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Robust Adaptive Finite-Time Fault-Tolerant Tracking Control for Uncertain Systems

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Abstract: The adaptive finite-time fault-tolerant tracking issue of a class of uncertain systems is taken into consideration in this paper. The unknown time-varying actuator faults and uncertainties of both the system matrix and the input matrix are considered simultaneously when designing the control scheme. The unknown control parameters and gains caused by the actuator faults and model uncertainties are estimated by the designed adaptive laws to construct the fault-tolerant controller. Lyapunov stability analysis shows that the finite-time convergence property of the closed-loop system can be accomplished with great robustness to actuator faults and uncertainties guaranteed. Simulation results of an application to an aircraft model are illustrated to highlight the effectiveness of the proposed methodology.

Key Words: Uncertain systems, Actuator faults, Adaptive control, Fault-tolerant control, Finite-time control.

1 Introduction

Fault-tolerant tracking control is a key technology of uncertain systems, which has been served as an interesting issue in the past decades. The unknown time-varying actuator faults and model uncertainties can cause accidents ranging from minor to catastrophic. Therefore, it is of great significance to design the effective fault-tolerant control (FTC) techniques to accommodate those actuator faults and uncertainties, and to improve system reliability and robustness.

There have been many constructive results in the literature on control of systems with actuator faults, such as sliding mode control [1], model predictive control [2], fault detection and diagnosis-based control [3], and so on. In addition to these methods, adaptive control, which has the capability to adapt to uncertain actuator faults directly, has attracted the attention of researchers over the past decades. In [4], the output tracking issue for multi-input and multi-output nonlinear systems with actuator faults was investigated by designing an adaptive neural control scheme. In [5] and [6], two robust adaptive fault-tolerant controllers were developed, which addressed the state stabilization and tracking problems of uncertain linear systems, respectively. In [7], a robust adaptive control scheme was studied using a control-separation-based linear quadratic design for solving the persistent actuator failure compensation problem.

All the aforementioned references, however, only designed the FTC laws based on the asymptotic stability analysis, which means that the convergence is at best exponential with infinite settling time. It is well known that the finite-time stability can not only guarantee the system trajectories converge to the equilibrium within a finite time, but also provide better robustness and uncertainty attenuate capabilities. Thus, it is more practical to design the finite-time FTC schemes. In [8], a distributed finite-time fault-tolerant formation control law was designed for multi-quadrotor systems by using the consensus protocol and sliding mode algorithm. To handle the fault-tolerant problem for strict-feedback switched system with finite-time convergence property, a neural networks-based adaptive controller was developed in [9]. In [10], a fixed-time observer was designed to estimate the information of actuator faults and model uncertainties, and a finite-time multivariable sliding mode controller was designed for hypersonic vehicles.

Different from actuator faults existing as an unexpected issue, model uncertainty is actually an inevitable problem for the system. For example, the uncertainty in dynamics of the aircraft may occur due to payload variation, fuel consumption, and so on. Many existing researches have studied the control issues of uncertain systems in the fault-free case [11–13]. In [11], the frequency selective learning approach was used for adaptation of the unknown uncertainty in the input matrix, and an adaptive optimal controller was presented to achieve exponential stability of the closed-loop system. An online reinforcement learning approach was developed in [12] for uncertain linear systems with uncertainties entering both the system matrix and the input matrix. In order to solve the FTC problem for linear systems with model uncertainties, several robust fault-tolerant controllers were developed in [5, 6] and our recent work [14], while only the uncertainty of the system matrix was considered. Obviously, a more challenging issue is the FTC under the input matrix uncertainty. In [15], the robust fault-tolerant constrained control problem for a class of uncertain linear systems was concerned, however, the input matrix uncertainty considered was in a special form.

Motivated by the above observations, the authors further study the finite-time FTC problem of a class of uncertain systems. The main contributions are as follows:

- A robust adaptive finite-time FTC scheme is developed for uncertain linear systems with unknown actuator faults including loss of effectiveness, outage, and time-varying bias, which guarantees the finite-time stability of the closed-loop system.
- The unknown time-varying uncertainties of both the system matrix and the input matrix are taken into consideration, which are more general compared to [15] and our recent work [14].

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The remainder of this paper is organized as follows. Section 2 states the problem formulation and main objective of this paper. The design of the adaptive finite-time fault-tolerant controller and stability analysis are presented in Section 3. Simulation results are studied in Section 4, followed by some conclusions drawn in Section 5.

Notations. $\mathbb{R}^n$ refers to the n-dimensional Euclidean space, while $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. $\|\cdot\|$ denotes the Euclidean norm of a vector or matrix. The superscript "T" represents matrix transposition. diag$\{\dots\}$ represents a block-diagonal matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively. I stands for the identity matrix.

