CS 584/684 Algorithm Design and Analysis

Homework 1

Portland State U, Winter 2021 *01/05/20*
Lecturer: Fang Song *Due: 01/12/19*

Instructions. This problem set contains 4 pages (including this cover page) and 4 questions. A random subset of problems will be graded.

- Your solutions will be graded on *correctness* and *clarity*. You should only submit work that you believe to be correct, and you will get significantly more partial credit if you clearly identify the gap(s) in your solution. It is good practice to start any long solution with an informal (but accurate) summary that describes the main idea. You may opt for the "I take 15%" option.
- You need to submit a PDF file before the deadline. Either a clear scan of you handwriting or a typeset document is accepted. You will get 5 bonus points for typing in LaTeX (Download and use the accompany TeX file).
- You may collaborate with others on this problem set. However, you must **write up your own solutions** and **list your collaborators and any external sources** for each problem. Be ready to explain your solutions orally to a course staff if asked.
- For problems that require you to provide an algorithm, you must give a precise description of the algorithm, together with a proof of correctness and an analysis of its running time. You may use algorithms from class as subroutines. You may also use any facts that we proved in class or from the book.

1. (5 points) (Survey) **Attention: this problem is due Friday, January 08, 11:59pm PST.** Complete the survey at `https://forms.gle/e56YsGkRnaiBHbYA6`.

2. (15 points) (Order of growth rate) Take the following list of functions and arrange them in ascending order of growth rate. That is, if function $g(n)$ immediately follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$. (Hint: Look up *Stirling's approximation* for factorials.)

   - $f_1(n) = n^{2.5}$
   - $f_2(n) = \sqrt{2n}$
   - $f_3(n) = n!$
   - $f_5(n) = 100^n$
   - $f_6(n) = n^2 \log n$
   - $f_7(n) = 2^{\sqrt{n}}$
   - $f_8(n) = 2^{2^n}$
   - $f_9(n) = n^{\log n}$
   - $f_{10}(n) = 0.01n$

3. (15 points) (Understanding big-O notation) Assume you have functions f and g such that $f(n)$ is $O(g(n))$. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.

   (a) $\log_2 f(n)$ is $O(\log_2 g(n))$.

   (b) $2^{f(n)}$ is $O(2^{g(n)})$.

   (c) $f(n)^2$ is $O(g(n)^2)$.

4. (Basic proof techniques) Read the chapter on `Proof by Induction` by Erickson (`http://jeffe.cs.illinois.edu/teaching/algorithms/notes/98-induction.pdf`), and note by Fleck (`http://mfleck.cs.illinois.edu/building-blocks/version-1.3/proofs.pdf`). Then do the following.

   (a) (10 points) Prove that for any positive $x \in \mathbb{R}$, $x + 1/x \geq 2$.

   (b) (10 points) . Prove that given an unlimited supply of 6-cent coins, 10-cent coins, and 15-cent coins, one can make any amount of change larger than 29 cents.