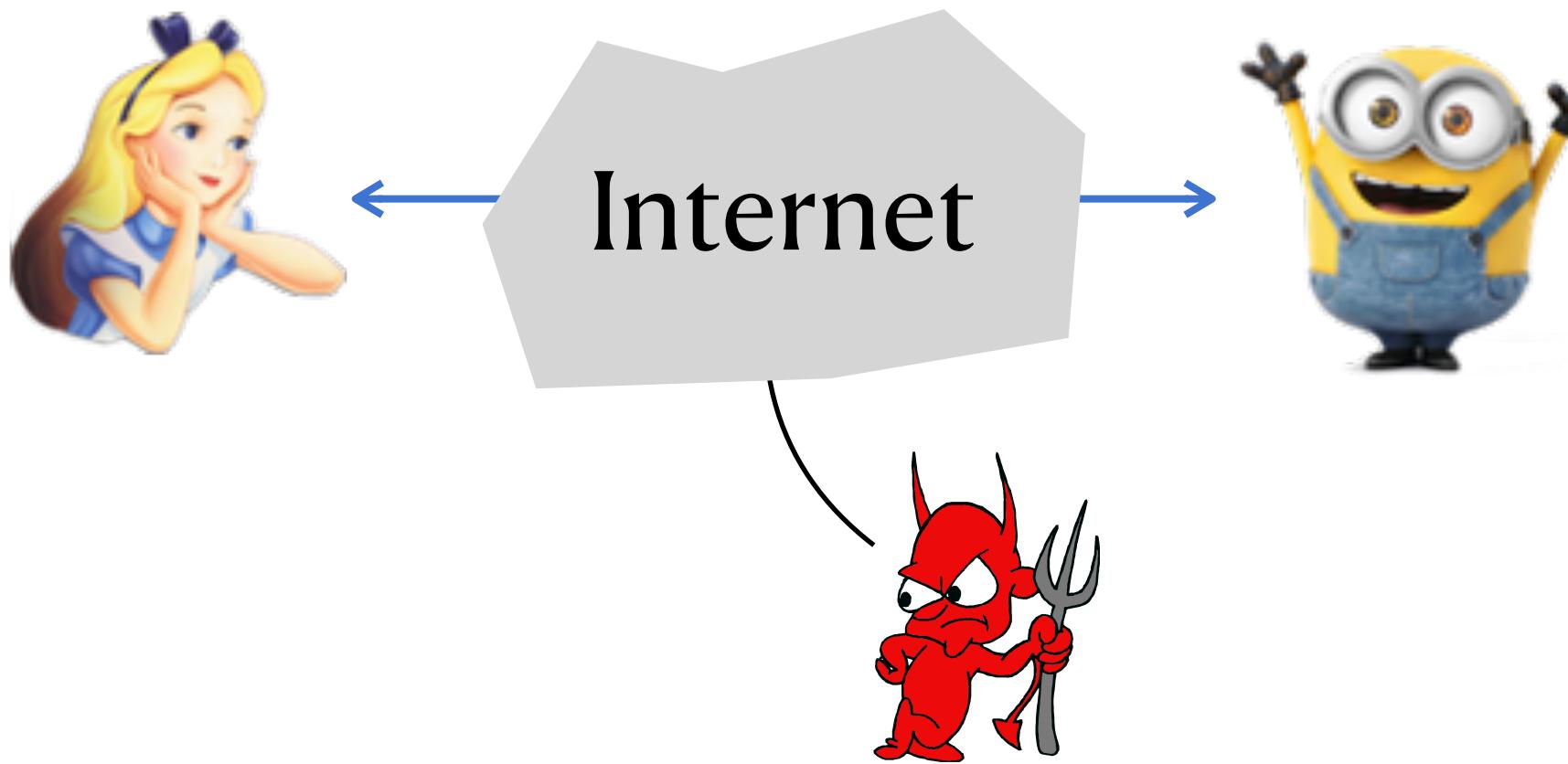
**Cryptography is the art of
writing or solving codes (ciphers)**

... for military activity and gossip

◎ 2 typical scenarios of “secret writing”

1949

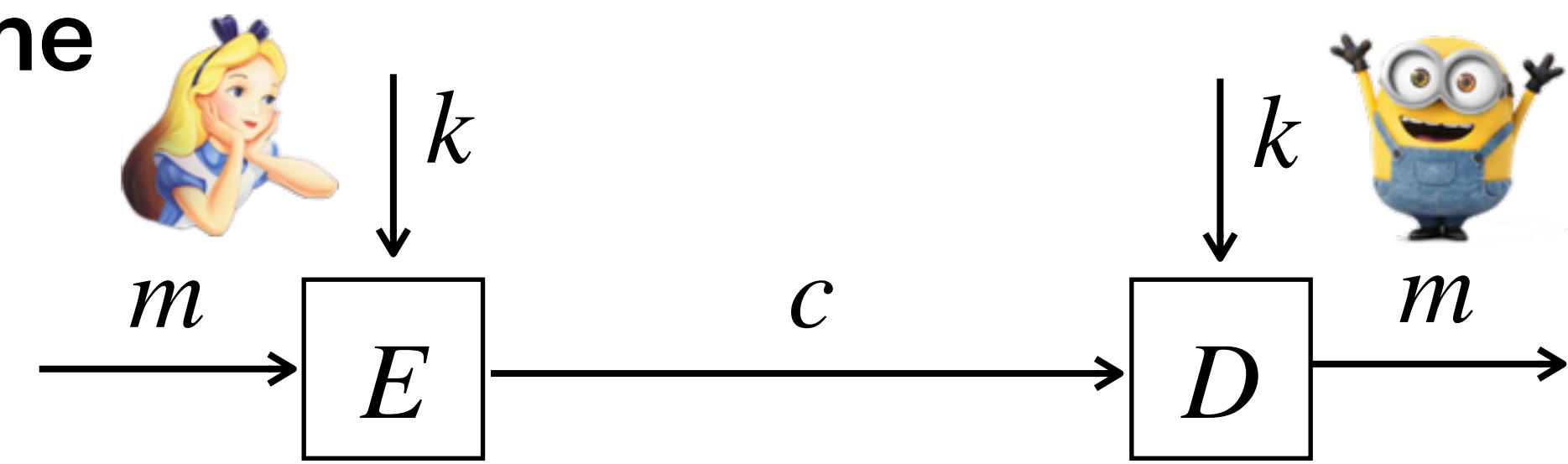
1976



Harddrive / cloud

Private-key (**symmetric**) encryption

- ◎ Call a **cipher** an **encryption scheme**
- ◎ Syntax of a private-key encryption scheme



- k : private key (secret key), shared between sender/receiver
- m : **plaintext** (message)
- c : **ciphertext**
- E : **encryption** (encode) algorithm, $(k, m) \mapsto c$
- D : **decryption** (decode/decipher) algorithm, $(c, k) \mapsto m$

Ceasar's cipher

- ◎ Example

$m = \text{cryptoisfun}$

 $c = \text{fubswlvixq}$

- ◎ Rule

a b c d ... x y z

d e f g ... a b c

$$\{a, \dots, z\} = \{0, \dots, 25\}$$

- $k = 3$ fixed
- $E(m_i) = (m_i + 3) \bmod 26$

★ Easy to break if we know it's encoded by Ceasar's cipher.

