Adaptive Finite-Time Fault-Tolerant Control of Uncertain Systems With Input Saturation

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***Abstract*—In this article, an adaptive finite-time fault-tolerant control (FTC) strategy is proposed for state tracking of uncertain systems subject to actuator faults and input saturation. The actuator faults, which include multiplicative and time-varying additive modes, are allowed to be unknown. To deal with the coupling of the unknown model uncertainties and the actuator faults, an adaptive mechanism is designed based on a wisely chosen Lyapunov function. The proposed finite-time FTC scheme, with adaptive laws of uncertain parameters, guarantees the stability of the closed-loop system, and the practical finite-time state tracking property. Then, the algorithm is applied to a flight control system, simulation and comparison results demonstrate the effectiveness and advantages of the designed control scheme.**

***Index Terms*—Adaptive fault-tolerant control, finite-time control, input saturation, tracking, uncertain systems.**

I. Introduction

The ACTUATOR component, which is used to execute control actions, plays an extremely important role in a practical control system. However, some undesirable factors, such as actuator faults and saturation, may cause a severe degradation of the system performance, instability, and even catastrophic accident. In many practical systems, especially in safety-critical ones (such as spacecrafts and aircrafts), redundant actuators are usually embedded to handle the actuator faults from the hardware level. Nowadays, it is definitely necessary to design effective fault-tolerant control (FTC) schemes from the algorithm level because of the complexity, sudden occurrence, and information unpredictability of actuator faults. Over the past decades, the FTC issues have achieved intensive investigations and considerable control methods have been proposed; see for example, adaptive control [1]–[6], H_∞ robust control [7], multiple-model control [8], and fault detection and diagnosis-based control [9]. Among which, adaptive FTC strategy has been a popular technique for its capability to deal with unknown parametric variations caused by the actuator faults without explicit fault detection and diagnostic block involved. To compensate for the unknown fixed or varying actuator stuck fault, an adaptive FTC design was developed in [1] for linear systems. Later on, some matching conditions were derived in [2] for adaptive state tracking of linear systems with unknown actuator faults. Based on two new integral sliding surfaces, an adaptive FTC scheme was proposed in [3] for uncertain linear systems with signals quantization and actuator faults. In [4], a robust adaptive FTC scheme was constructed for uncertain linear systems to track a closed-loop reference system in the presence of state matrix uncertainty and actuator faults. It is worth mentioning that the aforementioned FTC results focus on the infinite-time stability, that is, the desired system performance is considered only when the time variable goes to infinity. However, from the perspective of practical applications, the performance is expected to be guaranteed in finite time. For example, the aircraft must recover to its normal attitude within finite time after the failure occurring, otherwise the control accuracy maybe downgraded or even out of control. Because of its practical and theoretical importance, the finite-time FTC, which offers not only faster transient performance but also better robustness and disturbances attenuate ability, has attracted significant attention, see [10]–[13] and references therein. In [10] and [11], adaptive finite-time FTC strategies were designed for a class of nonlinear systems and spacecraft systems by combining the fuzzy logic and sliding-mode control technique, respectively. Recently in [12], a robust finite-time adaptive FTC strategy was proposed for uncertain linear systems, in which the state matrix uncertainty, multiplicative, and constant additive faults were taken into consideration.

On the other hand, from the view of physical constraints, the inevitable problem of input saturation should be considered during the controller design, especially for a finite-time FTC system. Normally, it may generate large control actions to maintain acceptable performance within finite time, thus the handling of input saturation is particularly necessary. To attenuate the saturation effects, auxiliary design systems were introduced in [14]–[19] for fault-free case, which modify the control signals by generating certain auxiliary variables. Taking the possible actuator faults into consideration, an adaptive FTC strategy was proposed in [20] for a class of nonlinear systems with control input and system state constrains, in

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which an auxiliary system was designed to handle the input saturation effect. In [21], an attitude tracking control scheme combined with an auxiliary system was developed for spacecrafts with actuator faults and input saturation. Based on an auxiliary control system, the FTC issue was studied in [22] for uncertain linear systems in the presence of actuator faults and saturation.

It is noted that most of the aforementioned results, unfortunately, are based on the condition that there exists no model uncertainty, which may beyond the realistic scenario. Several robust FTC schemes were designed for uncertain linear systems in [3], [4], and [12], in which only the uncertainty existing in the state matrix was considered. It is obvious that the uncertainty of the input matrix makes the controller design more challenging, which actually transforms the control signal into a part of the uncertainty. For systems with input matrix uncertainty, how to cope with the complex saturation nonlinearities and unknown actuator faults simultaneously under the framework of finite-time stability is a practical while challenging issue.

In this article, we explore the adaptive finite-time FTC issue of uncertain systems with actuator faults and input saturation. Both multiplicative and additive actuator faults are taken into consideration, which contain the partial loss of effectiveness, total loss of control, stuck in place and time-varying oscillatory. A more comprehensive model is established for actuators, which unifies the normal operation case, the different fault cases and the saturation case. In order to address the input saturation, a finite-time auxiliary system is introduced. Different from the conventional model reference adaptive designs [1], [2], [4], [8], and [23] which focus on the infinite-time stability issue, the proposed adaptive finite-time FTC scheme ensures the finite-time convergence property of the state tracking error. The main contributions of this article are highlighted as follows.

1) A time-varying actuator model is established, which integrates both multiplicative and additive faults and input saturation. In addition, the considered actuator faults are unknown, that is, the specific number of the failed actuators, the time of the fault occurred, and the mode of the faults are uncertain to the designer. Thus, the developed FTC strategy has the ability to deal with undetected faults.
2) The considered system is disturbed by uncertain factors from the system matrix and the input matrix simultaneously. The coupling of the unknown input matrix uncertainty and the actuator faults generates uncertain terms related to the control signal. Hence, the traditional FTC solutions depending on the uncertainty of the state matrix are no longer applicable. An adaptive mechanism is designed to estimate the unknown upper bounds of the uncertainties, due to which explicit information about faults and model uncertainties is not necessarily needed.
3) An adaptive finite-time FTC scheme is proposed for state tracking of uncertain systems subject to several uncertain factors in both the state and input matrices. By employing the proposed control strategy, the practical finite-time stability of the closed-loop system is guaranteed, and the state tracking error between the considered uncertain system and the reference model converges to a small region of the origin in finite time.

The remainder of this article is organized as follows. Section II presents the control problem statement. An adaptive finite-time FTC scheme is proposed in Section III with stability analysis and extended to nonfinite-time control case successfully. The algorithm has been applied to a flight control system and numerical comparative analysis is given in Section IV to demonstrate the effectiveness and advantages of the designed control strategy. Finally, Section V concludes this article.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional and $n \times m$-dimensional Euclidean space, respectively. $\|\cdot\|$ represents the Euclidean norm of vectors. K^{T} denotes the transpose of matrix K. For an vector $\varphi = [\varphi_1, \dots, \varphi_n]^{\mathrm{T}}$, $\varphi_i (i = 1, \dots, n)$ denotes the ith element of φ. The vector $\varphi^\alpha = [\varphi_1^\alpha, \dots, \varphi_n^\alpha]^{\mathrm{T}}$ is defined with a scalar $\alpha > 0$. $\mathrm{diag}\{\varphi_1, \dots, \varphi_n\}$ represents a diagonal matrix whose main diagonal elements are $\varphi_1, \dots, \varphi_n$. $\min\{\varphi_1, \dots, \varphi_n\}$ and $\max\{\varphi_1, \dots, \varphi_n\}$ are the minimum and maximum values of elements $\varphi_1, \dots, \varphi_n$, respectively. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of a matrix, respectively. I_n denotes an identity matrix with dimension n.

II. Problem Statement

A. Preliminaries

Definition 1 [10]: The equilibrium $\gamma = 0$ of system $\dot{\gamma} = f(\gamma)$ is practical finite-time stable, if for all $\gamma(t_0) = \gamma_0$, there exists a constant $\delta > 0$ and an adjustment time function $\mathrm{T}(\delta, \gamma_0) < \infty$ such that for all $t \geq t_0 + \mathrm{T}$, $\|\gamma(t)\| \leq \delta$.

Lemma 1 [10]: For system $\dot{\gamma} = f(\gamma)$, if there exist constants $\zeta > 0$, $0 < \alpha < 1$, $0 < \bar{\psi} < \infty$, and a positive-definite function $V(\gamma)$ such that

$$\dot{V}(\gamma) \leq -\zeta V^\alpha(\gamma) + \bar{\psi} \tag{1}$$

then the trajectory of $\dot{\gamma} = f(\gamma)$ is practical finite-time stable.

Lemma 2 [24]: For a positive-definite matrix $J \in \mathbb{R}^{n\times n}$ and a vector $\xi \in \mathbb{R}^n$, then

$$\lambda_{\min}(J)\|\xi\|^2 \leq \xi^{\mathrm{T}} J \xi \leq \lambda_{\max}(J)\|\xi\|^2. \tag{2}$$

Lemma 3 [25]: For any constant $\varepsilon > 0.5$, matrices Ψ, $\hat{\Psi}$, $\tilde{\Psi} \in \mathbb{R}^n$ with $\tilde{\Psi} = \hat{\Psi} - \Psi$, the following relation holds:

$$-\tilde{\Psi}^{\mathrm{T}}\hat{\Psi} \leq -\frac{2\varepsilon - 1}{2\varepsilon}\tilde{\Psi}^{\mathrm{T}}\tilde{\Psi} + \frac{\varepsilon}{2}\Psi^{\mathrm{T}}\Psi. \tag{3}$$

Lemma 4 [26]: For real variables ϕ and γ, and any positive constants ς_1, ς_2, and ς_3, the following inequality is satisfied:

$$|\phi|^{\varsigma_1}|\gamma|^{\varsigma_2} \leq \frac{\varsigma_1}{\varsigma_1 + \varsigma_2}\varsigma_3|\phi|^{\varsigma_1+\varsigma_2} + \frac{\varsigma_2}{\varsigma_1 + \varsigma_2}\varsigma_3^{-\frac{\varsigma_1}{\varsigma_2}}|\gamma|^{\varsigma_1+\varsigma_2}. \tag{4}$$

Lemma 5 [27]: For variables x_h, $h = 1, 2, \dots, n$, and constant $0 < a \leq 1$, then

$$\left(\sum_{h=1}^{n}|x_h|\right)^a \leq \sum_{h=1}^{n}|x_h|^a. \tag{5}$$

B. System Description

Consider a class of uncertain systems described by

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + B_1 d(t) \tag{6}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ and $u(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ denote the system state and the control input signal, respectively. $d(t) \in \mathbb{R}^l$ is the external disturbance and $\|d(t)\| \leq \bar{d}$ with $\bar{d}$ being an unknown scalar. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are the nominal matrices. $\Delta A(t)$ and $\Delta B(t)$ represent the unknown, time-varying uncertainties in state matrix and input matrix, respectively. For $\Delta A(t)$, $\Delta B(t)$, and B_1, the following matching conditions are valid:

$$\Delta A(t) = BE_1(t), \quad \Delta B(t) = BE_2(t), \quad B_1 = BF \tag{7}$$

where $E_1(t)$ and $E_2(t)$ are unknown matrices satisfying $\|E_1(t)\| \leq \epsilon_1$ and $\|E_2(t)\| \leq \epsilon_2$ with ϵ_1 and ϵ_2 being unknown positive constants, and $F \in \mathbb{R}^{m\times l}$ is a known matrix.

