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Adaptive Finite-Time Varying Fault Compensation Tracking Control for Linear Systems

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***Abstract*—In this paper, an adaptive finite-time fault-tolerant tracking control scheme is proposed for linear systems in the presence of actuator faults. The faults including outage, loss of effectiveness and time-varying bias are all taken into consideration. For those uncertain and varying faults, an adaptive mechanism is designed which promises the desired stability and practical finite-time state tracking properties. The simulation results on an application to an aircraft system illustrate the efficiency of the theoretical results.**

***Keywords*—linear systems; actuator faults; finite-time control; fault-tolerant control; adaptive control**

I. Introduction

In recent decades, the fault-tolerant control (FTC) issue of systems with actuator faults has been studied broadly due to its importance in the fields of flight control system, power system, and so on. In [1], a FTC scheme was proposed for spacecraft attitude control based on a fault detection observer and an indirect fault identification approach. By assuming that the fault information has been obtained, an integral sliding mode control based FTC strategy was designed in [2] for a class of uncertain nonlinear systems. It is noted that both the aforementioned control schemes require fault detection and diagnosis (FDD) to obtain accurate fault information, while false alarms and the time delay caused by on-line fault diagnosis may affect the FTC effect.

On the other hand, adaptive FTC has been widely applied to handle actuator faults without any fault information, which means that FDD is not required [3]–[8]. In [3], the backstepping adaptive FTC strategy with adaptive laws was designed for high-speed trains position tracking. By using an adaptive reference model, a hybrid direct–indirect adaptive controller was proposed in [4] for linear systems to obtain the asymptotic state tracking in the presence of actuator faults. An adaptive FTC scheme was proposed for a class of nonlinear systems with a possibly infinite number of unknown actuator faults in [5]. In [6], a direct adaptive FTC law was developed for linear systems which can compensate the actuator faults automatically.

It is worth noting that the aforementioned studies on FTC consider the stability without the convergence time limitation, which indicates that those control strategies guarantee the desired system performance with infinite settling time. However, it is obvious that FTC schemes with finite-time convergence are more practical due to its faster convergence rate and better robustness. In [9] and [10], the issues of finite-time control for nonlinear systems were addressed in fault-free case. Consider the finite-time FTC problem, many constructive results were established [11]–[15]. By using the radial basis function network technology, an adaptive finite-time FTC method was proposed in [11] for the attitude control of reentry vehicle. In [12], an integrated adaptive fault-tolerant controller was constructed for a nonlinear system by employing the approximation ability of fuzzy logic systems. A robust FTC scheme was derived in [13] for spacecraft finite-time attitude stabilization on the basis of the sliding mode control. In our recent work [14], a finite-time fault-tolerant tracking control scheme was developed for linear systems with unknown actuator faults including constant stuck, which are essentially not varying. However, for some cases, the bias fault exists, which may be a time-varying one. For example, the spacecraft wheels may produce time-varying bias torque even when the commanded attitude control torque is zero, due to the changes in coulomb friction and viscous friction of the bearings caused by aging, lubrication, etc. In [15], the stabilization problem of linear systems with actuator faults and saturations was studied and a finite-time robust compensation control strategy was proposed, while the state tracking problem has not been considered.

Motivated by [14] and [15], in this paper, we further study the finite-time tracking control problem for linear systems in the presence of unknown time-varying actuator faults. The main contributions are as follows:

- An adaptive finite-time FTC scheme is designed for linear systems with unknown actuator faults including outage, loss of effectiveness and time-varying bias.
- The time-varying bias fault considered can be stuck, drift, etc., which is no longer required to be a piecewise constant function as in [4], [14] and [16].

The remaining of this paper is organized as follows. In Section II, the control problem of the paper is formulated. Section III elaborates an adaptive finite-time fault-tolerant tracking control strategy. In Section IV, an application to an aircraft system is introduced to illustrate the theoretical result. Finally, the conclusion end this paper in Section V.

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II. Problem Formulation

Consider the linear systems modeled as follows:

$$\dot{x}(t) = Ax(t) + Bu(t)\,, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^T \in \mathbb{R}^m$ are state and input vectors, respectively, A, B are appropriate dimensional constant matrices and B is full-row rank.

