

Quantum Cryptography

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Centrum Wiskunde & Informatica

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1969: Man on the Moon

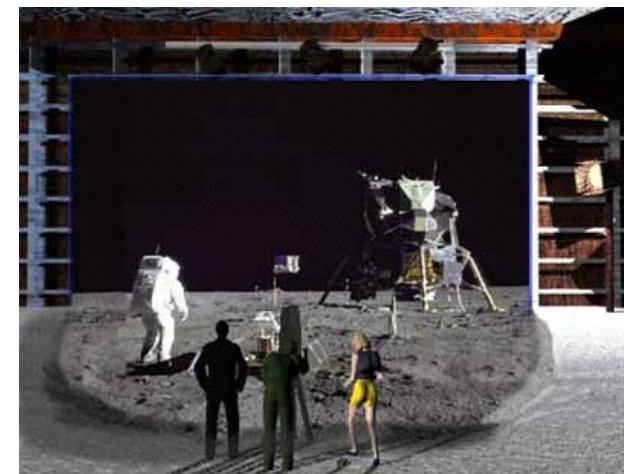


<http://www.unmuseum.org/moonhoax.htm>

- How can you prove that you are at a specific location?

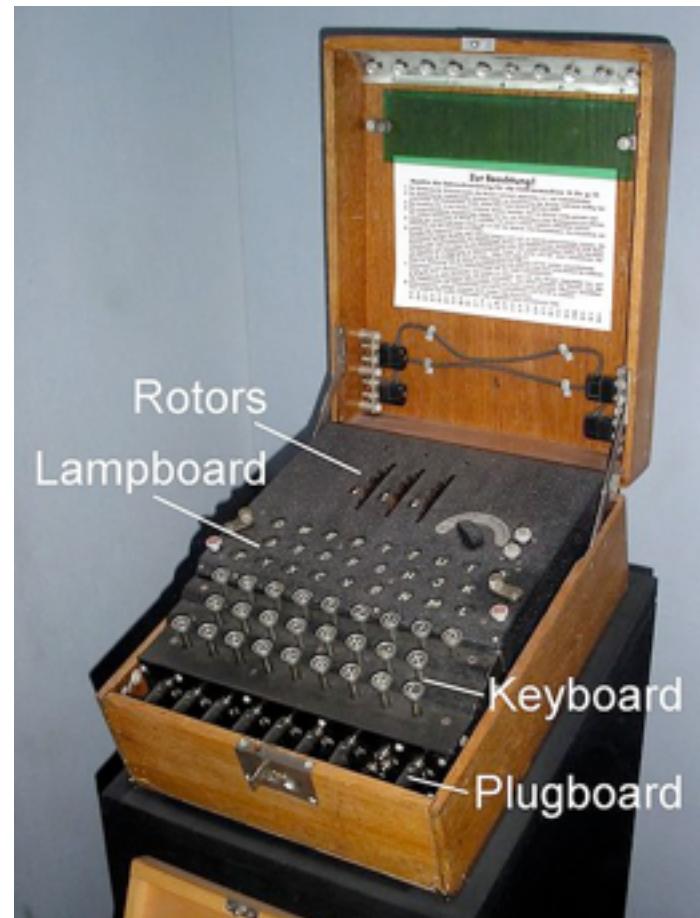
What will you learn from this Talk?

- Classical Cryptography
- Introduction to Quantum Mechanics
- Quantum Key Distribution
- Position-Based Cryptography



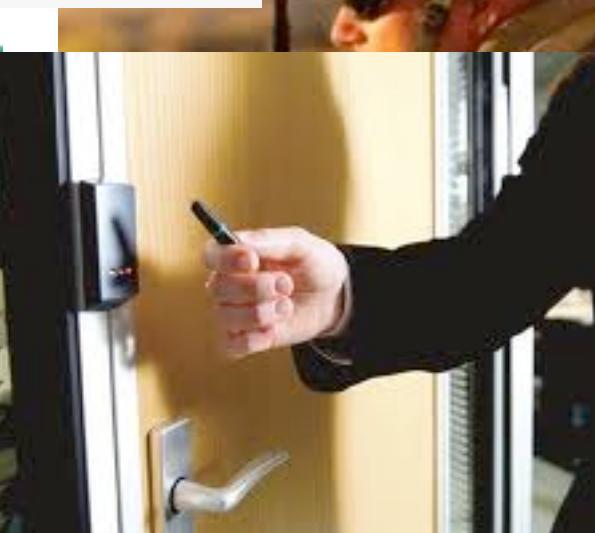
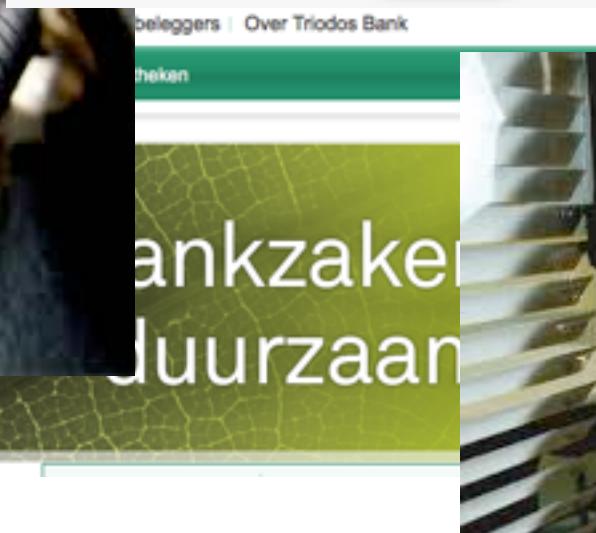
Ancient Cryptography

- 3000 years of fascinating history
- until 1970: **private communication** was the only goal



Modern Cryptography

- is **everywhere!**
- is concerned with all settings where people **do not trust** each other



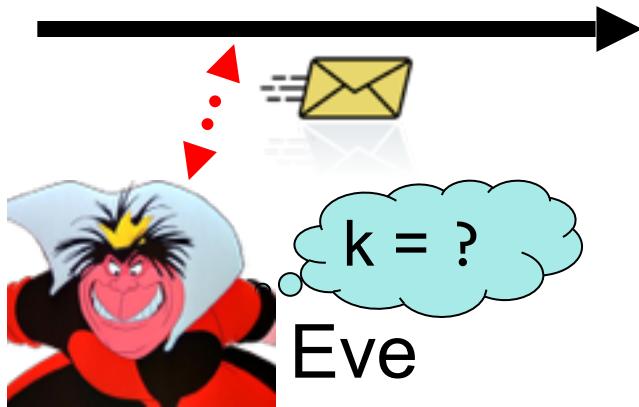
Secure Encryption

$m = \text{'I do not know you'}$

Alice



$k = 0101 \ 1011$



Bob



$k = 0101 \ 1011$

- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a secret key k

eXclusive OR (XOR) Function

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x	y	$x \oplus y$
0	0	0
1	0	1
0	1	1
1	1	0

- Some properties:

- $\forall x : x \oplus 0 = x$ $\Rightarrow \forall x, y : x \oplus y \oplus y = x$
- $\forall x : x \oplus x = 0$

One-Time Pad Encryption

$$m = 0000 \ 1111$$

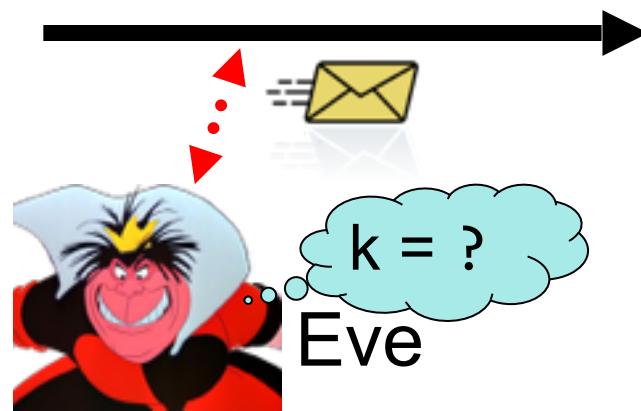
$$c = m \oplus k = 0101 \ 0100$$

$$m = c \oplus k = 0000 \ 1111$$

Alice



$$k = 0101 \ 1011$$



Bob



$$k = 0101 \ 1011$$

- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a key k
- Recipe:

$$m = 0000 \ 1111$$

$$c = 0101 \ 0100$$

$$k = 0101 \ 1011$$

$$k = 0101 \ 1011$$

$$c = m \oplus k = 0101 \ 0100$$

$$c \oplus k = 0000 \ 1111$$

- Is it secure?

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Perfect Security

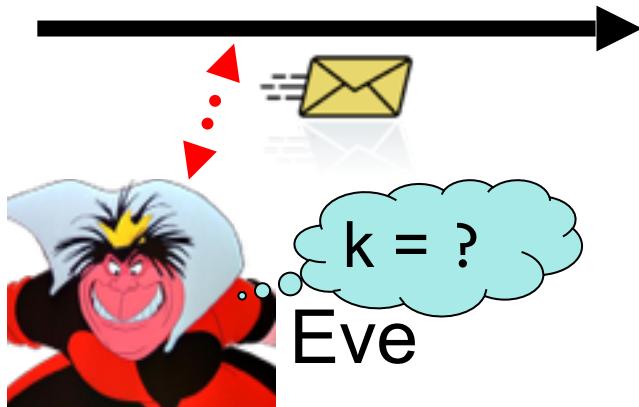
⁹ $m = ?$

Alice



$k = ?$

$$c = m \oplus k = 0101 \ 0100$$



$$m = c \oplus k = ?$$



Bob



$k = ?$

- Given that $c = 0101 \ 0100$,
 - is it possible that $m = 0000 \ 0000$?
 - Yes, if $k = 0101 \ 0100$.
 - is it possible that $m = 1111 \ 1111$?
 - Yes, if $k = 1010 \ 1011$.
 - it is possible that $m = 0101 \ 0101$?
 - Yes, if $k = 0000 \ 0001$

- In fact, every m is possible.

- Hence, the one-time pad is perfectly secure!

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Problems With One-Time Pad

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$$m = 0000 \ 1111$$

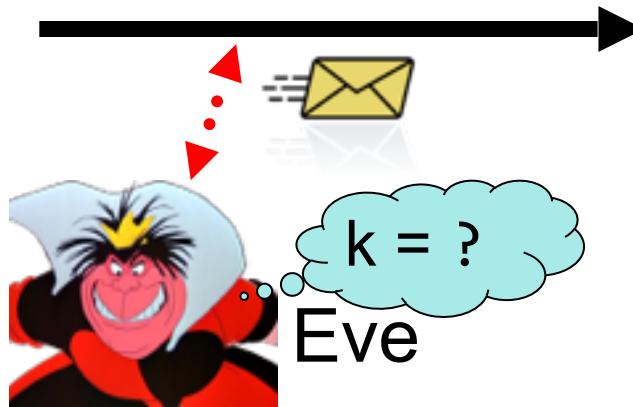
$$c = m \oplus k = 0101 \ 0100$$

$$m = c \oplus k = 0000 \ 1111$$

Alice



$$k = 0101 \ 1011$$



Bob



$$k = 0101 \ 1011$$

- The key has to be **as long as** the message (Shannon's theorem)
- The key can only be **used once**.
- In practice, other encryption schemes (such as [AES](#)) are used which allow to encrypt long messages with short keys.
- One-time pad does not provide [authentication](#):
Eve can easily flip bits in the message

Symmetric-Key Cryptography

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- Encryption insures **secrecy**:
Eve **does not learn** the message, e.g. [one-time pad](#)
- Authentication insures **integrity**:
Eve **cannot alter** the message
- General problem: players have to exchange a key to start with

What will you Learn from this Talk?