2 Preliminaries and Problem Formulation

In this section, the mathematical models of the considered uncertain system and actuator faults are given, and the finite-time FTC problem is formulated.

2.1 System Description

Consider the uncertain system described by

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t)\,, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, m is the number of actuators, A and B are appropriate dimensional constant matrices, respectively; and $\Delta A(t) = BL(t)$ and $\Delta B(t) = BG(t)$ are uncertain time-varying parameter matrices, respectively. The unknown matrix function $L(t)$ and $G(t)$ are bounded, i.e.,

$$\|L(t)\| \le \bar{l}, \quad \|G(t)\| \le \bar{g}\,, \quad (2)$$

where $\bar{l}$ and $\bar{g}$ are unknown constants.

Remark 1. *For a practical dynamic system, such as an aircraft, an active suspension, and a dual-motor driving, the system model is usually uncertain due to payload variation, fuel consumption, etc. Different from [6] and [14], both the uncertainties of the system matrix and the input matrix are taken into consideration in this paper, which is more practical. It is worth pointing out that when $G(t)$ is a diagonal matrix and $(G(t) + I)$ is positive definite, $BG(t)$ can represent the uncertainty of the input matrix modeled in [15], which means that the uncertainty considered in [15] is a special case of this paper.*

2.2 Actuator Fault Model

The actuator fault model is described by

$$u(t) = \sigma(t)v(t) + \rho(t)u^*(t), \quad (3)$$

where $v(t) \in \mathbb{R}^m$ is the designed control signal vector, $u^*(t) \in \mathbb{R}^m$ is the unknown time-varying bias vector, and $\sigma(t) = \text{diag}\{\sigma_1(t), \sigma_2(t), \dots, \sigma_m(t)\}$ and $\rho(t) = \text{diag}\{\rho_1(t), \rho_2(t), \dots, \rho_m(t)\}$ are the unknown fault pattern matrices, which satisfy that piecewise constant $\sigma_i(t) \in [0, 1]$, $\rho_i(t) \in \{0, 1\}$, and $\sigma_i(t)\rho_i(t) = 0$, $i = 1, 2, \dots, m$. The unknown bias fault $u^*(t)$ satisfies the following as:

$$\|u^*(t)\| \le \bar{u}\,, \quad (4)$$

where $\bar{u}$ is an unknown constant.

The considered actuator faults including loss of effectiveness, outage and time-varying bias, which have been summarized in Table 1.

Table 1: Actuator Fault Model

Fault Model	σ_i	ρ_i
Normal	1	0
Loss of Effectiveness	$0 < \sigma_i < 1$	0
Outage	0	0
Time-Varying Bias	0	1

2.3 Control Objective

The reference model is given as

$$\dot{x}_m(t) = A_m x_m(t) + B_m r_m(t)\,, \quad (5)$$

where $x_m(t) \in \mathbb{R}^n$ is the reference state vector, $r_m(t) \in \mathbb{R}^m$ is the reference input vector, A_m and B_m are the reference system matrix and input matrix, respectively, and A_m is Hurwitz, i.e., there exist positive definite matrices $P, Q \in \mathbb{R}^{n\times n}$ such that

$$A_m^T P + P A_m = -Q\,. \quad (6)$$

For the sake of simplicity, the notation (t) for time-dependency is omitted in the subsequent description.

Assumption 1. $B(I + G)\sigma B^T > 0$.

Remark 2. *In order to completely compensate for the effects of actuator faults and system model uncertainties, Assumption 1 is a reasonable requirement, which means that there is sufficient redundancy in the control surfaces and the control direction will not be changed by the input matrix uncertainty. Assumption 1 guarantees that there exist $K_x \in \mathbb{R}^{m\times n}$ and $K_r \in \mathbb{R}^{m\times m}$ such that*

$$A + B\sigma K_x = A_m\,, \quad B\sigma K_r = B_m\,. \quad (7)$$

The control objective here is to design a finite-time fault-tolerant controller v to make the states of system (1) track the states of reference system (5) in a finite time with the effects of unknown time-varying actuator faults and uncertainties of both the system matrix and the input matrix.

2.4 Some Lemmas

Lemma 1 ([16])**.** *For the system $\dot{x} = f(x, u)$, if there exist a continuous function $V(x) > 0$, constants $l > 0$, $0 < d < 1$ and $0 < h < \infty$ such that*

$$\dot{V}(x) \le -lV^d(x) + h, \quad (8)$$

then the system $\dot{x} = f(x, u)$ is practical finite-time stable. The trajectories of the system can reach the set $\left\{x \middle| V^d(x) \le h/((1-\iota)l)\right\}$ in a finite time $T_r \le V^{1-d}(0)/(l\iota(1-d))$, where $0 < \iota < 1$ and $V(0)$ is the initial value of $V(x)$.