Remark 1: Instead of considering the state matrix uncertainty only as in [3], [4], and [12], (6) represents a linearized physical system dynamics with uncertainties in both the state and input matrices. Therefore, theoretically speaking, the system studied in this article is more complex and the controller design is more challenging. It is noted that the handling of the uncertainty existing in the input matrix is a much more challenging problem, which actually transforms the control signal into a part of the uncertainty [22], [23], [28].

Remark 2: Note that the considered $\Delta A(t)$ and $\Delta B(t)$ are unknown while bounded, and in the range space of B. From the perspective of the practical system, it is reasonable to assume that the model uncertainties are bounded. In [22], it give a similar expression of $\Delta B(t) = B\kappa(t)$, while $\kappa(t) \triangleq \mathrm{diag}\{\kappa_1(t), \kappa_2(t), \ldots, \kappa_m(t)\}$ is restricted to $|\kappa_i(t)| \leq \bar{\kappa}_i$ with known $\bar{\kappa}_i < 1$ for $i = 1, \ldots, m$. That is, $\kappa(t)$ in [22] can only be a diagonal matrix with a known upper bound. In this article, $E_2(t)$ in (7) is no longer restricted to be a diagonal one, and is with an unknown bound. Then, uncertainty $\Delta B(t)$ considered in (6) is much more general.

C. Actuator Model

The output of the ith actuator in the presence of unknown faults and saturation is described as

$$u_i(t) = \rho_i(t)v_{si}(t) + u_i^*(t), \ \ i = 1, \ldots, m \tag{8}$$

where $\rho_i(t) \in [0, 1]$ is an unknown piecewise constant function of time, $u_i^*(t)$ denotes the bounded time-varying additive fault, and $v_{si}(t)$ is the control signal subjected to saturation nonlinearity which is described as follows:

$$v_{si}(t) = \mathrm{sat}(v_{ci}(t)) = \begin{cases} u_{i\max}, & v_{ci}(t) > u_{i\max} \\ v_{ci}(t), & u_{i\min} \leq v_{ci}(t) \leq u_{i\max} \\ u_{i\min}, & v_{ci}(t) < u_{i\min} \end{cases} \tag{9}$$

where $v_{ci}(t)$ is the commanded control signal generated by the designed control scheme, $u_{i\min}$ and $u_{i\max}$ are the known lower and upper bound constraints of the ith control input, respectively.

Remark 3: Many practical actuator faults in real-life systems can be described by (8), such as the stuck fault of the spacecraft's reaction wheel, the thrust loss fault of the autonomous underwater vehicle, and so on. More specifically:

1) if $\rho_i(t) = 1$ and $u_i^*(t) = 0$, the ith actuator is fault-free;
2) if $0 < \rho_i(t) < 1$, the ith actuator suffers from a partial loss of effectiveness, which is also named as multiplicative fault;
3) if $\rho_i(t) = 0$, the ith actuator suffers from a total loss of control;
4) if $u_i^*(t) \neq 0$, the ith actuator suffers from the additive fault, which contains the stuck in place and the time-varying oscillatory fault.

For the sake of convenience, the actual input of system (6) is expressed as

$$u(t) = \rho(t)v_s(t) + u^*(t) \tag{10}$$

where

$$\begin{aligned} v_s(t) &= [v_{s1}(t), v_{s2}(t), \ldots, v_{sm}(t)]^{\mathrm{T}} \\ \rho(t) &= \mathrm{diag}\{\rho_1(t), \rho_2(t), \ldots, \rho_m(t)\} \\ u^*(t) &= \left[u_1^*(t), u_2^*(t), \ldots, u_m^*(t)\right]^{\mathrm{T}} \end{aligned} \tag{11}$$

and the additive fault $u^*(t)$ is unknown and bounded, that is

$$\|u^*(t)\| \leq \bar{u} \tag{12}$$

with $\bar{u}$ being an unknown positive constant.

Remark 4: Note that the considered actuator faults are unknown, that is, the specific number of the failed actuators, the time of the fault occurred, and the mode of the faults are uncertain to the designer. Moreover, the coupling of the actuator faults and the input matrix uncertainty generates uncertain terms related to the control signal, which makes the controller design more challenging.

D. Control Problem

Consider the reference model as follows:

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t) \tag{13}$$

where $x_m(t) \in \mathbb{R}^n$ and $r(t) \in \mathbb{R}^m$ represent the reference state and input, respectively, $A_m \in \mathbb{R}^{n\times n}$ and $B_m \in \mathbb{R}^{n\times m}$ are both constant matrices, and A_m is Hurwitz, which means that there exist two positive definite matrices $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ such that

$$A_m^{\mathrm{T}} P + PA_m = -Q. \tag{14}$$

Our *control objective* is to design an adaptive finite-time FTC scheme for uncertain systems described by (6) with unknown model uncertainty (7), actuator faults (8), and input saturation (9), such that the system state $x(t)$ tracks the given reference model state $x_m(t)$ in finite time.

For the sake of brevity, the notation (t) for time-dependency is omitted in the subsequent description whenever no confusion would arise.

The following assumption is given.

Assumption 1: For the considered actuator faults and input matrix uncertainty, matrix $B(I + E_2)\rho B^{\mathrm{T}}$ is positive definite.

Remark 5: Assumption 1 indicates that B is full-row rank, that is, $\operatorname{rank}(B) = n$. Then, there exist matrices $K_x \in \mathbb{R}^{m\times n}$ and $K_r \in \mathbb{R}^{m\times m}$ such that

$$A + B\rho K_x = A_m, \quad B\rho K_r = B_m \tag{15}$$

which is the so-called model matching condition in classic model reference adaptive control theory [1], [2]. It has been proved in [29] that $\operatorname{rank}(B) = n$ is a sufficient and necessary condition for a linear system to completely track any given continuously differentiable reference signal, which is bounded and with bounded derivative, within finite time. Moreover, it can be concluded that $\operatorname{rank}(B, AB, \ldots, A^{n-1}B) = n$, and consequently (A, B) is controllable.

Remark 6: Assumption 1 is also an actuator redundancy condition, which implies that the designed control scheme can still achieve the desired control objective even in the presence of certain actuator faults and input matrix uncertainty. Therefore, Assumption 1 is reasonable and it can be met by many practical systems [30]–[32]. It is noted that the positive definite of matrix BB^{T} is a necessary condition for Assumption 1. Therefore, for a system with $\operatorname{rank}(B) = n$, such as the F-18 high-angle-of-attack research vehicle [31] and three-tank system [32], the designed control system can tolerate a certain degree of model uncertainty and actuator faults.

Remark 7: A basic assumption for the considered control problem is that the actuators under the saturation constraint can still guarantee the feasibility of control performance [14], [15]. Consider a simple system modeled as $\dot{x} = ax + \operatorname{sat}(v)$, where $x \in \mathbb{R}$ is a state variable, the lower and upper bound constraints of the control input are $-u_M$ and u_M with $u_M > 0$, respectively. If $a > 0$ and $u_M < ax(0)$ with $x(0) > 0$ being the initial value of x, there is no controller satisfying saturation constraint to stabilize the system. Therefore, in order to ensure the closed-loop stability, restrictions on the saturation values for the actuators are necessary if the controlled system is not required to be bounded input bounded state. However, the specific restriction values generally depend on itself characteristics of the practical system.

III. Adaptive Finite-Time FTC Scheme

A. Controller Design

Define the state tracking error as follows:

$$e = x - x_m. \tag{16}$$

According to Assumption 1, the following lemma is ready to be presented.

Lemma 6 [4]: There exists an unknown scalar $\eta > 0$, such that

$$\eta\|e^{\mathrm{T}}PB\|^2 \le \min\{e^{\mathrm{T}}PB\rho B^{\mathrm{T}}Pe, e^{\mathrm{T}}PB(I+E_2)\rho B^{\mathrm{T}}Pe\}. \tag{17}$$

The adaptive finite-time fault-tolerant controller is constructed as

$$v_c = v_1 + v_2 + v_3 + v_4. \tag{18}$$

The control function v_1 is designed as

$$v_1 = \hat{K}_x x + \hat{K}_r r \tag{19}$$

where $\hat{K}_x = [\hat{K}_{x1}, \hat{K}_{x2}, \ldots, \hat{K}_{xm}]^{\mathrm{T}} \in \mathbb{R}^{m\times n}$ and $\hat{K}_r = [\hat{K}_{r1}, \hat{K}_{r2}, \ldots, \hat{K}_{rm}]^{\mathrm{T}} \in \mathbb{R}^{m\times m}$ are the estimates of K_x and K_r in (15), which can be, respectively, obtained with the following adaptive laws:

$$\dot{\hat{K}}_{xi} = -\Pi_{xi}\left(2xe^{\mathrm{T}}Pb_i + \Upsilon_{xi}\hat{K}_{xi}\right) \tag{20}$$

$$\dot{\hat{K}}_{ri} = -\Pi_{ri}\left(2re^{\mathrm{T}}Pb_i + \Upsilon_{ri}\hat{K}_{ri}\right) \tag{21}$$

where $i = 1, \ldots, m$, b_i denotes the ith column of matrix B, $\Pi_{xi}, \Upsilon_{xi} \in \mathbb{R}^{n\times n}$, and $\Pi_{ri}, \Upsilon_{ri} \in \mathbb{R}^{m\times m}$ are positive-definite gain matrices.

The control signal v_2 in (18) is constructed as

$$v_2 = \begin{cases} -\frac{c_1\hat{\chi}_1 B^{\mathrm{T}}Pe(e^{\mathrm{T}}Pe)^{\alpha}}{2\|e^{\mathrm{T}}PB\|^2}, & \|e^{\mathrm{T}}PB\| > 0 \\ 0, & \|e^{\mathrm{T}}PB\| = 0 \end{cases} \tag{22}$$

where c_1 is a positive constant, α is expressed as

$$\alpha = \frac{p+q}{2q} \tag{23}$$

with odd integers $p > 0$ and $q > 0$ satisfying the following condition:

$$0 < p/q < 1 \tag{24}$$

and $\hat{\chi}_1$ is the estimate of unknown constant χ_1 which is defined as

$$\chi_1 = \frac{1}{\eta} \tag{25}$$

and the adaptive law of $\hat{\chi}_1$ is designed as

$$\dot{\hat{\chi}}_1 = \gamma_{11}\left(c_1\left(e^{\mathrm{T}}Pe\right)^{\alpha} + 2\|e^{\mathrm{T}}PB\|^2 - \gamma_{12}\hat{\chi}_1\right) \tag{26}$$

where γ_{11} and γ_{12} are positive constants and the initial value $\hat{\chi}_1(0) \ge 0$.

