

**Quantum  
attacks at door**

**Still  
NO  
action**

**How come,  
Chief?**

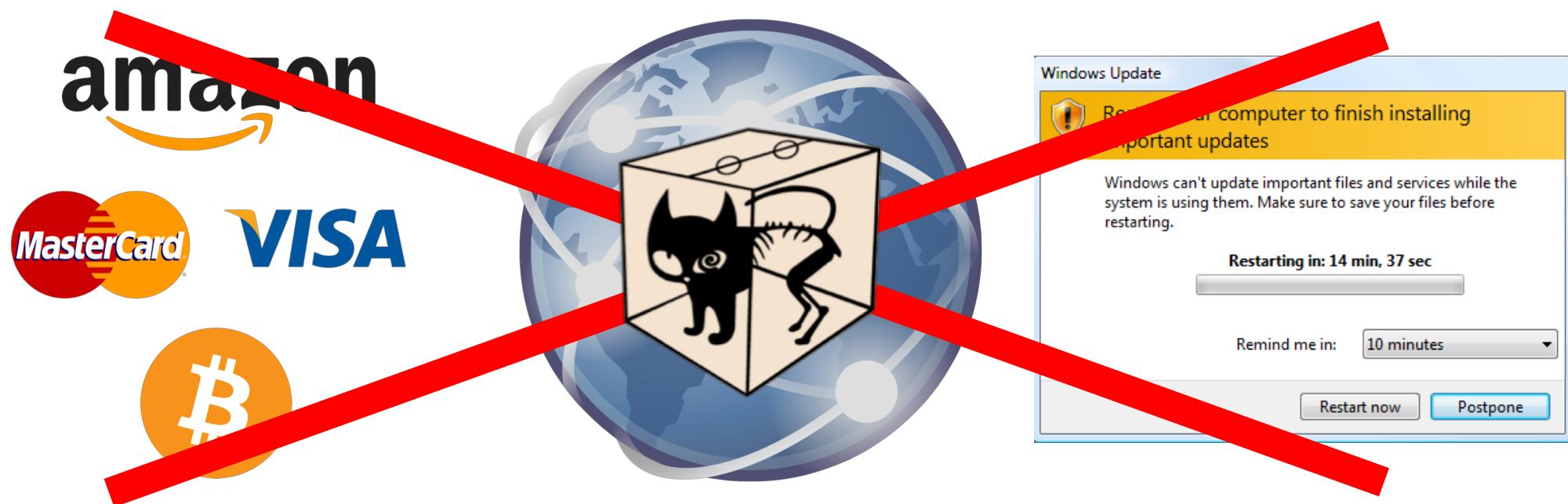
# **Quantum-safe cryptography & more**

Fang Song

~~Portland State U~~

Texas A&M U [fang.song@tamu.edu](mailto:fang.song@tamu.edu)

# Secure Internet will be shattered by quantum computers in $x$ years



# Break at a foundation: Cryptography





# How do quantum attackers break cryptography?



# 1. Crack hard problems

RON RIVEST, ADI SHAMIR & LEN ADLEMAN



RSA public-key cryptography

WHITFIELD DIFFIE & MARTIN HELLMAN



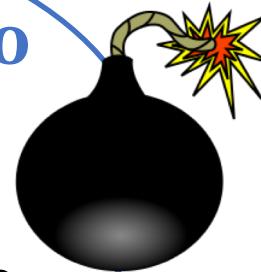
Invented public-key  
cryptography

Assumption: (better  
be) hard to solve

## Public-key crypto

- RSA encryption
- Digital signature
- DH key-exchange

...



Factoring  
Discrete Logarithm

Quantum computer  
solves them<sup>a</sup>, fast!



<sup>a</sup>[Shor94]



## 2. Transform security methodology

- Alert: unique quantum attacks

- A classical protocol proved “unconditionally” secure,
- Broken by quantum entanglement (vs. shared randomness) <sup>b</sup>

<sup>b</sup>[CSSTII]



“... created mathematical structures that turned cryptography from an art into a science”

- Formal framework of modern crypto





# How to secure cryptography against quantum attackers?

Hard problems cracked

Security framework failed

- 
- The diagram illustrates two paths from security challenges to solutions. On the left, a blue arrow points from the text "Hard problems cracked" down to a white box containing the number "1" and the text "Build on alternative problems". On the right, another blue arrow points from the text "Security framework failed" down to a white box containing the number "2" and the text "Quantum security framework".
- 1 Build on alternative problems
  - 2 Quantum security framework

# My work on Post-Quantum Crypto

## 1. Design efficient quantum algorithms<sup>5,8</sup>

- Solve algebraic problems **exponentially** faster
- **Break** candidate quantum-safe problem & cryptosystems
- Develop new quantum algorithmic tools



5.SODA16

8. STOC14, QIP15

## 2. Acquire quantum-safe crypto

### i. Develop formal methodology for quantum security<sup>4,7,10</sup>

- **Model** quantum security: composable protocols, ...
- Develop **general techniques**: “quantum-friendly” reductions, fine-grained quantum complexity ...



1.PQCrypto18

2.Crypto17

4.PKI16

6.TQC15

7.PQCrypto14

9.TCC13

10.Crypto11,QIP11

### ii. Construct and analyze quantum-secure schemes<sup>1,2,6,9,10</sup>

- Two-party computation protocols
- Hash functions and random-oracle heuristic
- Block ciphers and message-authentication

# What is $x$ , after all?

universal, fault-tolerant (e.g., run Shor's factoring alg.)

$x=?$ : born of a ~~quantum computer~~

$X = -10?$



$x$

2007

2018

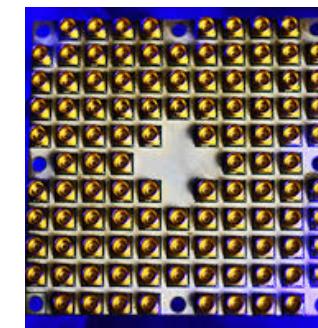
$\infty$



S. Aaronson (UT Austin)



G. Kalai (Hebrew U)



CES 2018: Intel's 49-Qubit Chip Shoots for Quantum Supremacy



# Quantum threat is pressing

- $x=?$ : universal quantum computer (breaking public-key crypto)

A: time to transit to new cryptosystems  
B: how long your data needs to be safe

<---- >>> your expected time!

Theorem. If  $A + B > X$ , then act now!



- $x \leq 0$ : quantum attack on cryptography ← Available now!