Lemma 2 ([5])**.** *There exists a constant $\eta > 0$, such that*

$$\min\{e^T PB\sigma B^T Pe, e^T PB(I+G)\sigma B^T Pe\} \ge \eta\|e^T PB\|^2\,, \quad (9)$$

where e is defined in (10).

3 Main Results

In this section, an adaptive finite-time FTC scheme is developed to achieve the state tracking property in the presence of unknown time-varying actuator faults and uncertainties of both the system matrix and the input matrix.

3.1 Error System

Let

$$e = x - x_m \tag{10}$$

be the tracking error, then, with (1), (3) and (5), the dynamic error system is described as

$$\begin{aligned}\dot{e} =&(A + BL)x + B(I + G)\sigma v \\ &+ B(I + G)\rho u^* - A_m x - B_m r_m\,.\end{aligned} \tag{11}$$

3.2 Adaptive Control Design

Introduce some unknown constants k_j, $j = 1, 2, 3, 4$ as

$$k_1 = \frac{1}{\eta}, \quad k_2 = \frac{(1 + \bar{g})\bar{u}}{\eta}, \quad k_3 = \frac{\bar{l}}{\eta}, \quad k_4 = \frac{\bar{g}}{\eta}. \tag{12}$$

The finite-time fault-tolerant controller is designed as

$$v = v_1 + v_2 + v_3 + v_4 + v_5, \tag{13}$$

with

$$v_1 = \hat{K}_x x + \hat{K}_r r_m, \tag{14}$$

$$v_2 = \begin{cases} -\frac{\mu_1 \hat{k}_1 B^T Pe(e^T Pe)^d}{2\|B^T Pe\|^2}\,, & \text{if } \|B^T Pe\| > 0\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \tag{15}$$

$$v_3 = -\frac{\hat{k}_2^2 B^T Pe}{\hat{k}_2 \|B^T Pe\| + \mu_2}, \tag{16}$$

$$v_4 = -\frac{\hat{k}_3^2 \|x\|^2 B^T Pe}{\hat{k}_3 \|x\| \|B^T Pe\| + \mu_3}, \tag{17}$$

$$v_5 = -\frac{\hat{k}_4^2 \|v_1\|^2 B^T Pe}{\hat{k}_4 \|v_1\| \|B^T Pe\| + \mu_4}, \tag{18}$$

where $\hat{K}_x = [\hat{K}_{x1}, \hat{K}_{x2}, \ldots, \hat{K}_{xm}]^T \in \mathbb{R}^{m\times n}$, $\hat{K}_r = [\hat{K}_{r1}, \hat{K}_{r2}, \ldots, \hat{K}_{rm}]^T \in \mathbb{R}^{m\times m}$ and $\hat{k}_j$, $j = 1, 2, 3, 4$ are the estimates of K_x, K_r and k_j, respectively, $\mu_j > 0$ and $0 < d < 1$ are the chosen constants.

Remark 3. *The controller (13) includes five items: v_1-v_5. v_1 inherits from the traditional model reference adaptive control. v_2 is actually an exponential item related to finite-time control. v_3 compensates the effect of the time-varying bias. v_4 compensates the effect of the system matrix uncertainty. v_5 compensates the effect of the input matrix uncertainty.*

To update the estimates $\hat{K}_x$, $\hat{K}_r$ and k_j, $j = 1, 2, 3, 4$ in controller (13), for $i = 1, 2, \ldots, m$, the adaptive laws are designed as

$$\dot{\hat{K}}_{xi} = -\Gamma_{xi}(2xe^T Pb_i + \Lambda_{xi}\hat{K}_{xi})\,, \tag{19}$$

$$\dot{\hat{K}}_{ri} = -\Gamma_{ri}(2r_m e^T Pb_i + \Lambda_{ri}\hat{K}_{ri})\,, \tag{20}$$

$$\dot{\hat{k}}_1 = c_{11}(\mu_1 (e^T Pe)^d - c_{12}\hat{k}_1)\,, \tag{21}$$

$$\dot{\hat{k}}_2 = c_{21}(2\|B^T Pe\| - c_{22}\hat{k}_2)\,, \tag{22}$$

$$\dot{\hat{k}}_3 = c_{31}(2\|B^T Pe\|\|x\| - c_{32}\hat{k}_3)\,, \tag{23}$$

$$\dot{\hat{k}}_4 = c_{41}(2\|B^T Pe\|\|v_1\| - c_{42}\hat{k}_4)\,, \tag{24}$$

where b_i is the ith column of B, $\Gamma_{xi}, \Lambda_{xi} \in \mathbb{R}^{n\times n}$ and $\Gamma_{ri}, \Lambda_{ri} \in \mathbb{R}^{m\times m}$ are chosen constant diagonal positive definite matrices, and c_{j1} and c_{j2} are chosen positive constants.

