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Fall'19 CSCE 629

Analysis of Algorithms

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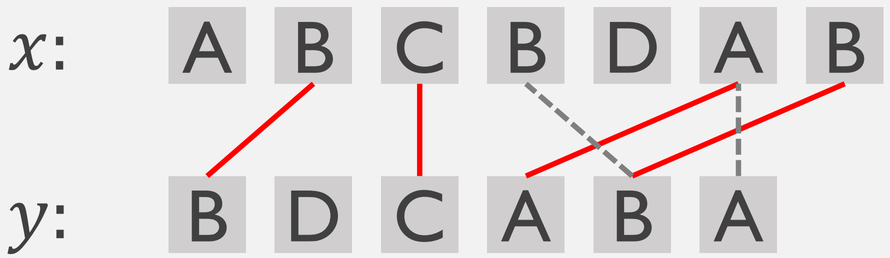
Lecture 10

- Longest common subsequence

Credit: based on slides by A.Smith and K.Wayne

Longest common subsequence (LCS)

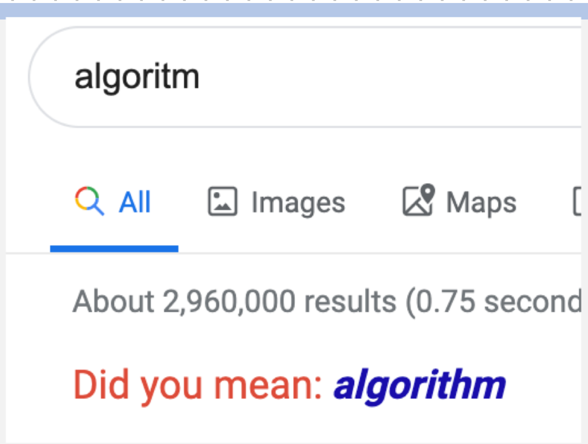
- Input. Two subsequences $x[1, \ldots, m]$ and $y[1, \ldots, n]$
- Output. A longest subsequence common to both.



- Other names you may heard of
  - Sequence alignment
  - Edit distance: $n - lenth(LCS(x, y))$
  - …

Motivation

- String matching [Levenshtein 1965]
  - Auto corrector
  - Spell checker
  - Speech recognition
  - Machine translation



- Computational biology [Needleman-Wunsch, 1970's]
  - simple measure of genome similarity

ACGTACGTACGTACGTACGTACGTATCGTACGT
AACGTACGTACGTACGTACGTACGTACGTACGT

 ACGTACGTACGTACGTACGTACGTA T ATCGTACGT
AACGTACGTACGTACGTACGTACGTA   ATCGTACGT

DP1: develop a recursion

- Input. Two subsequences $x[1, \ldots, m]$ and $y[1, \ldots, n]$
- Output. A longest subsequence common to both.
  - (Simplification) Look at the length of a longest-common subsequence
  - Extend the algorithm to find the LCS itself

Notation. Denote the length of a sequence s by $|s|$.

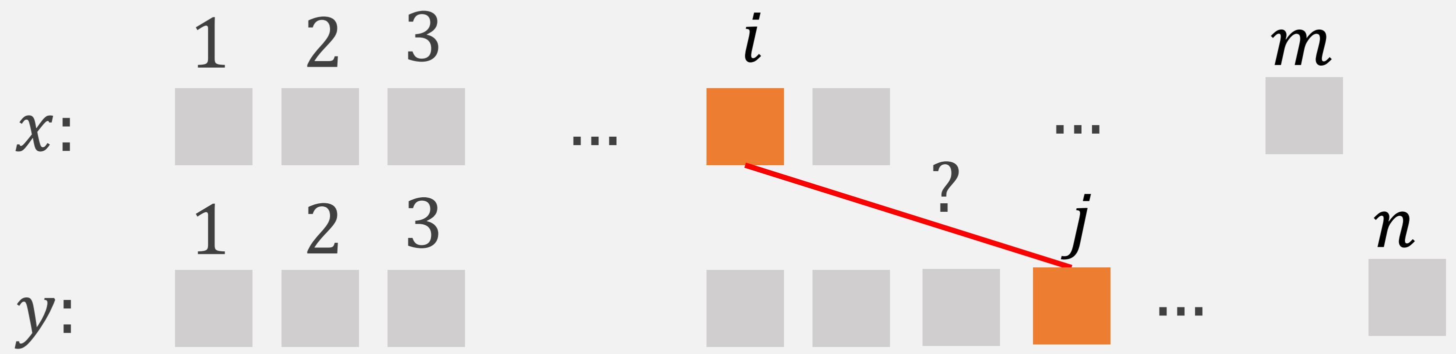
Def. $c(i,j) \coloneqq |\mathrm{LCS}(x[1, \ldots, i], y[1, \ldots, j])|$

- Goal. Find $c(m,n)$
- Basis: $c(i,j) = 0$ if $i = 0\ or\ j = 0$
- Recursion: how to define $c(i,j)$ recursively?

