

Cold fronts in galaxy clusters I: A case for the large-scale global eigen modes in unmagnetized and weakly magnetized cluster core

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ABSTRACT

Galaxy clusters show large-scale azimuthal X-ray surface brightness fluctuations known as cold fronts. These are overdense (average density jumps $\sim 30\%$ or post-jump density $\sim 130\%$) and have milder discontinuity in pressure. Cold fronts are argued to originate due to sloshing driven by sub-halo passage at close proximity to the cluster center. While this is a viable source of large-scale perturbations, the physical mechanisms that can sustain such density structures (of specific geometry) are not clear. In this work, we explore whether long wavelength thermal instability is an explanation for cold front formation in a cluster core which is perturbed by sub-halos or AGN activity. Using global linear perturbation analysis, we show that internal gravity waves (thermally unstable) can form large-scale three-dimensional spiral structures, akin to observed cold fronts. We explore if the presence of magnetic field (along spherical $\hat{\phi}$) may support such structures (by suppressing small scale Kelvin-Helmholtz modes) or disrupt them (by promoting additional thermal instability). We find that latter happens at shorter wavelengths and only at frequencies above the characteristic buoyancy or Brunt Väisälä frequency ($> N_{BV}$). Our work implies, firstly, that large-scale spirals may be formed and sustained over a long timescale ($> N_{BV}^{-1}$) even in presence of aligned magnetic fields that is otherwise supportive against mixing at the interface. Secondly, short-wavelength (but relatively longer along the field) unstable compressive modes may form within or in the vicinity of such spirals. The instability is an overstable slow wave, and grows in 2D at timescales $\gtrsim 2 - 3$ times longer than the spiral growth timescale (via thermal instability). Thus we claim that this instability cannot destroy the large scale coherence.

Key words: MHD – instabilities – methods: analytical – methods: numerical – galaxies: clusters: intracluster medium

1 INTRODUCTION

High resolution X-ray imaging of the intracluster medium of many galaxy clusters show sharp discontinuities in surface brightness, which translate to discontinuities in density (jumps of $\lesssim 100\%$ and $\sim 30\%$ on an average from outward to CF, which implies a fractional density fluctuation ~ 0.3), metallicity, temperature, and milder discontinuities in pressure. These are referred to as cold fronts (CFs; Markevitch & Vikhlinin 2007). Broadly CFs are classified into merger CFs (contact discontinuities between the ICM of the cluster and subcluster, e.g., Sarkar et al. 2023) and CFs seen near the core (e.g., Naor et al. 2020). In the latter case, thermal pressure discontinuities ($\sim 10 - 20\%$ jumps from CF outward) have been discussed, taking into account the prospect of non-thermal pressure support (implies moderate magnetic field for such a medium). The density contrast is always in the sense that the high density zone sits deeper in the gravitational potential well and, hence, these are considered stable against Rayleigh-Taylor instabilities. The origin of these cooling spirals has been mostly described by interaction of off-center fly-by satellite halos leading to gas sloshing (Dupke et al. 2007; Ascasibar & Markevitch 2006; ZuHone et al. 2013), suggesting the existence

of velocity shear and hence Kelvin-Helmholtz instabilities across the CFs. Other scenarios of formation include shear driven centripetal acceleration (Keshet et al. 2010).

Thermal instability (TI) on a background thermal equilibrium has been discussed widely in the context of interstellar medium (McKee & Ostriker 1977), solar prominences (Xia et al. 2012; Antolin & Froment 2022), accretion disks (Nayakshin et al. 2000), and in the intracluster/circumgalactic medium (McCourt et al. 2012; Sharma et al. 2012; Kunz et al. 2012; Esmerian et al. 2021; Donahue & Voit 2022) to describe the multiphase nature of the gaseous medium. The latter is the most relevant context for CFs. The idea of generating a radiative cooling driven dense phase from a hot diffuse medium, which is sufficiently heated intermittently, is compelling. The problem can be easily abstracted to instabilities and oscillations about an equilibrium if cooling and heating roughly balance one another (although that is not a necessary condition for thermal instability, see Balbus 1986; such an assumption makes the problem analytical to some extent). Even solar prominence has been explained using models of TI (Xia et al. 2012; Antolin & Froment 2022). There are also TI models for dynamical systems like outflows from accretion disk scales (e.g., Waters et al. 2022), outflows at halo scales (e.g., Huang et al. 2022), and cooling flows in clusters (e.g., Nulsen 1986). On the other hand, TI has also been discussed at cosmological scales

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(e.g., [Mandelker et al. 2021](#)). Given the widespread application, it is lucrative to explore if TI can also produce CF geometry (spirals and/or long wavelength in spherical $\hat{\phi}$ direction when line-of-sight is along \hat{z} without loss of generality).

In galaxy cluster cores, isobaric local thermal instability is often evoked to explain H α filaments seen in observations ([Fabian et al. 2001, 2016](#)). The current paradigm is that a cooler, dense phase condenses from the hot intracluster medium (ICM) due to local TI, with a morphology dictated by local magnetic field structure ([Ji et al. 2018](#)). Azimuthal large-scale density contrasts, if present, are expected to be global modes instead of local TI. CFs, described by global thermal instability, must also produce metallicity jumps ([Simionescu et al. 2010](#)). More recently, it is suggested that metals can be advected by spirals from inside and outside the arms ([Naor & Keshet 2020](#)). Notably, advection of metals, and a resultant staircase structure of metallicity profile (due to discontinuities at the location of the mode) have been discussed in isochoric/isobaric thermal instability ([Das et al. 2021](#)). Thus, global TI may exhibit the necessary properties of CFs.

In this work, we show that the large scale spirals are formed in stratified cluster cores due to thermally unstable buoyancy waves. Further we consider the effect of weak magnetic fields (high plasma β , the ratio of thermal to magnetic pressure) since β in cluster deduced from Faraday rotation is large (e.g., [Böhringer et al. 2016](#)). Magnetic fields are expected to be aligned with cold front so that thermal conduction is reduced in the direction perpendicular to the field to preserve the temperature jump seen in observations ([Markevitch et al. 2003](#) although the exact details are missing, e.g., [ZuHone et al. 2013](#)). There is no hint apriori that such a coherent weak field is conducive for or preventive against thermal instability. We analyse if the latter scenario is likely.