Kirchhoff's principle

The cipher method must **NOT** be required to be secret, and it must be able to fall into the hands of the **enemy** without inconvenience.



- © Security should rely solely on the **secrecy of the key**

1. Much easier to secure & update a **short** key than complex enc/dec **algorithms**.
2. Public **scrutiny** makes a cipher more trustworthy.
3. Easier to maintain at large-scale.

- ★ Only use **standardized cryptosystems whenever possible!**

Ceasar ++: shift & substitution cipher

◎ Shift cipher

$$\{a, \dots, z\} = \{0, \dots, 25\}$$

$$\{a, \dots, z\} = \{0, \dots, 25\}$$

- Pick $k \in \{0, \dots, 25\}$ and keep it secret
- $E(m_i) = (m_i + k) \bmod 26$
- $k = 3$ fixed
- $E(m_i) = (m_i + 3) \bmod 26$
- Only 26 possibilities, brute-force search a key! Ex. Decipher “dszqupjtgvo”.

◎ Substitution cipher

- k defines a **permutation** on the alphabet.
- Ex. encrypt “cryptoisfun” under this key.
- Subsumes Shift Cipher as a special case.

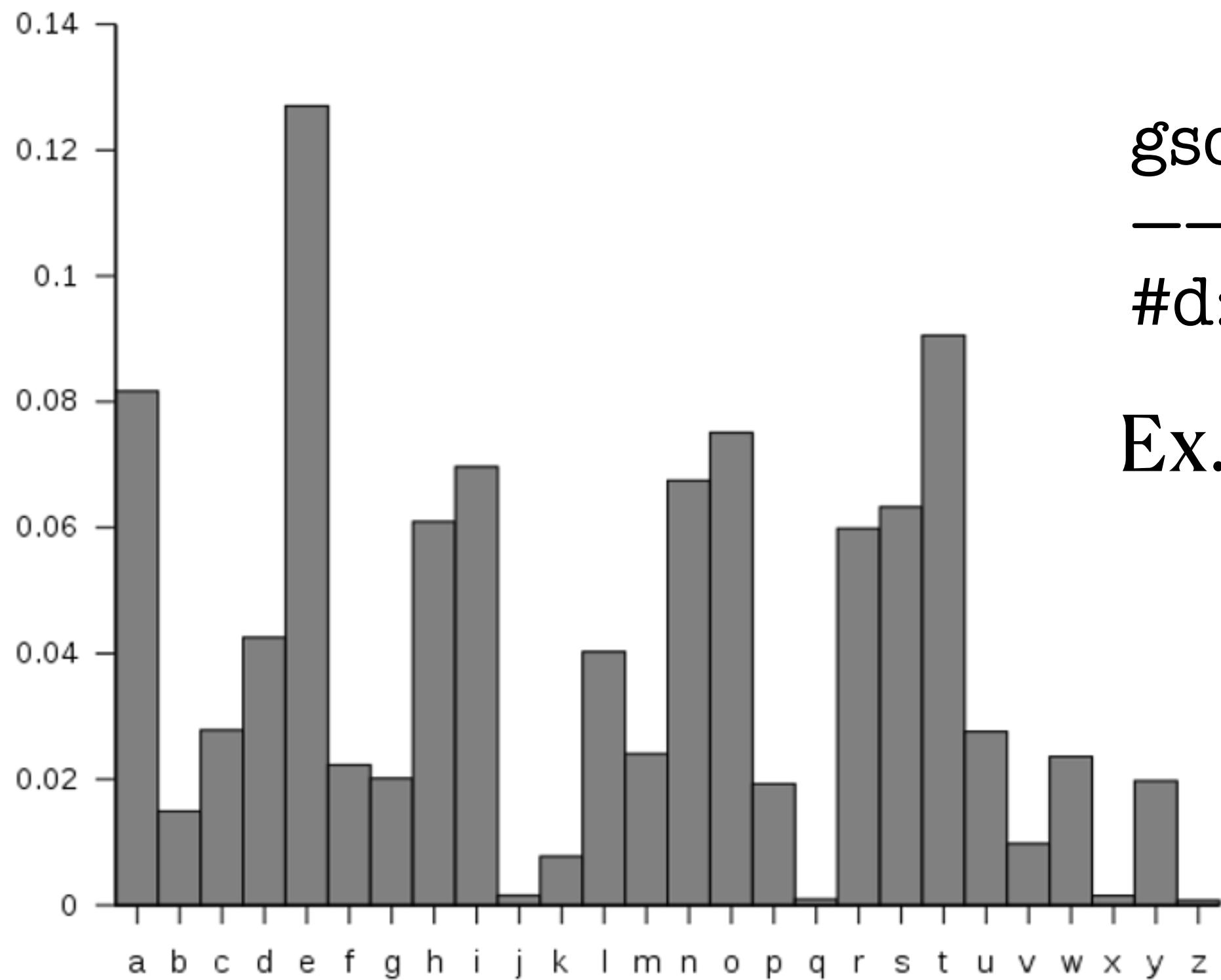
a b c d e f g h i j k l m n o p q r s t u v w x y z	-----	x e u a d n b k v m r o c q f s y h w g l z i j p t
---	-------	---

◎ How many possible keys?

$$26! \approx 2^{88}$$

Breaking sub cipher by frequency analysis

- Sub cipher preserves frequency: one-to-one correspondence.
- Frequency distribution in English language is publicly known.
- Typical sentences close to average frequency distribution.



gsd uvpsdh cdgsfa clwg qfg ed hdylvhda gf ed wduhdg

#d: 18, #g:14, #q: 9

Ex. Decipher it by hand or online solver.