3.3 Stability Analysis

Theorem 1. *Consider the system (1) with actuator faults (3) and time-varying uncertainties of both the system matrix and the input matrix, if Assumption 1 is satisfied, the developed control scheme (13) updated by the adaptive laws (19)-(24) can guarantee that the tracking error (10) converges to a small neighborhood of the origin in a finite time.*

Proof. Choose a Lyapunov function candidate as

$$\begin{aligned} V =&e^T Pe + \frac{1}{2}\sum_{i=1}^{m} \sigma_i \big(\tilde{K}_{xi}^T \Gamma_{xi}^{-1} \tilde{K}_{xi} + \tilde{K}_{ri}^T \Gamma_{ri}^{-1} \tilde{K}_{ri}\big) \\ &+ \frac{1}{2}\sum_{j=1}^{4} c_{j1}^{-1} \eta \tilde{k}_j^2\,, \end{aligned} \tag{25}$$

where $\tilde{K}_{xi} = \hat{K}_{xi} - K_{xi}$, $\tilde{K}_{ri} = \hat{K}_{ri} - K_{ri}$, and $\tilde{k}_j = \hat{k}_j - k_j$, $j = 1, 2, 3, 4$.

With (11) and (13), the time derivative of V is

$$\begin{aligned} \dot{V} =&2e^T P\Big[A_m e + B\sigma(\tilde{K}_x x + \tilde{K}_r r_m) + BLx + BG\sigma v_1 \\ &+ B(I + G)\sigma(v_2 + v_3 + v_4 + v_5) + B(I + G)\rho u^*\Big] \\ &+ \sum_{i=1}^{m} \sigma_i \big(\tilde{K}_{xi}^T \Gamma_{xi}^{-1} \dot{\hat{K}}_{xi} + \tilde{K}_{ri}^T \Gamma_{ri}^{-1} \dot{\hat{K}}_{ri}\big) + \sum_{j=1}^{4} c_{j1}^{-1} \eta \tilde{k}_j \dot{\hat{k}}_j. \end{aligned} \tag{26}$$

Case 1. If $\|B^T Pe\| > 0$, it follows that

$$2e^T PB(I + G)\sigma v_2 \le -\mu_1 \eta \hat{k}_1 (e^T Pe)^d. \tag{27}$$

Case 2. If $\|B^T Pe\| = 0$, according to Assumption 1, in this case $e = 0$, one has

$$2e^T PB(I + G)\sigma v_2 = -\mu_1 \eta \hat{k}_1 (e^T Pe)^d = 0. \tag{28}$$

Combining (27) and (28) results in

$$2e^T PB(I + G)\sigma v_2 \le -\mu_1 (e^T Pe)^d - \mu_1 \eta \tilde{k}_1 (e^T Pe)^d. \tag{29}$$

Considering the adaptive laws (19)-(24), it has

$$\begin{aligned} &2e^T PB\sigma(\tilde{K}_x x + \tilde{K}_r r_m) + \sum_{i=1}^{m} \sigma_i \big(\tilde{K}_{xi}^T \Gamma_{xi}^{-1} \dot{\hat{K}}_{xi} \\ &+ \tilde{K}_{ri}^T \Gamma_{ri}^{-1} \dot{\hat{K}}_{ri}\big) = -\sum_{i=1}^{m} \sigma_i \big(\tilde{K}_{xi}^T \Lambda_{xi} \hat{K}_{xi} + \tilde{K}_{ri}^T \Lambda_{ri} \hat{K}_{ri}\big), \\ &2e^T PB(I + G)\sigma v_2 + c_{11}^{-1} \eta \tilde{k}_1 \dot{\hat{k}}_1 \\ &\le -\mu_1 (e^T Pe)^d - \eta c_{12} \tilde{k}_1 \hat{k}_1, \\ &2e^T PB(I + G)\sigma v_3 + 2e^T PB(I + G)\rho u^* + c_{21}^{-1} \eta \tilde{k}_2 \dot{\hat{k}}_2 \\ &\le -\frac{2\eta \hat{k}_2^2 \|B^T Pe\|^2}{\hat{k}_2 \|B^T Pe\| + \mu_2} + 2\eta k_2 \|B^T Pe\| + c_{21}^{-1} \eta \tilde{k}_2 \dot{\hat{k}}_2 \\ &\le 2\eta \mu_2 - \eta c_{22} \tilde{k}_2 \hat{k}_2, \\ &2e^T PB(I + G)\sigma v_4 + 2e^T PBLx + c_{31}^{-1} \eta \tilde{k}_3 \dot{\hat{k}}_3 \\ &\le -\frac{2\eta \hat{k}_3^2 \|B^T Pe\|^2 \|x\|^2}{\hat{k}_3 \|B^T Pe\| \|x\| + \mu_3} + 2\eta k_3 \|B^T Pe\| \|x\| \end{aligned}$$