In the absence of magnetic field, the compressive modes are expected to propagate the fastest and typically these do not contribute to the thermal instability in hot galaxy cluster cores. This is easy to see from the instability threshold in which the isentropic modes never satisfy the threshold for linear instability ([Field 1965](#)). On the other hand, the internal gravity waves that are easily confined within the core contribute efficiently to non-linear random shear motions which provide the seed density contrasts to isobaric thermal instability. The threshold for latter is extremely easy to reach in cores. Linearly, these waves are unstable in a characteristic thermal instability (cooling) timescale ($\sim t_{\text{TI}}^{-1} \sim t_{\text{cool}}^{-1}$). As a result, often the buoyancy oscillations and thermal instability can be thought to be working in conjunction towards growth as well as saturation of condensed gas. The small-scale (relative to core size) density perturbations grow, saturate, and drop out of the hot phase into the gravitational potential well fast. With idealized magnetic field topology, local periodic box simulations of stratified gas shows elevated rates of growth in condensation (e. g., [Ji et al. 2018](#)) due to magnetic tension preventing buoyancy oscillations. For an initially oscillating and rapidly cooling clump, magnetic tension reduces the oscillation and supports further cooling *in-situ*.

The buoyancy oscillations are also critically dependent on the characteristic length scales perpendicular to the direction of gravity. For large perpendicular length scales, in principle, the saturation of thermal instability can be delayed since the buoyancy timescale increases (frequency $\sim \frac{k_{\perp} N_{\text{BV}}}{k}$, where N_{BV} is a characteristic Brunt-Väisälä frequency for buoyancy oscillations). Since the small-scale growth cannot completely destroy such larger structures (linear growth rate is scale independent), this is a viable mechanism to sustain long filamentary structures of ~ 100 s kpc. In practice, Kelvin-Helmholtz instability takes over fast. But this can be prevented with a magnetic

field aligned with such a filament of thermally unstable gas. This is expected to work perfectly if the alignment and gravity are perpendicular. The question that we address here is: can such a mode describe the cold fronts in clusters? In space plasma (e.g., earth's magnetosphere and the interstellar medium) such a problem has been explored earlier to understand hydromagnetic instability (e.g., [Gold 1959; Parker 1966](#)), albeit the field strengths are expected to be stronger than in cluster cores. For a global model of cluster, such an azimuthal field is not entirely justified unless the field is dragged and wrapped around by multiple minor mergers or AGN jets produce such coherent fields via Biermann battery. For the current purpose, we use an idealized azimuthal magnetic field.

The two main issues following the above discussions are (i) can TI be applied to produce azimuthally extended linear density contrasts at moderate amplitude ($\delta\rho/\rho_0 \lesssim 0.1 - 0.3$)? (ii) Does the presence of weak magnetic fields trigger any other instability to support/destroy such a density contrast if produced? In this first paper, we discuss global linear eigen spectra for thermal instability which produce perfect spirals in hydrodynamic case. We further discuss ideal MHD eigen spectra to assess if that introduces any invasive new instability. In a forthcoming paper, we will discuss a suite of non-linear MHD simulations focussing on the large-scale azimuthal density contrasts in galaxy cluster cores.

We organize this paper as follows. In section 2 we have two parts, section 2.1 presents the method and extension to solve for linear global overstable TI modes with and without magnetic field, and section 2.2 discusses the physical interpretation of magnetized modes. In section 3 we discuss the results with and without magnetic field in the global atmosphere, and the interpretation using local linear analysis that is introduced in section 2.2 first. In Section 4, we discuss and conclude.

2 PHYSICAL MODELS AND METHODS

We explore long wavelength modes by a global linear mode analysis for an intracluster medium with gradients in density, temperature, and pressure. In order to support and interpret the MHD results we also perform a local linear analysis and identify the relevant mode at play. In what follows, we first describe the full magnetohydrodynamic equations together with our assumptions about radiative cooling and heating. We then describe the method of global mode analysis in section 2.1, and the local analysis in section 2.2.

We consider the following ideal MHD conservation equations for the ICM:

$$\frac{D\rho}{Dt} = -\rho\nabla \cdot \mathbf{v}, \quad (1)$$

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla p - g\hat{\mathbf{r}} + \frac{1}{\rho}(\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (2)$$

$$\frac{p}{(\gamma - 1)} \frac{D}{Dt} \left[\ln \left(\frac{p}{\rho^{\gamma}} \right) \right] = -q^-(n, T) + q^+(r, t), \quad (3)$$

$$\frac{D\mathbf{B}}{Dt} = (\mathbf{B} \cdot \nabla)\mathbf{v} - \mathbf{B}(\nabla \cdot \mathbf{v}) \quad (4)$$

where D/Dt is the Lagrangian derivative, and ρ , \mathbf{v} and p are mass density, velocity and pressure. Radiative cooling is described by the term $q^-(n, T) \equiv n_e n_i \Lambda(T)$ is the radiative cooling (where $n_e \equiv \rho/[\mu_e m_p]$ and $n_i \equiv \rho/[\mu_i m_p]$ are electron and ion number densities, respectively; $\mu_e = 1.17$, $\mu_i = 1.32$, and m_p is proton mass, $\Lambda(T)$ is the temperature-dependent cooling function). We include a heating term that, by assumption, balances the radiative cooling in a shell-averaged sense, $q^+(r, t) \equiv \langle q^- \rangle$, and represents AGN feedback

via local turbulent heating (also see section 5.4 in McCourt et al. 2012 for an analysis of this assumption).

Assuming the magnetic field is sufficiently weak that the gas pressure dominates, we assume that background hydrostatic equilibrium implies $d\rho_0/dr = -\rho_0 g$, where a subscript ‘0’ refers to equilibrium quantities and acceleration due to gravity $g \equiv d\Phi/dr$ (Φ is the fixed NFW gravitational potential). The details of the atmosphere and the physical parameters are mentioned in Choudhury & Sharma 2016. These equations are closed by the divergence free condition, $\nabla \cdot \mathbf{B} = 0$.