DP1: develop a recursion

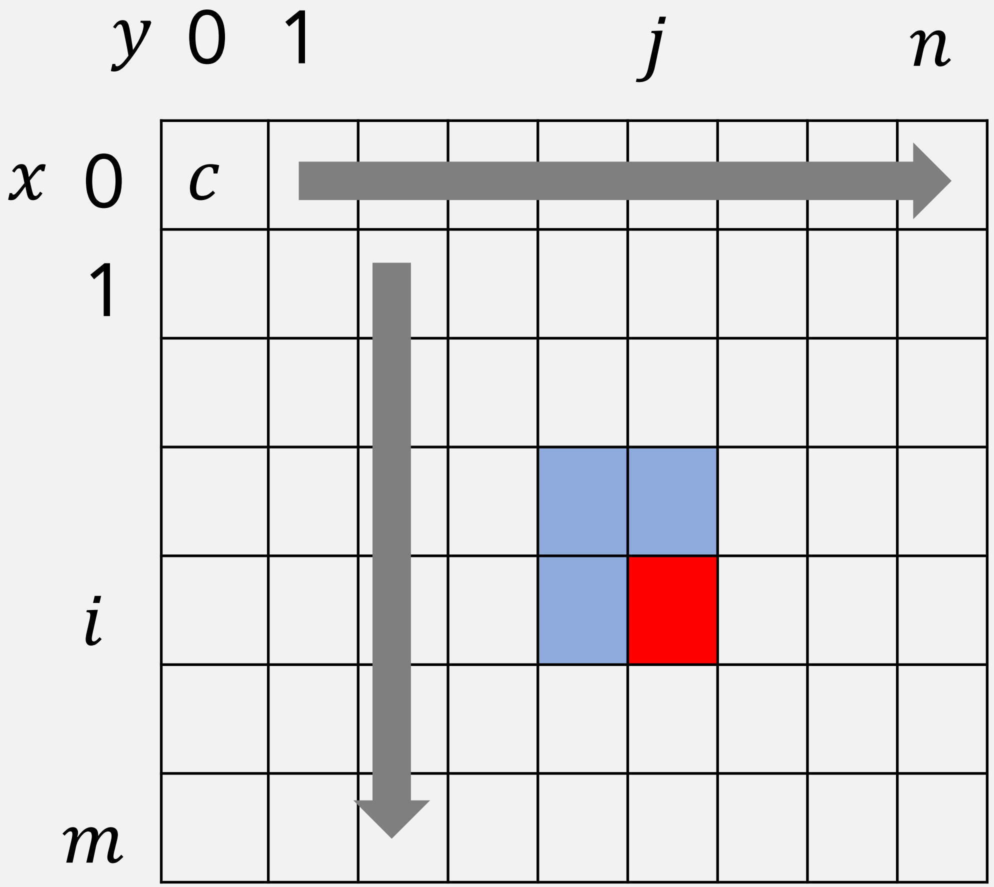
- Case1: $x[i] = y[j]$ $\quad c(i,j) = c(i-1, j-1) + 1$