$$+ c_{31}^{-1}\eta\tilde{k}_3\dot{\hat{k}}_3 \leq 2\eta\mu_3 - \eta c_{32}\tilde{k}_3\hat{k}_3,$$

and

$$2e^T PB(I+G)\sigma v_5 + 2e^T PBGv_1 + c_{41}^{-1}\eta\tilde{k}_4\dot{\hat{k}}_4$$
$$\leq -\frac{2\eta\hat{k}_4^2\|B^T Pe\|^2\|v_1\|^2}{\hat{k}_4\|B^T Pe\|\|v_1\| + \mu_4} + 2\eta k_4\|B^T Pe\|\|v_1\|$$
$$+ c_{41}^{-1}\eta\tilde{k}_4\dot{\hat{k}}_4 \leq 2\eta\mu_4 - \eta c_{42}\tilde{k}_4\hat{k}_4,$$

then it follows that

$$\dot{V} \leq -\mu_1(e^T Pe)^d + 2\eta(\mu_2+\mu_3+\mu_4) - \sum_{j=1}^{4}\eta c_{j2}\tilde{k}_j\hat{k}_j$$
$$- \sum_{i=1}^{m}\sigma_i\big(\tilde{K}_{xi}^T\Lambda_{xi}\hat{K}_{xi} + \tilde{K}_{ri}^T\Lambda_{ri}\hat{K}_{ri}\big) - e^T Qe. \quad (30)$$

For any $\omega_{xi}, \omega_{ri}, \omega_j > 0.5, i = 1, 2, \ldots, m, j = 1, 2, 3, 4,$ one has

$$-\tilde{K}_{xi}^T\hat{K}_{xi} \leq -\frac{2\omega_{xi}-1}{2\omega_{xi}}\tilde{K}_{xi}^T\tilde{K}_{xi} + \frac{\omega_{xi}}{2}K_{xi}^T K_{xi},$$
$$-\tilde{K}_{ri}^T\hat{K}_{ri} \leq -\frac{2\omega_{ri}-1}{2\omega_{ri}}\tilde{K}_{ri}^T\tilde{K}_{ri} + \frac{\omega_{ri}}{2}K_{ri}^T K_{ri},$$
$$-\tilde{k}_j\hat{k}_j \leq -\frac{2\omega_j-1}{2\omega_j}\tilde{k}_j^2 + \frac{\omega_j}{2}k_j^2, \quad (31)$$

and

$$\Big(\frac{2\omega_{xi}-1}{2\omega_{xi}}\sigma_i\tilde{K}_{xi}^T\Lambda_{xi}\tilde{K}_{xi}\Big)^d \leq \frac{2\omega_{xi}-1}{2\omega_{xi}}\sigma_i\tilde{K}_{xi}^T\Lambda_{xi}\tilde{K}_{xi} + \bar{d},$$
$$\Big(\frac{2\omega_{ri}-1}{2\omega_{ri}}\sigma_i\tilde{K}_{ri}^T\Lambda_{ri}\tilde{K}_{ri}\Big)^d \leq \frac{2\omega_{ri}-1}{2\omega_{ri}}\sigma_i\tilde{K}_{ri}^T\Lambda_{ri}\tilde{K}_{ri} + \bar{d},$$
$$\Big(\frac{2\omega_j-1}{2\omega_j}\eta c_{j2}\tilde{k}_j^2\Big)^d \leq \frac{2\omega_j-1}{2\omega_j}\eta c_{j2}\tilde{k}_j^2 + \bar{d}, \quad (32)$$

where $\bar{d} = (1-d)d^{d/(1-d)}$.