Before presenting the control signal v_3 in (18), and in order to deal with the input saturation effect, a finite-time auxiliary system is constructed as

$$\dot{z}_a = \begin{cases} -\Pi_z z_a - \frac{c_{z1}}{2} z_a^{p/q} + c_{z2}\Delta v \\ \quad -\frac{f(\Delta v, \hat{\chi}_2)}{\|z_a\|^2} z_a, & \|z_a\| > z_0 \\ -\Pi_z z_a - \frac{c_{z1}}{2} z_a^{p/q} + c_{z2}\Delta v, & \|z_a\| \le z_0 \end{cases} \tag{27}$$

where $f(\Delta v, \hat{\chi}_2) = 0.5(\hat{\chi}_2 + c_{z2}^2)\|\Delta v\|^2$, $z_a \in \mathbb{R}^m$ is an auxiliary variable, $\Delta v = v_s - v_c$, Π_z is a designed gain matrix, c_{z1}, c_{z2}, and z_0 are positive constants, and $\hat{\chi}_2$ is the estimate of an unknown scalar defined as follows:

$$\chi_2 = (1 + \epsilon_2)^2 \tag{28}$$

and the following adaptive law is designed to update $\hat{\chi}_2$:

$$\dot{\hat{\chi}}_2 = \gamma_{21}\left(\|\Delta v\|^2 - \gamma_{22}\hat{\chi}_2\right) \tag{29}$$

where γ_{21} and γ_{22} are positive constants.

Then, v_3 in (18) is designed as

$$v_3 = -\Upsilon_z z_a - \hat{\chi}_1 B^{\mathrm{T}}Pe \tag{30}$$

where Υ_z is chosen such that

$$\lambda_{\min}(\Pi_z) - \frac{1}{2}\lambda_{\max}^2(\Upsilon_z) - \frac{1}{2} \ge 0. \tag{31}$$

The control function v_4 is constructed as

$$v_4 = v_{41} + v_{42} + v_{43} \tag{32}$$

where v_{41}, v_{42}, and v_{43} are given as

$$v_{41} = -\frac{\hat{\chi}_3^2 B^{\mathrm{T}} Pe}{\hat{\chi}_3 \|e^{\mathrm{T}} PB\| + c_2} \tag{33}$$

$$v_{42} = -\frac{\hat{\chi}_4^2 \|x\|^2 B^{\mathrm{T}} Pe}{\hat{\chi}_4 \|x\| \|e^{\mathrm{T}} PB\| + c_3} \tag{34}$$

$$v_{43} = -\frac{\hat{\chi}_5^2 \|v_1 + v_3\|^2 B^{\mathrm{T}} Pe}{\hat{\chi}_5 \|v_1 + v_3\| \|e^{\mathrm{T}} PB\| + c_4} \tag{35}$$

where c_2, c_3, and c_4 are positive constants to avoid singularity, $\hat{\chi}_3$, $\hat{\chi}_4$, and $\hat{\chi}_5$ are estimates of unknown constants χ_3, χ_4, and χ_5 which are defined as

$$\chi_3 = \frac{(1+\epsilon_2)\bar{u} + \|F\|\bar{d}}{\eta}, \qquad \chi_4 = \frac{\epsilon_1}{\eta}, \qquad \chi_5 = \frac{\epsilon_2}{\eta} \tag{36}$$

and $\hat{\chi}_3$, $\hat{\chi}_4$, and $\hat{\chi}_5$ are, respectively, updated by the following adaptive laws:

$$\dot{\hat{\chi}}_3 = \gamma_{31}\left(2\|e^{\mathrm{T}} PB\| - \gamma_{32}\hat{\chi}_3\right) \tag{37}$$

$$\dot{\hat{\chi}}_4 = \gamma_{41}\left(2\|e^{\mathrm{T}} PB\| \|x\| - \gamma_{42}\hat{\chi}_4\right) \tag{38}$$

$$\dot{\hat{\chi}}_5 = \gamma_{51}\left(2\|e^{\mathrm{T}} PB\| \|v_1 + v_3\| - \gamma_{52}\hat{\chi}_5\right) \tag{39}$$

where γ_{31}, γ_{32}, γ_{41}, γ_{42}, γ_{51}, and γ_{52} are positive constants, and the initial values $\hat{\chi}_3(0)$, $\hat{\chi}_4(0)$, and $\hat{\chi}_5(0)$ are non-negative.

Remark 8: The designed control scheme consists of four components: v_1, v_2, v_3, and v_4. Among which, v_1 is a model matching term inherited from the classic model reference adaptive control to stabilize the system. For the purpose of finite-time convergence, v_2 is designed to introduce an exponential term. v_3 handles input saturation by incorporating a finite-time auxiliary system. v_4 compensates the influence of the actuator additive fault and the model uncertainties. More specifically, v_{41} is to compensate the influence of additive fault, v_{42} is to compensate the influence of uncertainty in input matrix, and v_{43} is to compensate the influence of uncertainty in output matrix.

Remark 9: In the auxiliary system (27), $(c_{z1}/2)z_a^{p/q}$ is introduced for finite-time convergence purpose. The parameters c_{z1}, c_{z2}, and Π_z can be designed to adjust the tracking performance and avoid the possible overcompensation of input saturation. z_0 is a quite small constant chosen to avoid the singularity. v_3 and the auxiliary system compensate for the input saturation by eliminating the non-negative terms related to saturation error Δv. More specifically:

1) if $\Delta v = 0$, then $\dot{z}_a = -\Pi_z z_a - (c_{z1}/2)z_a^{p/q}$, indicating that z_a will converge into set $\|z_a\| \le z_0$;
2) when $\|z_a\| > z_0$, if saturation occurs, i.e., $\|\Delta v\| \ne 0$, then saturation can be compensated by the auxiliary system and v_3;
3) after z_a getting into the set $\|z_a\| \le z_0$.
   a) If saturation occurs while $\|\Delta v\| < \bar{v}$ with $\bar{v}$ being a large scalar, then $\|z_a\|$ may: i) remain in the set $\|z_a\| \le z_0$ if $\|\Delta v\|$ is quite small or ii) increase into the set $\|z_a\| > z_0$ driven by Δv, saturation is compensated by the auxiliary system and v_3.
   b) If $\|z_a\| \le z_0$ while $\|\Delta v\| \ge \bar{v}$, which means that severe saturation occurs suddenly, we can reset z_a to $\|z_a\| > z_0$, and then, the saturation can be compensated swiftly.

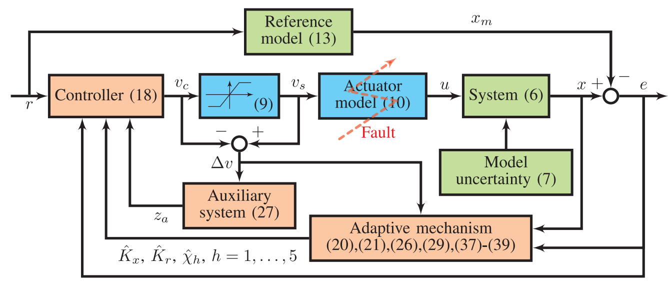

Fig. 1. Schematic of the proposed adaptive finite-time FTC system.

Remark 10: For a first-order differential equation $\dot{\chi} = \gamma_\chi(f_\chi - \kappa_\chi \chi)$, where γ_χ and κ_χ are positive scalars, and f_χ is a non-negative function. If the initial value $\chi(0) \ge 0$, then the solution

$$\chi(t) = e^{-\gamma_\chi \kappa_\chi t}\chi(0) + \int_0^t e^{-\gamma_\chi \kappa_\chi (t-\tau)} \gamma_\chi f_\chi d\tau \ge 0$$

holds for all $t \ge 0$. Thus, $\hat{\chi}_1(t)$, $\hat{\chi}_3(t)$, $\hat{\chi}_4(t)$, and $\hat{\chi}_5(t)$ remain non-negative for $t \ge 0$ as their initial values are chosen to be non-negative, which is useful in stability analysis.

For clarity, the overall structure of the proposed adaptive finite-time FTC system is illustrated in Fig. 1, and the designed control scheme is summarized in Table I.

B. Stability Analysis

Theorem 1: Consider the closed-loop system consisting of uncertain system (6), control law (18), and adaptive laws (20), (21), (26), (29), and (37)–(39) with possible input saturation and unknown actuator faults (10). Suppose that Assumption 1 holds. Then, the closed-loop system is practical finite-time sable, and the state tracking error between the considered uncertain system and the reference model converges to a small region of the origin in finite time.

Proof: By applying the proposed controller (18) to system (6), the closed-loop system is described by

$$\begin{aligned}
\dot{x} &= (A + BE_1)x + B(I+E_2)\rho v_c + B(I+E_2)\rho\Delta v \\
&\quad + B(I+E_2)u^* + B_1 d \\
&= A_m x + B_m r + BE_1 x + B\rho\left(\hat{K}_x - K_x\right)x \\
&\quad + B\rho\left(\hat{K}_r - K_r\right)r + BE_2\rho\left(\hat{K}_x x + \hat{K}_r r\right) \\
&\quad + B(I+E_2)\rho(v_2 + v_3 + v_4) + B(I+E_2)u^* \\
&\quad + B(I+E_2)\rho\Delta v + B_1 d
\end{aligned} \tag{40}$$

where ρ and u^* are defined in (10) and (11).