Poly-alphabetic shift cipher

◎ A.k.a. Vigenère cipher

- Key k : a string of letters
- Encrypt E :

Key	psu psu psu psu psu -----
Plaintext	cry pto isf una ndc ool -----
Ciphertext	rjs eli xkz jfu cvw dgf

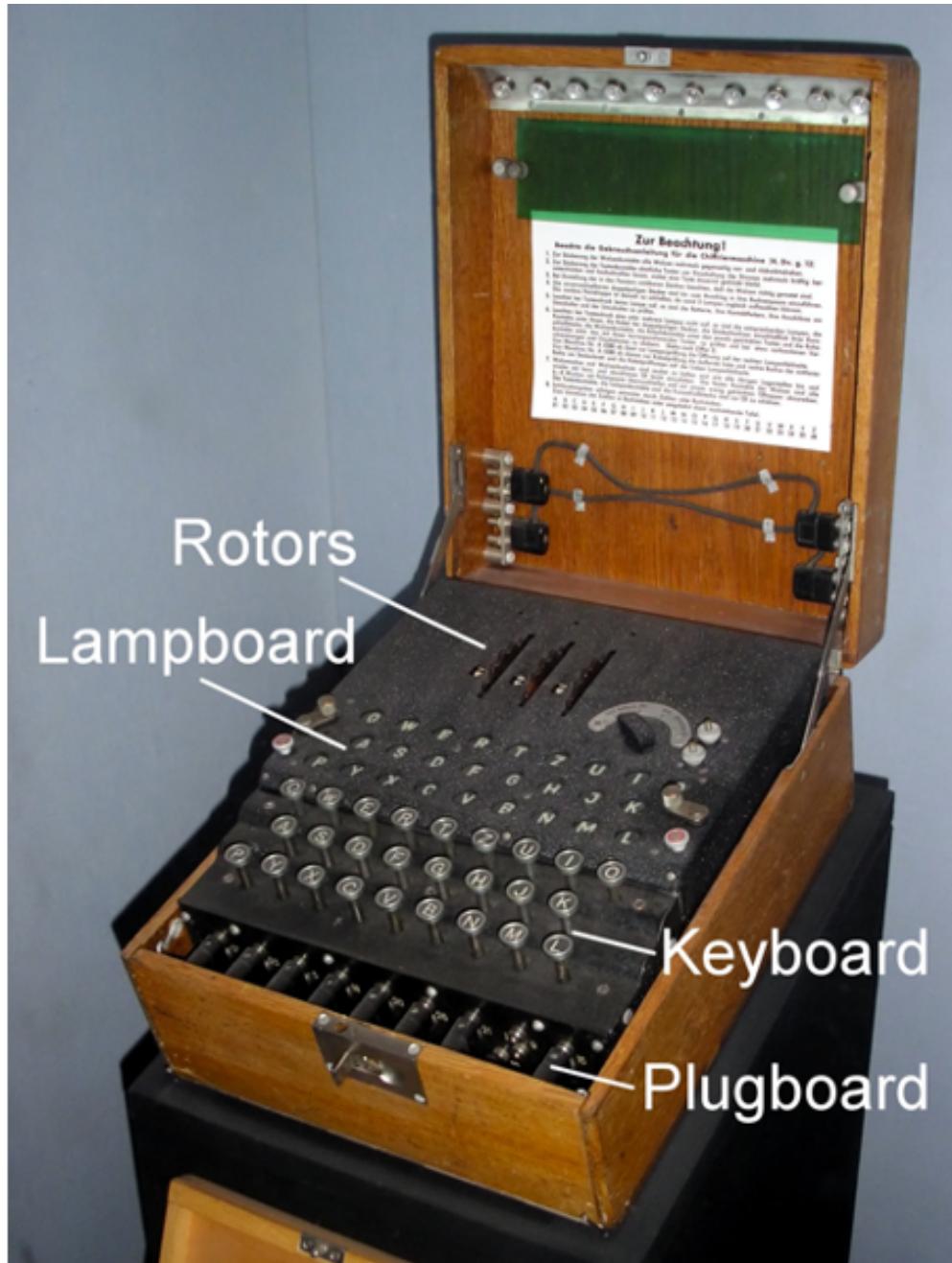
- Considered “unbreakable” for > 300 years.

