Homework 1

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Problem 1:
When $\alpha \to 0$, only the constant term is left in the equation and all nodes have the same centrality β.

As α increases, the centrality increases and eventually there is one point at which it diverges. It happens when

$$det(\mathbf{I} - \alpha\mathbf{A}) = 0$$

such that $(\mathbf{I} - \alpha\mathbf{A})$ is not invertible.

$det(\mathbf{I} - \alpha\mathbf{A}) = 0 \Rightarrow det(\alpha^{-1}\mathbf{I} - \mathbf{A}) = 0 \Rightarrow \alpha^{-1} =$ eigenvalues of the matrix $\mathbf{A}$

The smallest α to make $det(\mathbf{I} - \alpha\mathbf{A}) = 0$ is $\alpha = \frac{1}{\lambda_1}$, where λ_1 is the largest eigenvalue of the matrix $\mathbf{A}$.

Therefore, $0 < \alpha < \frac{1}{\lambda_1}$ guarantees the convergence of the Katz centrality. Beyond this point (i.e., $\frac{1}{\lambda_1}$), by the Perron-Frobenius theorem, there will be negative centralities that are meaningless to interpret. Therefore, in practice, most researchers employ values close to $\frac{1}{\lambda_1}$ to place maximum weight on the eigenvector term and smallest weight on the constant term.

Problem 2:
$|N(v_i) \cap N(v_j)| = [A^T A]_{ij} = [A^2]_{ij} =$ the number of walks of length 2 between v_i and v_j.

Problem 3: Please see the python script.