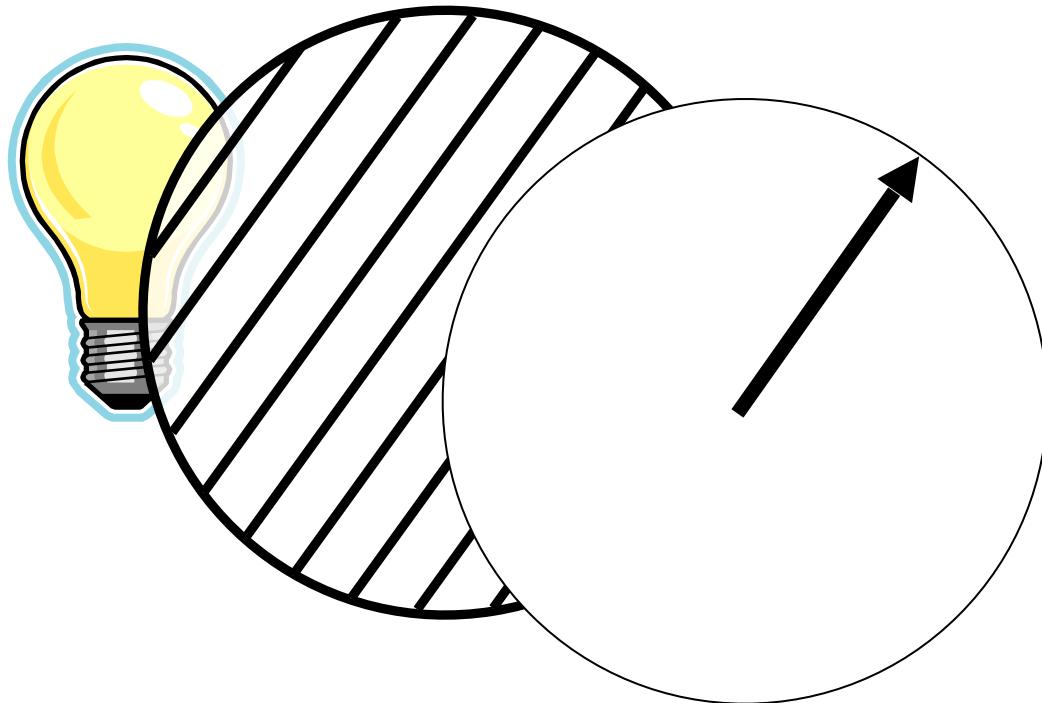
✓ Classical Cryptography



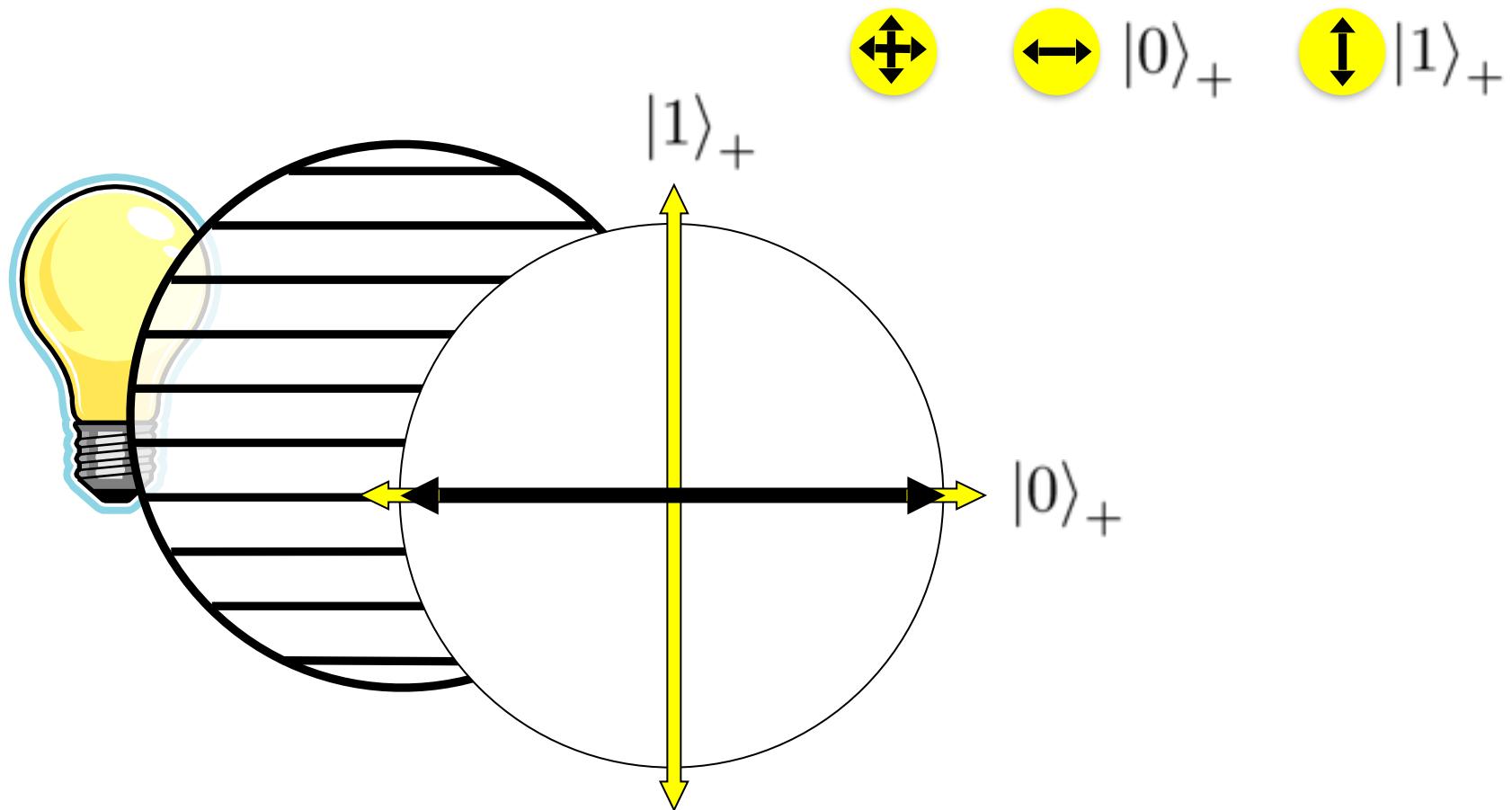
- Introduction to Quantum Mechanics
- Quantum Key Distribution
- Position-Based Cryptography