2.1 Global perturbations

We first write the linearised equations. The perturbations, the background magnetic field orientation and the specific considerations are described below the following linearized equations.

$$\sigma F_\rho = -\frac{1}{r^2} \frac{d}{dr} (r^2 \rho_0 F_r) + l(l+1) \underbrace{\frac{\rho_0 F_\theta}{r}}, \quad (5)$$

$$\begin{aligned} \sigma F_r &= -\frac{1}{\rho_0} \frac{d}{dr} \left[p_0 \left(\frac{F_s}{s_0} + \gamma \frac{F_\rho}{\rho_0} \right) \right] - \frac{g F_\rho}{\rho_0}, \\ &\quad \left[-\frac{1}{\rho_0} \frac{\partial}{\partial r} \left(\frac{B_0}{r} \frac{\partial(r F_A)}{\partial r} \right) + \left(m^2 F_A - \frac{2\partial(r F_A)}{\partial r} \right) \frac{B_0}{\rho_0 r^2} \right] \end{aligned} \quad (6)$$

$$\sigma F_\theta = -\frac{p_0}{r \rho_0} \left(\frac{F_s}{s_0} + \gamma \frac{F_\rho}{\rho_0} \right), \quad (7)$$

$$\left[-\frac{1}{r \rho_0} \left(\frac{B_0}{r} \frac{\partial(r F_A)}{\partial r} \right) \right]$$

$$\begin{aligned} \sigma F_\phi &= -\frac{p_0}{r \rho_0} \left(\frac{F_s}{s_0} + \gamma \frac{F_\rho}{\rho_0} \right) \\ &\quad \left[-\frac{1}{r \rho_0} \left(\frac{B_0}{r} \frac{\partial(r F_A)}{\partial r} \right) + \frac{B_0}{\rho_0 r} \frac{\partial(F_A)}{\partial r} - \frac{F_A}{\rho_0 r} \frac{\partial B_0}{\partial r} \right] \end{aligned} \quad (8)$$

$$\begin{aligned} \sigma F_s &= -\frac{\gamma s_0 N_{BV}^2}{g} F_r \\ &\quad - \frac{s_0}{t_{cool0}} \left[2 \frac{F_\rho}{\rho_0} + \frac{d \ln \Lambda}{d \ln T} \left(\frac{F_s}{s_0} + (\gamma - 1) \frac{F_\rho}{\rho_0} \right) \right] \end{aligned} \quad (9)$$

$$\sigma F_A = -F_r B_0 \quad (10)$$

In this section, we look for the three-dimensional perturbations in a global spherical cluster atmosphere but on the plane of $\theta = \pi/2$ (easily tractable and without loss of generality we basically take a plane with magnetic field and gravity; the equations are first linearized and then we use $\theta = \pi/2$).¹ Note that in the hydrodynamic case, this assumption of $\theta = \pi/2$ is not a necessity in this formalism. The background quantities describing the equilibrium are radial functions (see Choudhury & Sharma 2016) and the perturbations, in general, depend on all coordinates and time as $e^{i\omega t} \delta_{\rho,p,s,v}(r, \theta, \phi)$. Our global linear stability analysis is solved as a linear eigenvalue problem in radius. The radial component of all perturbed quantities (density, velocity, etc.) are expanded in a Chebyshev polynomial basis (Pseudospectral method; see Boyd 2001), and the matrix equation

¹ In the most general formulation of our global linearized equations, for any θ , trigonometric functions of θ appear. The same exercise can be trivially extended for various values of θ . It will reveal the entire 3D structure of the instability. Further, due to this reason, the instability disappears when we use poloidal field. The presence of background field on the plane ($\theta = \frac{\pi}{2}$ condition) is essential to assess the instability that we discuss in ideal MHD.

for eigenvalues and eigenfunctions is solved numerically on a Gauss-Lobatto (GL) grid of size n (sets the number of terms in the basis as well). The GL grid is a mapping of the original grid $[r_{in}, r_{out}]$ to a new variable $-1 \leq \zeta \leq 1$ and all boundary conditions are put at $\zeta = [1, -1]$.² In addition to the boundary conditions used in Choudhury & Sharma 2016 (section 3, last paragraph), we use a boundary condition for F_A as described later (below). The perturbed quantities are written in a spherical harmonic basis in the angular direction as follows,

$$\delta\rho = F_\rho(r) Y_l^m, \delta p = F_p(r) Y_l^m, \delta s = F_s(r) Y_l^m, \delta T = F_T(r) Y_l^m,$$

$$\delta v_r = F_r(r) Y_l^m, \delta v_\theta = F_\theta(r) \frac{\partial Y_l^m(\theta, \phi)}{\partial \theta}, \delta v_\phi = \frac{F_\phi(r)}{\sin \theta} \frac{\partial Y_l^m(\theta, \phi)}{\partial \phi},$$

$$\delta A = F_A(r) Y_l^m$$

where $Y_l^m(\theta, \phi)$ are the spherical harmonics of order (l, m) and F carries the radial dependence. Here, $s = \ln \left(\frac{p}{\rho^\gamma} \right)$ denotes the entropy index governed by Eq. 3 and $N_{BV} = \frac{g}{\gamma} \frac{\partial s}{\partial r}$ is the characteristic Brunt-Väisälä frequency for buoyancy oscillations (also mentioned in the introduction). These forms are obtained by comparing the r, θ, ϕ dependence of various terms in Eqs. 1-4. In hydrodynamics, if we write the equations corresponding to F_ϕ and F_θ , these have identical evolution (related to the pressure gradient term). Hence mathematically, solving for F_θ or F_ϕ in hydrodynamic case is equivalent (note that there are differences in θ and ϕ dependence for the δv_θ and δv_ϕ). In magnetohydrodynamics, this degeneracy of the evolution of the radial part of the angular velocity perturbations is broken by the magnetic field direction and we need to consider both equations in the dynamics. δA is the perturbation of the magnitude of background magnetic vector potential (\hat{A}) that we describe below.

We first perturb the equilibrium and then solve for the perturbed quantities only on the central plane ($\theta = \frac{\pi}{2}$). In eq 5, if $F_\theta = F_\phi$ (the two non-radial directions are indistinguishable at any given radial point in the unmagnetized case), then we can directly use

$$\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial Y_l^m}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 Y_l^m}{\partial \phi^2} = -l(l+1) Y_l^m \quad (11)$$

but if $F_\theta \neq F_\phi$ we can instead utilize another property of spherical harmonics and write,

$$\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial Y_l^m}{\partial \theta} \right) = \left(\frac{m^2}{\sin^2 \theta} - l(l+1) \right) Y_l^m \quad (12)$$

The latter is used in the first equation. In what follows, we describe the terms associated with ideal MHD. Note that we perturb the magnetic vector potential of the guide field in this analysis above to assure divergence-free condition (where $\nabla \times (A \hat{A}) = \mathbf{B}$, $A = A_0(r) + \delta A$). Also, eqns. 5-10 assume the background magnetic field is in \hat{A} ,

$$\nabla \times \mathbf{A}_0 = \mathbf{B}_0 = B_0(r) \hat{A} \quad (13)$$

where $B_0(r) = \sqrt{\frac{8\pi}{\beta}} \sqrt{p_0(r)}$. Due to the perturbation δA , there is perturbation in magnetic field along \hat{r} and $\hat{\phi}$, such that $\delta B_r =$

² This method is associated with application of Gaussian quadrature (integral) in spectral methods to solve eigen-problem and Gauss-Lobatto grid provides efficient quadrature rules. In this method, it is important to assess convergence; in other words, for sufficiently high values of n , only the physical eigenvalues for every n we try, must match at high precision.