The actuator fault can be described as

$$u(t) = \sigma v(t) + \rho u^*(t), \tag{2}$$

where $v(t) = [v_1(t), v_2(t), \ldots, v_m(t)]^T$ is the vector of control signals to be designed, $u^*(t) = [u_1^*(t), u_2^*(t), \ldots, u_m^*(t)]^T$ is the vector of bias faults, $\sigma = \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$ and $\rho = \text{diag}\{\rho_1, \rho_2, \ldots, \rho_m\}$ are unknown fault pattern matrices with piecewise constant $\sigma_i \in [0, 1]$, $\rho_i \in \{0, 1\}$ and $\sigma_i\rho_i = 0$, $i = 1, 2, \ldots, m$. The types of actuator faults are summarized in Table I.

TABLE I: Actuator fault type

Fault type	σ_i	ρ_i
Normal	1	0
Outage	0	0
Loss of effectiveness	$0 < \sigma_i < 1$	0
Bias	0	1

It is noted that $u^*(t)$ is the unknown and time-varying bias faults. $u^*(t)$ can represent not only the constant stuck faults in our previous work [14], but also the irregular drift faults, such as aircraft rudder drift, spacecraft wheel drift, and so on. Therefore, the faults considered in this paper are much more general and practical. Normally, $u^*(t)$ is an unknown and bounded vector function satisfying that

$$\|u^*(t)\| \leq \bar{u}\,, \tag{3}$$

where $\bar{u}$ is an unknown positive constant.

Give the reference model as

$$\dot{x}_m(t) = A_m x_m(t) + B_m r_m(t)\,, \tag{4}$$

where $x_m(t) \in \mathbb{R}^n$ and $r_m(t) \in \mathbb{R}^m$ are reference state and input vectors. A_m is Hurwitz, that is, there exist positive definite matrices $P, Q \in \mathbb{R}^{n\times n}$ such that

$$A_m^T P + PA_m = -Q\,. \tag{5}$$

Assumption 1. $\text{rank}(B\sigma) = \text{rank}(B)$.

Assumption 2. There exist $K_x^* \in \mathbb{R}^{m\times n}$ and $K_r^* \in \mathbb{R}^{m\times m}$ such that

$$A + BK_x^* = A_m\,, \quad BK_r^* = B_m\,. \tag{6}$$

Remark 1. As discussed in [7] and [14], Assumption 1 is an actuator redundancy condition which ensures that the system remains capable to be stabilized even with actuator faults. Assumption 2 is a standard state-feedback state tracking matching condition, i.e., [4] and [8]. Moreover, Assumptions 1 and 2 represent that there exist $K_x \in \mathbb{R}^{m\times n}$ and $K_r \in \mathbb{R}^{m\times m}$ such that

$$A + B\sigma K_x = A_m\,, \quad B\sigma K_r = B_m\,. \tag{7}$$

Lemma 1 ([17]). For $\lambda_k \in \mathbb{R}$, $k = 1, 2, \ldots, n$ and $0 < p < 1$, the following inequality holds:

$$\left(\sum_{k=1}^{n} |\lambda_k|\right)^p \leq \sum_{k=1}^{n} |\lambda_k|^p\,. \tag{8}$$

Lemma 2 ([18]). Consider the system $\dot{x} = f(x, u)$. If there exists a continuous positive definite function $V(x)$, real numbers $\eta > 0$, $0 < \alpha < 1$ and $0 < \bar{\psi} < \infty$, such that $\dot{V}(x) \leq -\eta V^\alpha(x) + \bar{\psi}$, then the system $\dot{x} = f(x, u)$ is practical finite-time stable. The trajectories of the system can reach the set $\left\{x \middle| V^\alpha(x) \leq \frac{\bar{\psi}}{(1-\iota)\eta}\right\}$ in a finite time $T_r \leq V^{1-\alpha}(0)/\eta\iota(1-\alpha)$, where $0 < \iota < 1$ and $V(0)$ is the initial value of $V(x)$.

III. Control Design And Stability Analysis

In this section, an adaptive finite-time FTC scheme is designed to achieve the desired stability and state tracking properties in the presence of unknown actuator faults including outage, loss of effectiveness and time-varying bias.