Quantum Bit: Polarization of a Photon

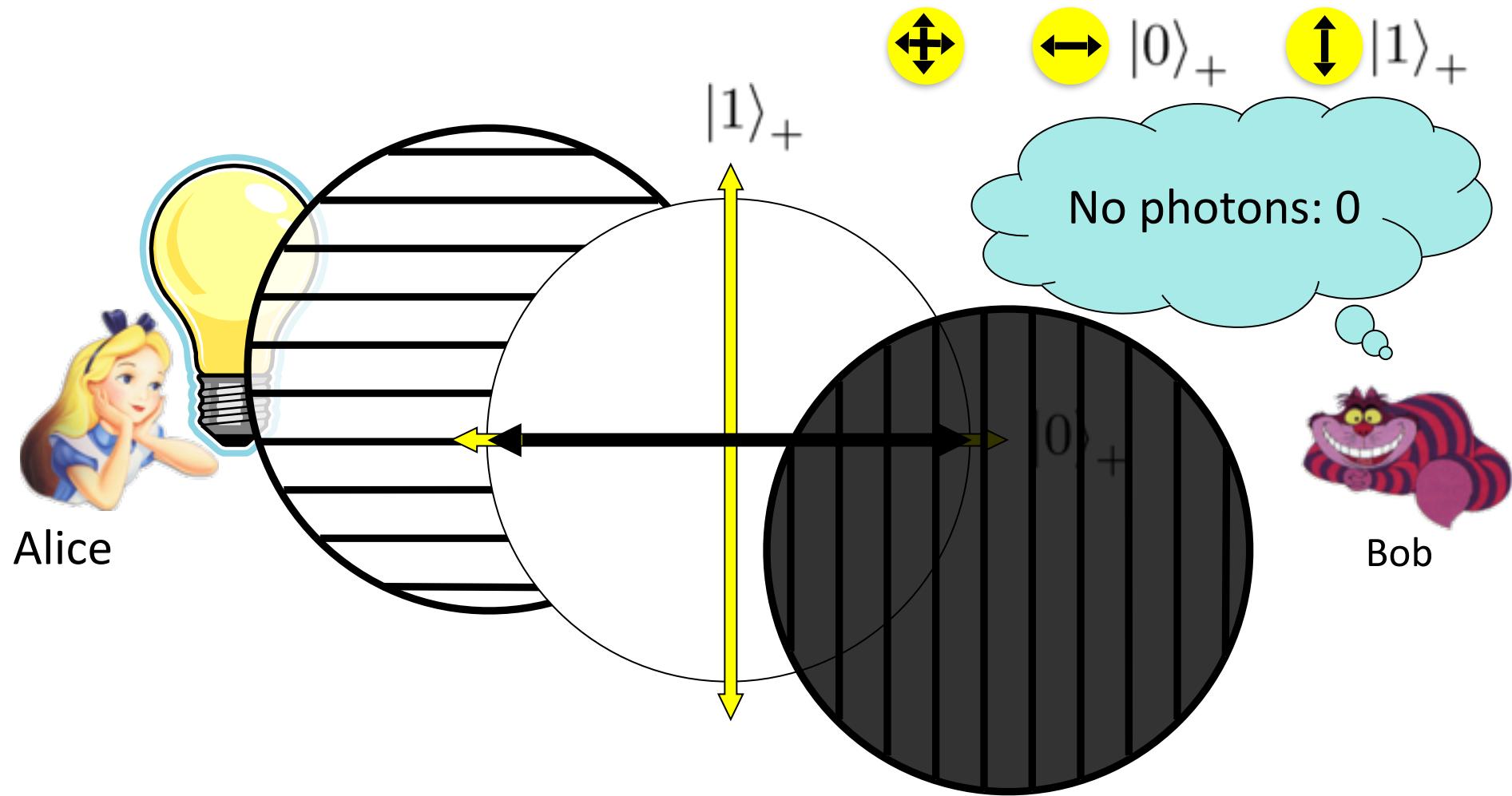
qubit as unit vector in \mathbb{C}^2



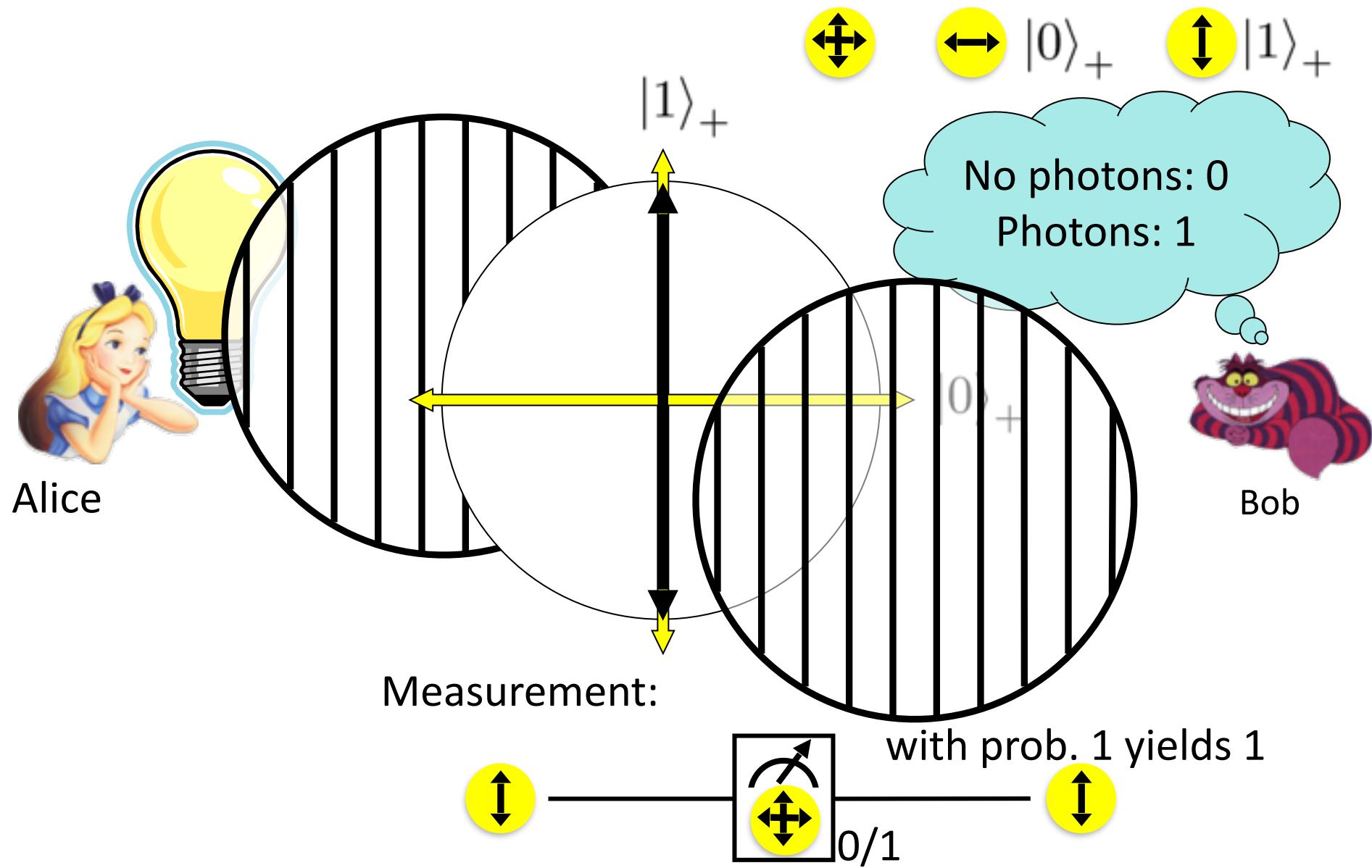
Qubit: Rectilinear/Computational Basis



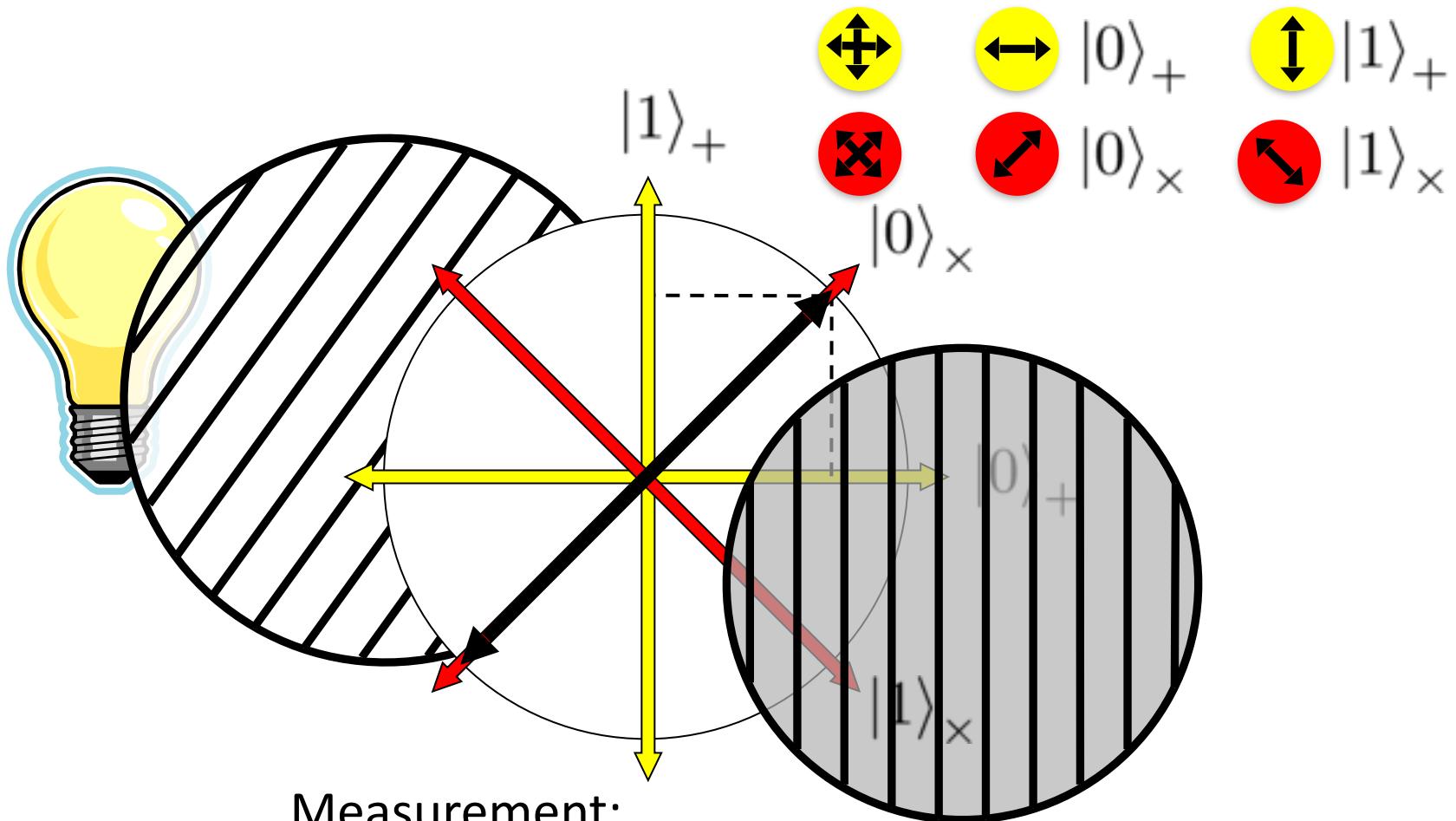
Detecting a Qubit



Measuring a Qubit



Diagonal/Hadamard Basis



$$\frac{\begin{array}{c} \leftrightarrow \\ + \end{array} + \begin{array}{c} \uparrow \\ \downarrow \end{array}}{\sqrt{2}} = \begin{array}{c} \leftrightarrow \\ \times \end{array}$$

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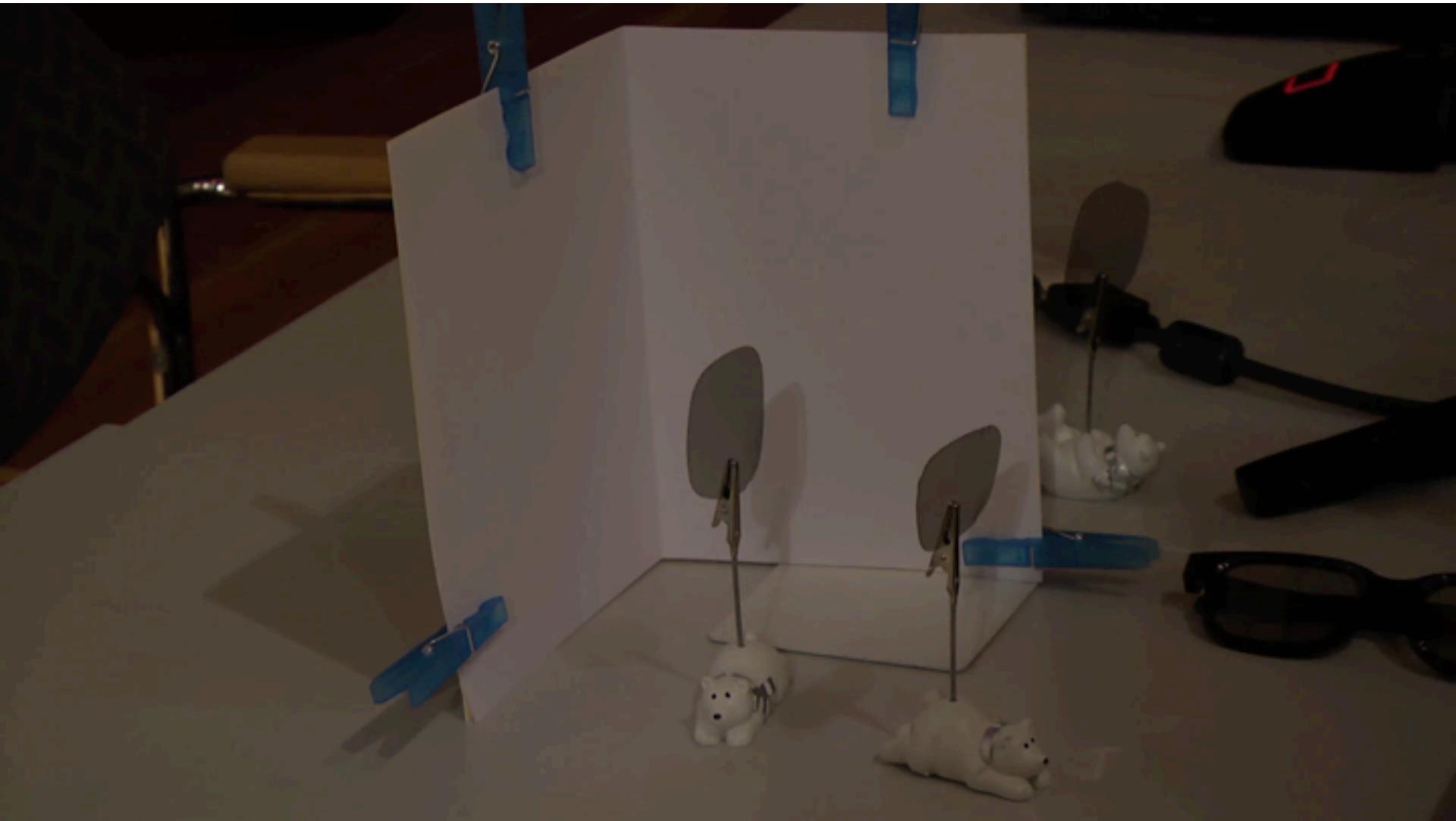
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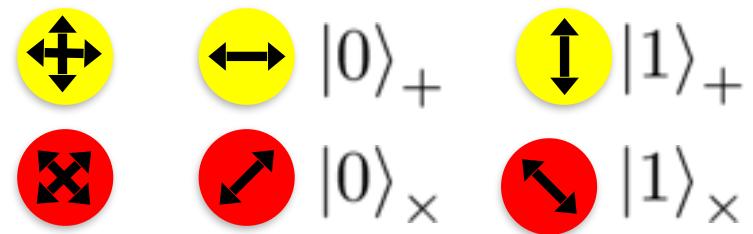
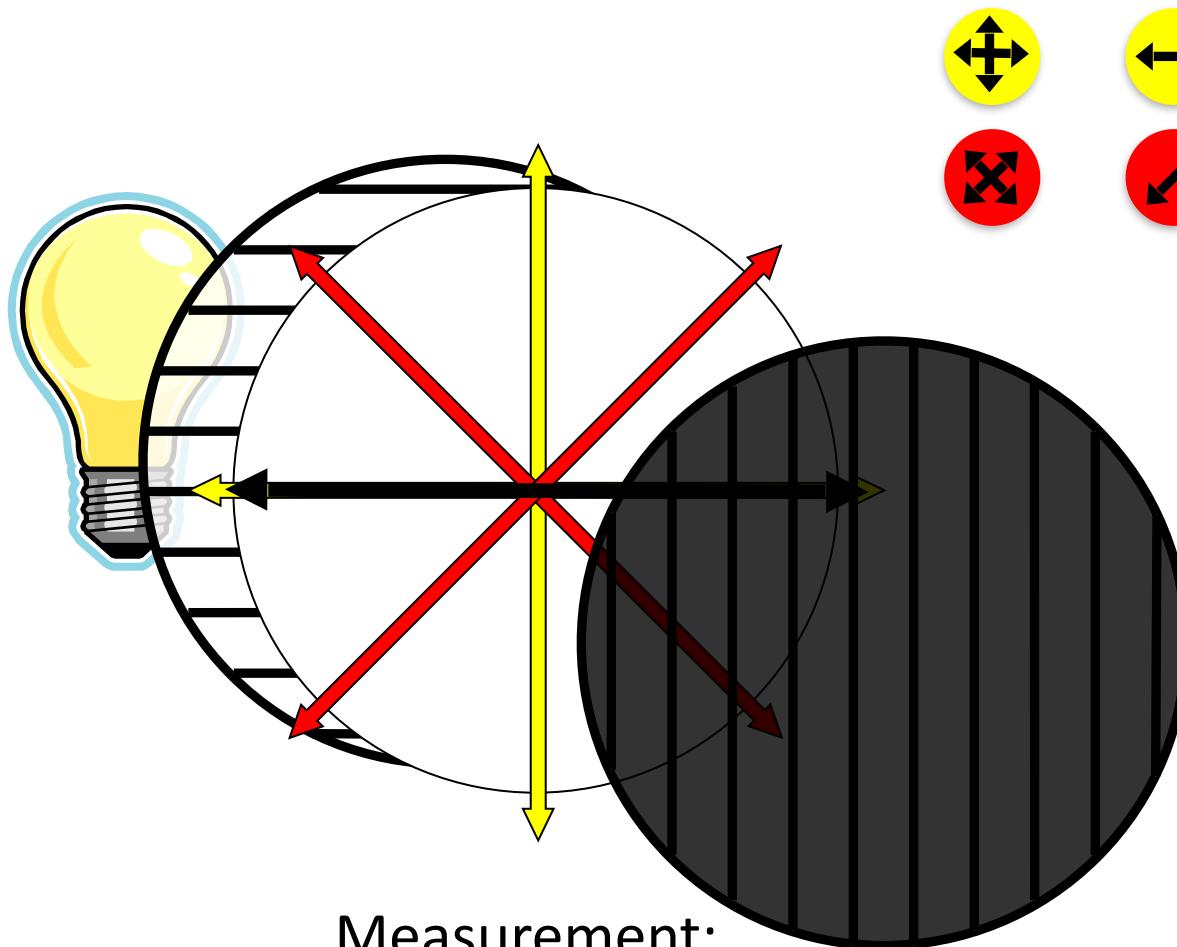
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Video



Measuring Collapses the State



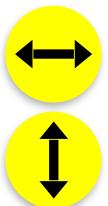
Measurement:

$$\frac{\left(\begin{smallmatrix} \leftrightarrow \\ \uparrow\downarrow \end{smallmatrix}\right) + \left(\begin{smallmatrix} \times\otimes \\ \rightarrow\leftarrow \end{smallmatrix}\right)}{\sqrt{2}} = \left(\begin{smallmatrix} \rightarrow\leftarrow \\ \times\otimes \end{smallmatrix}\right)$$

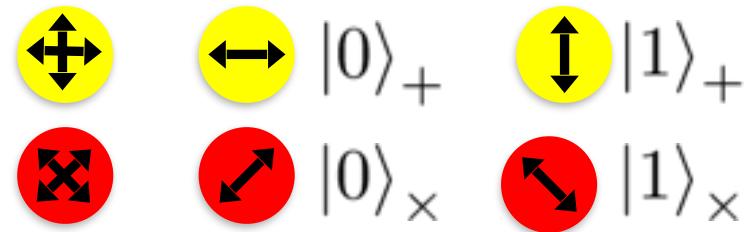
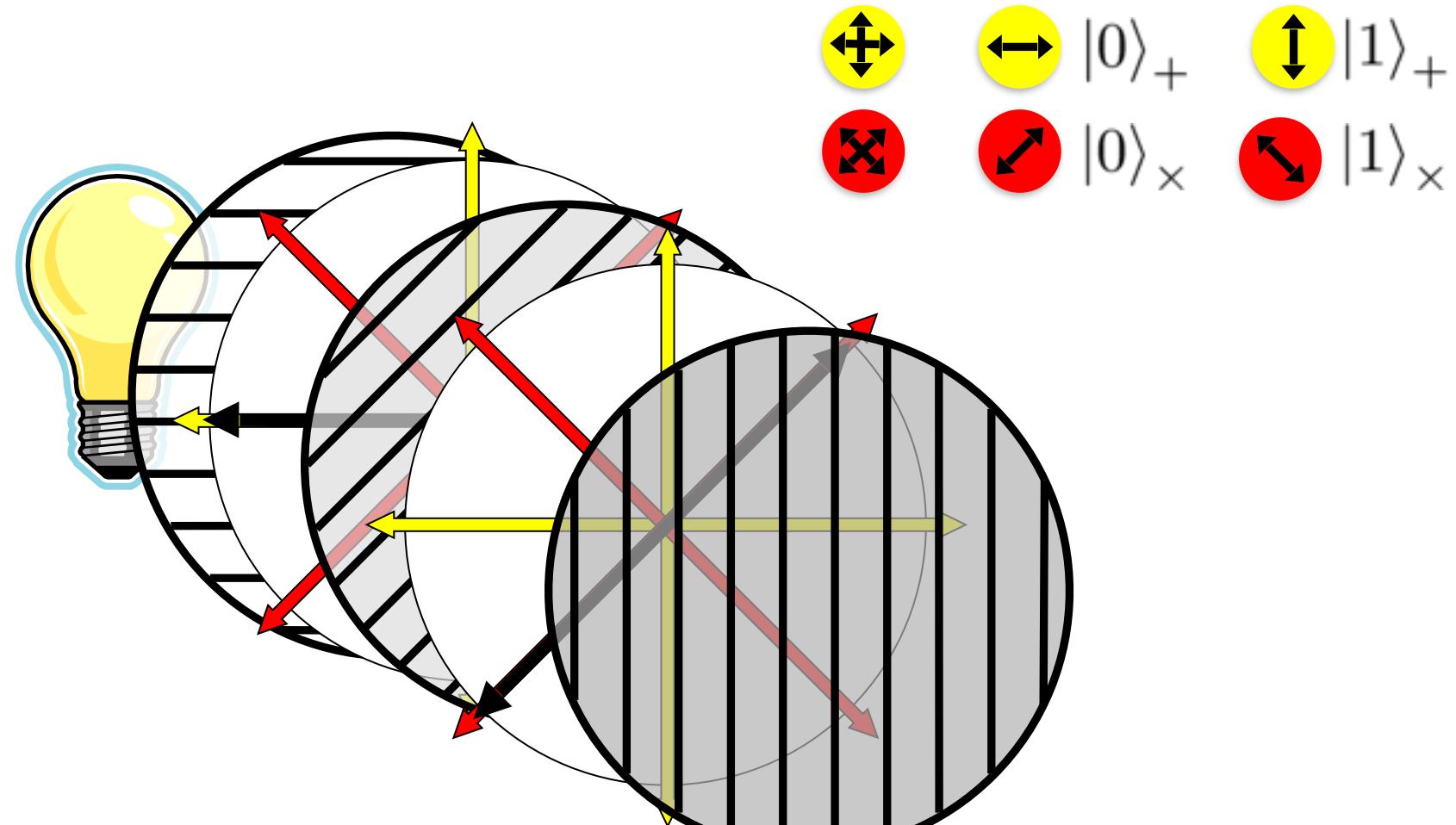
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$\boxed{\text{ } \curvearrowleft \text{ } \curvearrowright}$ $0/1$