Substituting (31) and (32) into (30) yields that

$$\dot{V} \leq -\sum_{i=1}^{m}\left(\frac{2\omega_{xi}-1}{2\omega_{xi}}\sigma_i\tilde{K}_{xi}^T\Lambda_{xi}\tilde{K}_{xi}\right)^d$$
$$-\sum_{i=1}^{m}\left(\frac{2\omega_{ri}-1}{2\omega_{ri}}\sigma_i\tilde{K}_{ri}^T\Lambda_{ri}\tilde{K}_{ri}\right)^d$$
$$-\sum_{j=1}^{4}\left(\frac{2\omega_j-1}{2\omega_j}\eta c_{j2}\tilde{k}_j^2\right)^d - \mu_1(e^T Pe)^d$$
$$+\sum_{i=1}^{m}\frac{\omega_{xi}}{2}\sigma_i K_{xi}^T\Lambda_{xi}K_{xi} + \sum_{i=1}^{m}\frac{\omega_{ri}}{2}\sigma_i K_{ri}^T\Lambda_{ri}K_{ri}$$
$$+\sum_{j=1}^{4}\frac{\omega_j}{2}\eta c_{j2}k_j^2 + 2\eta(\mu_2+\mu_3+\mu_4) + (2m+4)\bar{d}$$
$$\leq -l\left[(e^T Pe)^d + \sum_{i=1}^{m}\left(\frac{1}{2}\sigma_i\tilde{K}_{xi}^T\Gamma_{xi}^{-1}\tilde{K}_{xi}\right)^d + \sum_{i=1}^{m}\left(\frac{1}{2}\right.\right.$$
$$\left.\left.\times\sigma_i\tilde{K}_{ri}^T\Gamma_{ri}^{-1}\tilde{K}_{ri}\right)^d + \sum_{j=1}^{4}\left(\frac{1}{2}c_{j1}^{-1}\eta\tilde{k}_j^2\right)^d\right] + h$$
$$\leq -lV^d + h, \quad (33)$$

where

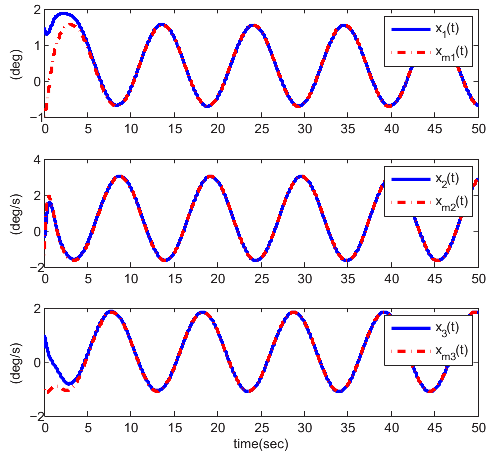

Fig. 1: State tracking trajectories for fault-free case.

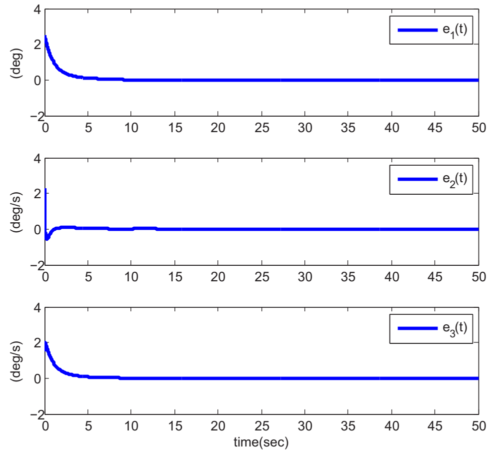

Fig. 2: Tracking errors for fault-free case.

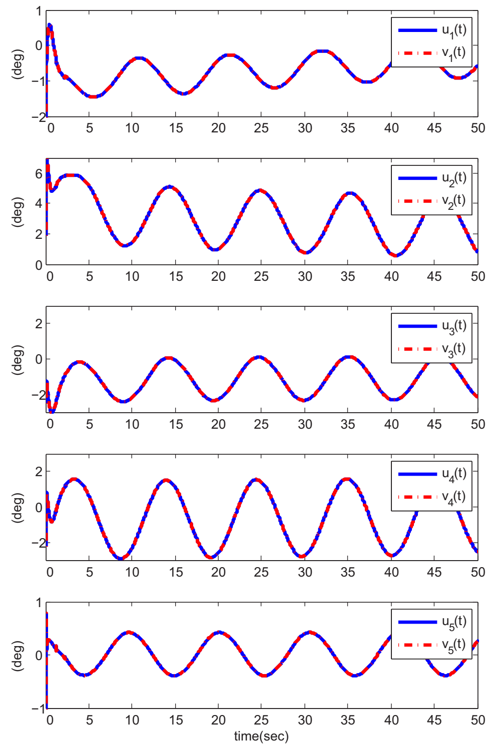

Fig. 3: Control signals for fault-free case.