The finite-time FTC controller is constructed as

$$v = \hat{K}_x x + \hat{K}_r r_m + k_1 + k_2 + k_3\,, \tag{9}$$

where $\hat{K}_x = [\hat{K}_{x1}, \hat{K}_{x2}, \ldots, \hat{K}_{xm}]^T \in \mathbb{R}^{m\times n}$ and $\hat{K}_r = [\hat{K}_{r1}, \hat{K}_{r2}, \ldots, \hat{K}_{rm}]^T \in \mathbb{R}^{m\times m}$ are the estimates of K_x and K_r, respectively, $k_q \in \mathbb{R}^m$, $q = 1, 2, 3$ will be designed later.

Substituting (2) and (9) into (1) yields that

$$\dot{x} = Ax + B\sigma(\hat{K}_x x + \hat{K}_r r_m + k_1 + k_2 + k_3) + B\rho u^*\,. \tag{10}$$

Denote the tracking error

$$e = x - x_m\,, \tag{11}$$

then together with (4), (7) and (10), the tracking error system can be formulated as

$$\begin{aligned}\dot{e} =& A_m e + B\sigma(\hat{K}_x - K_x)x + B\sigma(\hat{K}_r - K_r)r_m \\ &+ B\sigma(k_1 + k_2 + k_3) + B\rho u^*\,.\end{aligned} \tag{12}$$

Lemma 3 ([6]). For the diagonal matrix σ in (2), there exists a constant $0 < \chi \leq 1$, such that

$$\chi\|e^T PB\|^2 \leq e^T PB\sigma B^T Pe \leq \|e^T PB\|^2\,. \tag{13}$$

Introduce unknown positive constants

$$\xi_1 = \frac{1 - l_1\chi}{\chi}\,, \quad \xi_2 = \frac{\bar{u}}{\chi}\,, \tag{14}$$

where $0 < l_1 < 1$ is a chosen constant.

Then, k_q, $q = 1, 2, 3$ in (9) are designed as

$$k_1 = \frac{B^T Pee^T Qe}{2\|e^T PB\|^2 + 1}, \tag{15}$$

$$k_2 = \begin{cases} -\frac{l_2 B^T Pe(l_1+\hat{\xi}_1)(e^T Pe)^\alpha}{2\|e^T PB\|^2}\,, & \text{if } \|e^T PB\| > 0\,, \\ 0_{m\times 1}\,, & \text{if } \|e^T PB\| = 0\,, \end{cases} \quad (16)$$

$$k_3 = -\frac{\hat{\xi}_2^2 B^T Pe}{\hat{\xi}_2\|B^T Pe\| + l_3}\,, \quad (17)$$

where $\hat{\xi}_1$ and $\hat{\xi}_2$ are the estimates of ξ_1 and ξ_2, respectively, $l_2, l_3 > 0$ and $0 < \alpha < 1$ are chosen constants.

For $i = 1, 2, \dots, m$, the adaptive laws to update $\hat{K}_{xi}$, $\hat{K}_{ri}$, $\hat{\xi}_1$ and $\hat{\xi}_2$ are designed as

$$\dot{\hat{K}}_{xi} = -\Gamma_i(2xe^T Pb_i + \Lambda_i \hat{K}_{xi})\,, \quad (18)$$

$$\dot{\hat{K}}_{ri} = -\Upsilon_i(2r_m e^T Pb_i + \Theta_i \hat{K}_{ri})\,, \quad (19)$$

$$\dot{\hat{\xi}}_1 = c_1(l_2(e^T Pe)^\alpha - \varsigma_1\hat{\xi}_1)\,, \quad (20)$$

$$\dot{\hat{\xi}}_2 = c_2(2\|e^T PB\| - \varsigma_2\hat{\xi}_2)\,, \quad (21)$$

where b_i is the ith column of B, Γ_i, Λ_i, Υ_i, Θ_i, c_1, ς_1, c_2 and ς_2 are chosen positive gains. The initial values of $\hat{\xi}_1$ and $\hat{\xi}_2$ are positive, that is, $\hat{\xi}_1(0) > 0$ and $\hat{\xi}_2(0) > 0$.

Theorem 1. Consider the linear systems (1) with unknown actuator faults (2), suppose that Assumptions 1 and 2 hold, the tracking errors will converge to a small neighborhood of the origin in finite time under the proposed control signal (9) with update laws (18)-(21).