with prob. $\frac{1}{2}$ yields 0
with prob. $\frac{1}{2}$ yields 1



Measuring Collapses the State

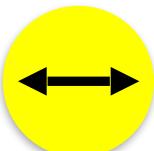


$$\begin{aligned} \leftrightarrow &= \frac{\leftrightarrow + \leftrightarrow}{\sqrt{2}} \rightarrow \leftrightarrow = \frac{\leftrightarrow + \downarrow}{\sqrt{2}} \rightarrow \downarrow \end{aligned}$$

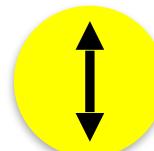
Quantum Mechanics



+ basis



$|0\rangle_+$



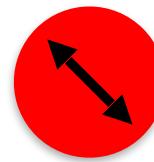
$|1\rangle_+$



\times basis



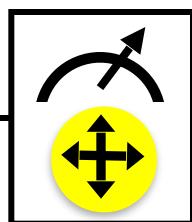
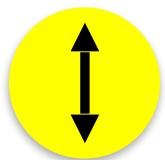
$|0\rangle_\times$



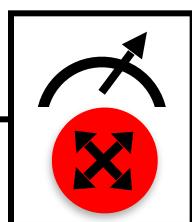
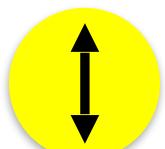
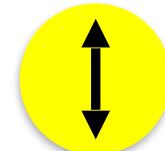
$|1\rangle_\times$

Measurements:

with prob. 1 yields 1



0/1



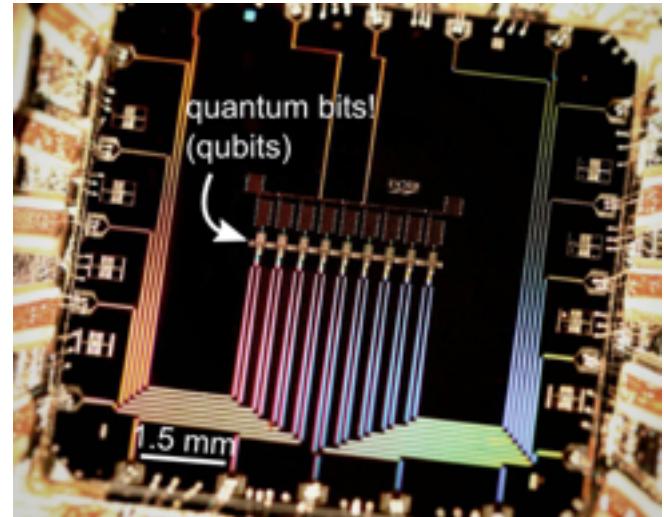
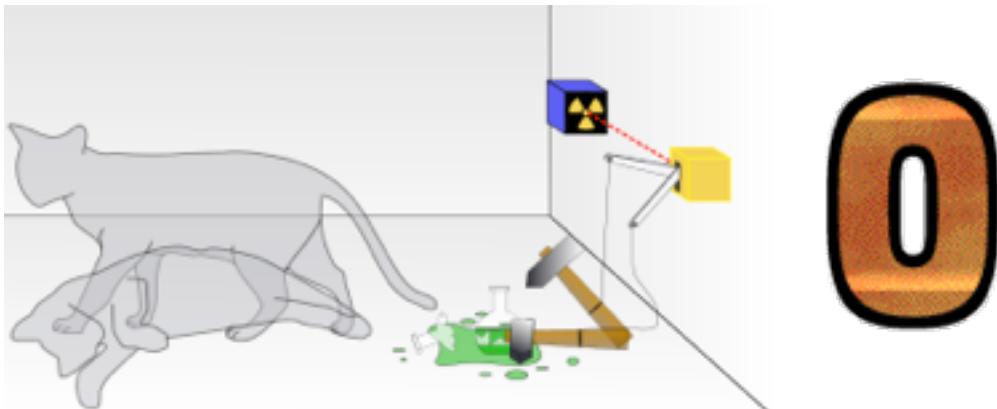
0/1

with prob. $\frac{1}{2}$ yields 0

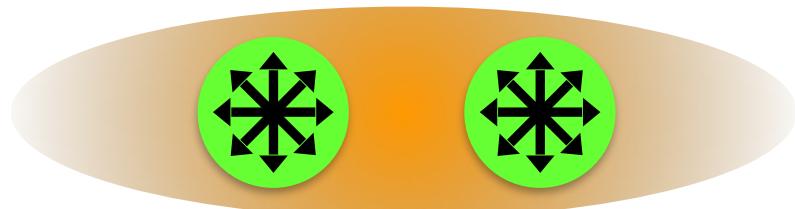


with prob. $\frac{1}{2}$ yields 1





Wonderland of Quantum Mechanics



What will you Learn from this Talk?

✓ Classical Cryptography



✓ Introduction to Quantum Mechanics

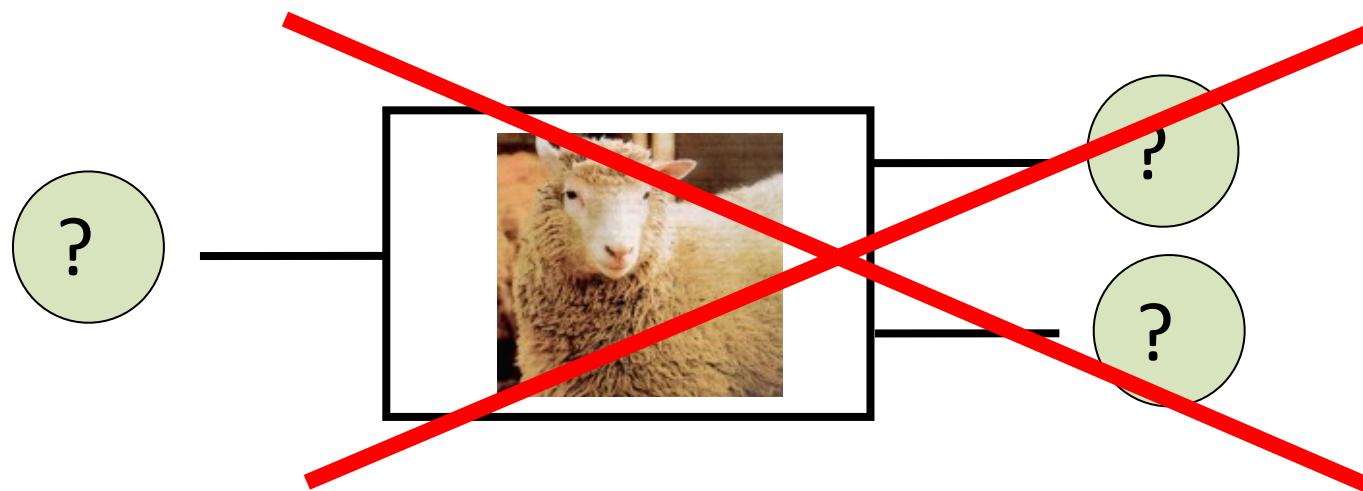
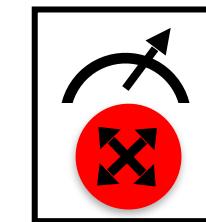
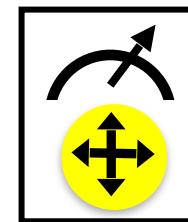
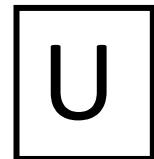
■ Quantum Key Distribution

■ Position-Based Cryptography

No-Cloning Theorem



Quantum operations:



Proof: copying is a **non-linear operation**

Quantum Key Distribution (QKD)

25

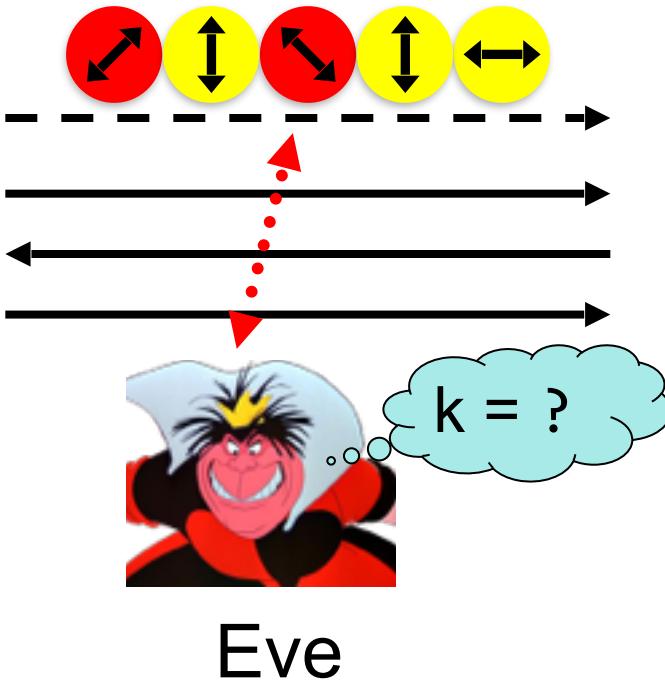
[Bennett Brassard 84]



Alice



$k = 0101 \ 1011$



Eve



Bob



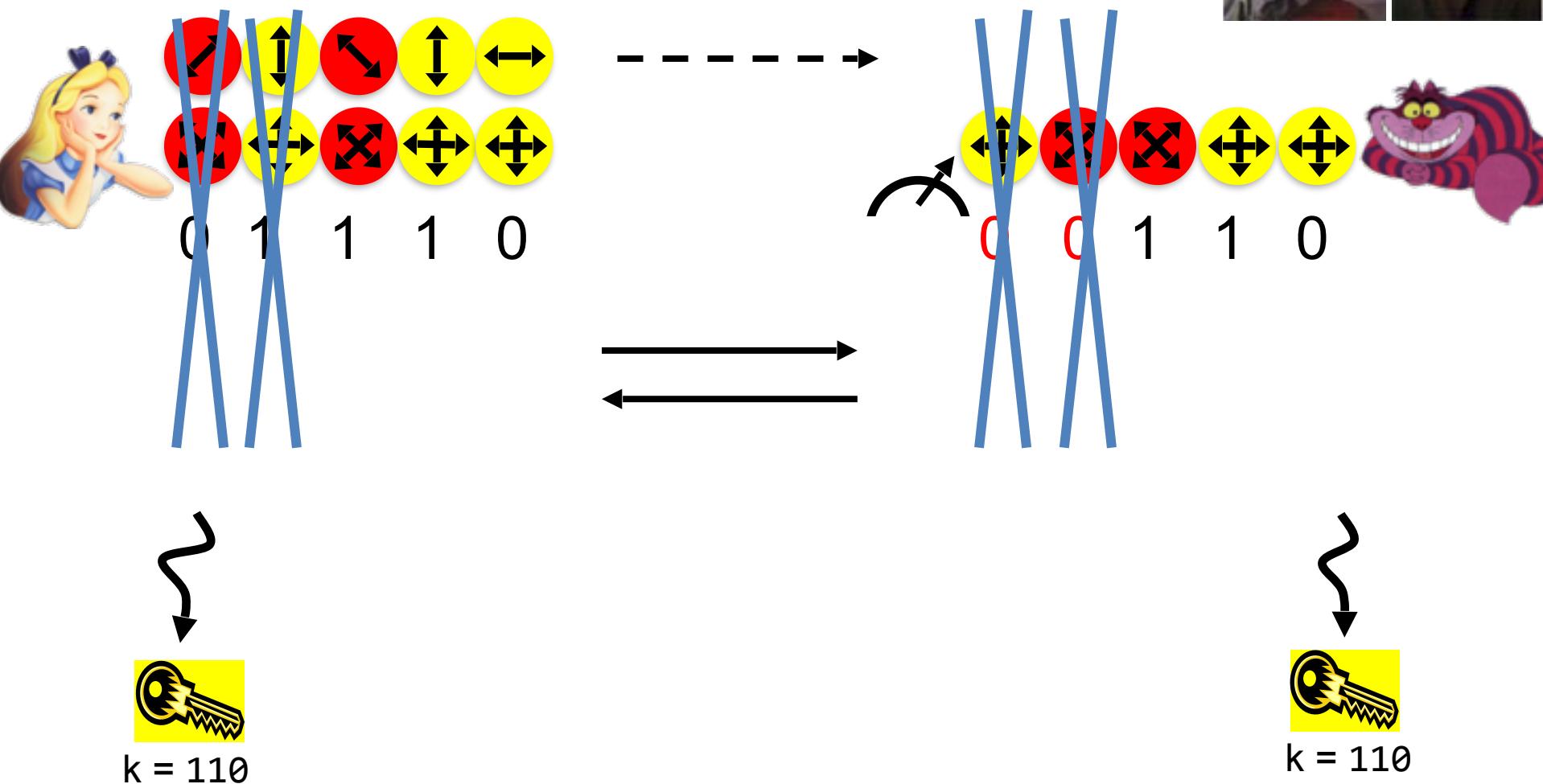
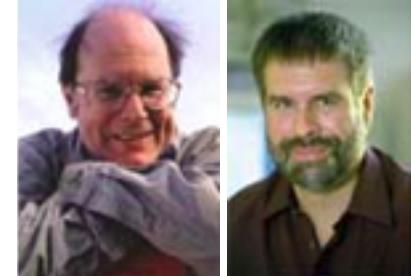
$k = 0101 \ 1011$