“... we announce preliminary plans for transitioning to **quantum resistant algorithms.**”

Aug 19, 2015

[www.nsa.gov/ia/programs/suiteb\\_cryptography/](http://www.nsa.gov/ia/programs/suiteb_cryptography/)



National Institute of  
Standards and Technology  
U.S. Department of Commerce

Post-Quantum Cryptography  
Standardization

Feb 24-26, 2016	NIST Presentation at PQCrypto 2016: <i>Announcement Dustin Moody</i>
April 28, 2016	NIST releases <a href="#">NISTIR 8105, Report on Post-Quantum Cryptography</a>
Dec 20, 2016	<a href="#">Formal Call for Proposals</a>
Nov 30, 2017	Deadline for submissions
Dec 4, 2017	NIST Presentation at AsiaCrypt 2017: <i>The Ship Has Sailed Dustin Moody</i>
Early 2018	Workshop - Submitter's Presentations
3-5 years	Analysis Phase - NIST will report findings <i>1-2 workshops during this phase</i>
2 years later	Draft Standards ready

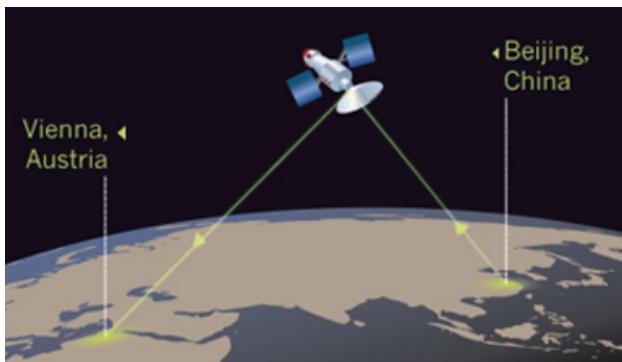
# Quantum cryptography: another approach



- Supplement (and outperform) classical cryptography
  - Quantum key distribution: secure against **unbounded** eavesdroppers
- Protect quantum information
  - Encrypt & authenticate quantum data ...

Impossible by  
classical crypto

Technology closer: commercial QKD products on the market



[https://en.wikipedia.org/wiki/Quantum\\_Experiments\\_at\\_Space\\_Scale](https://en.wikipedia.org/wiki/Quantum_Experiments_at_Space_Scale)

# My work on quantum cryptography

- Construct quantum protocols bypassing classical impossibility <sup>9</sup>

<sup>9</sup>TCC13

- Construct zero-knowledge proof systems for Quantum NP <sup>3</sup>

<sup>3</sup>FOCS16



Triumph of zero-knowledge proof, classically  
“... pioneered new methods for efficient  
verification of mathematical proofs”

- Construct pseudorandom quantum states and quantum money \*

Analogous to pseudorandom  
number generator

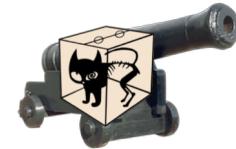
\* JLS'Preprint17

# Summary of my main work

## Post-Quantum Cryptography

### 1. Design efficient quantum algorithms

- Solve algebraic problems exponentially faster
- **Break** candidate quantum-safe problem & cryptosystems



5.SODA16  
8.STOC14, QIP15

### 2. Acquire quantum-safe crypto

i. Establish a formal methodology for quantum security

ii. Construct and analyze quantum-secure schemes

- Two-party computation, hash functions ...
- Block ciphers & message-authentication



1.PQCrypto18  
2.Crypto17  
4.PKC16  
6.TQC15  
7.PQCrypto14  
9.TCCI3  
10.Crypto11,QIP11

## Quantum Cryptography

- Quantum protocols bypassing classical impossibility
- Zero-knowledge for Quantum NP, quantum money ...



3.FOCS16  
7.TCCI3

# This Talk

---

## 1 Design efficient quantum algorithms

- Solve algebraic problems exponentially faster
- **Break** candidate quantum-safe problem & cryptosystems
- Develop new quantum algorithmic tools

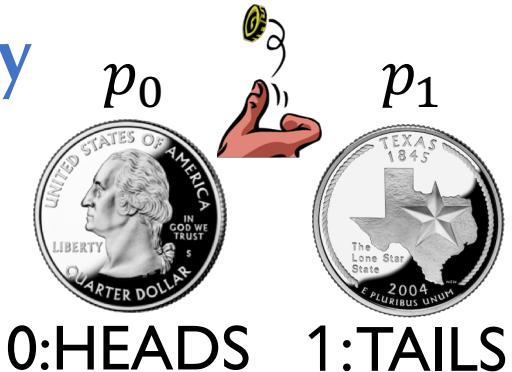
## 2 Acquire quantum-safe crypto-systems

- Challenges of analyzing quantum security
- Block cipher domain-extension & message-authentication

# Become a quantum expert in 3 minutes

## ■ Quantum = ++ Prob. theory

- $p_0, p_1 \in \mathbb{C}$  (negative OK)
- $|p_0|^2 + |p_1|^2 = 1$   
( $\ell_2$ -normalized)



## ■ Probability theory

- $0 \leq p_0, p_1 \leq 1$
- $p_0 + p_1 = 1$   
( $\ell_1$ -normalized)

## Physicists: Quantum weirdness

## ■ Quantum Superposition

→ No-cloning of unknown QState

### Qubit

$$= \frac{1}{\sqrt{2}} \left| \begin{array}{c} \text{QUARTER DOLLAR} \\ \text{HEADS} \end{array} \right\rangle - \frac{1}{\sqrt{2}} \left| \begin{array}{c} \text{QUARTER DOLLAR} \\ \text{TAILS} \end{array} \right\rangle$$

SCHRÖDINGER'S CAT IS  
ALIVE

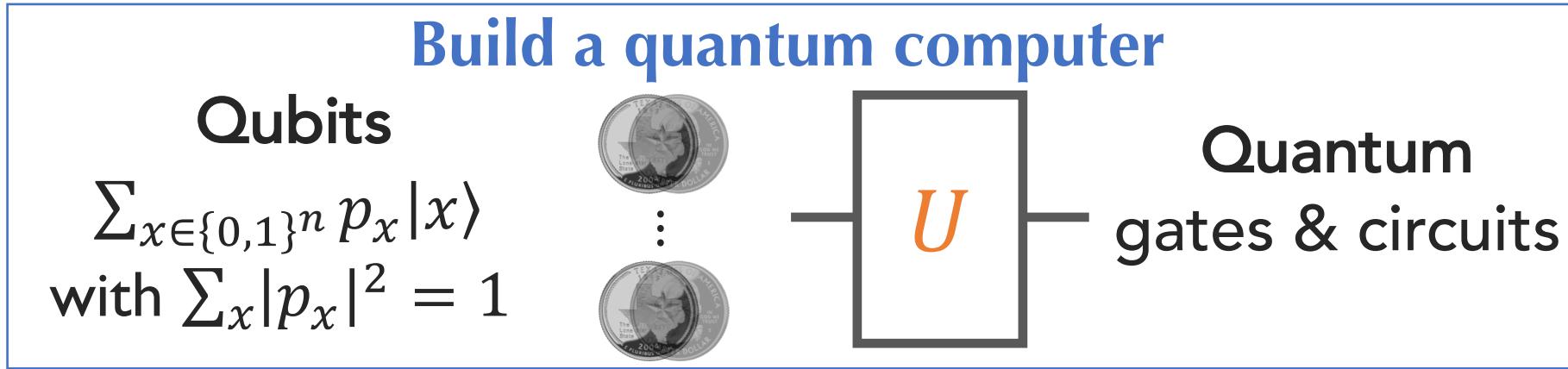


## ■ Quantum Entanglement

(A non-classical correlation)

$$\frac{1}{\sqrt{2}} \left| \begin{array}{c} \text{QUARTER DOLLAR} \\ \text{HEADS} \end{array} \right\rangle \left| \begin{array}{c} \text{QUARTER DOLLAR} \\ \text{HEADS} \end{array} \right\rangle - \frac{1}{\sqrt{2}} \left| \begin{array}{c} \text{QUARTER DOLLAR} \\ \text{TAILS} \end{array} \right\rangle \left| \begin{array}{c} \text{QUARTER DOLLAR} \\ \text{TAILS} \end{array} \right\rangle$$

# Computer scientists: a novel computer?



## ■ Is it any good?

- Classical computer cannot simulate (300 qubits  $\sim 2^{300}$  bits to describe)
- Solve hard problems **faster** by constructive interference ( $\neq$  mass parallelism)

exponentially

Which problems admit faster |quantum> algorithms than classical algorithms?