Then, according to (13), (16), and (40), the dynamic equation for the state tracking error is written as

$$\begin{aligned}
\dot{e} &= A_m e + B\rho\left(\hat{K}_x - K_x\right)x + B\rho\left(\hat{K}_r - K_r\right)r \\
&\quad + BE_2\rho\left(\hat{K}_x x + \hat{K}_r r\right) + B(I+E_2)u^* \\
&\quad + BE_1 x + B(I+E_2)\rho(v_2 + v_3 + v_4) \\
&\quad + B(I+E_2)\rho\Delta v + B_1 d.
\end{aligned} \tag{41}$$

TABLE I
PROPOSED ADAPTIVE FINITE-TIME FTC SCHEME

Finite-Time FTC Law:	
$v_c = v_1 + v_2 + v_3 + v_4$	(TI.1)
$v_1 = \hat{K}_x x + \hat{K}_r r$	(TI.2)
$v_2 = \begin{cases} -\frac{c_1 \hat{\chi}_1 B^T Pe\left(e^T Pe\right)^{\alpha}}{2\|e^T PB\|^2}, & \|e^T PB\|>0 \\ 0, & \|e^T PB\|=0 \end{cases}$	(TI.3)
$v_3 = -\Upsilon_z z_a - \hat{\chi}_1 B^T Pe$	(TI.4)
$v_4 = v_{41} + v_{42} + v_{43}$	(TI.5)
$v_{41} = -\frac{\hat{\chi}_3^2 B^T Pe}{\hat{\chi}_3\|e^T PB\| + c_2}$	(TI.6)
$v_{42} = -\frac{\hat{\chi}_4^2 \|x\|^2 B^T Pe}{\hat{\chi}_4\|x\|\|e^T PB\| + c_3}$	(TI.7)
$v_{43} = -\frac{\hat{\chi}_5^2 \|v_1 + v_3\|^2 B^T Pe}{\hat{\chi}_5\|v_1 + v_3\|\|e^T PB\| + c_4}$	(TI.8)
Auxiliary System:	
$\dot{z}_a = \begin{cases} -\Pi_z z_a - \frac{c_{z1}}{2} z_a^{p/q} + c_{z2}\Delta v - \frac{f(\Delta v, \hat{\chi}_2)}{\|z_a\|^2} z_a, & \|z_a\|>z_0 \\ -\Pi_z z_a - \frac{c_{z1}}{2} z_a^{p/q} + c_{z2}\Delta v, & \|z_a\|\le z_0 \end{cases}$	(TI.9)
Adaptive Laws:	
$\dot{\hat{K}}_{xi} = -\Pi_{xi}\left(2xe^T Pb_i + \Upsilon_{xi}\hat{K}_{xi}\right)$	(TI.10)
$\dot{\hat{K}}_{ri} = -\Pi_{ri}\left(2re^T Pb_i + \Upsilon_{ri}\hat{K}_{ri}\right)$	(TI.11)
$\dot{\hat{\chi}}_1 = \gamma_{11}\left(c_1(e^T Pe)^{\alpha} + 2\|e^T PB\|^2 - \gamma_{12}\hat{\chi}_1\right)$	(TI.12)
$\dot{\hat{\chi}}_2 = \gamma_{21}\left(\|\Delta v\|^2 - \gamma_{22}\hat{\chi}_2\right)$	(TI.13)
$\dot{\hat{\chi}}_3 = \gamma_{31}\left(2\|e^T PB\| - \gamma_{32}\hat{\chi}_3\right)$	(TI.14)
$\dot{\hat{\chi}}_4 = \gamma_{41}\left(2\|e^T PB\|\|x\| - \gamma_{42}\hat{\chi}_4\right)$	(TI.15)
$\dot{\hat{\chi}}_5 = \gamma_{51}\left(2\|e^T PB\|\|v_1 + v_3\| - \gamma_{52}\hat{\chi}_5\right)$	(TI.16)

Construct the following Lyapunov function:

$$V = e^{\mathrm{T}}Pe + z_a^{\mathrm{T}} z_a + \frac{1}{2}\sum_{i=1}^{m}\rho_i\left(\tilde{K}_{xi}^{\mathrm{T}}\Pi_{xi}^{-1}\tilde{K}_{xi} + \tilde{K}_{ri}^{\mathrm{T}}\Pi_{ri}^{-1}\tilde{K}_{ri}\right) + \frac{1}{2}\sum_{j=1,3,4,5}\gamma_{j1}^{-1}\eta\tilde{\chi}_j^2 + \frac{1}{2}\gamma_{21}^{-1}\tilde{\chi}_2^2 \tag{42}$$

where $\tilde{K}_{xi} = \hat{K}_{xi} - K_{xi}$, $\tilde{K}_{ri} = \hat{K}_{ri} - K_{ri}$, $\tilde{\chi}_j = \hat{\chi}_j - \chi_j$, and $\tilde{\chi}_2 = \hat{\chi}_2 - \chi_2$ are the parameter estimation errors.

Then consider (41), the derivative of V with respect to time is

$$\begin{aligned}\dot{V} = 2e^{\mathrm{T}}P\Big[&A_m e + B\rho\left(\tilde{K}_x x + \tilde{K}_r r\right) + BE_2\rho\left(\hat{K}_x x + \hat{K}_r r\right) \\ &+ B(I+E_2)\rho(v_2 + v_3 + v_4) + B(I+E_2)u^* + BE_1 x \\ &+ B_1 d + B(I+E_2)\rho\Delta v\Big] + 2z_a^{\mathrm{T}}\dot{z}_a + \sum_{j=1,3,4,5}\gamma_{j1}^{-1}\eta\tilde{\chi}_j\dot{\hat{\chi}}_j \\ &+ \gamma_{21}^{-1}\tilde{\chi}_2\dot{\hat{\chi}}_2 + \sum_{i=1}^{m}\rho_i\left(\tilde{K}_{xi}^{\mathrm{T}}\Pi_{xi}^{-1}\dot{\hat{K}}_{xi} + \tilde{K}_{ri}^{\mathrm{T}}\Pi_{ri}^{-1}\dot{\hat{K}}_{ri}\right).\end{aligned} \tag{43}$$

Then, according to the control signal v_2 in (22) and Lemma 6, the term $2e^{\mathrm{T}}PB(I+E_2)\rho v_2$ is discussed in two cases.

Case 1: When $\|e^{\mathrm{T}}PB\| > 0$, since $\hat{\chi}_1 \ge 0$, one obtains

$$\begin{aligned}&2e^{\mathrm{T}}PB(I+E_2)\rho v_2 \\ &\quad= -\frac{2c_1\hat{\chi}_1 e^{\mathrm{T}}PB(I+E_2)\rho B^{\mathrm{T}}Pe\left(e^{\mathrm{T}}Pe\right)^{\alpha}}{2\|e^{\mathrm{T}}PB\|^2} \\ &\quad\le -c_1\eta\hat{\chi}_1\left(e^{\mathrm{T}}Pe\right)^{\alpha}.\end{aligned} \tag{44}$$

Case 2: When $\|e^{\mathrm{T}}PB\| = 0$, according to Assumption 1, it has

$$2e^{\mathrm{T}}PB(I+E_2)\rho v_2 = -c_1\eta\hat{\chi}_1\left(e^{\mathrm{T}}Pe\right)^{\alpha} = 0. \tag{45}$$

Combining both cases, the following inequality can be obtained:

$$\begin{aligned}&2e^{\mathrm{T}}PB(I+E_2)\rho v_2 \\ &\quad\le -c_1\eta\hat{\chi}_1\left(e^{\mathrm{T}}Pe\right)^{\alpha} \\ &\quad= -c_1\left(e^{\mathrm{T}}Pe\right)^{\alpha} - c_1\eta\tilde{\chi}_1\left(e^{\mathrm{T}}Pe\right)^{\alpha}.\end{aligned} \tag{46}$$

Substituting (14) and (46) into (43), it yields

$$\begin{aligned}\dot{V} \le &-c_1\left(e^{\mathrm{T}}Pe\right)^{\alpha} - e^{\mathrm{T}}Qe + 2e^{\mathrm{T}}PB\rho\left(\tilde{K}_x x + \tilde{K}_r r\right) \\ &+ 2e^{\mathrm{T}}PBE_2\rho\left(\hat{K}_x x + \hat{K}_r r\right) - c_1\eta\tilde{\chi}_1\left(e^{\mathrm{T}}Pe\right)^{\alpha} \\ &+ 2e^{\mathrm{T}}PB(I+E_2)\rho(v_3 + v_4) + 2e^{\mathrm{T}}PB(I+E_2)u^* \\ &+ 2e^{\mathrm{T}}PBE_1 x + 2e^{\mathrm{T}}PB(I+E_2)\rho\Delta v + 2z_a^{\mathrm{T}}\dot{z}_a \\ &+ 2e^{\mathrm{T}}PBFd + \sum_{i=1}^{m}\rho_i\left(\tilde{K}_{xi}^{\mathrm{T}}\Pi_{xi}^{-1}\dot{\hat{K}}_{xi} + \tilde{K}_{ri}^{\mathrm{T}}\Pi_{ri}^{-1}\dot{\hat{K}}_{ri}\right) \\ &+ \sum_{j=1,3,4,5}\gamma_{j1}^{-1}\eta\tilde{\chi}_j\dot{\hat{\chi}}_j + \gamma_{21}^{-1}\tilde{\chi}_2\dot{\hat{\chi}}_2.\end{aligned} \tag{47}$$

If $\|z_a\| > z_0$, from the auxiliary system (27) and v_3 in (30), it implies that

$$\begin{aligned}&2e^{\mathrm{T}}PB\rho v_3 + 2z_a^{\mathrm{T}}\dot{z}_a + 2e^{\mathrm{T}}PB(I+E_2)\rho\Delta v \\ &\quad= -2e^{\mathrm{T}}PB\rho\Upsilon_z z_a - 2e^{\mathrm{T}}PB\rho\hat{\chi}_1 B^{\mathrm{T}}Pe - 2z_a^{\mathrm{T}}\Pi_z z_a \\ &\qquad- c_{z1}z_a^{\mathrm{T}}z_a^{p/q} - \left(\hat{\chi}_2 + c_{z2}^2\right)\|\Delta v\|^2 + 2c_{z2}z_a^{\mathrm{T}}\Delta v \\ &\qquad+ 2e^{\mathrm{T}}PB(I+E_2)\rho\Delta v.\end{aligned} \tag{48}$$

By using Lemma 2 and Young's inequality, the following inequalities are established:

$$\begin{aligned}-2e^{\mathrm{T}}PB\rho\Upsilon_z z_a &\le \|e^{\mathrm{T}}PB\|^2 + \lambda_{\max}^2(\Upsilon_z)\|z_a\|^2 \\ 2c_{z2}z_a^{\mathrm{T}}\Delta v &\le \|z_a\|^2 + c_{z2}^2\|\Delta v\|^2 \\ 2e^{\mathrm{T}}PB(I+E_2)\rho\Delta v &\le \|e^{\mathrm{T}}PB\|^2 + (1+\epsilon_2)^2\|\Delta v\|^2\end{aligned} \tag{49}$$

then, combining inequality (31), one has

$$\begin{aligned}&2e^{\mathrm{T}}PB\rho v_3 + 2z_a^{\mathrm{T}}\dot{z}_a + 2e^{\mathrm{T}}PB(I+E_2)\rho\Delta v \\ &\le -c_{z1}\left(z_a^{\mathrm{T}}z_a\right)^{\frac{p+q}{2q}} + 2\|e^{\mathrm{T}}PB\|^2 - 2\eta\hat{\chi}_1\|e^{\mathrm{T}}PB\|^2 \\ &\quad- \left(2\lambda_{\min}(\Pi_z) - \lambda_{\max}^2(\Upsilon_z) - 1\right)\|z_a\|^2 - \hat{\chi}_2\|\Delta v\|^2 \\ &\quad+ (1+\epsilon_2)^2\|\Delta v\|^2 \\ &\le -c_{z1}\left(z_a^{\mathrm{T}}z_a\right)^{\alpha} - 2\eta\tilde{\chi}_1\|e^{\mathrm{T}}PB\|^2 - \tilde{\chi}_2\|\Delta v\|^2.\end{aligned} \tag{50}$$