- Offers an **quantum solution** to the key-exchange problem which does **not** rely on **computational assumptions** (such as factoring, discrete logarithms, security of AES, SHA-3 etc.)
- Puts the players into the starting position to use symmetric-key cryptography (encryption, authentication etc.).

Quantum Key Distribution (QKD)

26

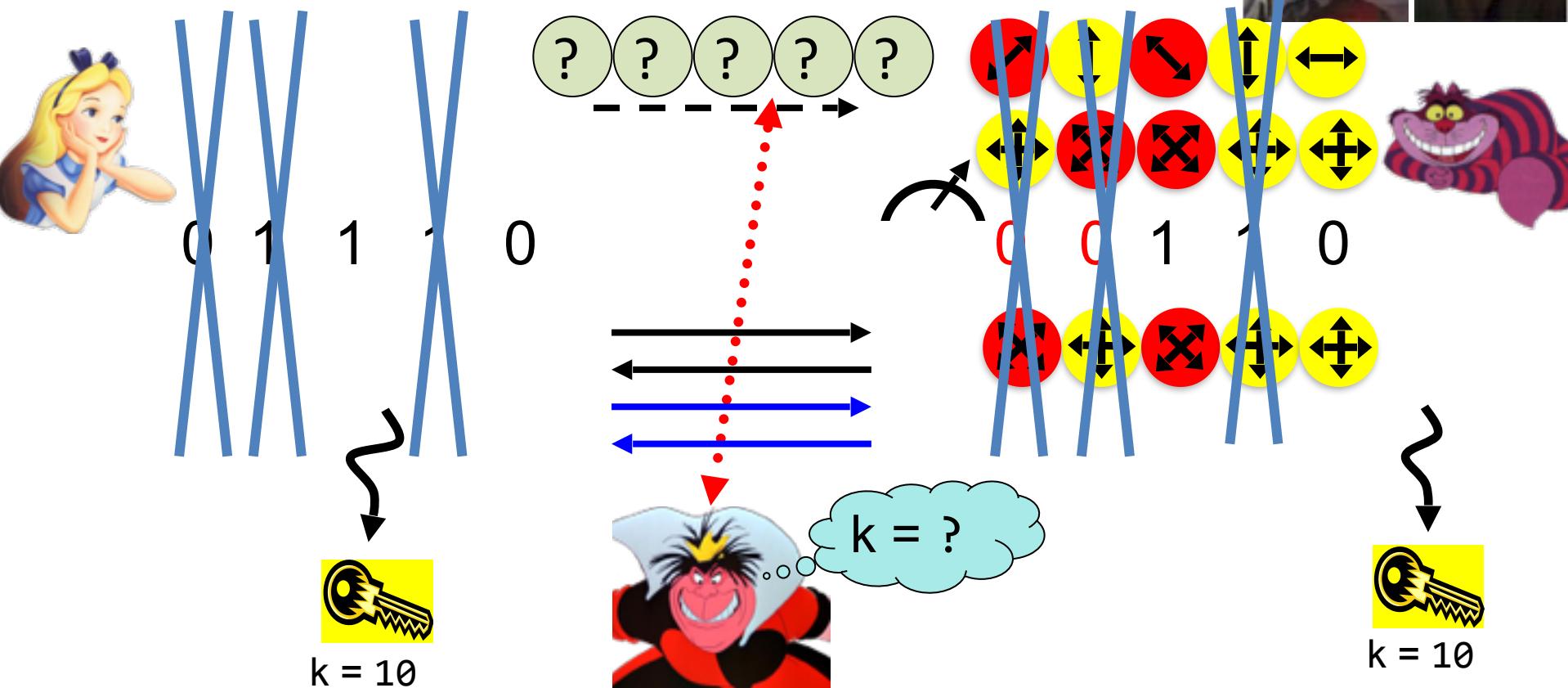
[Bennett Brassard 84]



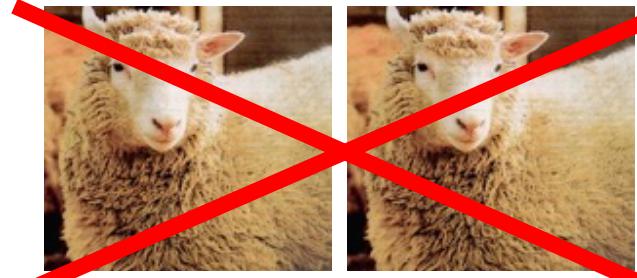
Quantum Key Distribution (QKD)

27

[Bennett Brassard 84]



- Quantum states are unknown to Eve, she **cannot copy them**.
- Honest players can **test** whether Eve interfered.

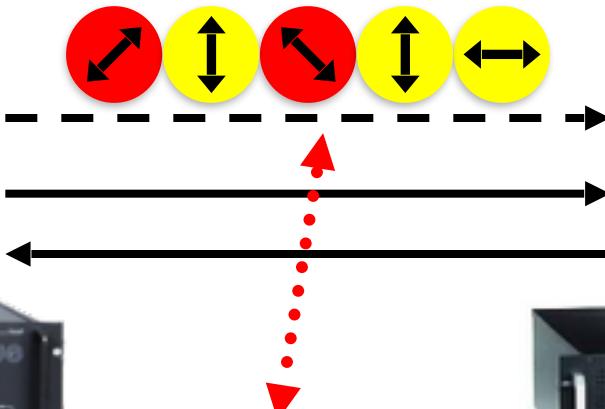


Quantum Key Distribution (QKD)

[Bennett Brassard 84]



Alice



Bob



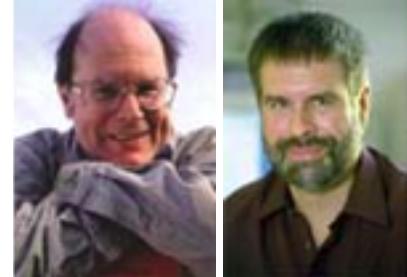
Eve

- **technically feasible:** no quantum computer required, only quantum communication

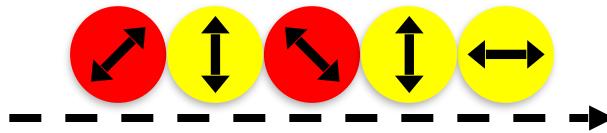
Quantum Key Distribution (QKD)

29

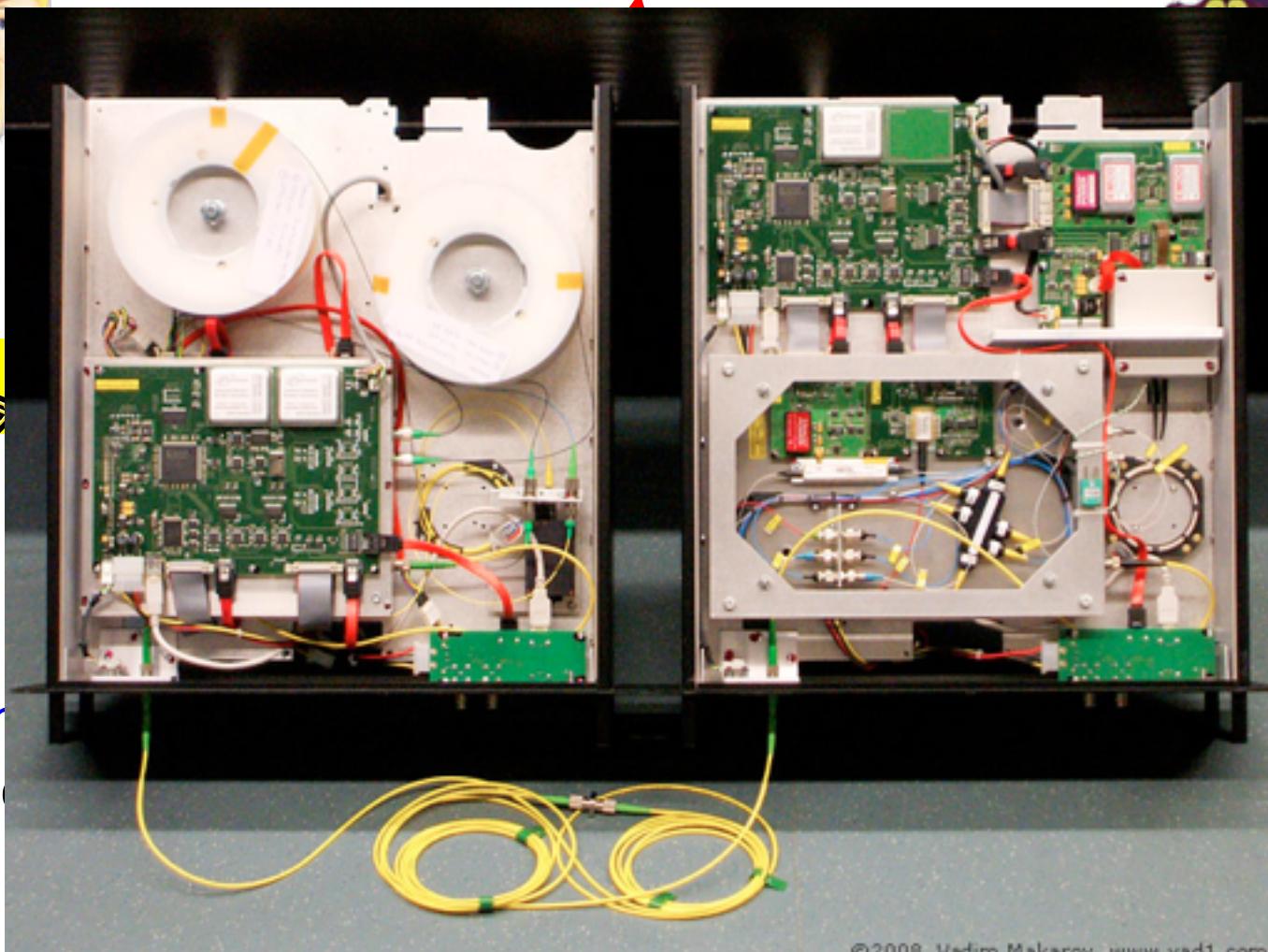
[Bennett Brassard 84]



Alice



Bob

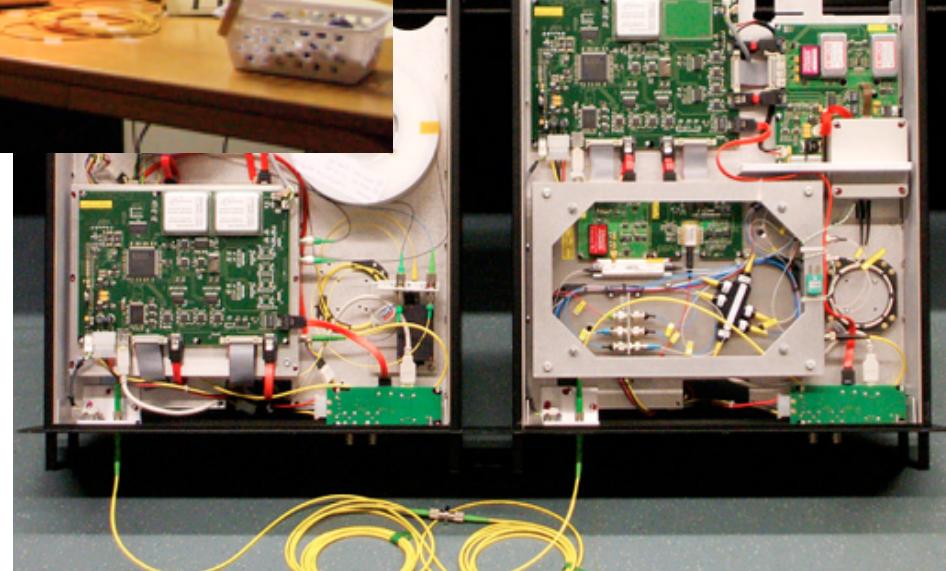


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only



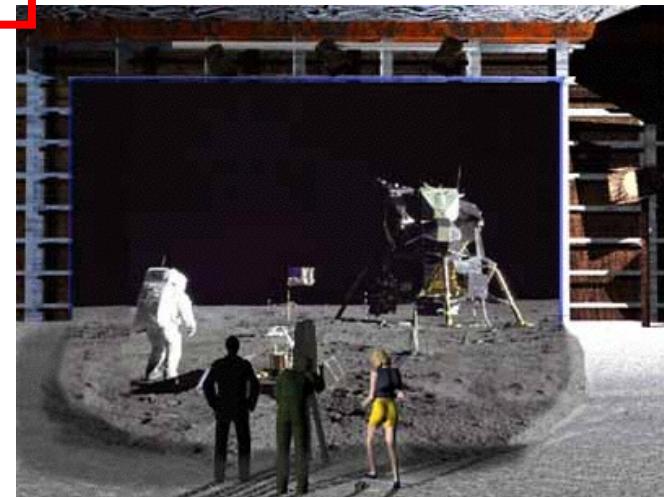
Quantum Hacking

e.g. by the group of [Vadim Makarov](#) (University of Waterloo, Canada)



What will you Learn from this Talk?