◎ Breaking Vigenère

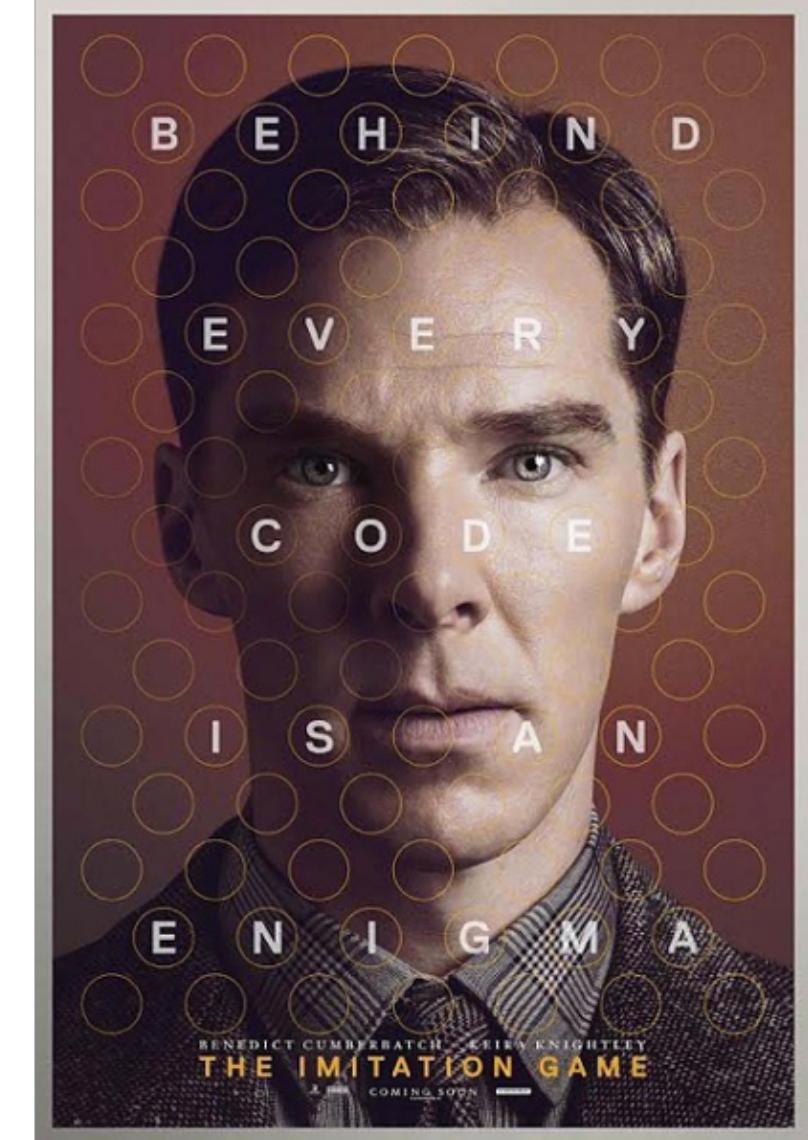
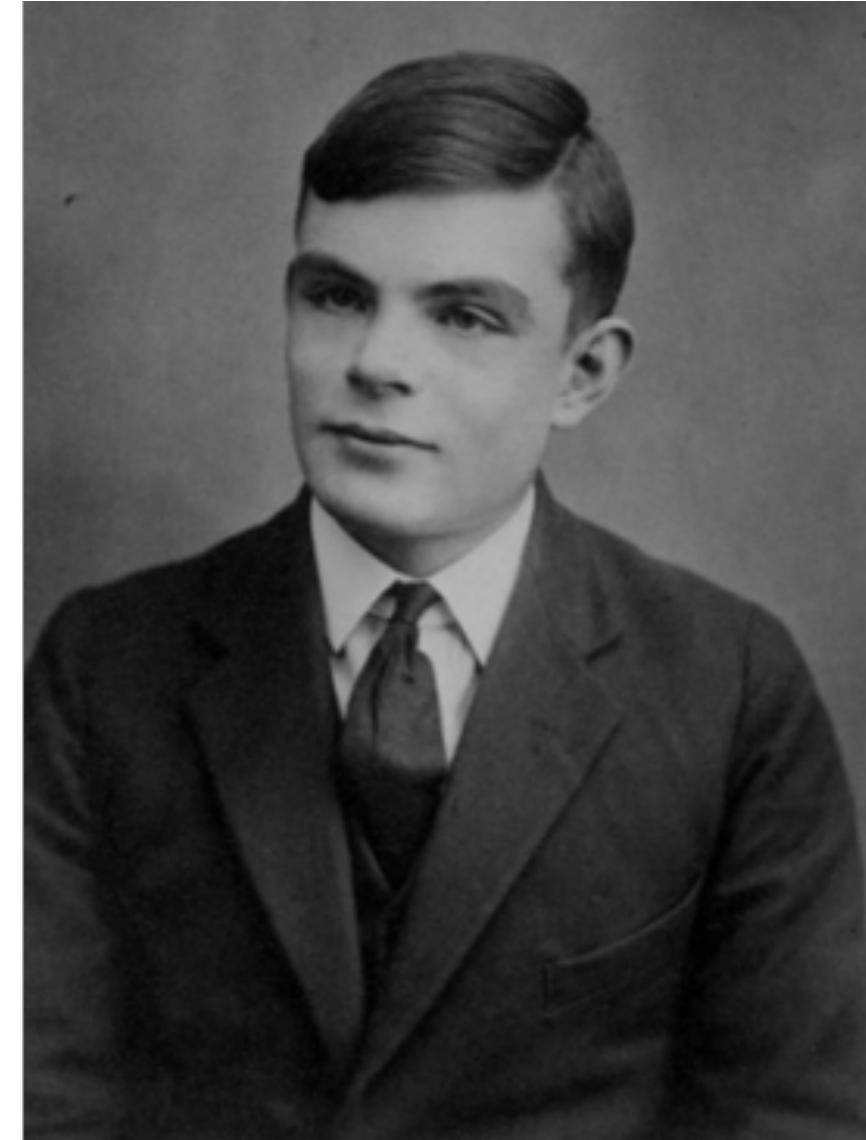
- Key length known: frequency analysis on each substring (under the same shift).
- How to determine the key length? Read KL.