- Case2: $x[i] \neq y[j]$ $\quad c(i,j) = \max\{c[i-1, j], c[i, j-1]\}$



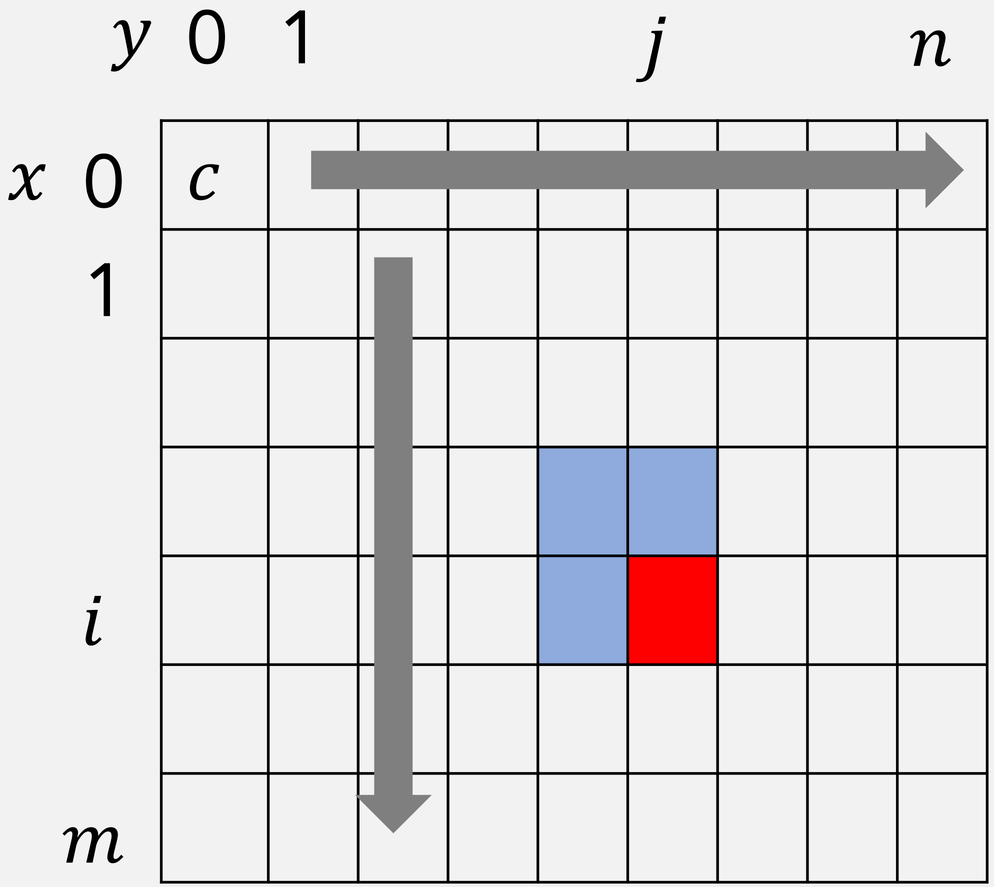
$$c(i,j) = \begin{cases} 0 \quad \text{if } i = 0 \text{ } or \text{ } j = 0 \\ c(i-1, j-1) + 1 \text{ if } x[i] \neq x[j] \\ \max\{c(i-1, j), c(i, j-1)\} \text{ if } x[i] \neq x[j] \end{cases}$$

DP2: build up solutions



- Subproblems: $O(mn)$
- Memoization data struture
  - 2-D array $c[0, \dots m, 0, \dots, n]$
- Dependencies
  - Each $c(i, j)$ depends on its three neighbors $c(i-1, j-1), c(i, j-1), c(i-1, j)$
- Evaluation order
  - Left-to-right, row by row

DP2: build up solutions



LCSLen $(\boldsymbol{x}[\mathbf{1},\ldots,\boldsymbol{m}],\boldsymbol{y}[\mathbf{1},\ldots,\boldsymbol{n}])$

// $\boldsymbol{c}(\boldsymbol{i},\boldsymbol{j})$ memoize subproblem values

For $j = 0, \ldots, n$

  $c[0, j] \leftarrow 0$

For $i = 1, \ldots, m$ // row by row

  $c[i, 0] \leftarrow 0$

  For $j =, \ldots, n$ //left to right

    If $x[i] = y[j]$

      $c(i, j) = c(i-1, j-1) + 1$

    Else

      $c(i, j) = min\{c(i, j-1), c(i-1, j)\}$

- Running time: $O(mn)$

Example

	y	A	B	C	B	D	A	B
x	0	0	0	0	0	0	0	0
B	0	0	1	1	1	1	1	1
D	0	0	1	1	1	2	2	2
C	0							
A	0							
B	0							
A	0							4

DP3: constructing an optimal solution

- Reconstruct LCS by tracing backwards

  $LCS(x, y) = \text{BCBA}$

  NB. Multiple solutions are possible.

	y	A	B	C	B	D	A	B
x	0	0	0	0	0	0	0	0
B	0	0	1	1	1	1	1	1
D	0	0	1	1	1	2	2	2
C	0	0	1	2	2	2	2	2
A	0	1	1	2	2	2	3	3
B	0	1	2	2	3	3	3	4
A	0	1	2	2	3	3	4	4

- Space: $O(mn)$
  - Can you do it in $\min\{m, n\}$? (Hint: divide and conquer)

Further development

[MasekPaterson'JCSS1980] $O(n^2/\log n)$

A Faster Algorithm Computing String Edit Distances*

How about $O(n^{1.9999})$?

Quadratic Barrier

[BI'STOC2015]

Edit Distance Cannot Be Computed in Strongly Subquadratic Time (unless SETH is false)

[BEG'SODA2018]

Approximating Edit Distance in Truly Subquadratic Time: Quantum and MapReduce*†

[CDGKS'FOCS2018]

2018 IEEE 59th Annual Symposium on Foundations of Computer Science

Approximating Edit Distance Within Constant Factor in Truly Sub-Quadratic Time

[HRS'STOC2019]

Near-Linear Time Insertion-Deletion Codes and (1+ε)-Approximating Edit Distance via Indexing

[BGHS'STOC2019]

$1+\epsilon$ Approximation of Tree Edit Distance in Quadratic Time*†