Ǝ Poly-time quantum algorithms for:

Factoring and discrete logarithm [Shor'94]

Basic problems in algebraic number theory

Unit group

Principal ideal problem

Class group

Constant degree number fields

[Hallgren'02'05,SV05]

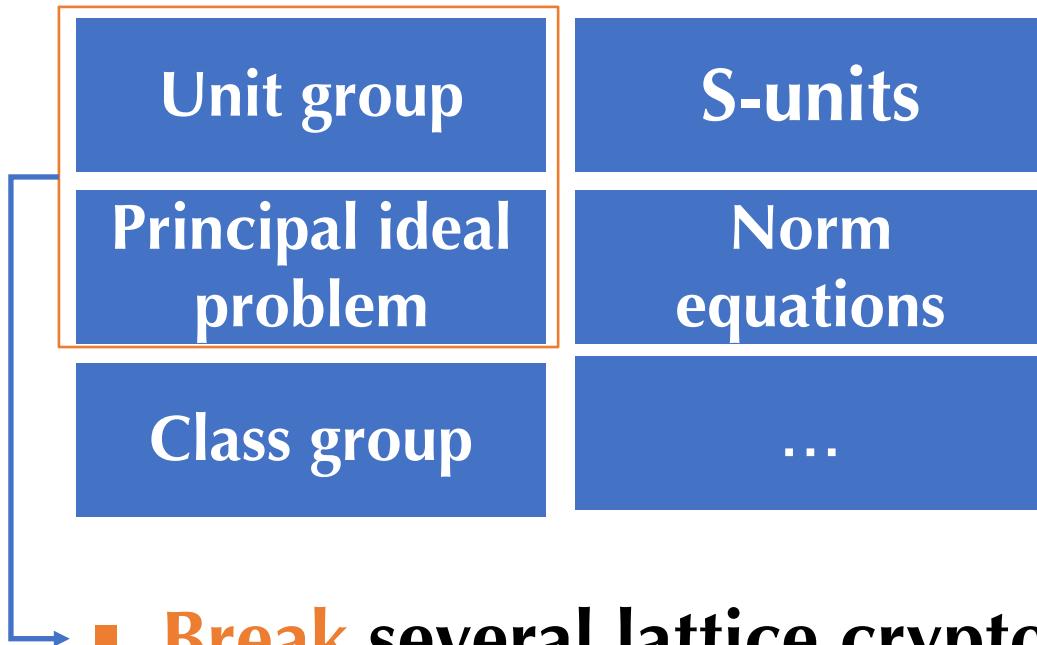
Arbitrary degree

[EHKS'STOCI4]

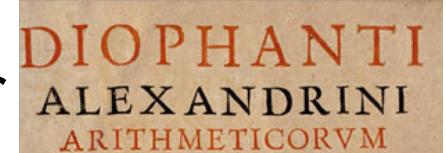
[BS'SODA16]

Best known classical algorithms need (at least) sub-exponential time

# Our contributions



- Efficient quantum algorithms for basic problems in number fields of **arbitrary**-degree
- More examples of quantum **exponential** speedup
- New **quantum** algorithmic tools



- **Break several lattice cryptosystems believed quantum-safe before**

QUANTA  
illuminating science  
MAGAZINE

CRYPTOGRAPHY

A Tricky Path to Quantum-Safe Encryption

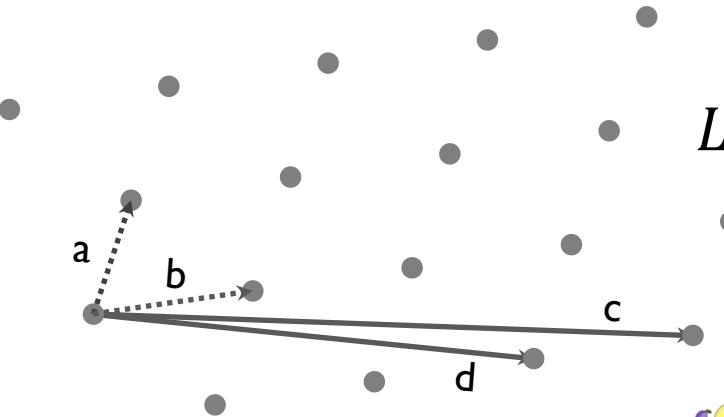
<https://www.quantamagazine.org/quantum-secure-cryptography-crosses-red-line-20150908/>

# Lattice-based cryptography

## ■ Lattice problems

- Shortest vector problem, ...

**Conjecture: hard even  
for quantum computers**



## ■ A neo-tree of crypto grows



Hash function, signature



Public-key encryption, ID-based encryption



Fully homomorphic encryption,  
program obfuscation ...