Poly-alphabetic substitution cipher

- Example: Enigma machine in WWII



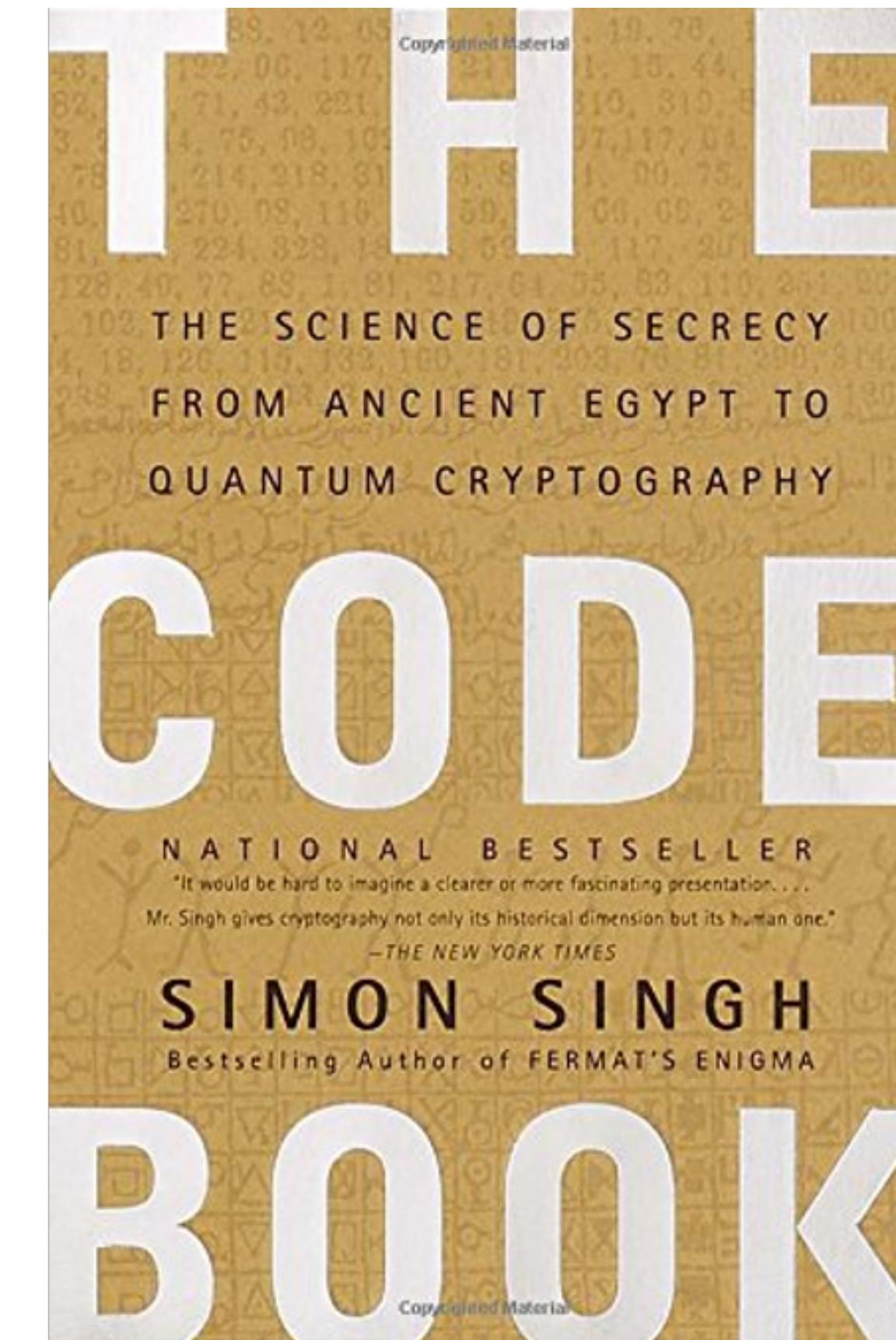
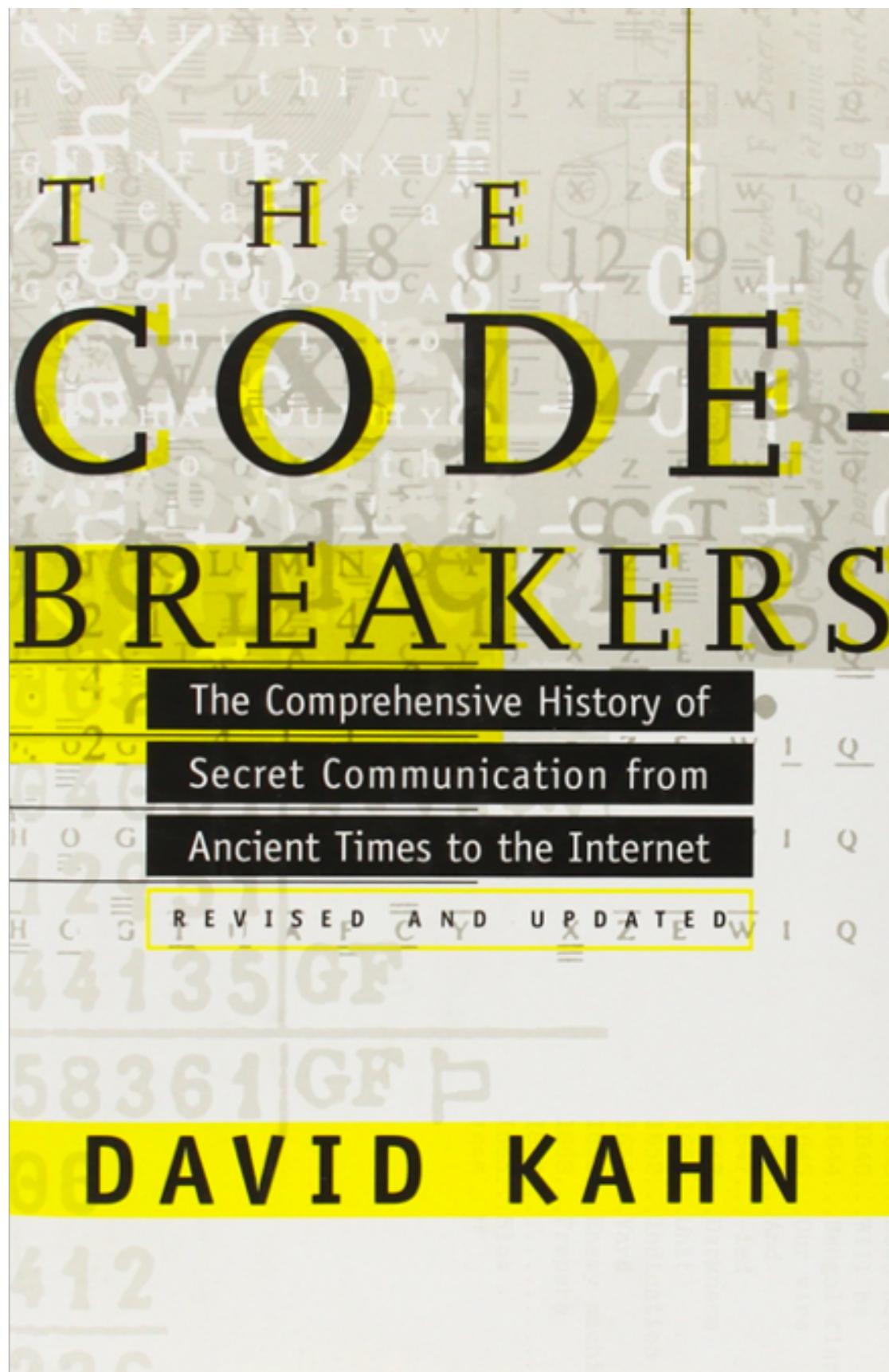
Alan Turing



Source: imdb

- Attack: same principle as before.