$-\frac{1}{r \sin \theta} \frac{\partial \delta A}{\partial \phi}$ and $\delta B_\phi = \frac{1}{r} \frac{\partial(r \delta A)}{\partial r}$. The induction equation in terms of vector potential is,

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times (\nabla \times \mathbf{A}) \quad (14)$$

which can be used to track the evolution of δA . Here we assume Weyl gauge or electric scalar potential to be zero. Note that the right hand side in the above equation is dependent only on the background magnetic field B_0 (in $\mathbf{v} \times B_0 \hat{\phi}$) since additional effects will be non-linear (velocities are linear perturbations). Thus in this system, we simply need to track the perturbation δA (δA_θ) in \mathbf{A}_0 (or the equivalent, \mathbf{A}_θ). However, one limitation is that δA_r , which can feed δB_θ and δB_ϕ via the derivatives $\frac{1}{r \sin \theta} \frac{\partial \delta A_r}{\partial \phi}$ and $-\frac{1}{r} \frac{\partial \delta A_r}{\partial \theta}$ is not evolved in our system. Firstly, this limitation makes the equations analytically tractable. Secondly, this is not problematic for our exploration of long wavelength modes along $\hat{\phi}$ in the $\theta = \pi/2$ plane. To elucidate the consequences of this limitation and why it is not an issue, we write the induction equation for δA_r and the complete $\delta \mathbf{B}$ (including the terms that are ignored in our approach) below:

$$\begin{aligned} \sigma F_{A_r} &= F_\theta B_0 \\ \delta \mathbf{B} &= -\frac{1}{r \sin \theta} \frac{\partial \delta A_\theta}{\partial \phi} \hat{\mathbf{r}} + \frac{1}{r \sin \theta} \frac{\partial \delta A_r}{\partial \phi} \hat{\phi} \\ &\quad + \frac{1}{r} \left(\frac{\partial}{\partial r} (r \delta A_\theta) - \frac{\partial \delta A_r}{\partial \theta} \right) \hat{\phi} \end{aligned} \quad (15)$$

Ignoring δA_r means we assume perturbation in velocity along $\hat{\phi}$ is much smaller than velocity perturbation along $\hat{\mathbf{r}}$ (verified for the relevant global modes we discuss in Figure A2). If we pursue long wavelength (along $\hat{\phi}$) modes in $r - \phi$ plane, in the spherically symmetric background, our limitation of assuming small δA_r compared to δA_θ is not unreasonable. We are choosing modes with velocity perturbations, $F_\theta \ll F_r$ and $\sigma F_{A_r} \approx 0$ (selectively capturing motions on the plane of gravity and magnetic field but retaining the parallel and perpendicular directions to the field). The divergence-free condition of $\delta \mathbf{B}$ remains unchanged. We also reduce the equations easily into analytically tractable form with our limitation.³ In eq 15, we thus include only the terms associated with δA_θ (alternately δA , the only component considered in magnetic potential perturbation).

In eq 2, we expand the Lorentz force term into the gradient of magnetic pressure and magnetic tension as following: $(\nabla \times \mathbf{B}) \times \mathbf{B} = (-\nabla(B^2) + (\mathbf{B} \cdot \nabla)\mathbf{B})$. Here, magnetic pressure ($p_B = B^2$) and magnetic tension [= $(\mathbf{B} \cdot \nabla)\mathbf{B}$] are linearised as $-\nabla(\mathbf{B}_0 \cdot \delta \mathbf{B}) + (\mathbf{B}_0 \cdot \nabla)\delta \mathbf{B} + (\delta \mathbf{B} \cdot \nabla)\mathbf{B}_0$. The perturbations δB_r and δB_ϕ fed by δA (δA_θ in case of field in $\hat{\phi}$ direction) can be expressed in terms of F_A (the radial part of the perturbation δA). Hence the perturbation associated with magnetic pressure is $\delta p_B = B_0 \delta B_\phi = \frac{B_0}{r} \frac{\partial(r \delta A)}{\partial r}$. If we perturb the tension term, we get the following vector (the components of which can again be expressed in terms of derivatives of F_A):

$$\begin{aligned} \left(\frac{B_0}{r \sin \theta} \frac{\partial \delta B_r}{\partial \phi} - \frac{2B_0 \delta B_\phi}{r} \right) \hat{\mathbf{r}} &- \left(\frac{2B_0 \delta B_\phi \cot \theta}{r} \right) \hat{\phi} \\ &+ \left(\frac{B_0}{r \sin \theta} \frac{\partial(\delta B_\phi)}{\partial \phi} + \frac{B_0 \delta B_r}{r} + \delta B_r \frac{\partial B_0}{\partial r} \right) \hat{\phi} \end{aligned}$$

The final expressions in which all MHD terms are written in terms of F_A are written in eqs 5 - 10.

³ On the other hand, if we take $\delta A_r = F_{A_r} \frac{\partial Y_l^m}{\partial \theta}$ in the same form as δv_θ , we end up obtaining higher θ derivatives of spherical harmonics in the associated magnetic pressure perturbations and tension terms. That makes the linear problem far more complex.

Lastly the boundary condition for F_A is $\frac{dF_A}{d\zeta} = 0$ at inner radial point where ζ denotes the mapped grid from the physical grid. The boundary conditions may not be unique to obtain same solutions (see last paragraph of section 3 in Choudhury & Sharma 2016). Moreover, $F_r = F_\phi = F_\rho = 0$ at the outer radial point, $\frac{dF_\theta}{d\zeta} = 0$, $F_s = 0$ at the inner radial location.