- ✓ Classical Cryptography
- ✓ Introduction to Quantum Mechanics
- ✓ Quantum Key Distribution
- Position-Based Cryptography



Position-Based Cryptography

- Typically, cryptographic players use **credentials** such as
 - secret information (e.g. password or secret key)
 - authenticated information
 - biometric features



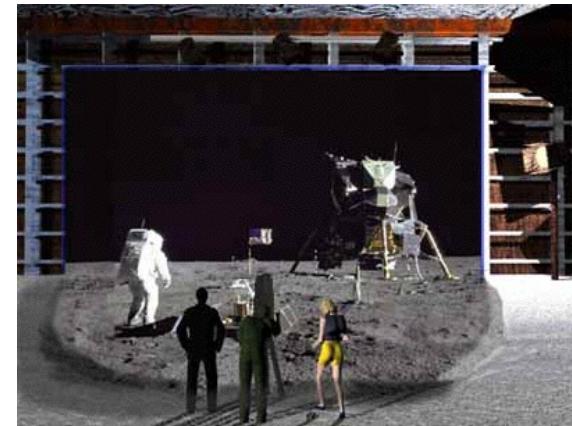
Can the geographical location of a player be used
as cryptographic credential ?



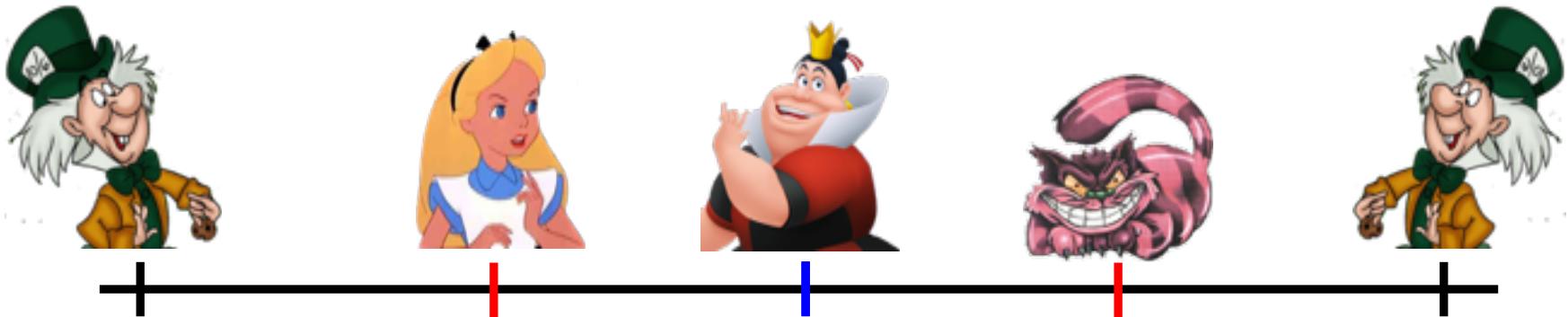
Position-Based Cryptography

Can the geographical location of a player be used as sole cryptographic credential ?

- Possible Applications:
 - Launching-missile command comes from within your military headquarters
 - Talking to the correct assembly
 - Pizza-delivery problem / avoid fake calls to emergency services
 - ...



Basic task: Position Verification



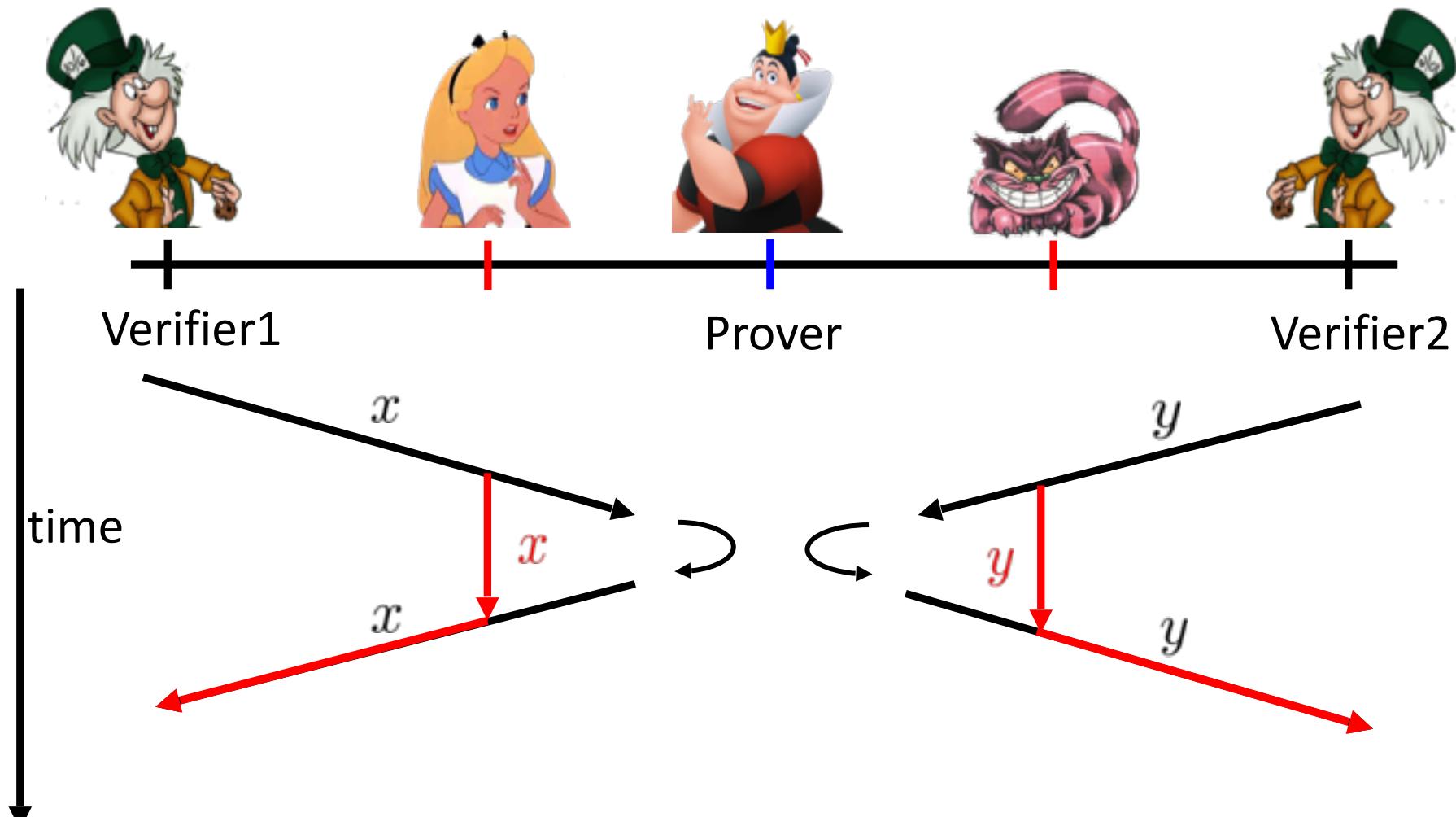
Verifier1

Prover

Verifier2

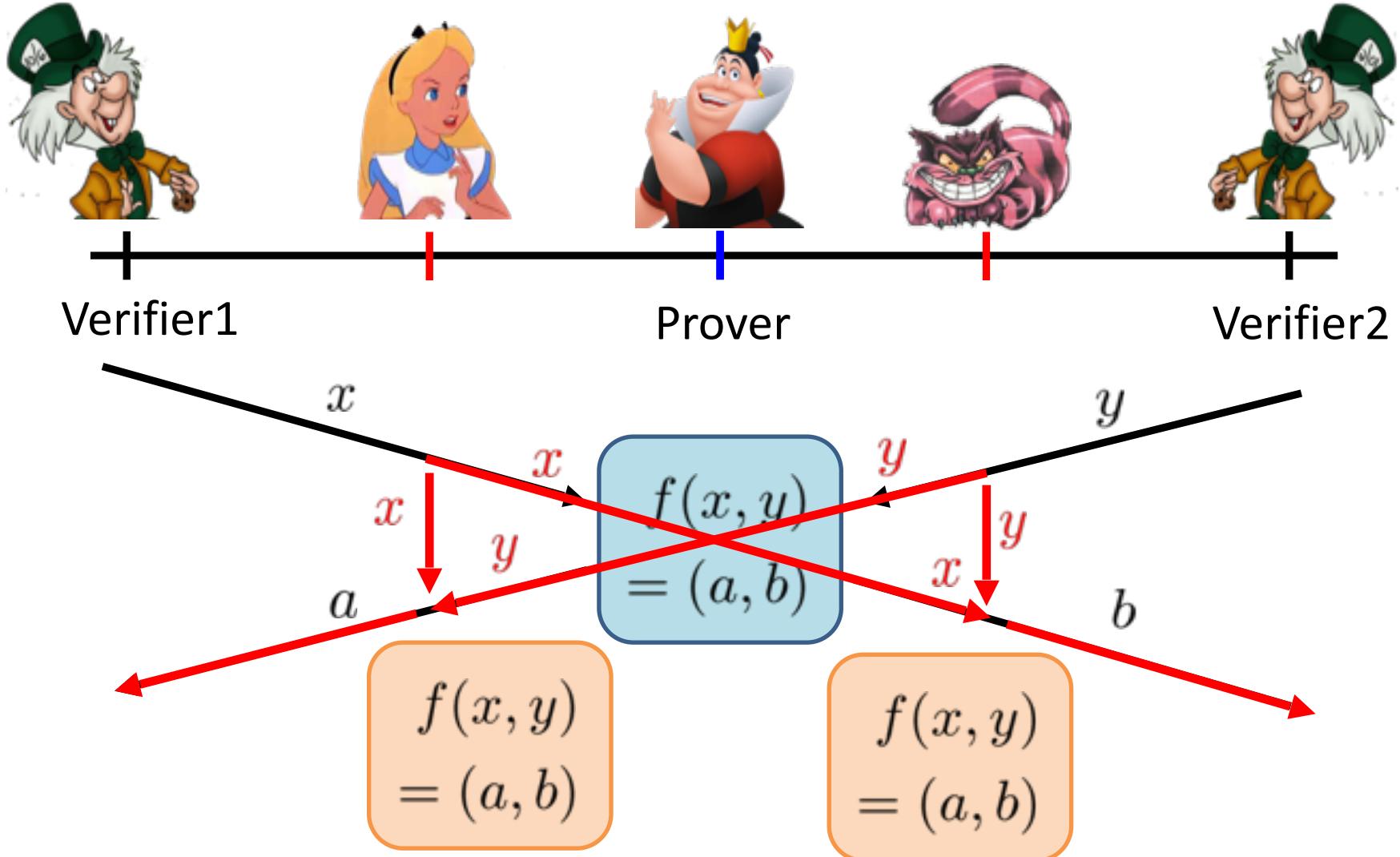
- Prover wants to convince verifiers that she is at a **particular position**
- no **coalition of (fake) provers**, i.e. not at the claimed position, can convince verifiers
- (over)simplifying assumptions:
 - communication at speed of light
 - instantaneous computation
 - verifiers can coordinate