Proof: Construct the Lyapunov function candidate as

$$\begin{aligned} V =& e^T Pe + \frac{1}{2}\sum_{i=1}^m \sigma_i \tilde{K}_{xi}^T \Gamma_i^{-1}\tilde{K}_{xi} + \frac{1}{2}\sum_{i=1}^m \sigma_i \tilde{K}_{ri}^T \Upsilon_i^{-1}\tilde{K}_{ri} \\ &+ \frac{1}{2}c_1^{-1}\chi\tilde{\xi}_1^2 + \frac{1}{2}c_2^{-1}\chi\tilde{\xi}_2^2\,, \end{aligned} \quad (22)$$

where $\tilde{K}_{xi} = \hat{K}_{xi} - K_{xi}$, $\tilde{K}_{ri} = \hat{K}_{ri} - K_{ri}$, $\tilde{\xi}_1 = \hat{\xi}_1 - \xi_1$ and $\tilde{\xi}_2 = \hat{\xi}_2 - \xi_2$.

The time derivative of V is

$$\begin{aligned} \dot{V} =& 2e^T P\Big[A_m e + B\sigma(\tilde{K}_x x + \tilde{K}_r r_m + k_1 + k_2 + k_3) \\ &+ B\rho u^*\Big] + \sum_{i=1}^m \sigma_i \tilde{K}_{xi}^T \Gamma_i^{-1}\dot{\hat{K}}_{xi} + \sum_{i=1}^m \sigma_i \tilde{K}_{ri}^T \Upsilon_i^{-1}\dot{\hat{K}}_{ri} \\ &+ \sum_{j=1}^2 c_j^{-1}\chi\tilde{\xi}_j\dot{\hat{\xi}}_j. \end{aligned} \quad (23)$$

Then, discuss the term $2e^T PB\sigma k_2$ in two cases.

Case 1. If $\|e^T PB\| > 0$, by using Lemma 3, it has

$$\begin{aligned} 2e^T PB\sigma k_2 =& -\frac{2l_2 e^T PB\sigma B^T Pe(l_1 + \hat{\xi}_1)(e^T Pe)^\alpha}{2\|e^T PB\|^2} \\ \le& -\chi l_2(l_1 + \hat{\xi}_1)(e^T Pe)^\alpha\,. \end{aligned} \quad (24)$$

Case 2. If $\|e^T PB\| = 0$, due to the fact of B and PB being full-row rank, in this case $e = 0_{n\times 1}$, hence

$$2e^T PB\sigma k_2 = 0\,. \quad (25)$$

Combining the above two cases results in

$$\begin{aligned} 2e^T PB\sigma k_2 \le& -l_2(\chi l_1 + \chi\hat{\xi}_1)(e^T Pe)^\alpha \\ =& -l_2(1 + \chi\hat{\xi}_1)(e^T Pe)^\alpha\,. \end{aligned} \quad (26)$$

Considering the definition of ξ_1 and ξ_2 in (14), and substituting (3), (5), (15), (17) and (26) into (23), it has

$$\begin{aligned} \dot{V} \le& e^T(A_m^T P + PA_m)e - l_2(e^T Pe)^\alpha - l_2\chi\hat{\xi}_1(e^T Pe)^\alpha \\ &+ 2\|e^T PB\|\|\rho\|\|u^*\| + 2e^T PB\sigma\tilde{K}_x x + 2e^T PB\sigma\tilde{K}_r r_m \\ &- \frac{2\chi\hat{\xi}_2^2\|e^T PB\|^2}{\hat{\xi}_2\|B^T Pe\| + l_3} + \frac{2\|e^T PB\|^2 e^T Qe}{2\|e^T PB\|^2 + 1} + \sum_{i=1}^m \sigma_i \tilde{K}_{xi}^T \\ &\times \Gamma_i^{-1}\dot{\hat{K}}_{xi} + \sum_{i=1}^m \sigma_i \tilde{K}_{ri}^T \Upsilon_i^{-1}\dot{\hat{K}}_{ri} + \sum_{j=1}^2 c_j^{-1}\chi\tilde{\xi}_j\dot{\hat{\xi}}_j \\ \le& - l_2(e^T Pe)^\alpha - l_2\chi\tilde{\xi}_1(e^T Pe)^\alpha + 2\chi\xi_2\|e^T PB\| \\ &+ 2e^T PB\sigma\tilde{K}_x x + 2e^T PB\sigma\tilde{K}_r r_m + \sum_{i=1}^m \sigma_i \tilde{K}_{xi}^T \Gamma_i^{-1}\dot{\hat{K}}_{xi} \\ &+ \sum_{i=1}^m \sigma_i \tilde{K}_{ri}^T \Upsilon_i^{-1}\dot{\hat{K}}_{ri} + \sum_{j=1}^2 c_j^{-1}\chi\tilde{\xi}_j\dot{\hat{\xi}}_j - \frac{2\chi\hat{\xi}_2^2\|e^T PB\|^2}{\hat{\xi}_2\|B^T Pe\| + l_3}. \end{aligned} \quad (27)$$