According to (36), it has

$$\begin{aligned} 2e^{\mathrm{T}}PB\big((I+E_2)u^*+Fd\big) &\le 2\big\|e^{\mathrm{T}}PB\big\|\,\big\|(I+E_2)u^*+Fd\big\| \\ &\le 2\eta\chi_3\big\|e^{\mathrm{T}}PB\big\| \\ 2e^{\mathrm{T}}PBE_1x &\le 2\big\|e^{\mathrm{T}}PB\big\|\|E_1\|\|x\| \\ &\le 2\eta\chi_4\big\|e^{\mathrm{T}}PB\big\|\|x\| \\ 2e^{\mathrm{T}}PBE_2\rho(v_1+v_3) &\le 2\big\|e^{\mathrm{T}}PB\big\|\|E_2\|\|v_1+v_3\| \\ &\le 2\eta\chi_5\big\|e^{\mathrm{T}}PB\big\|\|v_1+v_3\|. \end{aligned} \tag{51}$$

Substituting (50), (51) and the designed control signal v_4 in (32)–(35) into (47), and applying Lemma 6, one obtains

$$\begin{aligned} \dot{V} \le& -c_1\big(e^{\mathrm{T}}Pe\big)^{\alpha} + 2\eta\chi_3\big\|e^{\mathrm{T}}PB\big\| + 2\eta\chi_4\big\|e^{\mathrm{T}}PB\big\|\|x\| \\ &+ 2\eta\chi_5\big\|e^{\mathrm{T}}PB\big\|\|v_1+v_3\| - \frac{2\eta\hat{\chi}_3^2\big\|e^{\mathrm{T}}PB\big\|^2}{\hat{\chi}_3\big\|e^{\mathrm{T}}PB\big\|+c_2} \\ &- \frac{2\eta\hat{\chi}_4^2\|x\|^2\big\|e^{\mathrm{T}}PB\big\|^2}{\hat{\chi}_4\|x\|\big\|e^{\mathrm{T}}PB\big\|+c_3} - \frac{2\eta\hat{\chi}_5^2\|v_1+v_3\|^2\big\|e^{\mathrm{T}}PB\big\|^2}{\hat{\chi}_5\|v_1+v_3\|\big\|e^{\mathrm{T}}PB\big\|+c_4} \\ &- c_{z1}\big(z_a^{\mathrm{T}}z_a\big)^{\alpha} - 2\eta\tilde{\chi}_1\big\|e^{\mathrm{T}}PB\big\|^2 - \tilde{\chi}_2\|\Delta v\|^2 \\ &- c_1\eta\tilde{\chi}_1\big(e^{\mathrm{T}}Pe\big)^{\alpha} + 2e^{\mathrm{T}}PB\rho\big(\tilde{K}_x x+\tilde{K}_r r\big) \\ &+ \sum_{i=1}^{m}\rho_i\Big(\tilde{K}_{xi}^{\mathrm{T}}\Pi_{xi}^{-1}\dot{\hat{K}}_{xi} + \tilde{K}_{ri}^{\mathrm{T}}\Pi_{ri}^{-1}\dot{\hat{K}}_{ri}\Big) \\ &+ \sum_{j=1,3,4,5}\gamma_{j1}^{-1}\eta\tilde{\chi}_j\dot{\hat{\chi}}_j + \gamma_{21}^{-1}\tilde{\chi}_2\dot{\hat{\chi}}_2. \end{aligned} \tag{52}$$

Then, substituting the adaptive laws (20), (21), (26), (29), and (37)–(39) into (52) yields

$$\begin{aligned} \dot{V} \le& -c_1(e^{\mathrm{T}}Pe)^{\alpha} - c_{z1}\big(z_a^{\mathrm{T}}z_a\big)^{\alpha} + \frac{2\eta c_2\hat{\chi}_3\big\|e^{\mathrm{T}}PB\big\|}{\hat{\chi}_3\big\|e^{\mathrm{T}}PB\big\|+c_2} \\ &+ \frac{2\eta c_3\hat{\chi}_4\|x\|\big\|e^{\mathrm{T}}PB\big\|}{\hat{\chi}_4\|x\|\big\|e^{\mathrm{T}}PB\big\|+c_3} + \frac{2\eta c_4\hat{\chi}_5\|v_1+v_3\|\big\|e^{\mathrm{T}}PB\big\|}{\hat{\chi}_5\|v_1+v_3\|\big\|e^{\mathrm{T}}PB\big\|+c_4} \\ &- \sum_{i=1}^{m}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\hat{K}_{xi} - \sum_{i=1}^{m}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\hat{K}_{ri} \\ &- \sum_{j=1,3,4,5}\gamma_{j2}\eta\tilde{\chi}_j\hat{\chi}_j - \gamma_{22}\tilde{\chi}_2\hat{\chi}_2. \end{aligned} \tag{53}$$

According to the fact that $\hat{\chi}_3 \ge 0$, $\hat{\chi}_4 \ge 0$, and $\hat{\chi}_5 \ge 0$, one has

$$\begin{aligned} \frac{2\eta c_2\hat{\chi}_3\|e^{\mathrm{T}}PB\|}{\hat{\chi}_3\|e^{\mathrm{T}}PB\|+c_2} &\le 2\eta c_2 \\ \frac{2\eta c_3\hat{\chi}_4\|x\|\|e^{\mathrm{T}}PB\|}{\hat{\chi}_4\|x\|\|e^{\mathrm{T}}PB\|+c_3} &\le 2\eta c_3 \\ \frac{2\eta c_4\hat{\chi}_5\|v_1+v_3\|\|e^{\mathrm{T}}PB\|}{\hat{\chi}_5\|v_1+v_3\|\|e^{\mathrm{T}}PB\|+c_4} &\le 2\eta c_4 \end{aligned} \tag{54}$$

it follows that:

$$\begin{aligned} \dot{V} \le& -c_1\big(e^{\mathrm{T}}Pe\big)^{\alpha} - c_{z1}\big(z_a^{\mathrm{T}}z_a\big)^{\alpha} + 2\eta(c_2+c_3+c_4) \\ &- \sum_{i=1}^{m}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\hat{K}_{xi} - \sum_{i=1}^{m}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\hat{K}_{ri} \\ &- \sum_{j=1,3,4,5}\gamma_{j2}\eta\tilde{\chi}_j\hat{\chi}_j - \gamma_{22}\tilde{\chi}_2\hat{\chi}_2. \end{aligned} \tag{55}$$

By using Lemma 3, for any constants $\omega_{xi}, \omega_{ri}, \omega_j, \omega_2 > 0.5$, one obtains

$$\begin{aligned} -\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\hat{K}_{xi} &\le -\frac{2\omega_{xi}-1}{2\omega_{xi}}\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi} + \frac{\omega_{xi}}{2}K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi} \\ -\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\hat{K}_{ri} &\le -\frac{2\omega_{ri}-1}{2\omega_{ri}}\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri} + \frac{\omega_{ri}}{2}K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri} \\ -\eta\gamma_{j2}\tilde{\chi}_j\hat{\chi}_j &\le -\frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2 + \frac{\omega_j}{2}\eta\gamma_{j2}\chi_j^2 \\ -\gamma_{22}\tilde{\chi}_2\hat{\chi}_2 &\le -\frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2 + \frac{\omega_2}{2}\gamma_{22}\chi_2^2. \end{aligned} \tag{56}$$

Substituting the inequalities in (56) into (55) yields

$$\begin{aligned} \dot{V} \le& -\sum_{i=1}^{m}\bigg(\frac{2\omega_{xi}-1}{2\omega_{xi}}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi}\bigg)^{\alpha} - \sum_{i=1}^{m}\bigg(\frac{2\omega_{ri}-1}{2\omega_{ri}}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri}\bigg)^{\alpha} \\ &- \sum_{j=1,3,4,5}\bigg(\frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2\bigg)^{\alpha} \\ &- \bigg(\frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2\bigg)^{\alpha} + \sum_{i=1}^{m}\bigg(\frac{2\omega_{xi}-1}{2\omega_{xi}}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi}\bigg)^{\alpha} \\ &- \sum_{i=1}^{m}\frac{2\omega_{xi}-1}{2\omega_{xi}}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi} + \sum_{i=1}^{m}\bigg(\frac{2\omega_{ri}-1}{2\omega_{ri}}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri}\bigg)^{\alpha} \\ &- \sum_{i=1}^{m}\frac{2\omega_{ri}-1}{2\omega_{ri}}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri} \\ &+ \sum_{j=1,3,4,5}\bigg(\bigg(\frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2\bigg)^{\alpha} - \frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2\bigg) \\ &+ \bigg(\frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2\bigg)^{\alpha} - \frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2 - c_1(e^{\mathrm{T}}Pe)^{\alpha} \\ &- c_{z1}\big(z_a^{\mathrm{T}}z_a\big)^{\alpha} + \sum_{j=1,3,4,5}\frac{\omega_j}{2}\eta\gamma_{j2}\chi_j^2 + \frac{\omega_2}{2}\gamma_{22}\chi_2^2 \\ &+ \sum_{i=1}^{m}\frac{\omega_{xi}}{2}\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi} + \sum_{i=1}^{m}\frac{\omega_{ri}}{2}\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri} \\ &+ 2\eta(c_2+c_3+c_4). \end{aligned} \tag{57}$$

According to Lemma 4, the following inequalities can be obtained:

$$\begin{aligned} \bigg(\frac{2\omega_{xi}-1}{2\omega_{xi}}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi}\bigg)^{\alpha} &\le \frac{2\omega_{xi}-1}{2\omega_{xi}}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi} + \bar{\alpha} \\ \bigg(\frac{2\omega_{ri}-1}{2\omega_{ri}}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri}\bigg)^{\alpha} &\le \frac{2\omega_{ri}-1}{2\omega_{ri}}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri} + \bar{\alpha} \\ \bigg(\frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2\bigg)^{\alpha} &\le \frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2 + \bar{\alpha} \\ \bigg(\frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2\bigg)^{\alpha} &\le \frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2 + \bar{\alpha} \end{aligned} \tag{58}$$

where $\bar{\alpha} = (1-\alpha)\alpha^{\alpha/(1-\alpha)}$.