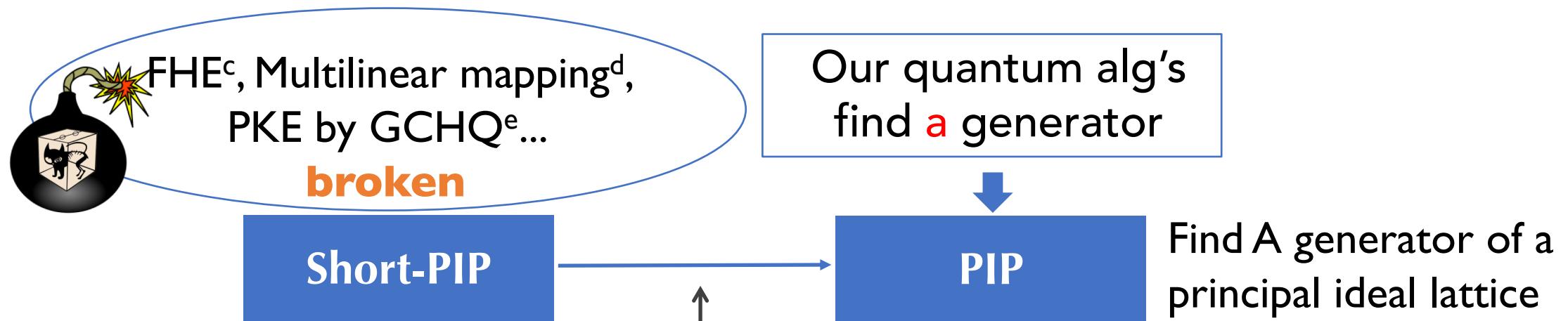
# Breaking some lattice crypto

- For efficiency, often use lattices with more **structures**



[CramerDW' Eurocrypt17]:  
extension to break more

- Short-PIP based cryptosystems **broken!**



<sup>c</sup>SmartV10

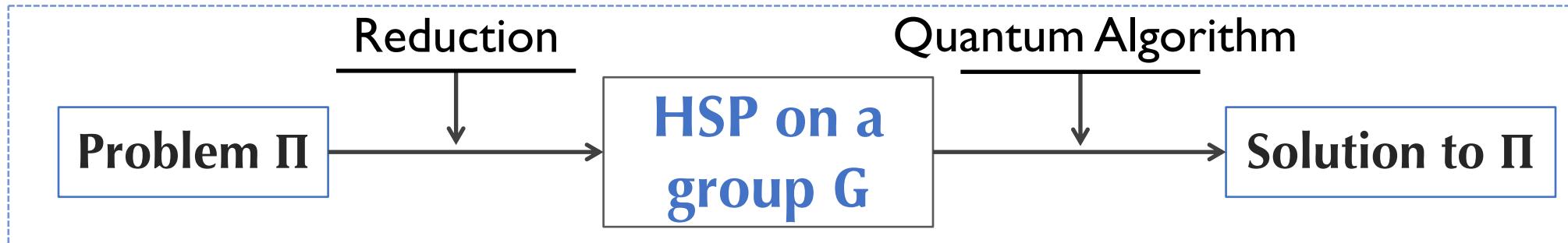
<sup>d</sup>GargGH13

<sup>e</sup>CampellGS15

<sup>f</sup>CramerDPR15

# Our algorithms: a generalized framework

- The **Hidden Subgroup Problem (HSP)** framework  
(captures most instances of quantum exponential speedup)



- Our contribution
  - a A continuous HSP framework
  - b Employ quantum power in reduction already



# Interesting HSP instances

Computational Problems	HSP on G	
Factoring	$\mathbb{Z}$	
Discrete logarithm	$\mathbb{Z}_N \times \mathbb{Z}_N$	<b>Abelian groups</b> $\exists$ efficient quantum algs
Number fields (PIP etc.)	Continuous $\mathbb{R}^{O(n)}$	
Simon's problem (Crypto app later)	$\mathbb{Z}_2^n$	
Graph isomorphism	Symmetric group	<b>Non-abelian</b>
Unique shortest vector problem	Dihedral group	Open question: ? efficient quantum algs

# This Talk

---

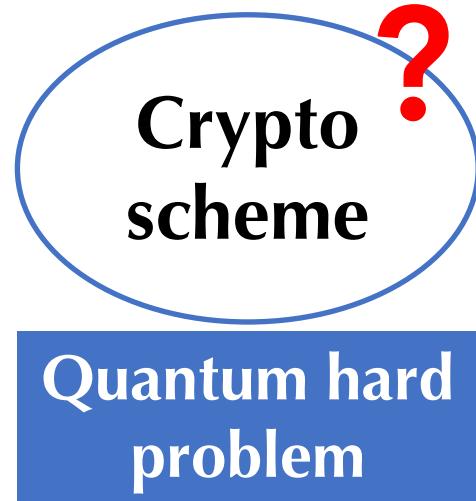
## 1 Design efficient quantum algorithms

- Solve algebraic problems exponentially faster
- **Break** candidate quantum-safe problem & cryptosystems
- Develop new quantum algorithmic tools

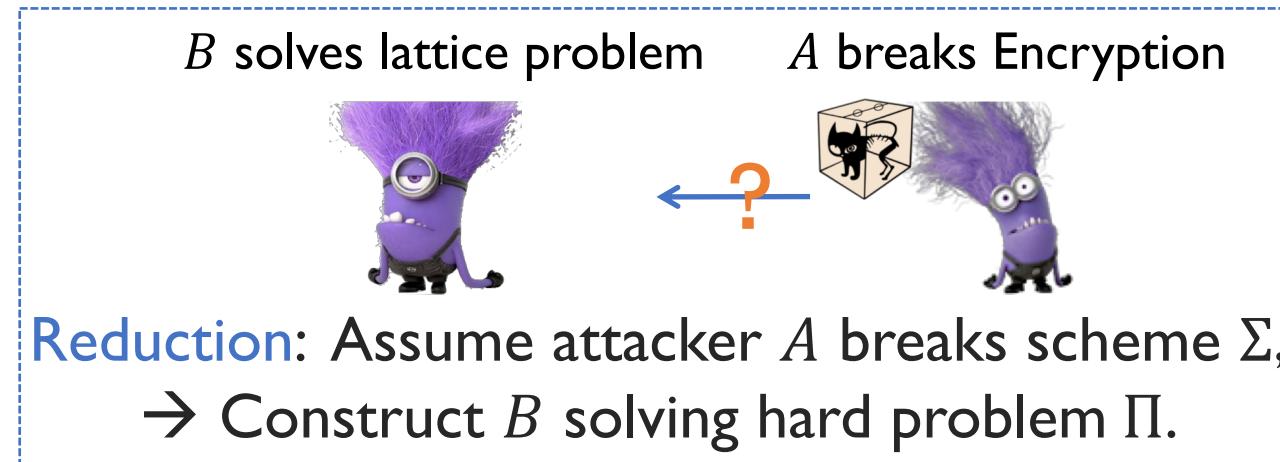
## 2 Acquire quantum-safe crypto-systems

- Challenges of analyzing quantum security
- Block cipher domain-extension & message-authentication

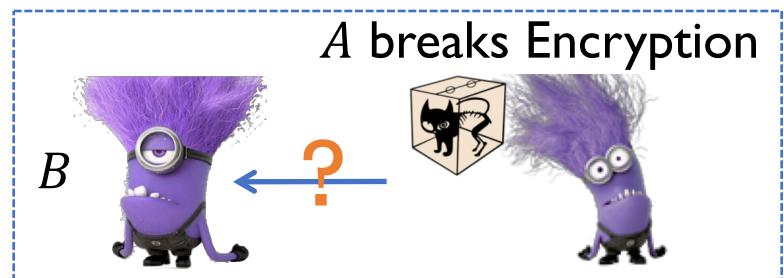
# Recall: classical security framework fails



- Security models: inadequate for Q attacks
  - Quantum security models: catching up and a lot to do
- Security analysis: can fail against Q attackers
  - Quantum security analysis: subtle & challenging



# Difficulties of analyzing quantum security



## (Classical) probability

Fix randomness of  $A$

If “ $A$  does  $Z$ ”,  $B$  works

**Rewinding:**  $B$  take snapshots of  $A$  at various points, run it back and forth

...

## Quantum

Quantum  $A$  no explicit random coins (inherently randomized)

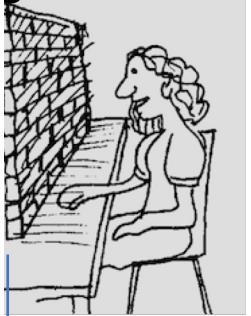
Event “ $A$  does  $Z$ ” ill-defined

Quantum rewinding, really?