Substituting adaptive laws (18)–(21) into (27) yields

$$\begin{aligned} \dot{V} \le& - l_2(e^T Pe)^\alpha - \sum_{i=1}^m \sigma_i \tilde{K}_{xi}^T \Lambda_i \hat{K}_{xi} - \sum_{j=1}^2 \varsigma_j\chi\tilde{\xi}_j\hat{\xi}_j \\ &- \sum_{i=1}^m \sigma_i \tilde{K}_{ri}^T \Theta_i \hat{K}_{ri} + \frac{2\chi l_3\hat{\xi}_2\|e^T PB\|}{\hat{\xi}_2\|B^T Pe\| + l_3} \\ \le& - l_2(e^T Pe)^\alpha - \sum_{i=1}^m \sigma_i \tilde{K}_{xi}^T \Lambda_i \hat{K}_{xi} - \sum_{j=1}^2 \varsigma_j\chi\tilde{\xi}_j\hat{\xi}_j \\ &- \sum_{i=1}^m \sigma_i \tilde{K}_{ri}^T \Theta_i \hat{K}_{ri} + 2\chi l_3\,. \end{aligned} \quad (28)$$

Then, considering the term $-\sum_{i=1}^m \sigma_i \tilde{K}_{xi}^T \Lambda_i \hat{K}_{xi}$, for any $\vartheta_i > \frac{1}{2}, i = 1, 2, \dots, m$, it has

$$-\sigma_i \tilde{K}_{xi}^T \Lambda_i \hat{K}_{xi} \le -\frac{2\vartheta_i - 1}{2\vartheta_i}\sigma_i \tilde{K}_{xi}^T \Lambda_i \tilde{K}_{xi} + \frac{\vartheta_i}{2}\sigma_i K_{xi}^T \Lambda_i K_{xi}\,, \quad (29)$$

$$\left(\frac{2\vartheta_i - 1}{2\vartheta_i}\sigma_i \tilde{K}_{xi}^T \Lambda_i \tilde{K}_{xi}\right)^\alpha \le \frac{2\vartheta_i - 1}{2\vartheta_i}\sigma_i \tilde{K}_{xi}^T \Lambda_i \tilde{K}_{xi} + \bar{\alpha}\,, \quad (30)$$

where $\bar{\alpha} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}$.

Similarly, for any $\gamma_i > \frac{1}{2}, i = 1, 2, \dots, m$ and $\varrho_j > \frac{1}{2}, j = 1, 2$, following inequalities can be derived when considering terms $-\sum_{i=1}^m \sigma_i \tilde{K}_{ri}^T \Theta_i \hat{K}_{ri}$ and $-\sum_{j=1}^2 \varsigma_j\chi\tilde{\xi}_j\hat{\xi}_j$:

$$-\sigma_i \tilde{K}_{ri}^T \Theta_i \hat{K}_{ri} \le -\frac{2\gamma_i - 1}{2\gamma_i}\sigma_i \tilde{K}_{ri}^T \Theta_i \tilde{K}_{ri} + \frac{\gamma_i}{2}\sigma_i K_{ri}^T \Theta_i K_{ri}\,, \quad (31)$$