2.2 The physical nature of global modes

In this section we present a physical interpretation of the global modes (discussed in next section) in the light of a local analysis and previous literature. We take a simplified local model to compare with the results of global modes. The background (not the perturbations) thermodynamic variables like density, pressure, temperature, and magnetic field are imposed to be constants in this local analysis. We will use the spatial location of the global instability and use the respective t_{cool} , N_{BV} , and similar parameters in this local analysis to understand which mode appears globally. We now describe the formulation of this local analysis.

We reduce the MHD equations to a local form with all the background variables constant except the presence of a local entropy gradient leading to buoyancy oscillations characterized by the oscillation frequency N_{BV} (thus our implication is that there is local entropy gradient but N_{BV} is constant). We consider a simple analytic set-up with gravity, magnetic field, and magnetic vector potential to be, $g\hat{x}$, $B_0\hat{y}$, $A_0x\hat{z}$. We obtain the following local equations for solutions $\propto e^{i\sigma t + ik \cdot x}$ ($\mathbf{k} = k_x\hat{x} + k_y\hat{y}$, such that we explore both the direction of gravity and magnetic field exactly like the plane in global analysis and simultaneously retain the tractability of 2D geometry):

$$\begin{aligned} \sigma \frac{\delta \rho}{\rho_0} &= -i\mathbf{k} \cdot \mathbf{v} \\ \sigma \mathbf{v} &= -iv_t^2 \mathbf{k} \frac{\delta p}{\rho_0} - \frac{i}{\rho_0} \mathbf{k} (\mathbf{B}_0 \cdot (\mathbf{k} \times \delta \mathbf{A})) \\ &\quad + \frac{i}{\rho_0} (\mathbf{B}_0 \cdot \mathbf{k}) (\mathbf{k} \times \delta \mathbf{A}) - g\hat{x} \frac{\delta \rho}{\rho_0} \\ (\sigma + \frac{1}{t_{\text{isc}}} \frac{\delta p}{\rho_0}) &= -\frac{\gamma N_{\text{BV}}^2}{g} \hat{g} \cdot \mathbf{v} + \gamma(\sigma - \frac{1}{t_{\text{isb}}} \frac{\delta \rho}{\rho_0}) \\ \sigma \delta A &= v_x B_0 \end{aligned}$$

where the standard isobaric and isochoric growth/decay timescales are $t_{\text{isb}} = \frac{\gamma t_{\text{cool}}}{2 - \Lambda_T}$, $t_{\text{isc}} = \frac{t_{\text{cool}}}{\Lambda_T}$, $\gamma v_t^2 = c_s^2$ and $\Lambda_T = d \ln \Lambda / d \ln T$. Here, N_{BV} is the Brunt-Väisälä frequency (which is the largest frequency of stable buoyant oscillations), and B_0 is the background magnetic field with $\beta = \frac{8\pi\rho_{\text{th}}}{B_0^2} \gg 1$ so that the background equilibrium is closely approximated by the hydrodynamic case. Note that the second and the third terms in the velocity/moment equation stand for the magnetic pressure $[-\frac{i}{\rho_0} \mathbf{k} (\mathbf{B}_0 \cdot \delta \mathbf{B})]$ and magnetic tension $[\frac{i}{\rho_0} (\mathbf{B}_0 \cdot \mathbf{k}) \delta \mathbf{B}]$. Before delving into the system, we briefly discuss previous exploration of similar magnetospheres.

Ferrière 2001 (Fig. 2) shows that both slow wave and Alfvén wave become overstable in presence of magnetic field due to buoyancy driven instabilities which have diverse names in the literature. Mathematically, modes associated with strictly transverse motions are called type I and those associated with strictly longitudinal motions are called type II interchange modes. The first case physically means a whole flux tube is displaced (or interchanged) and is often considered a generalization of hydrodynamic gravity mode; while in the second case the field line ripples and generates gravity and pressure-driven forces despite line aligned motions of the medium itself. However, motions in different directions are coupled and no