Substituting (58) into (57) and using Lemma 5 yields

$$\begin{aligned} \dot{V} \le& -c_1\big(e^{\mathrm{T}}Pe\big)^{\alpha} - c_{z1}\big(z_a^{\mathrm{T}}z_a\big)^{\alpha} - \sum_{i=1}^{m}\bigg(\frac{2\omega_{xi}-1}{2\omega_{xi}}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi}\bigg)^{\alpha} \\ &- \sum_{i=1}^{m}\bigg(\frac{2\omega_{ri}-1}{2\omega_{ri}}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri}\bigg)^{\alpha} \\ &- \sum_{j=1,3,4,5}\bigg(\frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2\bigg)^{\alpha} - \bigg(\frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2\bigg)^{\alpha} \end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{m}\frac{\omega_{xi}}{2}\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi}+\sum_{i=1}^{m}\frac{\omega_{ri}}{2}\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri}\\
&+\sum_{j=1,3,4,5}\frac{\omega_j}{2}\eta\gamma_{j2}\chi_j^2+\frac{\omega_2}{2}\gamma_{22}\chi_2^2+2\eta(c_2+c_3+c_4)\\
&+(2m+5)\bar{\alpha}\\
\le& -\zeta\Bigg[\left(e^{\mathrm{T}}Pe\right)^{\alpha}+\left(z_a^{\mathrm{T}}z_a\right)^{\alpha}+\sum_{i=1}^{m}\left(\frac{1}{2}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Pi_{xi}^{-1}\tilde{K}_{xi}\right)^{\alpha}\\
&+\sum_{i=1}^{m}\left(\frac{1}{2}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Pi_{ri}^{-1}\tilde{K}_{ri}\right)^{\alpha}+\sum_{j=1,3,4,5}\left(\frac{1}{2}\gamma_{j1}^{-1}\eta\tilde{\chi}_j^2\right)^{\alpha}\\
&+\left(\frac{1}{2}\gamma_{21}^{-1}\tilde{\chi}_2^2\right)^{\alpha}\Bigg]+\bar{\psi}_1\\
\le& -\zeta V^{\alpha}+\bar{\psi}_1
\end{aligned}\tag{59}
$$

where

$$
\zeta=\min\left\{\left(\frac{(2\omega_{xi}-1)\lambda_{\min}(\Upsilon_{xi})}{\omega_{xi}\lambda_{\max}(\Pi_{xi}^{-1})}\right)^{\alpha},\left(\frac{(2\omega_2-1)\gamma_{21}\gamma_{22}}{\omega_2}\right)^{\alpha}\right.
$$
$$
\left.\left(\frac{(2\omega_j-1)\gamma_{j1}\gamma_{j2}}{\omega_j}\right)^{\alpha},\left(\frac{(2\omega_{ri}-1)\lambda_{\min}(\Upsilon_{ri})}{\omega_{ri}\lambda_{\max}\left(\Pi_{ri}^{-1}\right)}\right)^{\alpha},c_1,c_{z1}\right\}
$$

and $\bar{\psi}_1=\sum_{j=1,3,4,5}(\omega_j/2)\eta\gamma_{j2}\chi_j^2+(\omega_2/2)\gamma_{22}\chi_2^2+\sum_{i=1}^{m}(\omega_{xi}/2)\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi}+\sum_{i=1}^{m}(\omega_{ri}/2)\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri}+2\eta(c_2+c_3+c_4)+(2m+5)\bar{\alpha}$.

If $\|z_a\|\le z_0$, then $\|\Delta v\|\le\bar{v}$, which indicates that $f(\Delta v,\hat{\chi}_2)$ is bounded. With a similar calculation as in the case of $\|z_a\|>z_0$, it follows that:

$$
\begin{aligned}
\dot{V}\le& -\zeta\Bigg[\left(e^{\mathrm{T}}Pe\right)^{\alpha}+\left(z_a^{\mathrm{T}}z_a\right)^{\alpha}+\sum_{i=1}^{m}\left(\frac{1}{2}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Pi_{xi}^{-1}\tilde{K}_{xi}\right)^{\alpha}\\
&+\sum_{i=1}^{m}\left(\frac{1}{2}\rho_i\tilde{K}_{ri}^{\mathrm{T}}\Pi_{ri}^{-1}\tilde{K}_{ri}\right)^{\alpha}+\sum_{j=1,3,4,5}\left(\frac{1}{2}\gamma_{j1}^{-1}\eta\tilde{\chi}_j^2\right)^{\alpha}\\
&+\left(\frac{1}{2}\gamma_{21}^{-1}\tilde{\chi}_2^2\right)^{\alpha}\Bigg]+\sum_{i=1}^{m}\frac{\omega_{xi}}{2}\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi}\\
&+\sum_{i=1}^{m}\frac{\omega_{ri}}{2}\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri}+\sum_{j=1,3,4,5}\frac{\omega_j}{2}\eta\gamma_{j2}\chi_j^2\\
&+\frac{\omega_2}{2}\gamma_{22}\chi_2^2+2\eta(c_2+c_3+c_4)\\
&+2f\left(\Delta v,\hat{\chi}_2\right)+(2m+5)\bar{\alpha}\\
\le& -\zeta V^{\alpha}+\bar{\psi}_2
\end{aligned}\tag{60}
$$

where $\bar{\psi}_2=\sum_{j=1,3,4,5}(\omega_j/2)\eta\gamma_{j2}\chi_j^2+(\omega_2/2)\gamma_{22}\chi_2^2+(2m+5)\bar{\alpha}+2\eta(c_2+c_3+c_4)+2\sup_{\|\Delta v\|<\bar{v}}f+\sum_{i=1}^{m}(\omega_{xi}/2)\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi}+\sum_{i=1}^{m}(\omega_{ri}/2)\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri}$.

Combining (59) and (60), it has

$$
\dot{V}\le -\zeta V^{\alpha}+\bar{\psi}\tag{61}
$$

where $\bar{\psi}=\max\{\bar{\psi}_1,\bar{\psi}_2\}$, and let

$$
\mathrm{T}^*=\frac{V^{1-\alpha}(0)}{\zeta\sigma(1-\alpha)}\tag{62}
$$

where $0<\sigma<1$ and $V(0)$ is the initial value of V. Then, according to Lemma 1, it can be concluded that for $\forall t\ge\mathrm{T}^*$, $V^{\alpha}\le(\bar{\psi}/(1-\sigma)\zeta)$, which means that the closed-loop system is practical finite-time stable.

Based on (61), it is clear that $\dot{V}\le 0$ for $V^{\alpha}\ge\bar{\psi}/\zeta$. Thus, V is bounded. Then, the signals included in V, i.e., e, z_a, $\tilde{K}_{xi}$, $\tilde{K}_{ri}$, and $\tilde{\chi}_j$ are bounded. As A_m is Hurwitz, x_m is bounded, which further implies that x is bounded as well. From the definition of $\tilde{K}_{xi}$, $\tilde{K}_{ri}$, and $\tilde{\chi}_j$, the boundedness of $\hat{K}_{xi}$, $\hat{K}_{ri}$, and $\hat{\chi}_j$ can be deduced. Thus, the designed control signal v is bounded. According to the input saturation and the boundedness of additive fault, v_s and u are bounded. Hence, all the closed-loop signals are bounded. Moreover, according to the definition of V, for $\forall t\ge\mathrm{T}^*$, it has

$$
\|e\|\le\frac{1}{\sqrt{\lambda_{\min}(P)}}\left(\frac{\bar{\psi}}{(1-\sigma)\zeta}\right)^{\frac{1}{2\alpha}}\tag{63}
$$

which implies that the state tracking error between the considered uncertain system and the reference model converges to a small region of the origin in finite time.

This completes the proof. ■

Remark 11: Parameters ω_{xi}, ω_{ri}, ω_j, and ω_2 appear during the stability analysis only, and are not included in the designed control strategy. It can be seen from (62) and (63) that the system tracking performance is determined by the initial value of V and the designed parameters α, $\bar{\psi}$, and ζ. By selecting the size of the corresponding control parameters (e.g., decreasing c_2, c_3, and c_4) to increase ζ and decrease $\bar{\psi}$, the smaller tracking error and the faster convergence rate can be achieved, and vice versa. However, large control action should be avoided for practical systems. Therefore, there is a tradeoff between the tracking performance and the control action when choosing these parameters.

Remark 12: Prescribed-time control is a kind of typical finite-time control method [33], [34]. By introducing certain prescribed time functions, the prescribed-time control system achieves the desired control objective within prescribed finite time. While in our paper, the practical finite-time convergence has been achieved and the upper bound of the settling time is derived by designing a fractional-power feedback-based control scheme instead of being prescribed. By selecting the controller parameters properly, the settling time can be adjusted and the large control actions in the initial moment can be avoided. For uncertain systems with unknown actuator faults, how to design a prescribed-time FTC scheme should be an interesting while challenging topic, which is worthy of further investigation.

C. Extension to Nonfinite-Time Control Case

Consider the FTC issue without finite-time convergence, the proposed control scheme can be modified to ensure that the closed-loop system is uniformly ultimately bounded (UUB). The corresponding results can be summarized in the following corollary.

Corollary 1: Consider the closed-loop system consisting of uncertain system (6) with possible input saturation and unknown actuator faults (10), and control scheme presented in Table II. Suppose that $\mathrm{rank}(B(I+E_2)\rho)=\mathrm{rank}(B)$ holds. Then, all the signals in the closed-loop system are UUB,

TABLE II
MODIFIED ADAPTIVE FTC SCHEME FOR NONFINITE-TIME CONTROL CASE

FTC Law:		
$v_c = v_1 + v_2 + v_3$		(TII.1)
$v_1 = \hat{K}_x x + \hat{K}_r r$		(TII.2)
$v_2 = -\Upsilon_z z_a - \hat{\chi}_1 B^T Pe$		(TII.3)
$v_3 = v_{31} + v_{32} + v_{33}$		(TII.4)
$v_{31} = -\dfrac{\hat{\chi}_3^2 B^T Pe}{\hat{\chi}_3\|e^T PB\| + c_2}$		(TII.5)
$v_{32} = -\dfrac{\hat{\chi}_4^2 \|x\|^2 B^T Pe}{\hat{\chi}_4\|x\|\|e^T PB\| + c_3}$		(TII.6)
$v_{33} = -\dfrac{\hat{\chi}_5^2 \|v_1+v_2\|^2 B^T Pe}{\hat{\chi}_5\|v_1+v_2\|\|e^T PB\| + c_4}$		(TII.7)
Auxiliary System:		
$\dot{z}_a = \begin{cases} -\Pi_z z_a + c_{z2}\Delta v - \frac{f(\Delta v, \hat{\chi}_2)}{\|z_a\|^2} z_a, & \|z_a\| > z_0 \\ -\Pi_z z_a + c_{z2}\Delta v, & \|z_a\| \le z_0 \end{cases}$		(TII.8)
Adaptive Laws:		
$\dot{\hat{K}}_{xi} = -\Pi_{xi}\left(2xe^T Pb_i + \Upsilon_{xi}\hat{K}_{xi}\right)$		(TII.9)
$\dot{\hat{K}}_{ri} = -\Pi_{ri}\left(2re^T Pb_i + \Upsilon_{ri}\hat{K}_{ri}\right)$		(TII.10)
$\dot{\hat{\chi}}_1 = \gamma_{11}\left(2\|e^T PB\|^2 - \gamma_{12}\hat{\chi}_1\right)$		(TII.11)
$\dot{\hat{\chi}}_2 = \gamma_{21}\left(\|\Delta v\|^2 - \gamma_{22}\hat{\chi}_2\right)$		(TII.12)
$\dot{\hat{\chi}}_3 = \gamma_{31}\left(2\|e^T PB\| - \gamma_{32}\hat{\chi}_3\right)$		(TII.13)
$\dot{\hat{\chi}}_4 = \gamma_{41}\left(2\|e^T PB\|\|x\| - \gamma_{42}\hat{\chi}_4\right)$		(TII.14)
$\dot{\hat{\chi}}_5 = \gamma_{51}\left(2\|e^T PB\|\|v_1+v_2\| - \gamma_{52}\hat{\chi}_5\right)$		(TII.15)

and the state tracking error between the considered uncertain system and the reference model converges to a small region of the origin.