- Quantum no-cloning
- Observations destructive

# We are not totally clueless...

ge barriers”



## Quantum (difficulties)

Quantum  $A$  no explicit random coins (inherently randomized)

Event “ $A$  does  $Z$ ” ill-defined

Quantum rewinding, really?

## Some progress (in my work <sup>1,4,6,9,10</sup>)

- Hardness of search → security
  - Restore **random-oracle** heuristic & hash function security in the quantum world
- 
- Special Q-rewinding [Watrous09]
  - Quantum-secure zero-knowledge proofs of knowledge & 2-party computation

??? Constant round Zero-Knowledge, ...

Fine, I know you can

- Solve factor, DL, some lattice problem, and break most **public-key** crypto
- Make my life miserable to struggle with new security models & analysis methodology

Apparently, you haven't broken **symmetric-key** crypto yet

Quantum attackers,  
is that the best you can do?



I've still got AES, try it...

# Break symmetric-key crypto (block-cipher related)



## ■ Block Cipher

- **Apps:** hash functions (SHA-2), authentication, encryption, ...
- **Examples:** DES, AES (Advanced Encryption Standard)



# Broken!

Core of DES

3-round Feistel Cipher

Easy on a quantum computer

Simplified AES

Even-Mansour Cipher

Message authentication

[KM10'12]  
[KLL+16]

Simon's Problem  
≡  
HSP on  $\mathbb{Z}_2^n$

Authenticated  
encryption

CBC-MAC

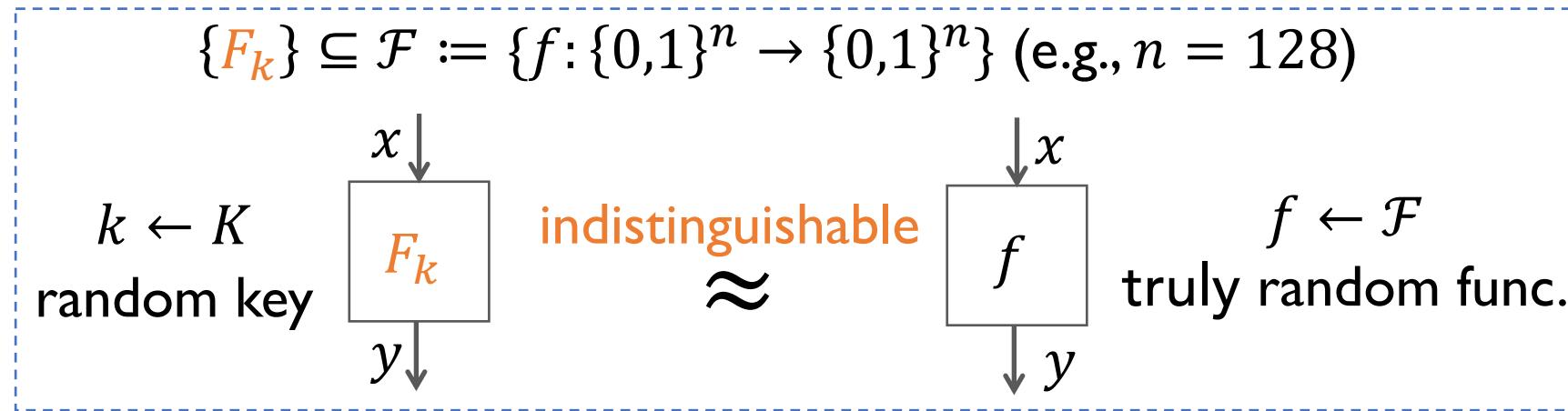
Galois/Counter mode

...

# Block cipher and domain extension

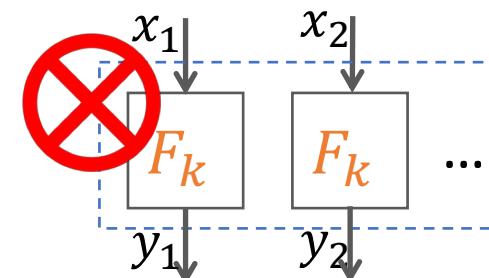
## ■ Block Cipher $\{F_k\}$

- “Efficiently computable permutation that looks random (with random key  $k$ )”



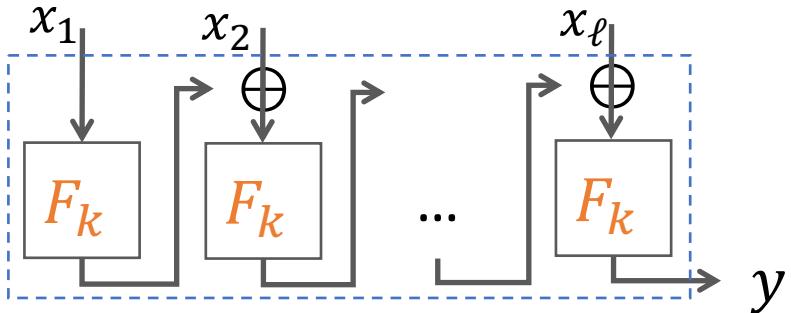
## ■ How about long inputs: domain-extension

- Given psodorandom  $\{F_k\}$  on  $n$ -bit input
- Construct pseudorandom  $\{G_k\}$  on  $\ell \cdot n$ -bit input



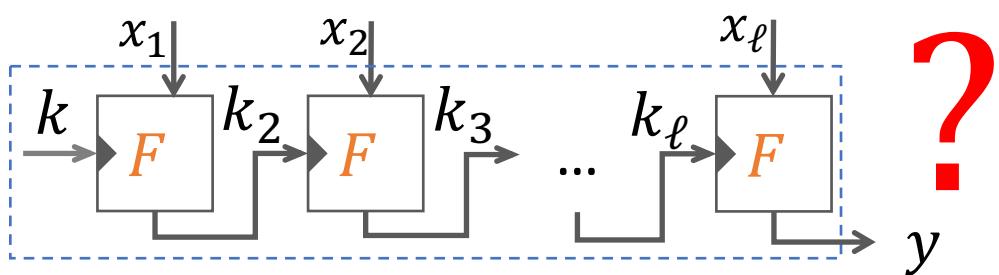
# Classically secure domain-extension

- CBC-MAC ANSI X9.19, ISO/IEC 9797



Broken by quantum!

