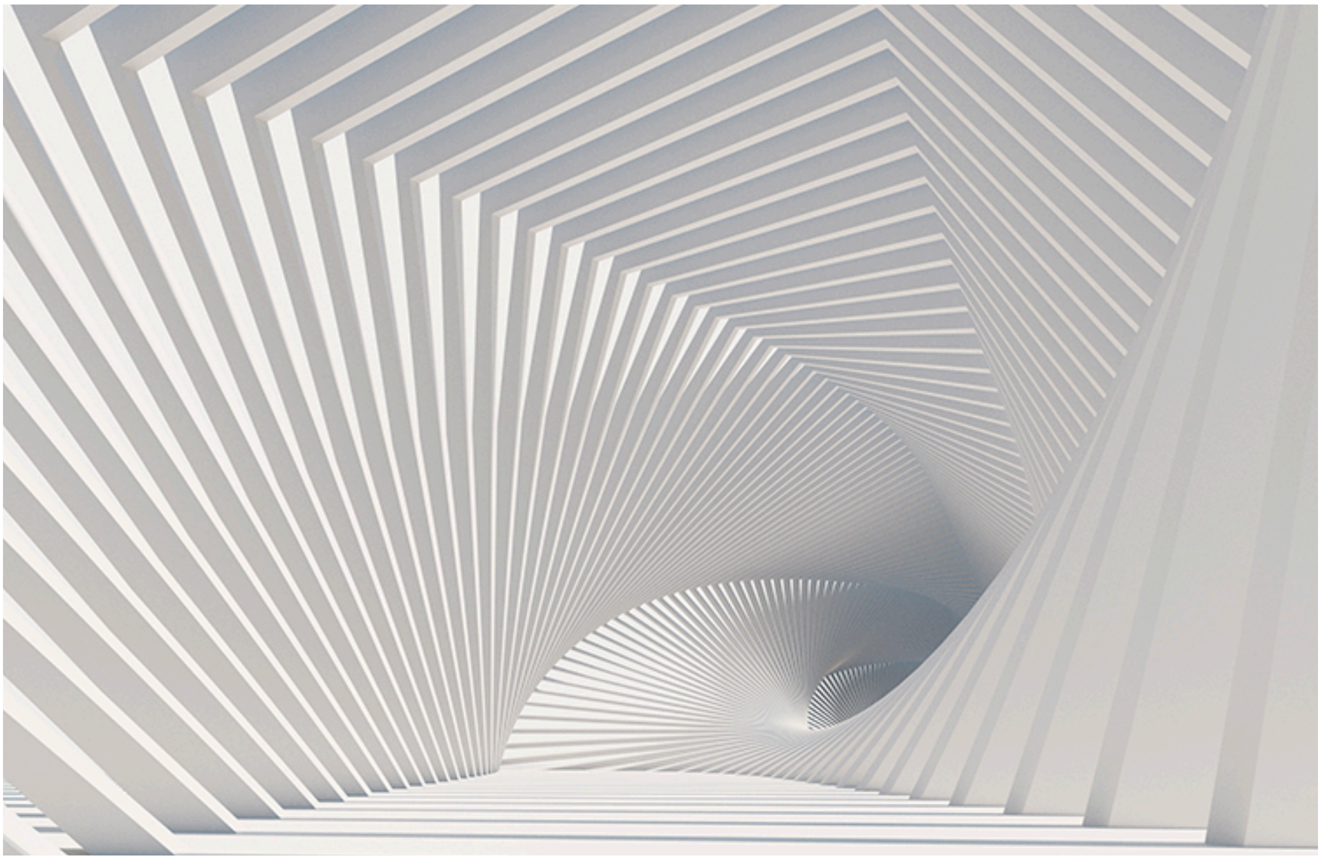
CS294 - DEEP RL



HW1. Behavior Cloning

There are quite a lot of materials to cover, we need to split the work to daily routine.

Found: In terms of the clarity of explanation, the slide merely guides your intuition, but the rigor of the displayed equation is horrible — however, it does the job of referencing people to the correct source. For me, the paper '***A Reduction of Imitation Learning and Structured Prediction to No-Regret Online Learning***' does a much better job at explaining even the basic concept in a flawless way.

The key question, (even the lecture does not explain this well, the slides also not doing good job) is why does

$$\pi_\theta(a \neq \pi^*(s) \,|\, s) \leq \epsilon \,.\, \forall s \in \mathcal{D}_{train}$$

Implies

$$\mathbb{E}[\sum_t c(s_t, a_t)] \leq \epsilon T + (1-\epsilon)(\epsilon(T-1) + (1-\epsilon(\ldots))) = \mathcal{O}(\epsilon T)$$

Solved:

The proof actually resides in a index-3 assemble reference. The supplementary proof of the paper '***Efficient Reductions for Imitation Learning***' (Stephen Ross)

π_θ, π^* — learned policy, demo (expert) policy (assumed to be deterministic !)