Position Verification: First Try



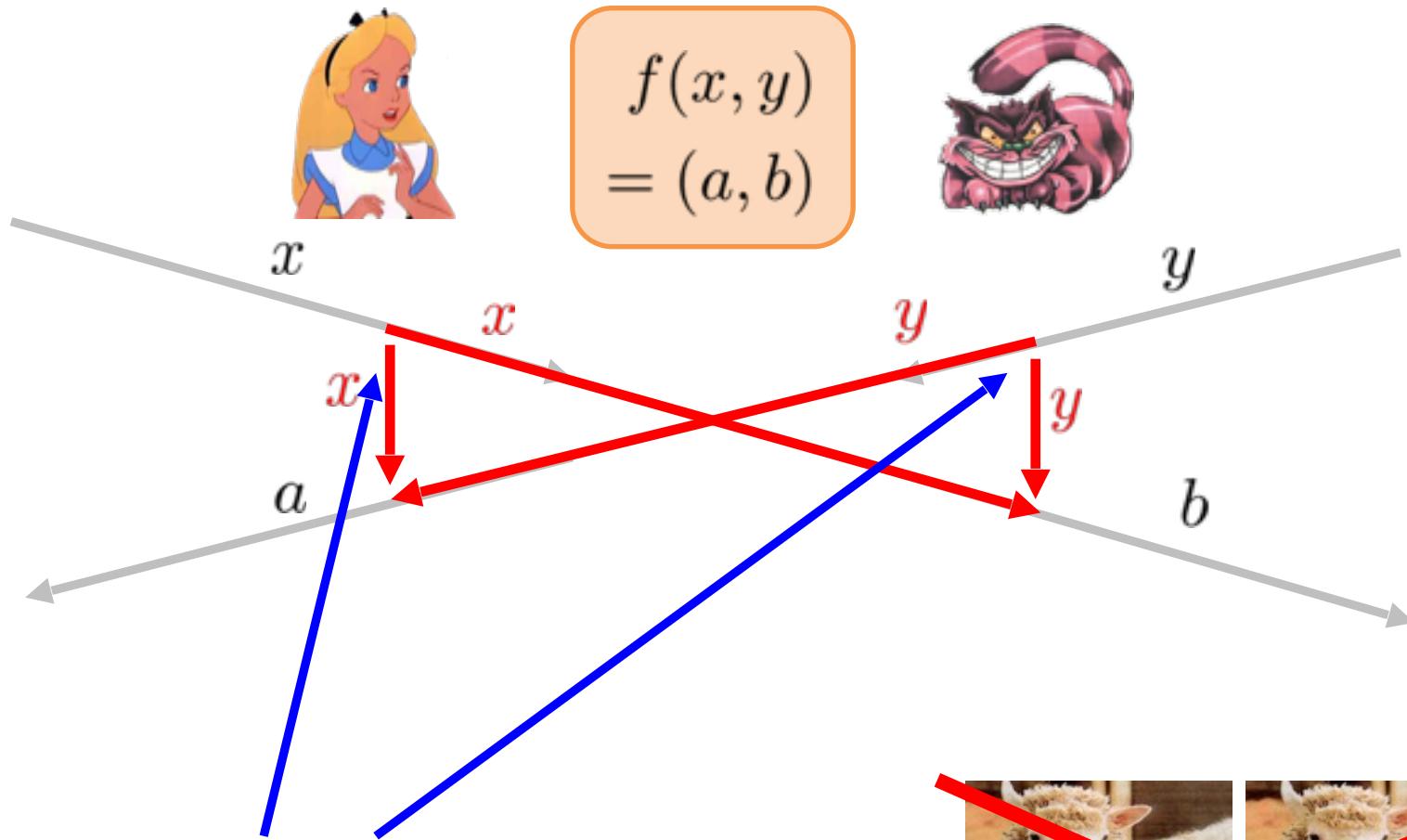
- distance bounding [\[Brands Chaum '93\]](#)

Position Verification: Second Try

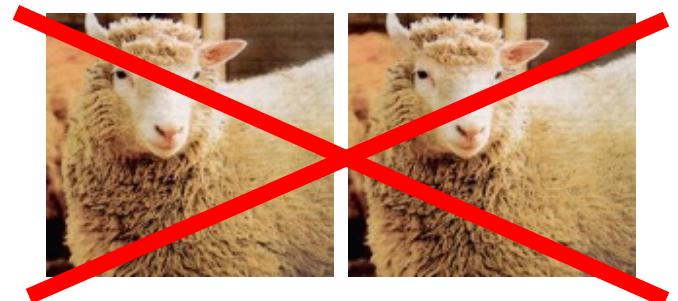


position verification is classically impossible !

The Attack

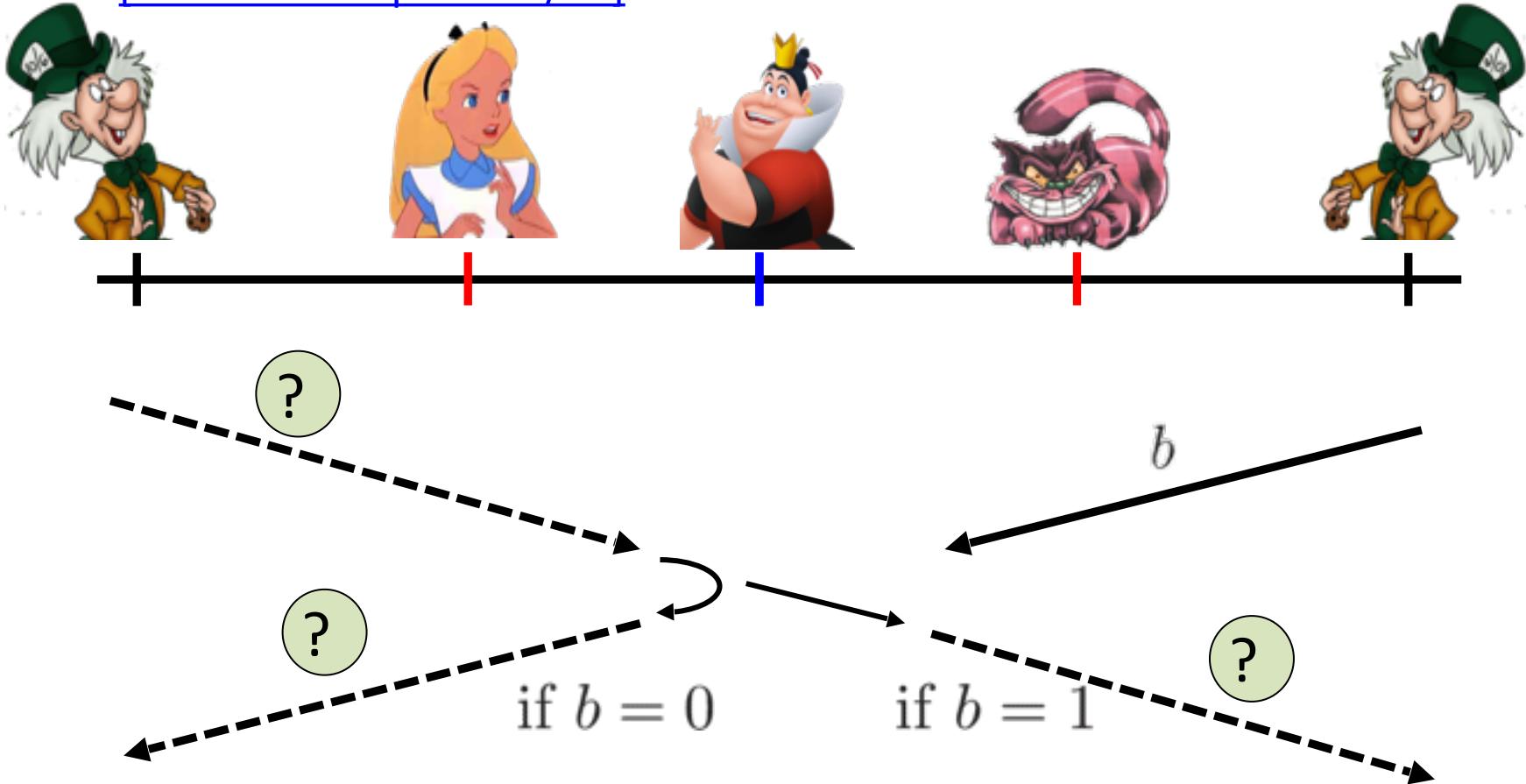


- copying classical information
- this is **impossible** quantumly



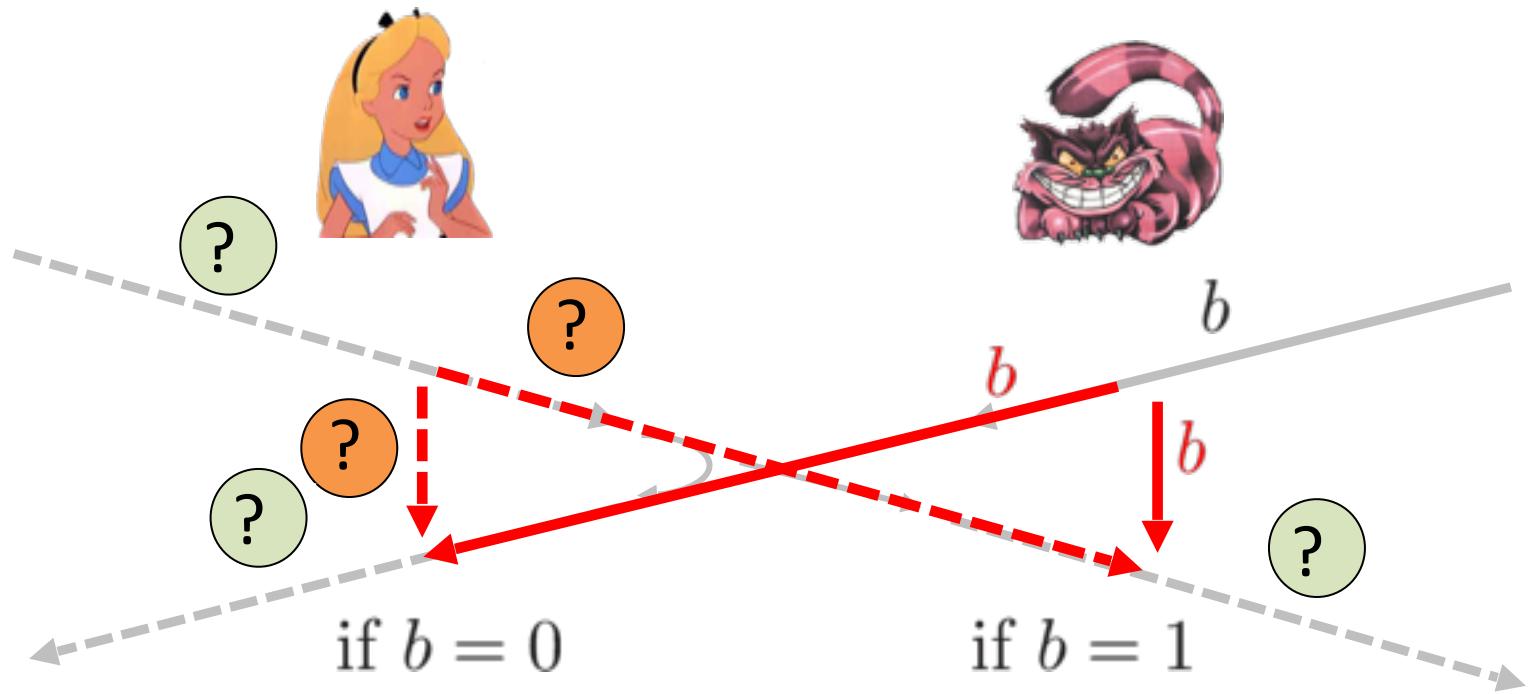
Position Verification: Quantum Try

[Kent Munro Spiller 03/10]

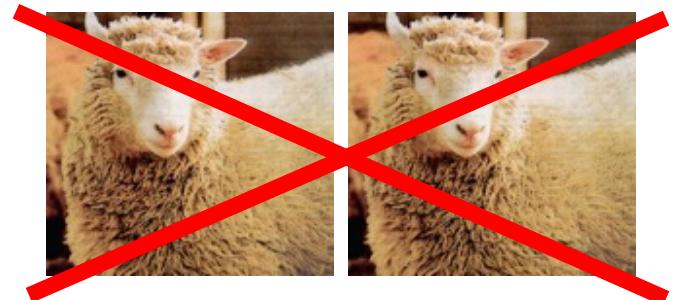


- Can we break the scheme now?

Attacking Game

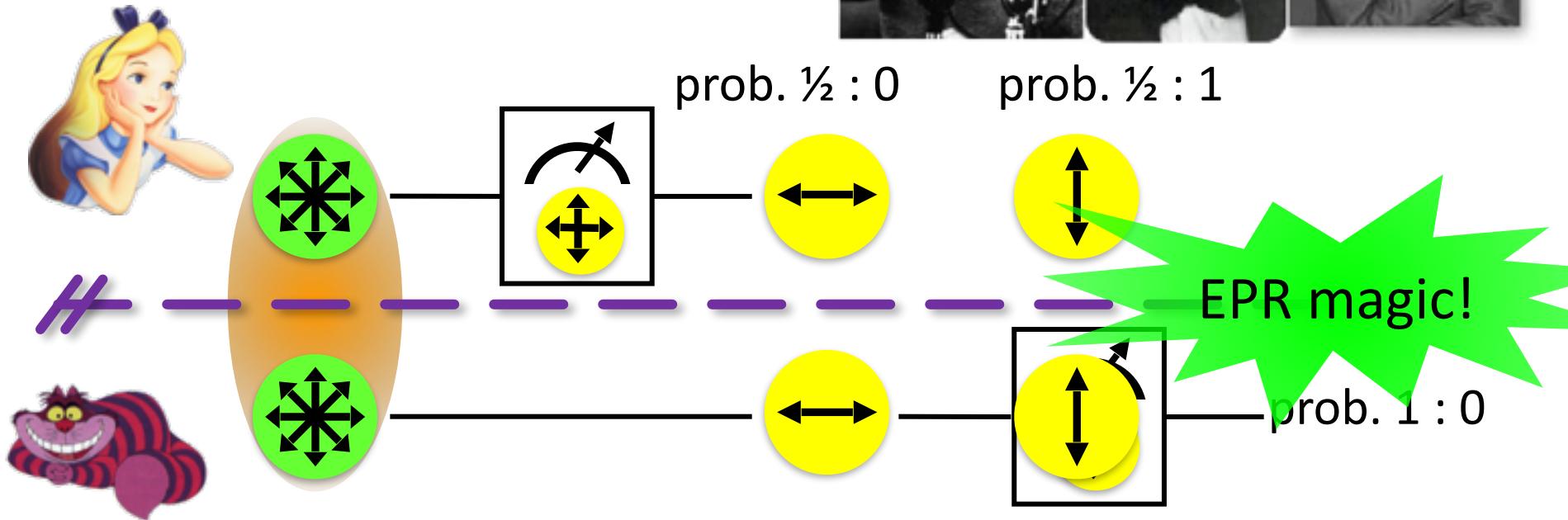


- Impossible to cheat due to no-cloning theorem
- Or not?