$$k_y = 0.1, k = 1.0, c_s = 1.0$$

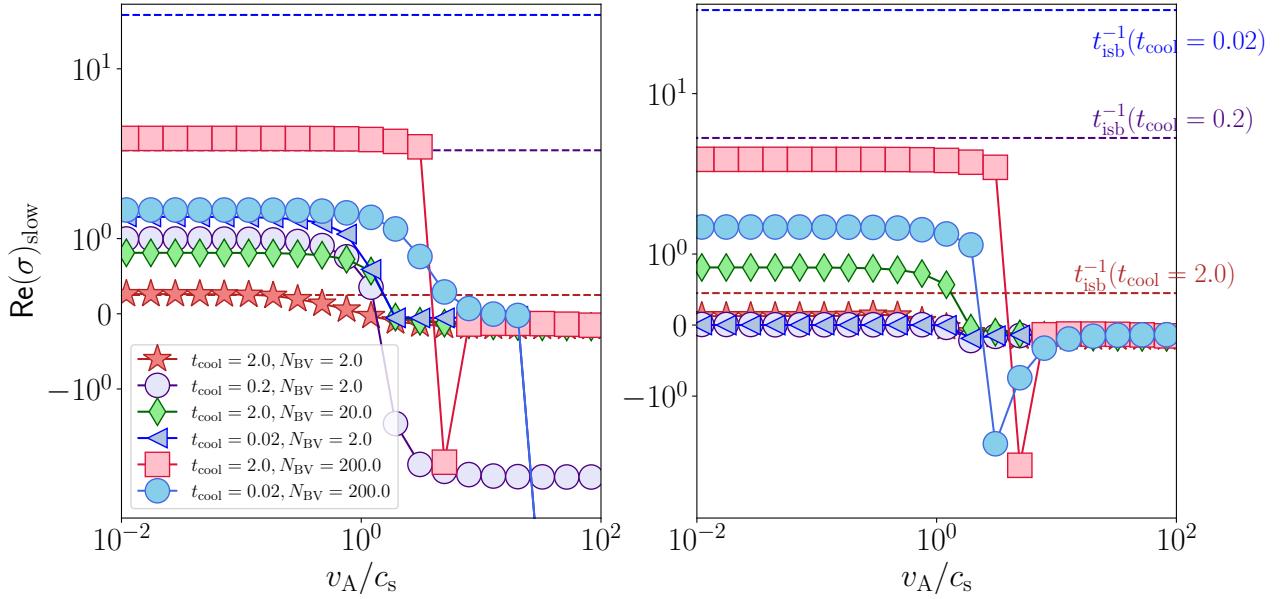


Figure 1. The local growth rates of the slow waves versus v_A/c_s as solved using eq. 23 with arbitrary physical parameters like t_{cool} , N_{BV} , etc (Figure 7 shows with realistic local physical parameters for a cluster core). The purpose is to demonstrate the growth rates relative to t_{isc}^{-1} (in left panel) and t_{isb}^{-1} (in right panel); the three horizontal dashed lines in red, purple, and blue to be compared to red star, purple circle and blue triangle; $t_{\text{isb}}^{-1}/t_{\text{isc}}^{-1}$ is $\frac{2-\Lambda_T}{\gamma\Lambda_T} = \frac{9}{5}$). k_y is chosen to be small to allow for longer modes along the local magnetic field. If $k_y \rightarrow k$, the growth rates are higher. If $k_y \rightarrow 0$, left mode is unstable at smaller rates and the right transitions to stability. At $v_A/c_s \gg 1$ no growth happens.

such strictly type I or II exist in real magnetospheres. Doing away with the formal definitions, Ferrière et al. 1999 analyse a system with horizontal field (including or excluding the field curvature $\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}$), and vertical stratification to understand unstable modes driven by buoyancy in presence/absence of rotation. While we do not consider rotation in the medium, thermal instability (uneven local radiative cooling) can give rise to the coupling terms that support growth of instability, similar to what rotation does.

Since motions in parallel and perpendicular directions (to background field) are used in literature to categorize this class of buoyancy instabilities, we consider the momentum equations closely. The projections of vector momentum equation along the (i) propagation direction, (ii) direction of stratification and gravity, and (iii) background magnetic field direction, reveal the driving forces as following:

$$(\sigma^2 - i\mathbf{k} \cdot \mathbf{g}) \frac{\delta\rho}{\rho_0} = -k^2 \delta p_{\text{tot}} + \mathbf{k} \cdot \delta \mathbf{F}_T \quad (16)$$

$$\frac{\delta\rho}{\rho_0} = \left[\frac{(\sigma + t_{\text{isc}}^{-1}) \frac{\delta\rho}{\rho_0}}{\gamma(\sigma - t_{\text{isb}}^{-1})} + \frac{\mathbf{v} \cdot \nabla s_0}{\gamma(\sigma - t_{\text{isb}}^{-1})} \right]$$

$$\begin{aligned} \left(\sigma + \frac{N_{\text{BV}}^2}{\sigma - t_{\text{isb}}^{-1}} \right) \mathbf{v} \cdot \nabla s_0 &= -i(\mathbf{k} \cdot \nabla s_0) \delta p_{\text{tot}} \\ - \frac{N_{\text{BV}}^2 (\sigma + t_{\text{isc}}^{-1}) \frac{\delta\rho}{\rho_0}}{(\sigma - t_{\text{isb}}^{-1})} &+ i\delta \mathbf{F}_T \cdot \nabla s_0 \end{aligned} \quad (17)$$

$$\sigma \mathbf{v} \cdot \mathbf{B}_0 = -i\mathbf{k} \cdot \mathbf{B}_0 \delta p_{\text{tot}} + i\delta \mathbf{F}_T \cdot \mathbf{B}_0 \quad (18)$$

where $s_0 = \ln \frac{p_0}{\rho_0^\gamma}$, perturbations in magnetic pressure and tension are included as $\delta p_{\text{tot}} = \gamma^{-1} c_s^2 \frac{\delta\rho}{\rho_0} + \frac{\mathbf{B}_0 \cdot \delta \mathbf{B}}{\rho_0}$ and $\delta \mathbf{F}_T = \frac{(\mathbf{B}_0 \cdot \mathbf{k}) \delta \mathbf{B}}{\rho_0}$. The first equation emphasizes that the velocity along the direction

of wave propagation is sourced by change in density contrast (thermal instability) which leads to a magnetic tension as well. Along the direction of stratification, an imbalance between buoyancy, pressure and tension can produce motions. Along the background field (last equation), there is less chance of motions without the intervention of radiative cooling (via the thermal pressure term). In fact, for a pure isobaric case, only motions along gravity are expected leading to a generalized buoyancy overstability (what should also be categorized in literature as type I). We can see this by simply considering $\delta p_{\text{tot}} \approx 0$, $\delta p/p_0 \approx 0$, such that the first two equations give a slight modification of hydrodynamic thermal instability:

$$\sigma^2 - t_{\text{isb}}^{-1} \sigma + N_{\text{BV}}^2 \left(\frac{k_y \delta F_T}{\mathbf{k} \cdot \delta \mathbf{F}_T} \right) = 0 \quad (19)$$

While this appears to be a simple modification, this may have importance consequences for the saturation of local thermal instability in non-linear multidimensional simulation since this reduces the stabilizing impact of buoyancy (or removes effect of entropy gradient, see Choudhury et al. 2019b). For this modified overstability, the exact magnitude of the magnetic field may not be relevant when the wave number is large along stratification or otherwise if the tension along parallel direction is negligible (although that implies large amplitude ripples in transverse direction that eventually may lead to type II). If the field aligned length scale is longer (large k_x), the effect is more prominent. This is equivalent to the picture of flux tubes moving transversely under buoyancy. Unless there is a convection, there is no other linear stability problem. However, in a multidimensional atmosphere, motions in different directions are coupled and radiative cooling aids that in this 2D case.

Now let us assume that uneven non-isobaric radiative cooling

generates a motion along the field (last velocity projection equation). The velocity generated along field is $\mathbf{v} \cdot \mathbf{B}_0$ in terms of which we derive the possible modes assuming $\mathbf{v} \cdot \nabla s_0 = 0$. Note that by this definition, these modes strictly become type II in a formal sense. We now only use the first and last equations to express growth rate as,

$$\sigma(\sigma + t_{\text{isc}}^{-1}) = (\sigma - t_{\text{isb}}^{-1}) \left(\frac{(\mathbf{k} \cdot \delta \mathbf{F}_T - k^2 \delta P_M) c_s^2 (\mathbf{k} \cdot \mathbf{B}_0)}{(\mathbf{k} \cdot \mathbf{g})(\mathbf{v} \cdot \mathbf{B}_0)} \right) \quad (20)$$