$C(s,a)$ — Immediate cost of doing action a in state s

$C_\pi(s) := \int C(s,a)\pi(a\,|\,s)da = \mathbb{E}_{a\sim\pi}[C(s,a)]$—expected immediate cost of performing policy in s

d_π^i — state distribution at time step i if we follow policy π from the initial time step

$d_\pi = \frac{1}{T}\sum_{i=1}^{T} d_\pi^i$ — average state visiting distribution (more often we use a discounted version of this)

$e(s,a) = I(a \neq \pi^*(s))$ — 0-1 loss of executing action a in state s

$e_\pi = \mathbb{E}_{a\sim\pi}(e(s,a))$ just like the definition of expected immediate cost, except for the 0-1 loss case

$J(\pi) = T\mathbb{E}_{s\sim d_\pi}[C_\pi(s)] = \mathbb{E}[\sum_{t=1}^{T} C_\pi(s_t)]$ — expected T-step cost of executing policy π

$\mathcal{R}_{\prod}(\pi) = J(\pi) - \min_{\pi'\in\prod} J(\pi')$ — regret of policy w.r.t the best policy in a particular policy class $\prod$

Theorem 2.1.

Let $\hat{\pi}$ be such that $\mathbb{E}_{s\sim d_{\pi^*}}[e_{\hat{\pi}}(s)] \leq \epsilon$. Then $J(\hat{\pi}) \leq J(\pi^*) + T^2\epsilon$

Key Missing part (took ~ 3 hours to figure out) from the proof is to show that

$$\mathbb{E}_{s\sim d_t}(C_{\hat{\pi}}(s)) \leq \mathbb{E}_{s\sim d_t}(C_{\pi^*}(s)) + e_t \qquad \text{(eq. 1)}$$

Here we note that by definition

$$e_t := \mathbb{E}_{s\sim d_t}[1 - \hat{\pi}(a = \pi^*(s)\,|\,s)]$$

In order to show (eq. 1), It suffices to show

$$C_{\hat{\pi}}(s) \leq C_{\pi^*}(s) + \sum_{a\neq\pi^*(s)} \pi(a\,|\,s)$$

$$\begin{aligned} C_{\hat{\pi}}(s) &= \sum_a C(s,a)\hat{\pi}(a\,|\,s) \\ &\leq \sum_{a\neq\pi^*(s)} \hat{\pi}(a\,|\,s) + \sum_{a=\pi^*(s)} C(a,s)\hat{\pi}(a\,|\,s) \\ &\leq \sum_{a\neq\pi^*(s)} \hat{\pi}(a\,|\,s) + \sum_{a=\pi^*(s)} C(a,s)\mathbb{I}(a = \pi^*(s)) \end{aligned}$$

$$\leq \sum_{a \neq \pi^*(s)} \hat{\pi}(a|s) + \sum_{a = \pi^*(s)} C(a,s)\pi^*(a|s)$$

$$\leq \sum_{a \neq \pi^*(s)} \hat{\pi}(a|s) + C_{\pi^*}(s)$$

End of proof ...

An Alternative argument is if we just use the 1-0 loss, then this inequality looks redundant. Another useful law is

$$\mathbb{P}(\bigcup_i E_i) \leq \sum_i \mathbb{P}(E_i)$$

Note that the lecture note uses 1-0 loss as the cost function, this would implies a much tighter upper-bound exists, and some of the process above is redundant, albeit the inequality still holds.

— Explain of In-Class reasoning on the 1-0 loss case: (Intuition adopted to speeds up proving)
Here is a few *mentality trick* to get there quickly
—

Corollary 1. Assume $\pi_\theta(a \neq \pi^*(s)|s) \leq \epsilon, \forall s \in \mathcal{S}$, we have

$$\mathbb{E}[\sum_t c(s_t, a_t)] = \mathcal{O}(\epsilon T^2)$$

Proof:

We split the expectation into summation of conditional expectation: when the first $t-1$ actions from learned policy are different from expert actions, and the t^{th} action matches with the expert action.

$$\mathbb{E}[\sum_t^T c(s_t, a_t)] = \mathbb{E}[\sum_t^T c(s_t, a_t) | c(s_1, a_1) = 1]\mathbb{P}(c(s_1, a_1) = 1)$$

$$+\mathbb{E}[\sum_{t=1}^T c(s_t, a_t) | c(s_1, a_1) = 0, c(s_2, a_2) = 1]\mathbb{P}(c(s_1, a_1) = 0, c(s_2, a_2) = 1) + \dots$$

Here is a trick, we need only an upper-bound, and we do not want any exact calculation here, so basically

$$\mathbb{P}(\dots, c(s_t, a_t) = 1) \leq \mathbb{P}(c(s_t, a_t) = 1) \leq \epsilon$$

(Marginal distribution always bigger than the joint distribution)

$$\sum_{t=1}^T c(s_t, a_t) \leq T$$

(By definition of cost function, it ranges between [0, 1])

Above two argument tells me the corollary holds. Completed.