Proof: Consider the Lyapunov function (42). When $\|z_a\| > z_0$, one has

$$\begin{aligned}\dot{V} \le& -e^{\mathrm{T}}Qe - \left(2\lambda_{\min}(\Pi_z) - \lambda_{\max}^2(\Upsilon_z) - 1\right)z_a^{\mathrm{T}}z_a \\ &- \sum_{i=1}^{m}\frac{2\omega_{xi}-1}{2\omega_{xi}}\rho_i\tilde{K}_{xi}^{\mathrm{T}}\Upsilon_{xi}\tilde{K}_{xi} - \sum_{i=1}^{m}\frac{2\omega_{ri}-1}{2\omega_{ri}} \\ &\times \rho_i\tilde{K}_{ri}^{\mathrm{T}}\Upsilon_{ri}\tilde{K}_{ri} - \sum_{j=1,3,4,5}\left(\frac{2\omega_j-1}{2\omega_j}\eta\gamma_{j2}\tilde{\chi}_j^2\right) \\ &- \frac{2\omega_2-1}{2\omega_2}\gamma_{22}\tilde{\chi}_2^2 + \sum_{j=1,3,4,5}\frac{\omega_j}{2}\eta\gamma_{j2}\chi_j^2 + \frac{\omega_2}{2}\gamma_{22}\chi_2^2 \\ &+ \sum_{i=1}^{m}\frac{\omega_{xi}}{2}\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi} + \sum_{i=1}^{m}\frac{\omega_{ri}}{2}\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri} \\ &+ 2\eta(c_2+c_3+c_4). \end{aligned} \tag{64}$$

When $\|z_a\| \le z_0$, similar results can be obtained and the details are omitted here. Thus, it can be concluded that

$$\dot{V} \le -CV + D \tag{65}$$

where

$$\begin{aligned}C = \min\Bigg\{&\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \frac{(2\omega_{xi}-1)\lambda_{\min}(\Upsilon_{xi})}{\omega_{xi}\lambda_{\max}\left(\Pi_{xi}^{-1}\right)}, 2\lambda_{\min}(\Pi_z) \\ &- \lambda_{\max}^2(\Upsilon_z) - 1, \frac{(2\omega_{ri}-1)\lambda_{\min}(\Upsilon_{ri})}{\omega_{ri}\lambda_{\max}\left(\Pi_{ri}^{-1}\right)}, \frac{(2\omega_j-1)\gamma_{j1}\gamma_{j2}}{\omega_j}, \\ &\frac{(2\omega_2-1)\gamma_{21}\gamma_{22}}{\omega_2}\Bigg\}\end{aligned}$$

and $D = \max\{D_1, D_2\}$ with $D_1 = \sum_{j=1,3,4,5}(\omega_j/2)\eta\gamma_{j2}\chi_j^2 + (\omega_2/2)\gamma_{22}\chi_2^2 + \sum_{i=1}^{m}(\omega_{xi}/2)\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi} + \sum_{i=1}^{m}(\omega_{ri}/2)\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri} + 2\eta(c_2 + c_3 + c_4)$ and $D_2 = \sum_{j=1,3,4,5}(\omega_j/2)\eta\gamma_{j2}\chi_j^2 + (\omega_2/2)\gamma_{22}\chi_2^2 + 2\eta(c_2 + c_3 + c_4) + 2\sup_{\|\Delta v\|<\bar{v}} f + \sum_{i=1}^{m}(\omega_{xi}/2)\rho_i K_{xi}^{\mathrm{T}}\Upsilon_{xi}K_{xi} + \sum_{i=1}^{m}(\omega_{ri}/2)\rho_i K_{ri}^{\mathrm{T}}\Upsilon_{ri}K_{ri}$.

By integrating both sides of (65), one has

$$V(t) \le V(0)e^{-Ct} + \frac{D}{C}(1 - e^{-Ct}) \le V(0) + \frac{D}{C} \tag{66}$$

which shows that V is UUB. Similar to the analysis below (62), it can be concluded that all the signals in the closed-loop system are bounded. Moreover, from (42) and (66), it yields

$$\lambda_{\min}(P)\|e\|^2 \le e^{\mathrm{T}}Pe \le V(0)e^{-Ct} + \frac{D}{C}\left(1 - e^{-Ct}\right) \tag{67}$$

thus the state tracking error in the Euclidean norm will converge into a residual set $\Omega = \{e \mid \|e\| \le \sqrt{(D/[\lambda_{\min}(P)C])}\}$ exponentially. Therefore, Corollary 1 can be derived. ■

Remark 13: Although nonfinite-time FTC has been studied in many literatures (e.g., [1], [2], [4], and [35]), the FTC for the considered system in the presence of unknown actuator faults, model uncertainty and input saturation is still open in nonfinite-time control case. Note that it is more difficult to achieve (61) for finite-time control case than (65) for nonfinite-time one. However, the advantages of finite-time control, such as faster transient performance, better robustness, and disturbances attenuate ability, are much more meaningful for practical systems in most situations. Nevertheless, in the nonfinite-time control case, c_2, c_3, and c_4 are still contained in D, thus these constants do not lead to a larger bound in the finite-time control case compared to nonfinite-time one.

IV. SIMULATION RESULTS

In this section, two simulation examples are presented to demonstrate the effectiveness of the proposed FTC scheme and show its advantages through comparison.

A. Application to Flight Control

In this section, the proposed scheme is employed for the control of an F-18 high-angle-of-attack research vehicle. The state space representation of the decoupled linearized longitudinal motion model is given as [31]

$$A = \begin{bmatrix} -1.175 & 0.9871 \\ -8.458 & -0.8776 \end{bmatrix} B = \begin{bmatrix} -0.194 & -0.03593 \\ -19.29 & -3.803 \end{bmatrix}. \tag{68}$$

The states are $x = [\alpha', q']^{\mathrm{T}}$, where α' and q' represent the attack angle (deg) and the pitch rate (deg/s), respectively. The control surfaces are $u = [\delta_E, \delta_{PTV}]^{\mathrm{T}}$, where δ_E and δ_{PTV} denote the symmetric elevator position (deg) and the symmetric pitch thrust velocity nozzle position (deg), respectively. The saturation constraints of actuators are set as $u_{1\max} = u_{2\max} = 3.5$ and $u_{1\min} = u_{2\min} = -3.5$.

The reference model is given as

$$A_m = \begin{bmatrix} -3 & 1 \\ -1 & -3 \end{bmatrix} \quad B_m = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \tag{69}$$

and the input is $r = [-1.15, 0.8]^{\mathrm{T}}$.

The model uncertainties and external disturbance are described by

$$E_1 = \begin{bmatrix} \sin(1.5t) & 0 \\ 0 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} \exp(-t) & 1 \\ 0 & 1 \end{bmatrix}$$
$$F = I_2 \qquad d = [0, 0.1\sin(t)]^{\mathrm{T}}. \tag{70}$$

The following faulty case is discussed:

$$\begin{cases} u_1 = 0.5v_{s1} - 1.8, & 10\,\mathrm{s} \le t < 20\,\mathrm{s} \\ u_2 = v_{s2} + 0.5\sin(2t), & 30\,\mathrm{s} \le t < 40\,\mathrm{s} \\ u_i = v_{si},\ i = 1, 2 & \text{otherwise.} \end{cases} \tag{71}$$

In (71), $u_1 = 0.5v_{s1} - 1.8$ denotes that the effectiveness of the first actuator, which generate a needless constant bias, loses 50%; and $u_2 = v_{s2} + 0.5\sin(2t)$ denotes that the second actuator generate a needless time-varying bias. Then, it can be concluded in the following.

1) *Fault-Free:* $u_i = v_{si}$, $i = 1, 2$ for $0 \le t < 10\,\mathrm{s}$.
2) *u_1 Fault:* $u_1 = 0.5v_{s1} - 1.8$, $u_2 = v_{s2}$ for $10\,\mathrm{s} \le t < 20\,\mathrm{s}$.
3) *u_1 Becomes Normal, Fault-Free:* $u_i = v_{si}$, $i = 1, 2$ for $20\,\mathrm{s} \le t < 30\,\mathrm{s}$.
4) *u_2 Fault:* $u_2 = v_{s2} + 0.5\sin(2t)$, $u_1 = v_{s1}$ for $30\,\mathrm{s} \le t < 40\,\mathrm{s}$.
5) *u_2 Becomes Normal, Fault-Free:* $u_i = v_{si}$, $i = 1, 2$ for $t \ge 40\,\mathrm{s}$.

It is easy to verify that the simulated system, uncertainties, and actuator faults satisfy Assumption 1. The practical context is considered during the simulation, that is, some Gaussian white noise is superimposed on the measured state. The parameters, gains, and initial values of the controller and adaptive update laws are listed in Table III. The simulation results are given in Figs. 2–4.

Fig. 2 shows the state tracking trajectories, that is, the reference model state trajectory $x_m(t)$ (solid) and the plant state trajectory $x(t)$ (dashed), and Fig. 3 shows the state tracking error $e(t)$. It can be seen from Figs. 2 and 3 that the desired system stability and practical finite-time tracking properties can be guaranteed by the proposed finite-time adaptive FTC scheme, despite there exist some transit responses caused by the model uncertainties or the occurrences of the actuator faults. Fig. 4 shows the actual control signal $u(t)$ (solid) and the designed control signal $v_c(t)$ (dashed). It is interesting to note that the control signal is saturated during the actuator fault phase.

