

Homework 1

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Problem 1:

When $\alpha \rightarrow 0$, only the constant term is left in the equation and all nodes have the same centrality β .

As α increases, the centrality increases and eventually there is one point at which it diverges. It happens when

$$\det(\mathbf{I} - \alpha \mathbf{A}) = 0$$

such that $(\mathbf{I} - \alpha \mathbf{A})$ is not invertible.

$$\det(\mathbf{I} - \alpha \mathbf{A}) = 0 \Rightarrow \det(\alpha^{-1} \mathbf{I} - \mathbf{A}) = 0 \Rightarrow \alpha^{-1} = \text{eigenvalues of the matrix } \mathbf{A}$$

The smallest α to make $\det(\mathbf{I} - \alpha \mathbf{A}) = 0$ is $\alpha = \frac{1}{\lambda_1}$, where λ_1 is the largest eigenvalue of the matrix \mathbf{A} .

Therefore, $0 < \alpha < \frac{1}{\lambda_1}$ guarantees the convergence of the Katz centrality. Beyond this point, there is little guidance for the value that α should take. In practice, most researchers employ values close to $\frac{1}{\lambda_1}$ to place maximum weight on the eigenvector term and smallest weight on the constant term.

Problem 2:

$|N(v_i) \cap N(v_j)| = [A^T A]_{ij} = [A^2]_{ij}$ = the number of walks of length 2 between v_i and v_j .

Problem 3: Please see the python script.