If the term in the second bracket in RHS is positive, or in other words the projected magnetic tension along the propagation direction is larger than projected magnetic pressure gradient, an overstability occurs. In order of magnitude, the inverse timescale is $\sim \frac{k^2 v_A \gamma H_p}{\epsilon} \frac{\delta B}{B_0}$ $\propto \frac{k^2 c_s H_p}{\epsilon \sqrt{\beta}} \left[\frac{\delta B}{B_0} / \frac{\delta p}{p_0} \right]$ assuming from the last projected momentum equation, $v_y \sim \frac{k_y c_s^2}{\sigma} \frac{\delta p}{p_0} \sim v_A \left(\frac{c_s^2}{v_A^2} \frac{\delta p}{p_0} \right) = \epsilon v_A$ (for weak magnetic field) with $\epsilon \lesssim 1$ and H_p is the pressure scale height. In fact, in the above analysis, the difference in the magnetic forces (numerator) is $v_A^2 k_x k_y \left(k_y \frac{\delta B_x}{B_0} - k_x \frac{\delta B_y}{B_0} \right)$. Thus $k_y \rightarrow 0$ is stabilized/decaying (unlike type I above). These modes, if growing, must have spatial periodicity along the background magnetic field. Note that the growth may also depend on the angle between the direction of propagation of the wave and the background field. Both types of modes described above are present in our global analysis and . In both global and local analysis we avoid the effect of background field curvature (not associated with the field perturbations) which, in the absence of gravity, produces type II ballooning modes.

After understanding the driving mechanisms of idealized type I or II overstabilities, we write the following complete dispersion relation in our local analysis:

$$\begin{aligned} \sigma^5 + \frac{\sigma^4}{t_{\text{isc}}} &+ \sigma^3 (k^2 c_{\text{ms}}^2 - i k_x c_s^2 N_{\text{BV}}^2 / g - i k_x g) \\ &+ \sigma^2 \left[\frac{-k^2 c_s^2}{t_{\text{isb}}} + \frac{k^2 v_A^2}{t_{\text{isc}}} - \frac{i k_x g}{t_{\text{isc}}} \right] \\ &+ \sigma (k_y^2 c_s^2 N_{\text{BV}}^2 + k^2 k_y^2 c_s^2 v_A^2) - \frac{k^2 k_y^2 c_s^2 v_A^2}{t_{\text{isb}}} = 0 \end{aligned} \quad (21)$$

where $c_{\text{ms}}^2 = c_s^2 + v_A^2$, $v_A^2 / c_s^2 = B_0^2 / \gamma p_0$ denote the fast magnetosonic speed and Alfvén wave speed relative to sound speed respectively. In the absence of gravity and associated stratification, we get a simpler form,

$$\begin{aligned} \sigma^5 + \frac{\sigma^4}{t_{\text{isc}}} &+ \sigma^3 k^2 c_{\text{ms}}^2 + \sigma^2 \left[\frac{-k^2 c_s^2}{t_{\text{isb}}} + \frac{k^2 v_A^2}{t_{\text{isc}}} \right] \\ &+ \sigma k^2 k_y^2 c_s^2 v_A^2 - \frac{k^2 k_y^2 c_s^2 v_A^2}{t_{\text{isb}}} = 0 \end{aligned} \quad (22)$$

The above dispersion relation has been well known in the MHD and in the limit of $\sigma \ll k c_s$ reduces to the cubic equation only. Conventionally, the solutions of the cubic in the regime $v_A \ll c_s$ are obtained as the classic isobaric thermal instability ($\sigma_* \sim t_{\text{isb}}^{-1}$) aka the non-propagating condensation mode and two conjugate purely propagating Alfvén waves ($\sigma_* \sim \pm k_y v_A$).

However we are probably in a regime in which N_{BV} is dominant compared to $\sim k v_A$ since magnetic field is weak while buoyancy is not necessarily weak. The relevant dispersion relation is rather,

$$\begin{aligned} \sigma^5 + \frac{\sigma^4}{t_{\text{isc}}} &+ \sigma^3 k^2 c_{\text{ms}}^2 + \sigma^2 \left[\frac{-k^2 c_s^2}{t_{\text{isb}}} + \frac{k^2 v_A^2}{t_{\text{isc}}} \right] \\ &+ \sigma (k^2 k_y^2 c_s^2 v_A^2 + k_y^2 c_s^2 N_{\text{BV}}^2) - \frac{k^2 k_y^2 c_s^2 v_A^2}{t_{\text{isb}}} = 0 \end{aligned} \quad (23)$$

The slow (and fast) wave frequencies in presence of buoyancy can be quite different. To explain this, we now estimate oscillation frequency in the adiabatic case (no growth/damping due to cooling or heating). In eq 23, if we take slow cooling regime $t_{\text{cool}} \rightarrow \infty$, the conventional magnetoacoustic wave dispersion is modified as,

$$\sigma^4 + \sigma^2 k^2 c_{\text{ms}}^2 + (k^2 k_y^2 c_s^2 v_A^2 + k_y^2 c_s^2 N_{\text{BV}}^2) = 0$$

The fast and slow waves now have the following frequencies ($\sigma = -i\omega$) -

$$\omega^2 = \frac{1}{2} k^2 c_{\text{ms}}^2 \pm \frac{1}{2} k^2 c_{\text{ms}}^2 \sqrt{1 - \frac{4(k_y^2 v_A^2 + k_y^2 N_{\text{BV}}^2 / k^2)}{k^2 c_{\text{ms}}^2}}$$

Particularly for the slow mode it implies that N_{BV} dominates the oscillatory/propagating part if $k c_{\text{ms}} \gg N_{\text{BV}}$. Thus, eq 23 must have a pair of fast magnetosonic waves, a pair of slow magnetosonic waves (both modified by buoyancy oscillation rates) and a growing mode. Figure 1 shows the growth rates of two slow modes from the dispersion relation eq. 23. While uneven radiative cooling can seed parallel motions, cooling rate may not dominate the overstability for large relative N_{BV} . We will see in our result section that the short wavelength overstability appears in global analysis above frequency N_{BV} . This is essentially a mixed mode (motions in different directions are coupled) but has mainly type II characteristics (in effective wave numbers). Non-zero pressure perturbations trigger type II at shorter timescale. However, a type I large-scale mode, which is spiralling like in pure hydrodynamic case, also appears below N_{BV} in global analysis and small k_y in our tailored local analysis described in section 3.2.1.