$$\left(\frac{2\gamma_i - 1}{2\gamma_i}\sigma_i \tilde{K}_{ri}^T \Theta_i \tilde{K}_{ri}\right)^\alpha \le \frac{2\gamma_i - 1}{2\gamma_i}\sigma_i \tilde{K}_{ri}^T \Theta_i \tilde{K}_{ri} + \bar{\alpha}\,, \quad (32)$$

$$-\varsigma_j\chi\tilde{\xi}_j\hat{\xi}_j \le -\frac{2\varrho_j-1}{2\varrho_j}\varsigma_j\chi\tilde{\xi}_j^2 + \frac{\varrho_j}{2}\varsigma_j\chi\xi_j^2\,, \tag{33}$$

$$\left(\frac{2\varrho_j-1}{2\varrho_j}\varsigma_j\chi\tilde{\xi}_j^2\right)^\alpha \le \frac{2\varrho_j-1}{2\varrho_j}\varsigma_j\chi\tilde{\xi}_j^2 + \bar{\alpha}\,. \tag{34}$$

From (29)-(34) and using Lemma 1, it has

$$\begin{aligned}
\dot{V} \le & -l_2(e^TPe)^\alpha - \sum_{i=1}^m\left(\frac{2\vartheta_i-1}{2\vartheta_i}\sigma_i\tilde{K}_{xi}^T\Lambda_i\tilde{K}_{xi}\right)^\alpha \\
& - \sum_{i=1}^m\left(\frac{2\gamma_i-1}{2\gamma_i}\sigma_i\tilde{K}_{ri}^T\Theta_i\tilde{K}_{ri}\right)^\alpha - \sum_{j=1}^2\left(\frac{2\varrho_j-1}{2\varrho_j}\varsigma_j\chi\tilde{\xi}_j^2\right)^\alpha \\
& + \sum_{i=1}^m\left(\frac{2\vartheta_i-1}{2\vartheta_i}\sigma_i\tilde{K}_{xi}^T\Lambda_i\tilde{K}_{xi}\right)^\alpha - \sum_{i=1}^m\frac{2\vartheta_i-1}{2\vartheta_i}\sigma_i\tilde{K}_{xi}^T\Lambda_i\tilde{K}_{xi} \\
& + \sum_{i=1}^m\left(\frac{2\gamma_i-1}{2\gamma_i}\sigma_i\tilde{K}_{ri}\Theta_i\tilde{K}_{ri}\right)^\alpha - \sum_{i=1}^m\frac{2\gamma_i-1}{2\gamma_i}\sigma_i\tilde{K}_{ri}^T\Theta_i\tilde{K}_{ri} \\
& + \sum_{j=1}^2\left(\frac{2\varrho_j-1}{2\varrho_j}\varsigma_j\chi\tilde{\xi}_j^2\right)^\alpha - \sum_{j=1}^2\frac{2\varrho_j-1}{2\varrho_j}\varsigma_j\chi\tilde{\xi}_j^2 + 2\chi l_3 \\
& + \sum_{i=1}^m\frac{\vartheta_i}{2}K_{xi}^T\sigma_i\Lambda_iK_{xi} + \sum_{i=1}^m\frac{\gamma_i}{2}K_{ri}^T\sigma_i\Theta_iK_{ri} + \sum_{j=1}^2\frac{\varrho_j}{2}\varsigma_j\chi\xi_j^2 \\
\le & -\sum_{i=1}^m\left(\frac{(2\vartheta_i-1)\Lambda_i\Gamma_i}{\vartheta_i}\right)^\alpha\left(\frac{1}{2}\sigma_i\tilde{K}_{xi}^T\Gamma_i^{-1}\tilde{K}_{xi}\right)^\alpha \\
& - \sum_{i=1}^m\left(\frac{(2\gamma_i-1)\Theta_i\Upsilon_i}{\gamma_i}\right)^\alpha\left(\frac{1}{2}\sigma_i\tilde{K}_{ri}^T\Upsilon_i^{-1}\tilde{K}_{ri}\right)^\alpha \\
& - \sum_{j=1}^2\left(\frac{(2\varrho_j-1)c_j\varsigma_j}{\varrho_j}\right)^\alpha\left(\frac{1}{2}c_j^{-1}\chi\tilde{\xi}_j^2\right)^\alpha - l_2\left(e^TPe\right)^\alpha \\
& + \sum_{i=1}^m\frac{\vartheta_i}{2}K_{xi}^T\sigma_i\Lambda_iK_{xi} + \sum_{i=1}^m\frac{\gamma_i}{2}K_{ri}^T\sigma_i\Theta_iK_{ri} + \sum_{j=1}^2\frac{\varrho_j}{2}\varsigma_j\chi\xi_j^2 \\
& + 2\chi l_3 + (2m+2)\bar{\alpha} \\
\le & -\eta\Bigg[\left(e^TPe\right)^\alpha + \sum_{i=1}^m\left(\frac{1}{2}\sigma_i\tilde{K}_{xi}^T\Gamma_i^{-1}\tilde{K}_{xi}\right)^\alpha \\
& + \sum_{i=1}^m\left(\frac{1}{2}\sigma_i\tilde{K}_{ri}^T\Upsilon_i^{-1}\tilde{K}_{ri}\right)^\alpha + \sum_{j=1}^2\left(\frac{1}{2}c_j^{-1}\chi\tilde{\xi}_j^2\right)^\alpha\Bigg] + \bar{\psi}\,, \\
\le & -\eta V^\alpha + \bar{\psi}\,,
\end{aligned} \tag{35}$$