Good reads on crypto history



Source: [amazon.com](https://www.amazon.com)

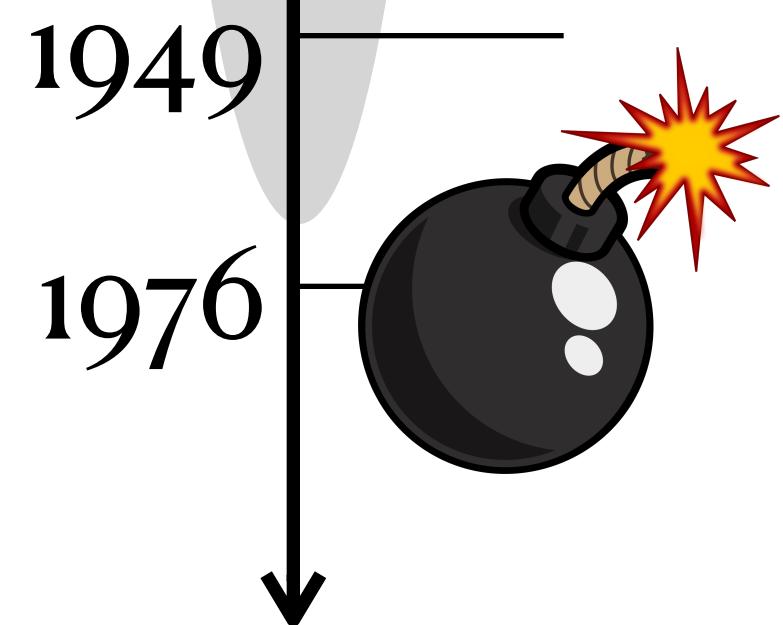
Lessons from historical ciphers

- ◉ Designing good ciphers is hard
- ◉ Looks unbreakable ≠ is unbreakable
- ◉ Intelligent but mostly an art

Not clear about

- Is a cipher secure?
- ... under what circumstances?
- ... and wait, what does “secure” mean precisely?

3000 BC



Revolution of **Modern** Cryptography

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(Historical) Cryptography is the art of
writing or solving codes (ciphers)

1. Much more rigorous: security via mathematics
2. Much more than “secret writing”: public-key crypto, ...

1949

Modern Cryptography involves the study of
mathematical techniques for securing
{digital information, systems and computations}
against adversarial attacks – KL

1976



Revolution of Modern Cryptography

What this course is about

A **conceptual** and **theoretical** tour
to **modern** cryptography

Yes

- Ideas
- Formal approach to security: define, construct, prove.

No

- Implementations
- Engineering skills

(Important but not our focus)

◎ Goal: a cryptographer's mind

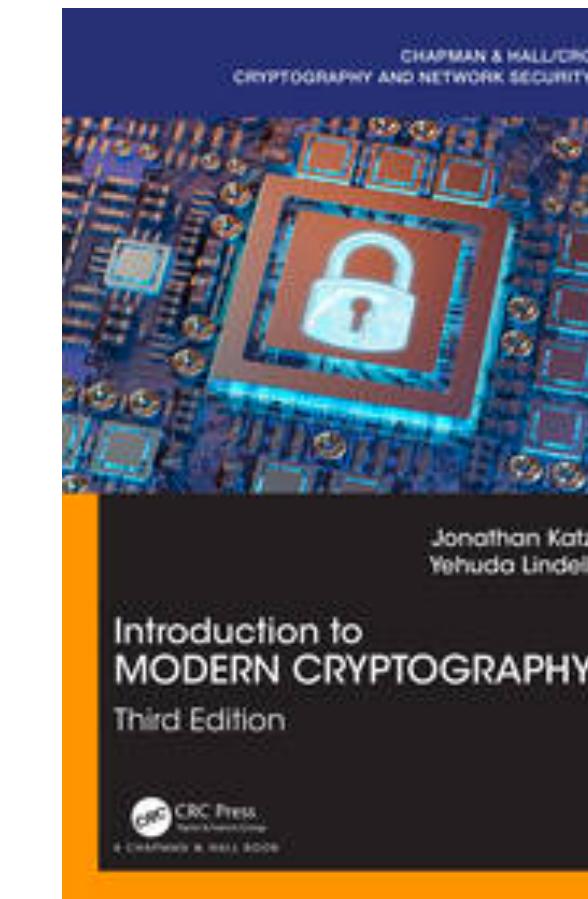
- A solid foundation for real-world security.
- Appreciate the intellectual beauty.
- Beneficial far beyond: differential privacy, ML, algorithms, ...

Logistics

- **Meetings:** M/W 2 - 3:50 pm @ KMC 480 (Zoom participation available)
- **Instructor:** Prof. Fang Song (fang.song@pdx.edu).

● **Texts**

- **Required:** KL
- Supplement: BS + More on Resource page



KL

A Graduate Course in Applied Cryptography

Dan Boneh and Victor Shoup

Version 0.5, Jan. 2020

BS

Prerequisite

Comfortable with **READING & WRITING** mathematical **Proofs**

- CS 350 or equivalent
- Some math helpful
 - Combinatorics, **probability**, linear algebra, number theory ...
 - “Big-Oh notation, random variable, independence, matrices, eigenvalue, congruence...”
- Programming not required