- Cascade (NMAC, HMAC) NIST.FIPS.198, IPSec, TLS



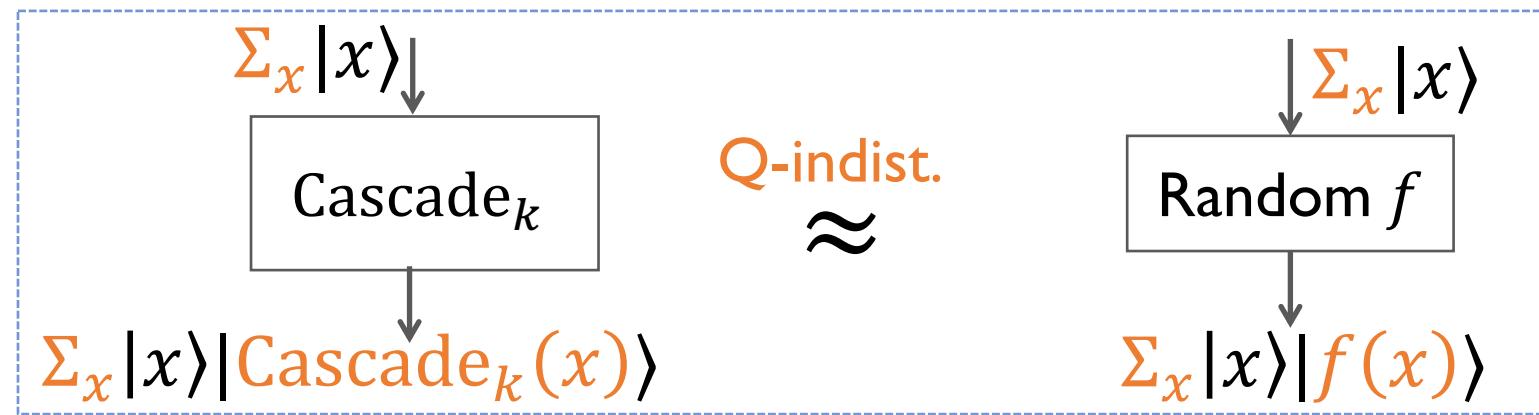
- Cascade (NMAC & HMAC) quantum-secure?
- Is quantum-secure domain-extension even possible?

# Our main result

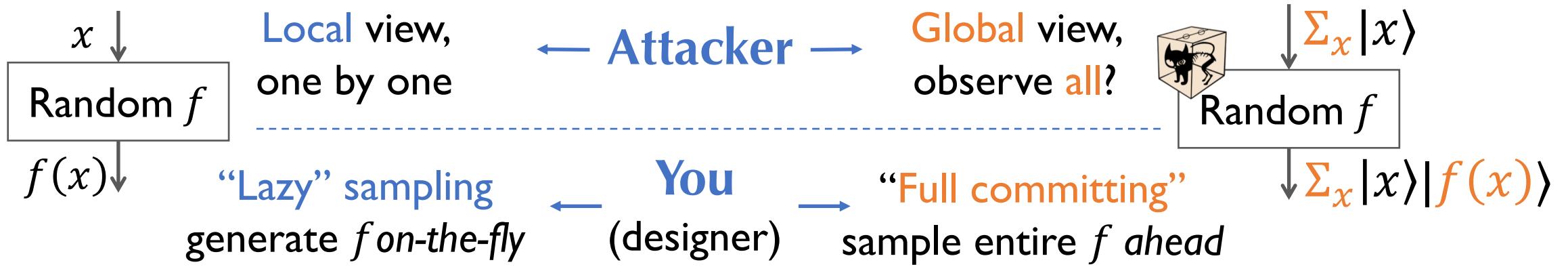
2. SY'Crypto17

Cascade, NMAC, HMAC (and more) are quantum-secure

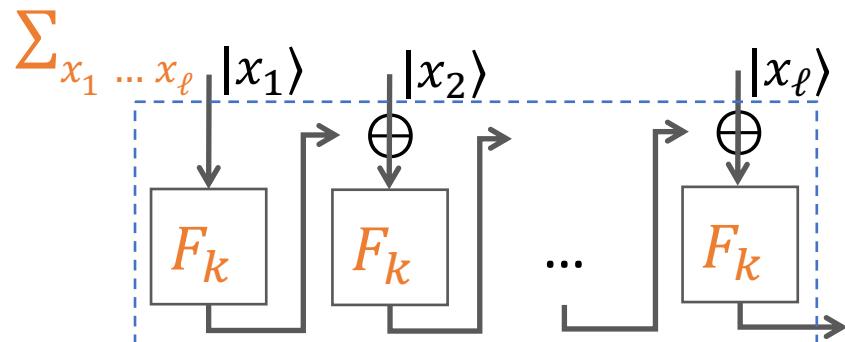
i.e., quantum-indistinguishable from a truly random function



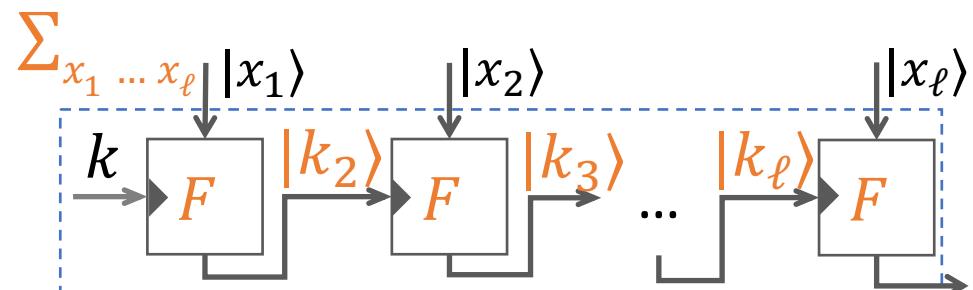
# Power of quantum-superposition attack



**CBC-MAC Broken!**  
global view on  $F_k$  of a random  $k$



**Cascade**  
global view on  $F_k$  of all  $k$ !



we proved it's quantum secure

# Future directions

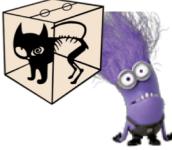
---

Quantum-safe  
Crypto

Emerging security  
issues

Power of quantum  
computing

# Future work: quantum-safe crypto



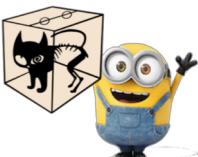
## Post-quantum crypto

- Are new candidates indeed hard?

- Break other lattice problems? Multivariate-equations? ...

- How to get quantum-secure cryptosystems?

- Fine-grained analysis of hash functions & block ciphers
- Quantum-safe Bitcoin & blockchain



## Quantum crypto

- What else are possible? & a formal framework?

- Quantum money, quantum tokenized crypto...

**NSF:Medium:Collaborative**

- New complexity foundation
- Q-secure auth., signature ...



# Future work: securing advancing technology

## What do they bring?

IoT, 5G, Cloud, Big Data, ML ...

- Diverse players of imbalanced resource

### ■ Lightweight Cryptography

- ☺ Memory-friendly, energy-friendly ...
- ☹ Attack-friendly too?