where

$$\begin{aligned}
\eta = \min\Bigg\{ & l_2, \left(\frac{(2\vartheta_i-1)\Lambda_i\Gamma_i}{\vartheta_i}\right)^\alpha, \left(\frac{(2\gamma_i-1)\Theta_i\Upsilon_i}{\gamma_i}\right)^\alpha, \\
& \left(\frac{(2\varrho_j-1)c_j\varsigma_j}{\varrho_j}\right)^\alpha\Bigg\}\,,
\end{aligned} \tag{36}$$

$$\begin{aligned}
\bar{\psi} = & \sum_{i=1}^m\frac{\vartheta_i}{2}K_{xi}^T\sigma_i\Lambda_iK_{xi} + \sum_{i=1}^m\frac{\gamma_i}{2}K_{ri}^T\sigma_i\Theta_iK_{ri} \\
& + \sum_{j=1}^2\frac{\varrho_j}{2}\varsigma_j\chi\xi_j^2 + 2\chi l_3 + (2m+2)\bar{\alpha}\,.
\end{aligned} \tag{37}$$

Therefore, let

$$T_r = \frac{V^{1-\alpha}(0)}{\eta\iota(1-\alpha)}, \tag{38}$$

according to Lemma 2, for $\forall t \ge T_r$, $V^\alpha \le \bar{\psi}/\eta(1-\iota)$ with $0 < \iota < 1$.

Thus, the proof is completed ■

IV. Simulation Results

In this section, we consider a lateral-directional dynamic model of the F-18 high-angle-of-attack research vehicle system. The linear plant is described by (1) with $x(t) = [\beta, p, r]^T$ and $u(t) = [\delta_{DT}, \delta_{AI}, \delta_{RU}, \delta_{RTV}, \delta_{YTV}]^T$, where β (deg), p (deg /s) and r (deg /s) stand for side-slip angle, roll rate and yaw rate, respectively; δ_{DT} (deg), δ_{AI} (deg), δ_{RU} (deg), δ_{RTV} (deg) and δ_{YTV} (deg) denote the differential tail deflection, aileron deflection, rudder deflection, roll thrust vector deflection and yaw thrust vector deflection, respectively. As in [19], the system and input matrices of system (1) are given as

$$\begin{aligned}
A &= \begin{bmatrix} -0.059 & 0.496 & -0.868 \\ -5.513 & -0.939 & 0.665 \\ 0.068 & 0.026 & -0.104 \end{bmatrix}, \\
B &= \begin{bmatrix} 0.006 & 0.006 & 0.004 & 0.000 & 0.090 \\ 1.879 & 1.328 & 0.029 & 0.675 & 0.217 \\ -0.109 & -0.096 & -0.084 & 0.007 & -2.974 \end{bmatrix}.
\end{aligned} \tag{39}$$

The reference model is chosen as (4) with

$$\begin{aligned}
A_m &= \begin{bmatrix} -0.153 & 0.486 & -0.778 \\ -6.310 & -2.971 & 0.331 \\ 3.139 & 0.199 & -3.065 \end{bmatrix}, \\
B_m &= B\,, \\
r_m &= [0, 0, 0, 0, 2\cos(0.8t)]^T.
\end{aligned} \tag{40}$$