Main topics

1. Overview. (1 week)

- History, principles of modern crypto, perfect secrecy

2. Private-key (*symmetric*) crypto (4 weeks)

- Encryption, message authentication, hash functions

3. Public-key (*asymmetric*) crypto (3 weeks)

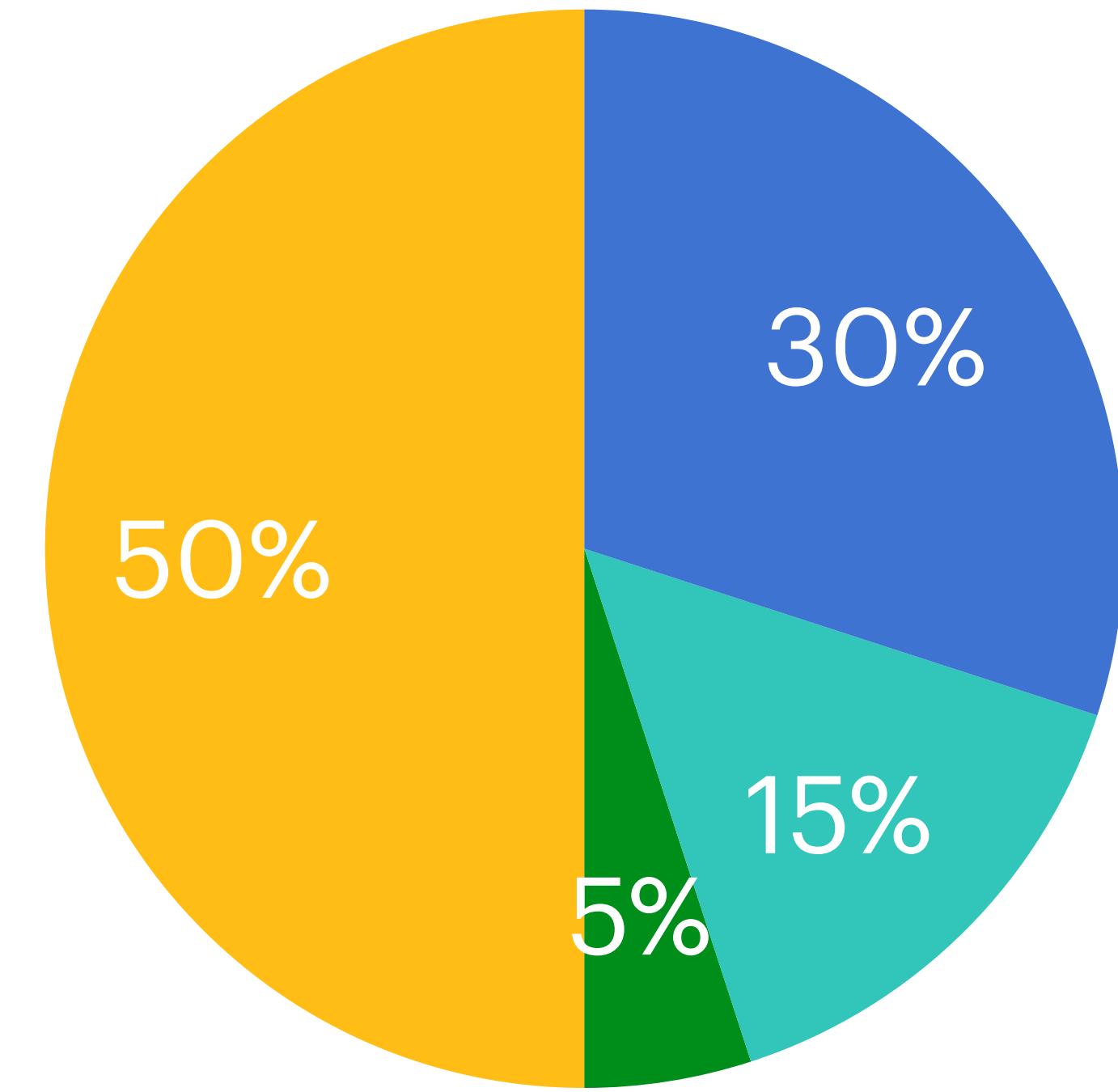
- Encryption, digital signature

4. Selected topics (2 weeks)

- Ethics, Bitcoin, quantum-safe crypto, ...

Policy: grading

- ◎ Homework (biweekly): 50%.
- ◎ Project: 30%.
- ◎ Quiz (biweekly): 15%.
- ◎ Participation: 5%.



Policy: homework

- **Late submission**

- 5 late days in total at your dispense.

- **Collaboration is encouraged.**

- Form study groups of ≤ 3 people, brainstorm etc.
 - Write up your solutions independently.
 - Mark the names of collaborators on each problem.
 - External resources NOT permitted.

- **Your solutions must be intelligible:**

- Be ready to explain your soln's, and convince others & **yourself**.

Policy cont'd

- Academic Integrity

- PSU Student Code of Conduct



- Academic accommodation

- Contact DRC (503-725-4150, drc@pdx.edu) and notify me.

- Covid

- Lecture recordings

- Comply with FERPA and PSU's Student Code of Conduct.
 - Sharing outside this class not permitted.

How to succeed?

- Study the reading materials in advance.
- Ask a lot of questions.
- Form study groups.
- Start on assignments **EARLY!**
 - Make baby steps every day >> leave everything till last minute.
 - Review lecture notes & reading materials **multiple** iterations!

To-do

1. Course webpage https://fangsong.info/teaching/w23_4585_icrypto/
 - “Schedule” page: reading materials.
 - “Resource” page: additional materials.
 - Check regularly!
2. Google Classroom: lecture notes, homework, quizzes
 - Join with code: *biqddg3* (<https://classroom.google.com/c/NTgwMTAwMDU4MjEw?cjc=biqddg3>)
 - A calendar “W23-CS-4585-iCrypto” will appear in your PSU Google Calendar.

To-do, cont'd

3. Slack([w23-4585-icrypto](#)): announcements, discussions, Q&A

- Invitations sent. Important information in Pinned msg.
- Post questions **publicly**, except for private concerns (DM me).
- (Less efficient): email and start your subject line with “w23-4585-icrypto”

4. Getting to know each other:

- Mingle in Slack. Post a short self intro. Form study groups.

5. HW 1 is out

- Short practice on some math/algorithms.
- Due in one week (others will be biweekly).

Today

1. ~~History & course info.~~
2. Principles of modern Cryptography

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Revolution of Modern Cryptography

Principles of modern crypto

1. Formal definitions of security

- What “security” do you want to achieve exactly?
- Guide the design and assess of a construction.
- Know better what you need.

Principles of modern crypto, cont'd

2. Rigorous proofs of security

- The only known method to reason against (infinitely) many possible attacking strategies.
- Never rely on your pure impression.

Principles of modern crypto, cont'd

3. Precise assumptions

- Unconditional security is often **impossible** to attain.
- Be precise, for validating and comparing schemes.

Assume “my construction satisfies the definition.”

Vs.

Assume “factoring 1000-bit integer cannot be done in less than 1000 steps”. 

- Well-studied >> ad hoc: test-of-time.
 - Neat >> vague: easy to assess/falsify.
- ★ **Modularity**: replace a building block when needed.

Recap: principles of modern crypto

1. Formal definitions of security
2. Rigorous proofs of security
3. Precise assumptions

In contrast, historical crypto is not careful about

- is a cipher secure?
- ... under what circumstances?
- ... and wait, what does “secure” mean precisely?

Provable security & real-world

A scheme has been proven secure



security in the real world

- Are the definitions / assumptions right?

- Not match what is needed.
- Not capture attackers' true abilities.

★ You (the designer/defender) have more in charge, instead of attackers.

- Seek improvements proactively: refine defs, test assumptions, ...