TABLE III
PARAMETERS AND INITIAL VALUES OF THE CONTROLLER AND ADAPTIVE LAWS

Parameter	Value
Control gain, Υ_z	$0.8I_2$
Positive constants, c_j, $j = 1, 2, 3, 4$	0.01
Auxiliary system gain, Π_z	I_2
Auxiliary system gains, c_{z1}, c_{z2}	1
Auxiliary system constant, z_0	0.01
Positive odd integer, p	3
Positive odd integer, q	5
Adaptive gains, Π_{xi}, Π_{ri}, Υ_{xi}, Υ_{ri}, $i = 1, 2$	$2I_2$
Adaptive gains, γ_{h1}, γ_{h2}, $h = 1, \ldots, 5$	2
Initial value, $z_a(0)$	$[-0.5, 0.5]^T$
Initial value, $x(0)$	$[-0.9, 0.6]^T$
Initial value, $x_m(0)$	$[0.9, -0.9]^T$
Initial values, $\hat{K}_x(0)$, $\hat{K}_r(0)$	I_2
Initial value, $\hat{\chi}_h(0)$	0.1

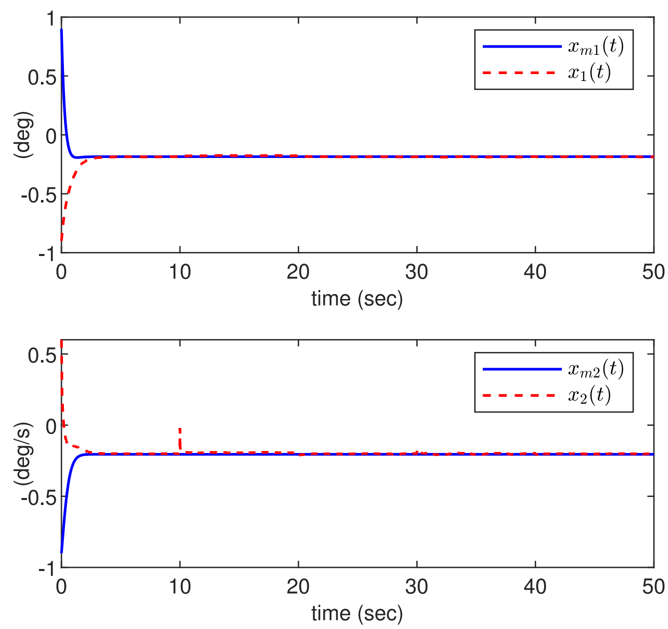

Fig. 2. Trajectories of the reference model state $x_m(t)$ and the plant state $x(t)$.

From Figs. 2–4, it can be seen that the proposed FTC method can compensate for the model uncertainties, input saturation, and multiplicative and additive actuator faults simultaneously. Therefore, the effectiveness and the performance of the proposed FTC strategy, as expected, has been illustrated by the simulation results.

B. Numerical Comparative Analysis

In this section, comparative analysis of the proposed scheme with the results in [4] is presented. In [4], the FTC problem of a linear system with state matrix uncertainty and external disturbances was studied. However, the input matrix uncertainty and the input saturation were not considered, thus these two

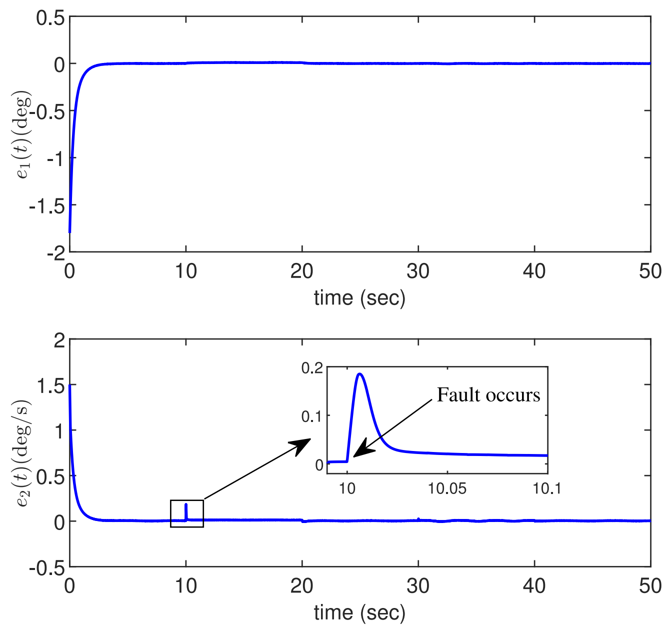

Fig. 3. State tracking error $e(t)$.

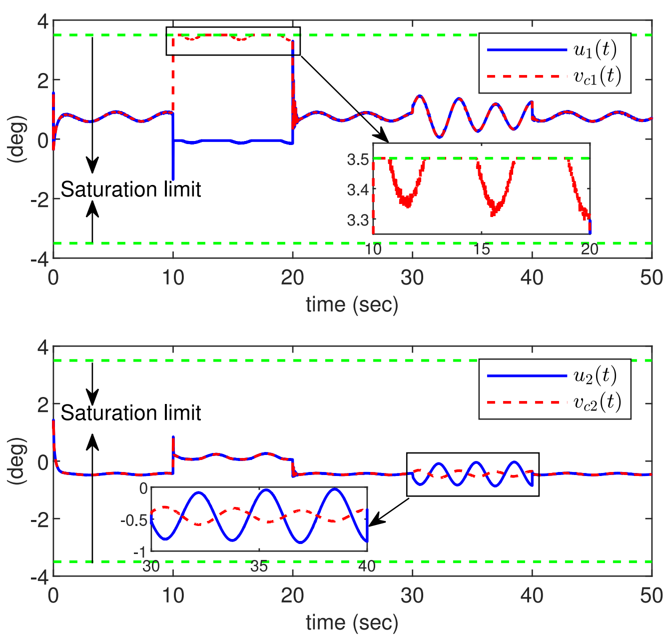

Fig. 4. Components of the actual control signal $u(t)$ and the designed control signal $v_c(t)$.

factors are ignored in the numerical comparison for the sake of fairness.

Consider an uncertain system described by (6) with

$$A = \begin{bmatrix} -2 & 4 & 0 \\ 0 & -7 & -4 \\ 0 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \tag{72}$$

and the settings of state matrix uncertainty and external disturbance are the same as in [4], i.e., $d = [0, 0, 0.1\sin(t)]^{\mathrm{T}}$

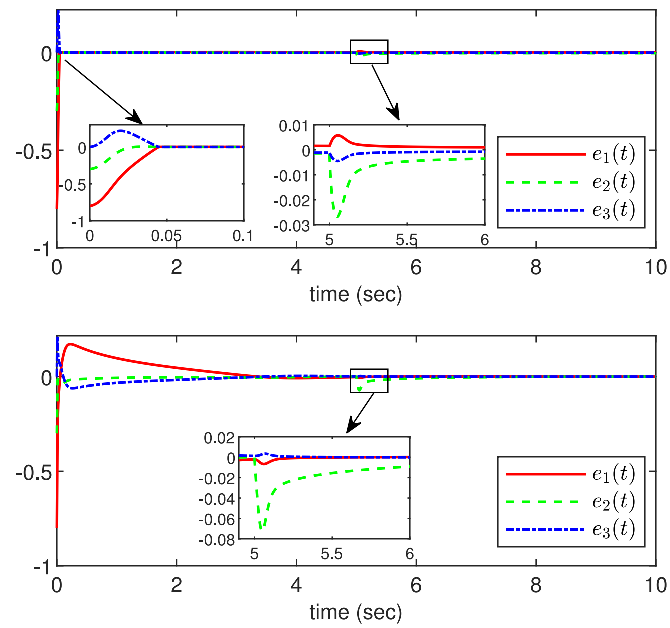

Fig. 5. State tracking error $e(t)$. Top: scheme in this article. Bottom: scheme in [4].

and

$$E_1 = \begin{bmatrix} 0.1\sin(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad F = \begin{bmatrix} 0.15 & 0 & 0 \\ 0 & 0 & 0.2 \\ 0 & 0 & 0 \end{bmatrix}. \tag{73}$$

The reference model is given as

$$A_m = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \quad B_m = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{74}$$

and $r = [0.5, 0.5, 0.5]^{\mathrm{T}}$.

Consider the following faulty case:

$$\begin{cases} u_2 = 0.3 v_{s2}, & t \geq 5\,\mathrm{s} \\ u_i = v_{si},\ i = 1, 2, 3 & \text{otherwise}. \end{cases} \tag{75}$$

The parameters and initial values of our control scheme are $\Pi_{xi} = \Pi_{ri} = 10 I_3$, $\Upsilon_{xi} = \Upsilon_{ri} = 0.05 I_3$, $i = 1, 2, 3$, $\gamma_{h1} = 10$, $\gamma_{h2} = 0.01$, $h = 1, 2, 3, 4, 5$, $c_j = 0.01$, $j = 1, 2, 3, 4$, $\hat{K}_x(0) = \hat{K}_r(0) = I_3$, and $\hat{\chi}_h(0) = 0.1$. The parameters and initial values of the compared control scheme are set to be the same as in [4].

The comparison results are given in Figs. 5 and 6. Fig. 5 shows the state tracking error $e(t)$ by using our scheme (top) and the one given in [4] (bottom); and the corresponding control signals are shown in Fig. 6. It is obvious that the proposed finite-time FTC scheme has better transient response and faster convergence rate despite the smaller peak value of the control signal. These results further illustrate the effectiveness and advantages of the proposed finite-time FTC strategy.

V. Conclusion

In this article, an adaptive finite-time FTC strategy is developed for state tracking of uncertain systems subject to unknown actuator faults and input saturation. Considering the

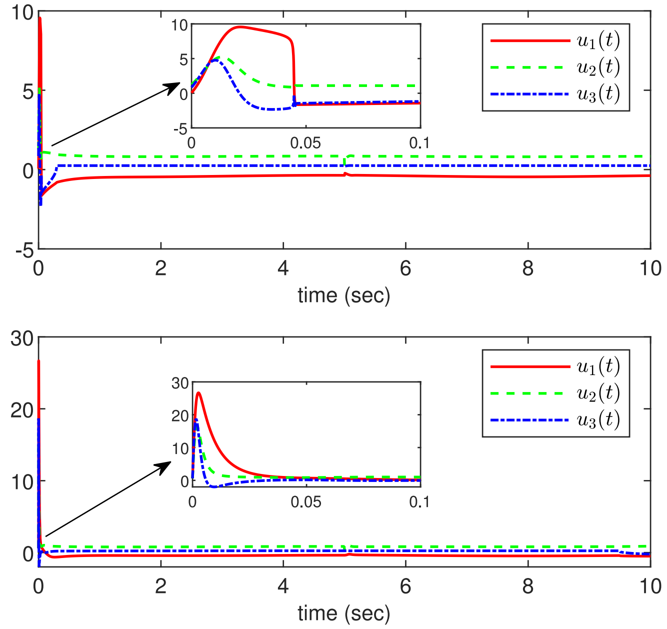

Fig. 6. Actual control signal $u(t)$. Top: scheme in this article. Bottom: scheme in [4].

physical constraints of the actuators, a finite-time auxiliary system is designed to address the input saturation. In order to construct the adaptive FTC scheme, an adaptive mechanism is proposed to provide online estimation information of unknown parameters caused by model uncertainties and actuator faults. Furthermore, the desired stability of the closed-loop system and the finite-time state tracking property are proven with the use of the finite-time Lyapunov theory. Finally, simulation results of an F-18 flight control system and the numerical comparative analysis with an existing result demonstrate the effectiveness and advantages of the designed FTC scheme. Based on the results of [36]–[39], future works will be concentrated on adaptive output feedback FTC of uncertain systems with input/output constraints and possibly infinite number of actuator faults.

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