$$
\begin{aligned}
l = &\min\Bigg\{\mu_1, \left(\frac{(2\omega_{xi}-1)\lambda_{\min}(\Lambda_{xi})}{\omega_{xi}\lambda_{\max}(\Gamma_{xi}^{-1})}\right)^d, \\
&\left(\frac{(2\omega_{ri}-1)\lambda_{\min}(\Lambda_{ri})}{\omega_{ri}\lambda_{\max}(\Gamma_{ri}^{-1})}\right)^d, \left(\frac{(2\omega_j-1)c_{j1}c_{j2}}{\omega_j}\right)^d\Bigg\}, \\
h = &\sum_{i=1}^{m}\frac{\omega_{xi}}{2}\sigma_i K_{xi}^T\Lambda_{xi}K_{xi} + \sum_{i=1}^{m}\frac{\omega_{ri}}{2}\sigma_i K_{ri}^T\Lambda_{ri}K_{ri} \\
&+ \sum_{j=1}^{4}\frac{\omega_j}{2}\eta c_{j2}k_j^2 + 2\eta(\mu_2+\mu_3+\mu_4) + (2m+4)\bar{d}.
\end{aligned}
$$

Therefore, according to Lemma 1, the trajectories of the system can reach the set $\left\{x\middle|V^d(x)\le h/((1-\iota)l)\right\}$ in a finite time $T_r \le V^{1-d}(0)/(l\iota(1-d))$, where $0<\iota<1$ and $V(0)$ is the initial value of $V(x)$.

The proof is completed. □

4 Simulation Study

To verify the effectiveness of the proposed adaptive finite-time FTC scheme, a simulation study on the F-18 lateral-directional dynamic model is given.

4.1 Simulation Condition

The state vector of the model is $x = [\beta, p, r]^T$, where β(deg), p(deg/s) and r(deg/s) represent side-slip angle, roll rate and yaw rate, respectively. The control input variables are differential tail deflection δ_{DT} (deg), aileron deflection δ_{AI} (deg), rudder deflection δ_{RU} (deg), roll thrust vector deflection δ_{RTV} (deg) and yaw thrust vector deflection δ_{YTV} (deg), that is, $u = [\delta_{DT}, \delta_{AI}, \delta_{RU}, \delta_{RTV}, \delta_{YTV}]^T$. The nominal system and input matrices are given as [17]

$$
\begin{aligned}
A &= \begin{bmatrix} -0.059 & 0.496 & -0.868 \\ -5.513 & -0.939 & 0.665 \\ 0.068 & 0.026 & -0.104 \end{bmatrix}, \\
B &= \begin{bmatrix} 0.006 & 0.006 & 0.004 & 0 & 0.090 \\ 1.879 & 1.328 & 0.029 & 0.675 & 0.217 \\ -0.109 & -0.096 & -0.084 & 0.007 & -2.974 \end{bmatrix}.
\end{aligned} \tag{34}
$$

The time-varying matrices L and G are given by

$$
\begin{aligned}
L &= \begin{bmatrix} 0.1\sin(0.1t) & 0 & 0 & 0.1 & 0.1\mathrm{e}^{-2t} \\ 0 & 0.1 & 0 & 0 & -0.1 \\ 0 & 0 & 0.1 & 0.1 & 0 \end{bmatrix}^T, \\
G &= \begin{bmatrix} 0.3 & 0 & 0 & 0 & 0.3\mathrm{e}^{-t} \\ 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}.
\end{aligned} \tag{35}
$$

The reference model is chosen as (5) with

$$
\begin{aligned}
A_m &= \begin{bmatrix} -0.153 & 0.486 & -0.778 \\ -6.310 & -2.971 & 0.331 \\ 3.139 & 0.199 & -3.065 \end{bmatrix}, \\
B_m &= B, \\
r_m &= [0, 3.5, 0, 0, 2.5\sin(0.6t)]^T.
\end{aligned} \tag{36}
$$

The actuator faults are simulated as

$$
\begin{cases} u_2(t) = 0.4v_2(t), & \text{for } 10 \text{ s} \le t < 35 \text{ s}, \\ u_3(t) = 1+\sin(t), & \text{for } t \ge 20 \text{ s}, \\ u_i(t) = v_i(t), & i = 1,2,3,4,5, \text{otherwise}. \end{cases} \tag{37}
$$

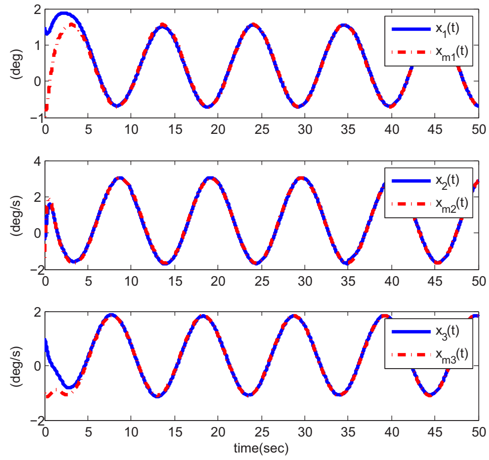

Fig. 4: State tracking trajectories for actuator faults (37).