Now we look at the HW problem. BTW, the argument provided in-class is wrong and is quite misleading, — although the intuition is probably fine there... Next time we get stuck on a CS course, try an alternative approach as the lecture is not rigid.

Homework Q1.1

Assume random demo policy $\pi*(a|s)$, prove

$$\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}_{p_{\pi*}(s_t)}[\pi_\theta(a_t \neq \pi*(s_t)|s_t)] \leq \epsilon \implies \sum_{s_t}|p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t)| \leq 2T\epsilon$$

Intuition: Closeness in policy leads to closeness in trajectories — average visitation policy closeness leads to linearly-bad bounds on trajectory difference.

(eq.1)

$$\pi_\theta(a_t \neq \pi*(s_t)|s_t) = \sum_{a_t \neq \pi^*(s_t)} \pi_\theta(a_t) = \sum_{a_t}\pi_\theta(a_t|s_t)(1-\pi*(s_t)) \geq \sum_{a_t}(\pi_\theta(a_t|s_t) - \pi*(a_t|s_t))$$

This equations shows how the likelihood of difference between policy relates to prob density diff

$$\mathbb{E}_{p_{\pi^*}(s_t)}[\sum_{a_t}(\pi_\theta(a_t|s_t) - \pi*(a_t|s_t))] \leq \mathbb{E}_{p_{\pi^*}(s_t)}[\pi_\theta(a_t \neq \pi*(s_t)|s_t)] \leq T\epsilon$$

Now we use induction to conclude, since the initial distribution $p(s_1)$ is decided by the environment (and not the policy) the $t = 1$ case naturally holds.

Assume the conclusion holds for t, we will prove the case for $t+1$

$$\sum_{s_{t+1}}|p_{\pi_\theta}(s_{t+1}) - p_{\pi^*}(s_{t+1})| = \sum_{s_{t+1}}|\sum_{s_t,a_t}\left(p_{\pi_\theta}(s_t)\pi_\theta(a_t|s_t)T(s_{t+1}|s_t,a_t) - p_{\pi^*}(s_t)\pi^*(a_t|s_t)T(s_{t+1}|s_t,a_t)\right)|$$

Here we can apply a classic decomposition

$$a_1b_1 - a_2b_2 = (a_1 - a_2)b_1 + (b_1 - b_2)a_2$$

To obtain:

$$p_{\pi_\theta}(s_t)\pi_\theta(a_t|s_t) - p_{\pi^*}(s_t)\pi^*(a_t|s_t) = (p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t))\pi_\theta(a_t|s_t) + p_{\pi^*}(s_t)(\pi_\theta(a_t|s_t) - \pi^*(a_t|s_t))$$

Now we decompose the full equation

$$\sum_{s_{t+1}}|p_{\pi_\theta}(s_{t+1}) - p_{\pi^*}(s_{t+1})| = \sum_{s_{t+1}}|\sum_{s_t,a_t}\left(p_{\pi_\theta}(s_t)\pi_\theta(a_t|s_t) - p_{\pi^*}(s_t)\pi*(a_t|s_t)\right)T(s_{t+1}|s_t,a_t)|$$

$$= \sum_{s_{t+1}}|\sum_{s_t,a_t}\left((p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t))\pi_\theta(a_t|s_t)\right)T(s_{t+1}|s_t,a_t)| \text{ — (term 1)}$$

$$+\sum_{s_{t+1}}|\sum_{s_t,a_t}\left(p_{\pi^*}(s_t)(\pi_\theta(a_t|s_t) - \pi^*(a_t|s_t))\right)T(s_{t+1}|s_t,a_t)| \text{ — (term 2)}$$

For term 1, we have

$$\sum_{s_{t+1}} |\sum_{s_t,a_t} \big((p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t))\pi_\theta(a_t|s_t)\big)T(s_{t+1}|s_t,a_t)| \le \sum_{s_{t+1}} |\sum_{s_t,a_t} |(p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t))|\pi_\theta(a_t|s_t)T(s_{t+1}|s_t,a_t)|$$
$$\le \sum_{s_t,a_t} |(p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t))|\pi_\theta(a_t|s_t) \le \sum_{s_t} |(p_{\pi_\theta}(s_t) - p_{\pi^*}(s_t))| \le T\epsilon$$

For term 2, we have

$$\sum_{s_{t+1}} \sum_{s_t,a_t} \big(p_{\pi^*}(s_t)(\pi_\theta(a_t|s_t) - \pi^*(a_t s_t))\big)T(s_{t+1}|s_t,a_t) \le \sum_{s_t,a_t} p_{\pi^*}(s_t)|\pi_\theta(a_t|s_t) - \pi^*(a_t s_t)| \le T\epsilon$$

As a result, we have

$$\sum_{s_{t+1}} |p_{\pi_\theta}(s_{t+1}) - p_{\pi^*}(s_{t+1})| \le 2\epsilon T$$

End of proof.