### ■ Update security model

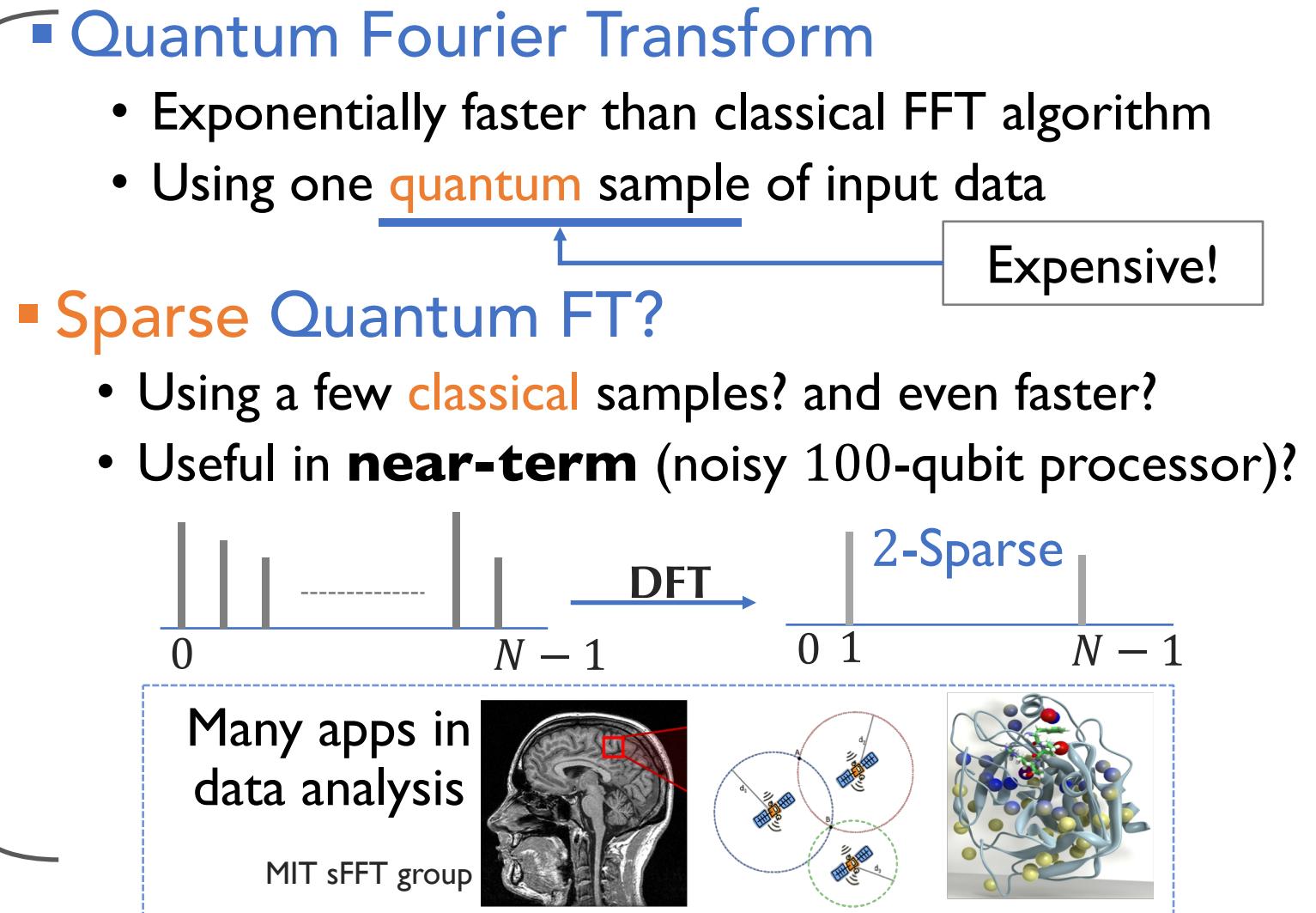
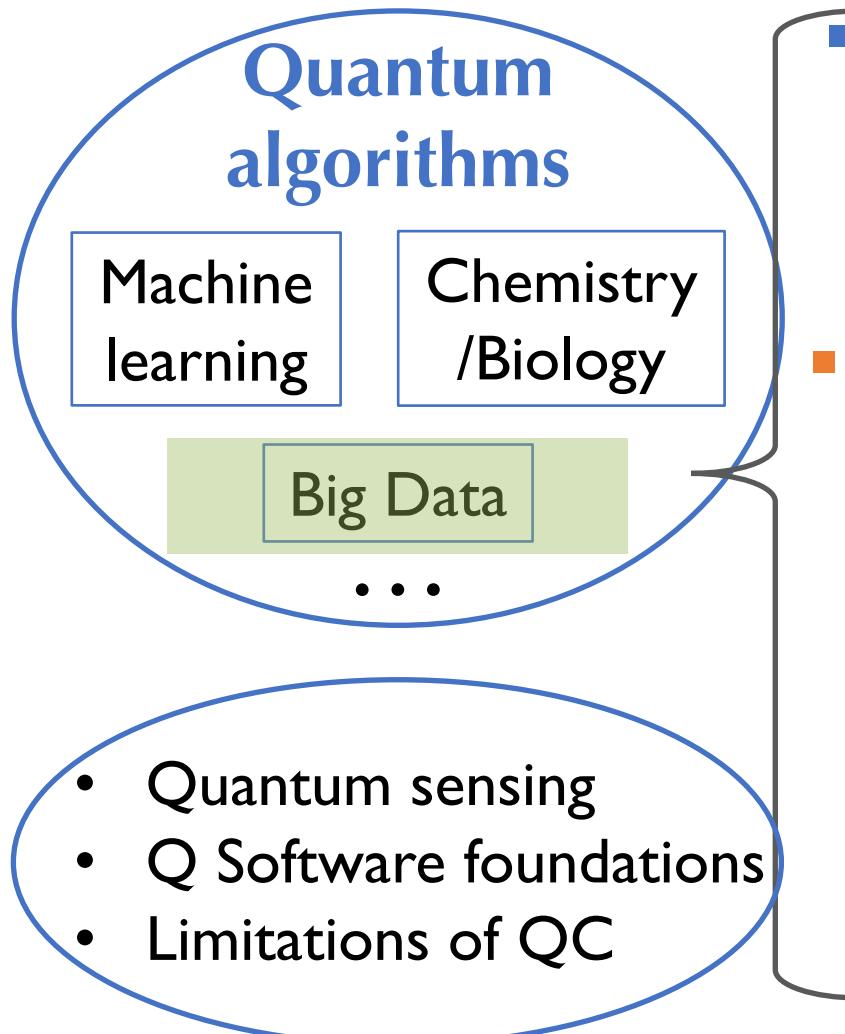
- Always poly-time adversary too coarse

### ■ Design protocols that better fit

- Workload respects capacity



# Future work: embracing quantum power



Quantum-safe  
Crypto

Emerging security  
issues

Power of quantum  
computing



Thank you!

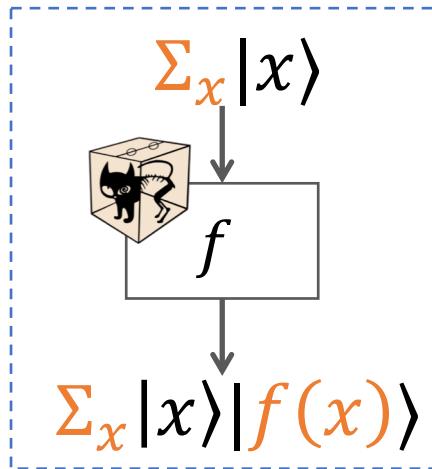
# References (my own)