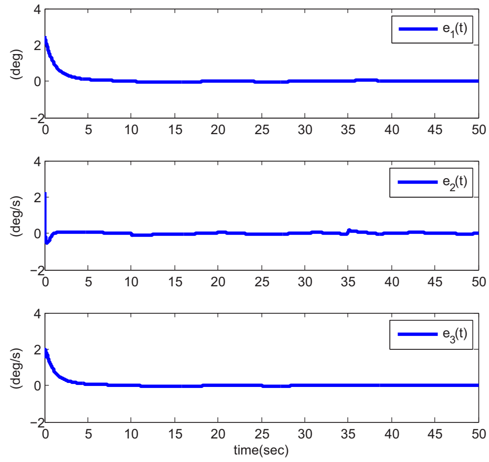

Fig. 5: Tracking errors for actuator faults (37).

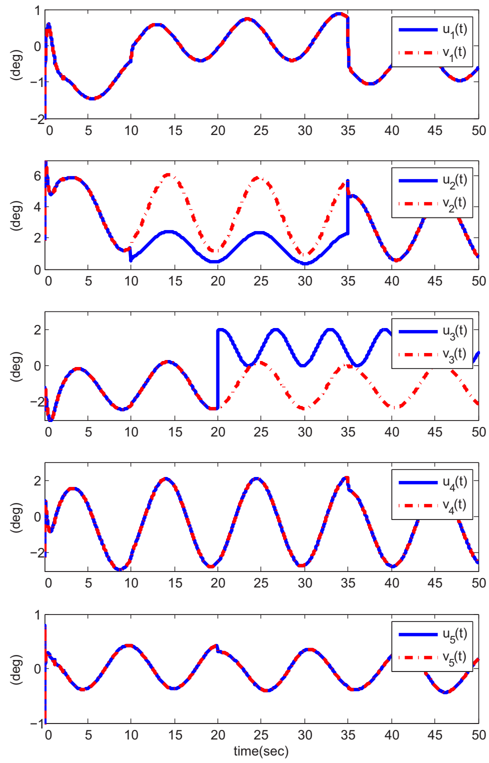

Fig. 6: Control signals for actuator faults (37).

Not that 1) $u_2(t) = 0.4v_2(t)$ represents that the aileron deflection δ_{AI} loses its 60% effectiveness; 2) $u_3(t) = 1+\sin(t)$ denotes that the rudder deflection δ_{RU} completely loses of effectiveness and has a time-varying bias fault.

The initial conditions are $x(0) = [1.5, 1, 0.5]^T$, $x_m(0) = [-1, -1.5, -1]^T$, $\hat{K}_{x1}(0) = [-1, 0, -1]^T$, $\hat{K}_{x2}(0) = [1, -1, 1]^T$, $\hat{K}_{x3}(0) = [-1, -1, 0]^T$, $\hat{K}_{x4}(0) = [0, -1, 0]^T$, $\hat{K}_{x5}(0) = [-1, 0, 1]^T$, $\hat{K}_r(0) = I_5$, and $\hat{k}_j(0) = 1, j = 1, 2, 3, 4$.

The control and adaptation parameters are chosen as $\Gamma_{xi} = 0.1I_3$, $\Lambda_{xi} = 0.01I_3$, $\Gamma_{ri} = 0.1I_5$, and $\Lambda_{ri} = 0.01I_5$ for $i = 1, 2, 3, 4, 5$, $c_{j1} = c_{j2} = 0.01$ for $j = 1, 2, 3, 4$, $\mu_1 = 0.01$, $\mu_2 = \mu_3 = \mu_4 = 5$, $\alpha = 0.8$.

4.2 Simulation Results

Figs.1-3 show the state tracking trajectories, tracking errors and control input signals for fault-free case, respectively, while Figs.4-6 show the state tracking trajectories, tracking errors and control input signals in the presence of unknown actuator faults (37), respectively. It can be seen that a great tracking effect can be achieved in the fault-free case. The tracking error fluctuates in the fault case, but the desired tracking effect can still be achieved. The simulation results show that the proposed adaptive finite-time FTC scheme can achieve the closed-loop stability and finite-time tracking properties both for healthy and faulty system with uncertainties of the system matrix and the input matrix.

5 Conclusion

In this paper, an adaptive finite-time FTC scheme is developed for tracking control of a class of uncertain systems with unknown time-varying actuator faults and uncertainties of both the system matrix and the input matrix. The considered actuator faults including loss of effectiveness, outage and time-varying bias. The unknown control parameters and gains caused by the actuator faults and model uncertainties are estimated by the designed adaptive laws to construct the fault-tolerant controller, which guarantees the closed-loop stability and finite-time tracking property. Finally, the effectiveness of the proposed adaptive control scheme is verified by simulation results.

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