Scratch

Supplement

1. ~~History & course info.~~
2. ~~Principles of modern Cryptography~~
3. **Review: mathematical background (on separate note)**
 - Sets
 - Asymptotic notations
 - Probability 101

Asymptotic notations

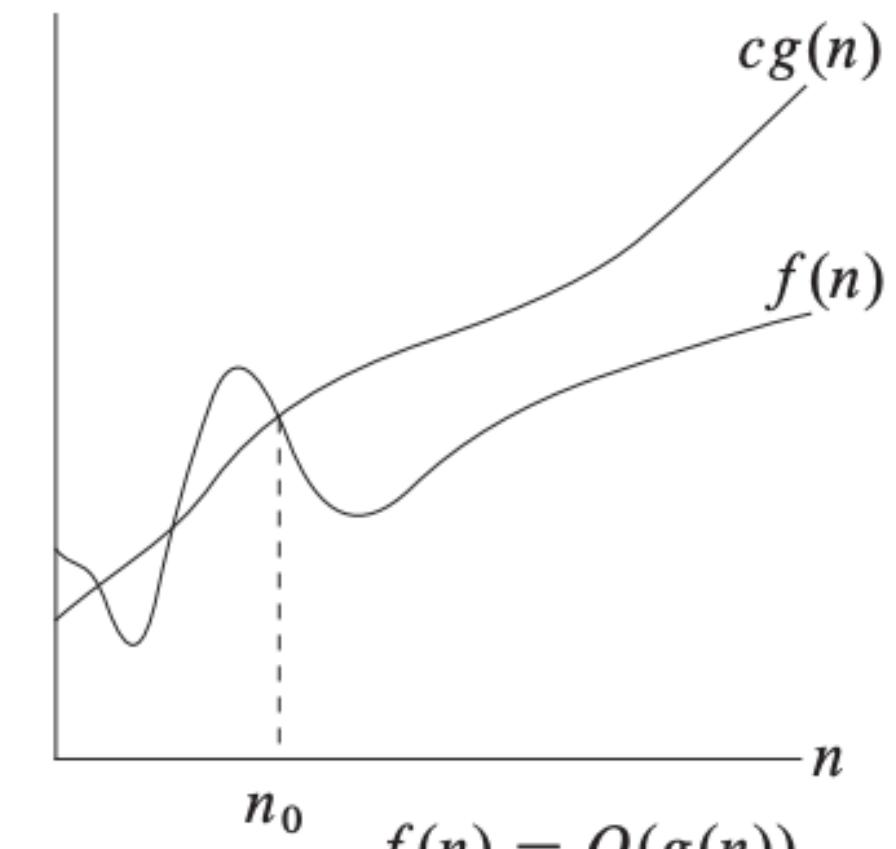
- $O(\cdot), \Omega(\cdot), \Theta(\cdot), o(\cdot), \omega(\cdot)$
 - Measure algorithm behaviors (by functions on integers) as problem size grows.

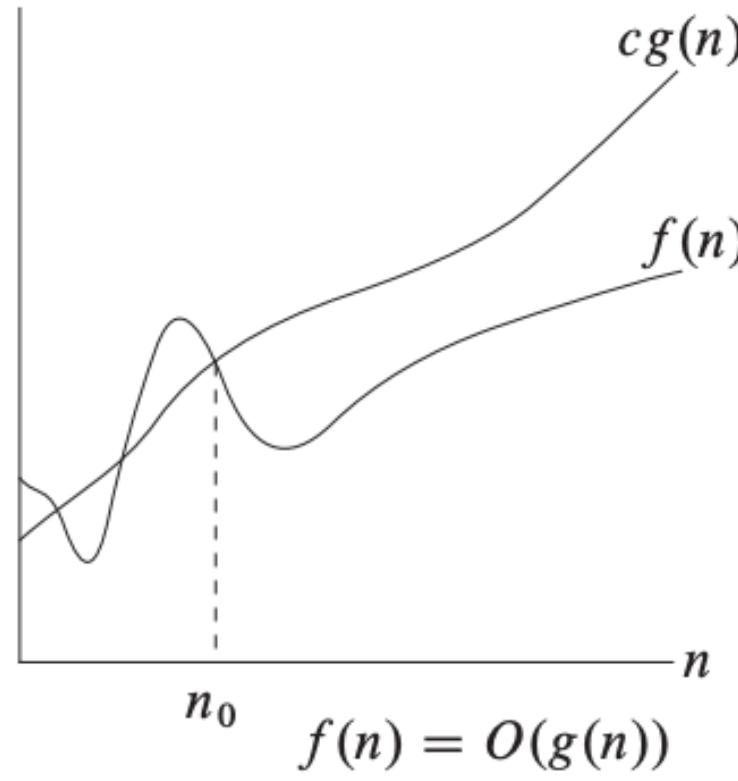
- Defining $O(\cdot)$: asymptotic upper bound

We write $f(n) = O(g(n))$ if there exist constants $c > 0, n_0 > 0$, such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$.

- $O(g(n))$ as a set

$$O(g(n)) := \{f(n) : \exists c > 0, n_0 > 0, \text{ such that } 0 \leq f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0\}$$





Examples

$f(n) = O(g(n))$ if there exist constants $c > 0, n_0 > 0$, such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$.

- $2n^2 = O(n^3)$
 - $c = 1, n_0 = 2$.
 - I.e., $2n^2 \in O(n^3)$
- $f(n) = n^3 + O(n^2)$
 - Meaning $f(n) = n^3 + h(n)$ for some $h(n) \in O(n^2)$

Exercise: sort by asymptotic order of growth

1. $n \log n$

2. \sqrt{n}

3. $\log n$

4. n^2

5. 2^n

6. n

7. $n!$

8. $n^{1,000,000}$

9. $n^{1/\log n}$

10. $\log(n!)$

List them in **ascending** order: if f appears before g , then $f = O(g)$

9, 3, 2, 6, 1=10, 4, 8, 5, 7



Summary

Notation	... means ...	Think...	E.g.	$\lim f(n)/g(n)$
$f(n)=O(g(n))$	$\exists c>0, n_0>0, \forall n > n_0 : 0 \leq f(n) < cg(n)$	Upper bound	$100n^2 = O(n^3)$	If it exists, it is $< \infty$
$f(n)=\Omega(g(n))$	$\exists c>0, n_0>0, \forall n > n_0 : 0 \leq cg(n) < f(n)$	Lower bound	$n^{100} = \Omega(2^n)$	If it exists, it is > 0
$f(n)=\Theta(g(n))$	both of the above: $f=\Omega(g)$ and $f=O(g)$	Tight bound	$\log(n!) = \Theta(n \log n)$	If it exists, it is > 0 and $< \infty$
$f(n)=o(g(n))$	$\forall c>0, n_0>0, \forall n > n_0 : 0 \leq f(n) < cg(n)$	Strict upper bound	$n^2 = o(2^n)$	Limit exists, $=0$
$f(n)=\omega(g(n))$	$\forall c>0, n_0>0, \forall n > n_0 : 0 \leq cg(n) < f(n)$	Strict lower bound	$n^2 = \omega(\log n)$	Limit exists, $=\infty$

Review: Chapter 3 [Introduction to Algorithms](#), By Cormen, Leiserson, Rivest and Stein.