---

1. Quantum Collision-Finding in Non-Uniform Random Functions. Marko Balogh and Edward Eaton and Fang Song. PQCrypto 2018.
2. Quantum Security of NMAC and Related Constructions. Fang Song and Aaram Yun. Crypto 2017.
3. Zero-knowledge proof systems for QMA. Anne Broadbent, Zhengfeng Ji, Fang Song and John Watrous. FOCS 2016.
4. Mitigating multi-target attacks in hash-based signatures. Andreas Hülsing, Joost Rijneveld and Fang Song. PKC 2016.
5. Efficient quantum algorithms for computing class groups and solving the principal ideal problem in arbitrary degree number fields. Jean-François Biasse and Fang Song. SODA 2016.
6. Making existentially unforgeable signatures strongly unforgeable in the quantum-random oracle model Authors: Edward Eaton and Fang Song. TQC 2015.
7. A note on quantum security for post-quantum cryptography. Fang Song. PQCrypto 2014.
8. A quantum algorithm for computing the unit group of an arbitrary degree number field. Kirsten Eisenträger, Sean Hallgren, Alexei Kitaev and Fang Song. STOC 2014.
9. Feasibility and completeness of cryptographic tasks in the quantum world. Serge Fehr, Jonathan Katz, Fang Song, Hong-Sheng Zhou and Vassilis Zikas TCC 2013.
10. Classical cryptographic protocols in a quantum world. Sean Hallgren, Adam Smith and Fang Song. Crypto 2011.
11. Pseudorandom states and unitaries, and applications to quantum money. Zhengfeng Ji, Yi-Kai Liu, Fang Song. arXiv:1711.00385, 2017
12. Basing cryptography on NP-hardness using quantum reductions. Nai-Hui Chia, Sean Hallgren, Fang Song. October 2017

# Is superposition attack realistic?

$f$ : crypto-algorithm  
(Enc, Sign, etc..) with  
**secret key**



Attacker can implement  
 $f$  as a quantum circuit

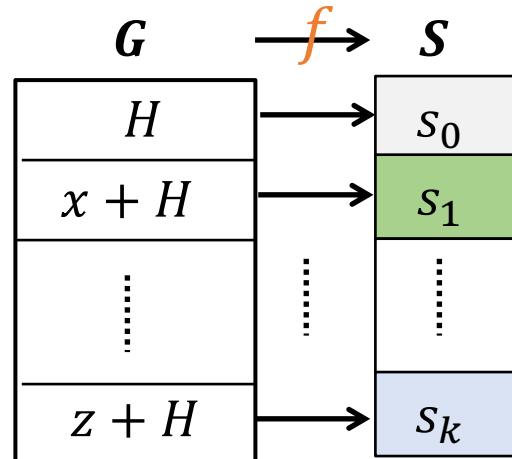
- Could occur in public-key setting
  - Ex. Block cipher to Pub-key Enc via obfuscation
- Building block in big system
  - Be conservative (quantum & classical hybrid Internet)
- Makes our **POSITIVE** result **stronger!**

# The Hidden Subgroup Problem (HSP) framework



Captures most quantum exponential speedup

- Standard Def.: HSP on finite group  $G$



**Given:** oracle function  $f: G \rightarrow S$ , s.t.  $\exists H \leq G$ ,

1. (Periodic on  $H$ )  $x - y \in H \Rightarrow f(x) = f(y)$
2. (Injective on  $G/H$ )  $x - y \notin H \Rightarrow f(x) \neq f(y)$

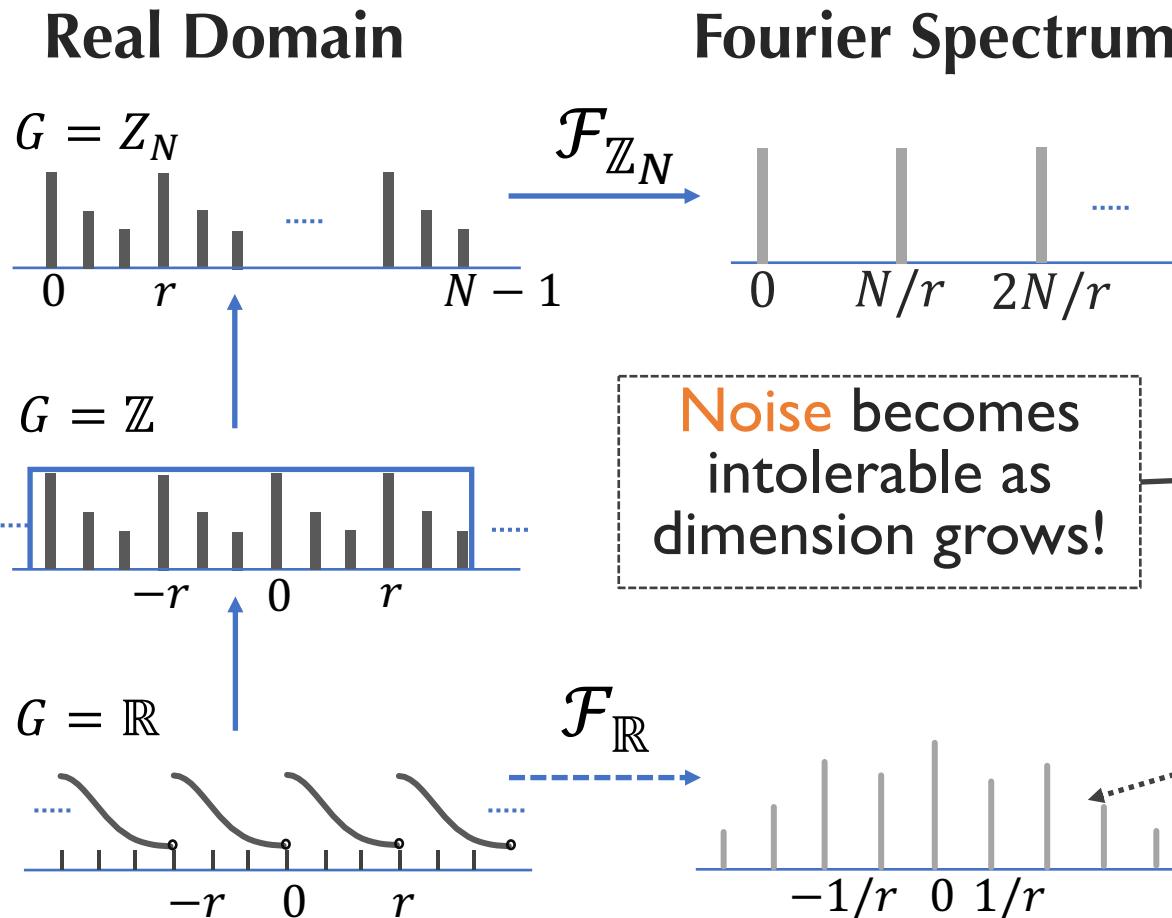
**Goal:** Find (hidden subgroup)  $H$ .

- Continuous  $G$  (e.g.,  $\mathbb{R}^n$ ) tricky, but we can handle [EHKS14]

# Solving HSP: quantum Fourier sampling

Given: oracle  $f: G \rightarrow S$  periodic on  $H$  & ...

Goal: find  $H$



Standard method for finite  $G$

- Quantum Fourier Sampling: quantum Fourier transform & measure
- Recover  $H$  from samples

Old method for  $\mathbb{R}^{\text{constant}}$

- Discretize & Truncate
- Reduce to finite  $G$

Our method for continuous  $\mathbb{R}^m$

- Informal: try to approx. sample the ideal Fourier spectrum directly!