The faults are modeled as

$$\begin{cases} u_1(t) = 2\sin(0.8t), & \text{for } 10 \text{ s} \le t < 35 \text{ s}, \\ u_4(t) = 0.4v_4(t), & \text{for } t \ge 20 \text{ s}, \\ u_i(t) = v_i(t), i = 1,2,3,4,5, & \text{otherwise}. \end{cases} \tag{41}$$

It is noted that $u_1(t) = 2\sin(0.8t)$ is actually a time-varying drift fault, which means that the first actuator is completely loss of control while the control signal changes over time; $u_4(t) = 0.4v_4(t)$ represents that the fourth actuator loses its 60% effectiveness.

The parameters of system are chosen as: $x(0) = [1, 2, 1]^T$, $x_m(0) = [-1, 1, 0.5]^T$, $\Gamma_i = \Upsilon_i = 1$, $\Lambda_i = \Theta_i = 0.01$ for $i = 1, 2, 3, 4, 5$, $c_1 = c_2 = 1$, $\varsigma_1 = \varsigma_2 = 0.01$, $l_1 = 0.01$, $l_2 = 0.1$, $l_3 = 1.5$, $\alpha = 0.8$.

Figure 1 shows the state tracking trajectories, while the actual and designed control signals are shown in Figure 2. It can be seen that 1) tracking performance is guaranteed when the system is fault-free; 2) tracking performance can be recovered after a transient response whenever a new fault occurs or recovers, and the system can tolerate the time-varying bias fault. Therefore, the proposed adaptive controller can achieve the stability and finite-time tracking properties of the linear system even in the presence of actuator faults.

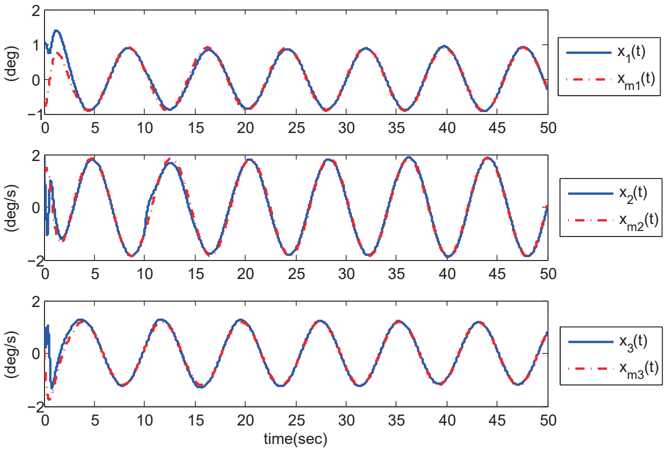

Fig. 1: State tracking trajectories.

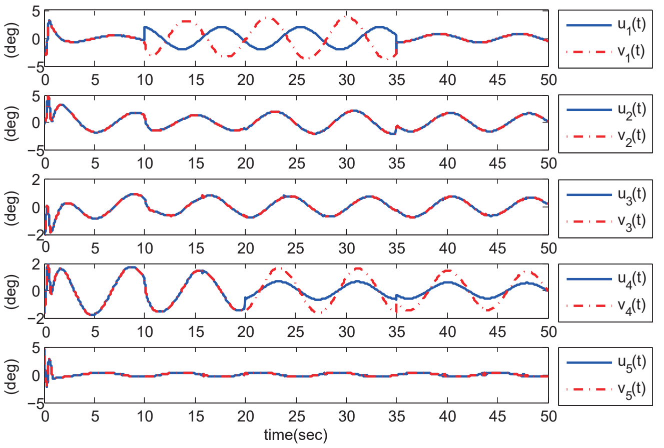

Fig. 2: Control signals.

V. Conclution

In this paper, we study the finite-time FTC scheme for linear systems. Time-varying bias fault, together with outage and loss of effectiveness are taken into consideration. The proposed adaptive controller ensures the finite-time tracking properties of the closed-loop system in the presence of unknown actuator faults. Simulation results on an aircraft system illustrate the performance of the designed control algorithm.

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