EPR Pairs

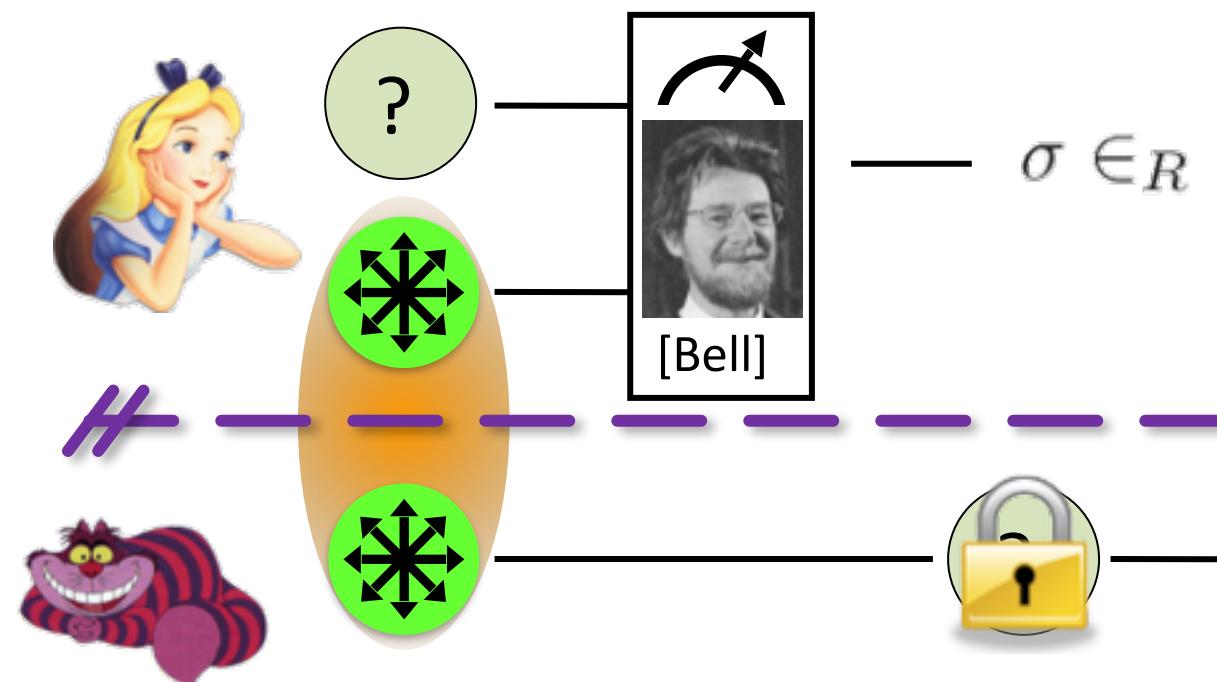
[Einstein Podolsky Rosen 1935]



- “spukhafte Fernwirkung” (spooky action at a distance)
- EPR pairs **do not allow to communicate** (**no contradiction** to relativity theory)
- can provide a shared random bit

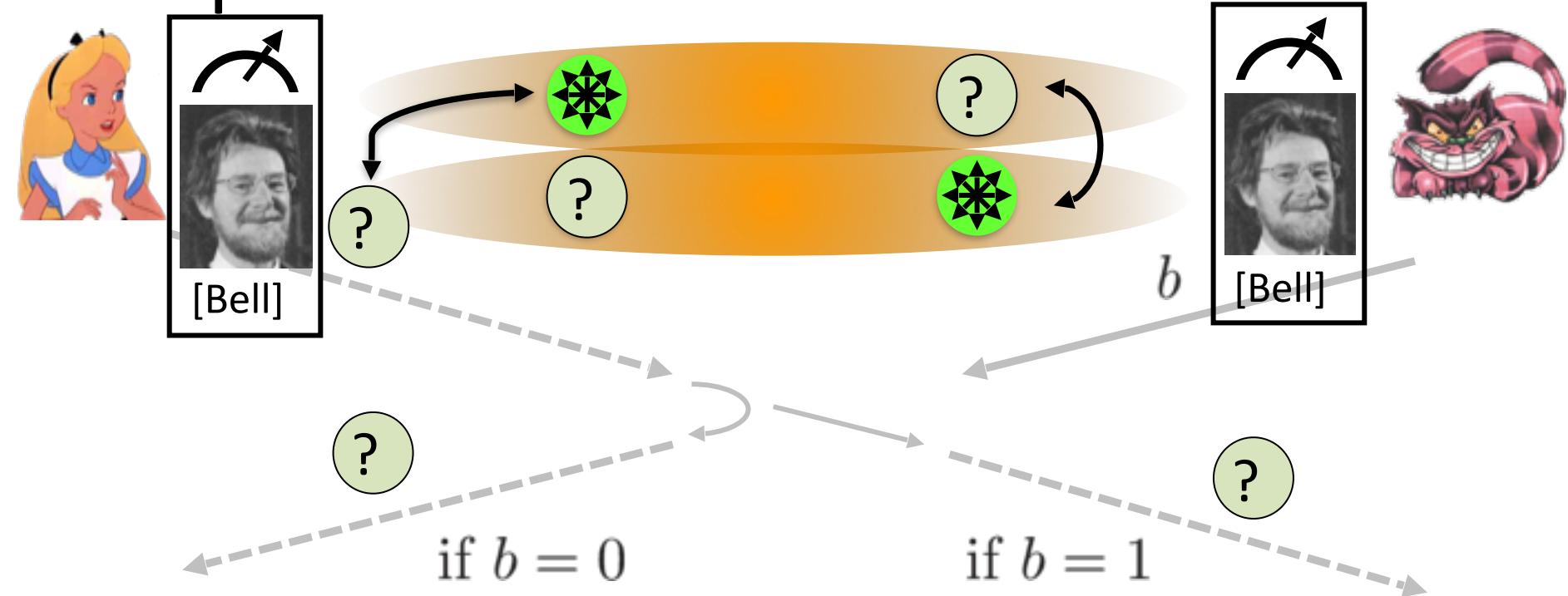
Quantum Teleportation

[Bennett Brassard Cr  peau Jozsa Peres Wootters 1993]



- does not contradict relativity theory
- Bob can only recover the teleported qubit after receiving the classical information σ

Teleportation Attack



- It is possible to cheat with entanglement !!
- Quantum teleportation allows to break the protocol perfectly.



No-Go Theorem

[Buhrman, Chandran, Fehr, Gelles, Goyal, Ostrovsky, Schaffner 2010] [Beigi Koenig 2011]

- Any position-verification protocol **can be broken** using an exponential number of entangled qubits.



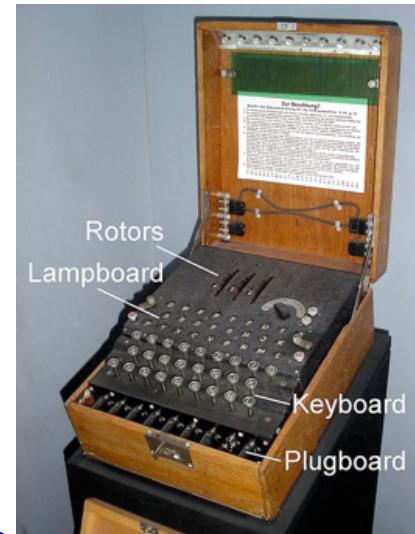
- **Question:** Are so many quantum resources really necessary?
- Does there exist a protocol such that:
 - **honest prover and verifiers are efficient, but**
 - **any attack requires lots of entanglement**



see <http://homepages.cwi.nl/~schaffne/positionbasedqcrypto.php> for recent developments

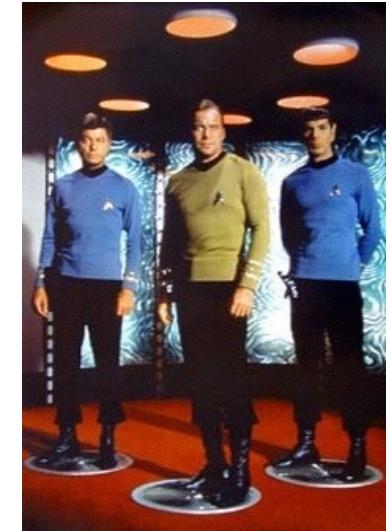
What Have You Learned from this Talk?

✓ Classical Cryptography



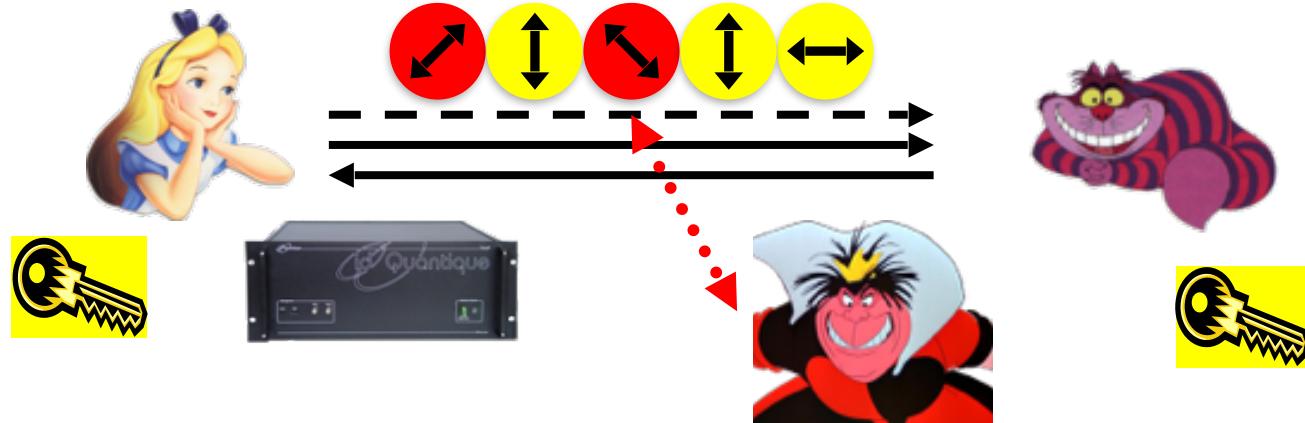
✓ Quantum Computing & Teleportation

$$\begin{array}{ll} \text{Yellow circle with double arrows} & |0\rangle_+ \\ \text{Red circle with double arrows} & |1\rangle_+ \\ \text{Yellow circle with diagonal arrows} & |0\rangle_\times \\ \text{Red circle with diagonal arrows} & |1\rangle_\times \end{array}$$

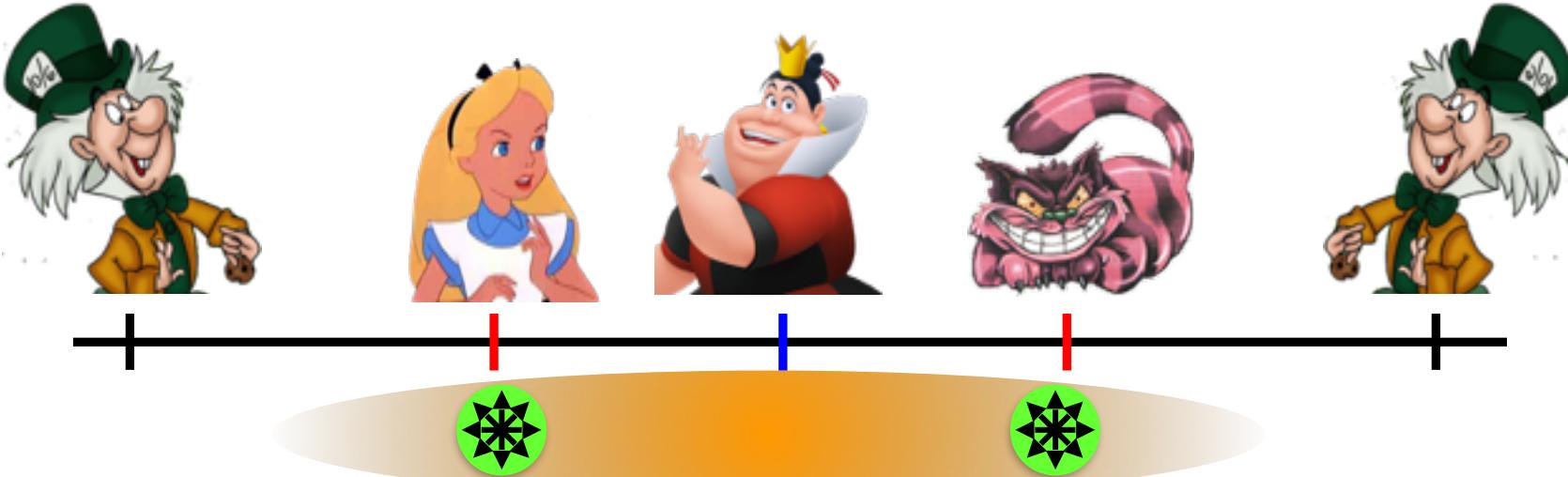


What Have You Learned from this Talk?

✓ Quantum Key Distribution ([QKD](#))



✓ Position-Based Cryptography



Thank you for your attention!

Questions



check <http://arxiv.org/abs/1510.06120> for
a survey about quantum cryptography
beyond key distribution

[Postdoc position in Amsterdam](#) available!

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