3 LINEAR EIGEN MODES AND IMPLICATIONS

In this section we present the results from the global mode analysis. The key characteristic to discuss in the following cases is the sustenance of long wavelength modes. First we present globally spiralling overstable buoyancy modes in the hydrodynamic case and then the possible longitudinal modes (along magnetic field) in the ideal MHD case.

In section 3.2, we discuss how the modes modify in presence of a weak magnetic field in the MHD global linear analysis on a 2D plane. Magnetized plasma can form global spirals only below buoyancy oscillation frequency. Above N_{BV} , structures of shorter wavelength form. Hence in cluster cores, both types of modes can be produced. Realistically, fragmented spiral structures or structures of azimuthal long and short wavelengths are expected to coexist in surface brightness, depending on what the background plasma β is at the time of formation of any given azimuthal length scale.

3.1 Large scale spiralling g-modes in hydrodynamic global analysis: perfect candidates for cold fronts

In the hydrodynamic case, we explore modes with $l = 10$ (related to the spherical harmonics and defined in section 2.1) . The value only sets the coherence scale along $\hat{\theta}$ (for all scales along $\hat{\phi}$ since the hydrodynamic equations are independent of m as discussed in section 2.1) and any other value of l is expected to produce similar solutions (tested in part (b) of upper left panels in Figure 2; although growth rate diminishes slightly for smaller l). While the value of global parameter l is not exactly comparable with local k_θ , we can make a qualitative comparison of the two by matching dimension

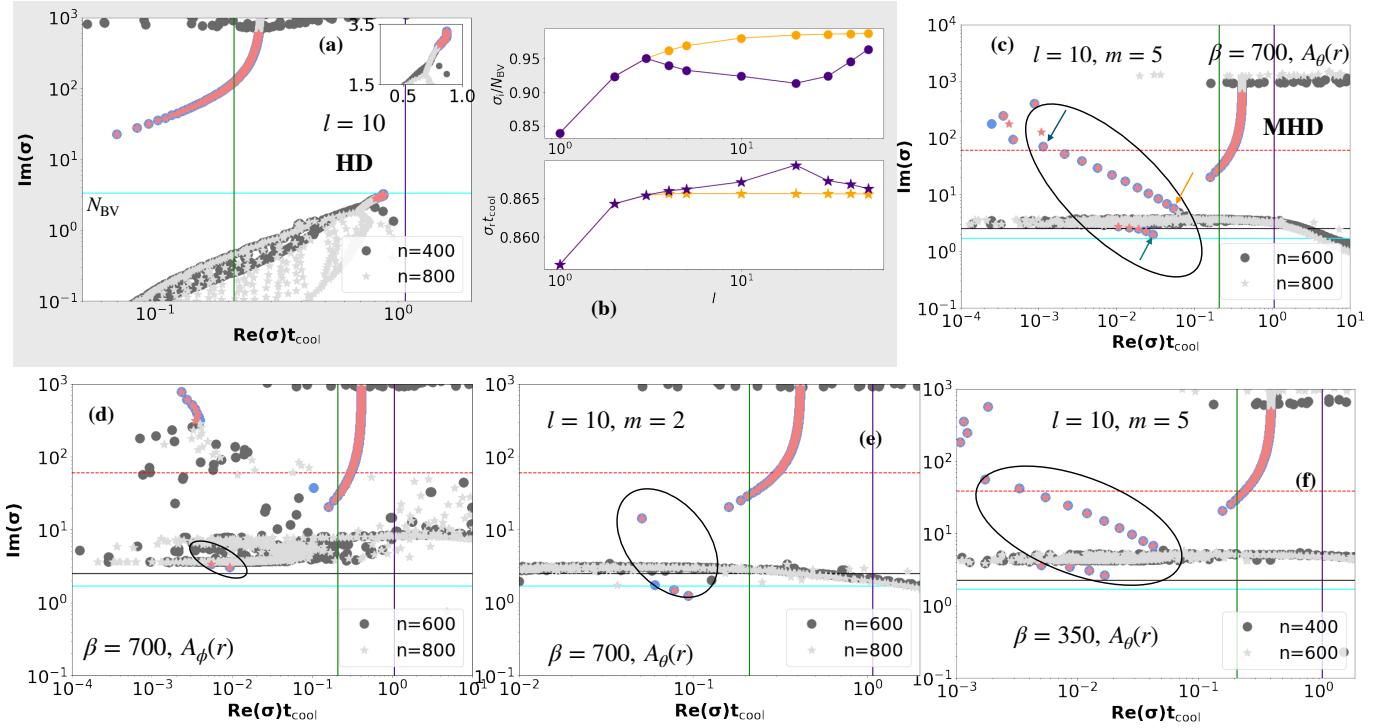


Figure 2. Demonstration of convergence in pseudospectral method and the global growth/oscillation rates in the ICM. In the grey region, (a) shows hydrodynamic overstable buoyancy modes (cyan line corresponds to maximum N_{BV} , the vertical lines show maximum t_{isc}^{-1} in green and maximum t_{isb}^{-1} in purple, which are characteristic buoyancy frequency, isochoric TI growth rate and isobaric TI growth rate). (b) shows the change in growth rate and frequencies for the fastest hydrodynamic mode with l that sets wave number along $\hat{\theta}$. The four remaining panels show the MHD cases with overstable branches circled in black. Horizontal lines show mean of characteristic radial magnetosonic frequency (red dashed; $\sqrt{c_s^2 + v_A^2}/dr$) and alfvén frequency (black solid; v_A/dr) in the global box.

to assess a characteristic length scale, as $\frac{2\pi}{k_\theta} \sim \frac{2\pi r}{l}$, or $k_\theta \sim \frac{l}{r}$ which implies at $r \sim 10$ kpc, an equivalent $k_\theta \sim 1$ or equivalently $\hat{\theta}$ length scale $l_\theta \sim 6$ kpc. For thermally overstable buoyancy modes, we expect mild dependence on local wave number in the oscillation frequency ($\sim \sqrt{\frac{k_\theta^2 + k_\phi^2 N_{BV}}{k}}$) and none for the growth rate ($\sim t_{\text{isb}}^{-1}$). We expect these long wavelength modes to have frequencies below N_{BV} .

In Figure 2, the gray part (background of first two panels in the top row) shows the converged eigenvalues for a hydrodynamic galaxy cluster in the complex plane (*left*) and the fastest mode's growth and oscillation rate with l (*right*). The former has convergence shown in red and blue colors while all the eigenvalues are shown in light and dark gray. The x-axis and y-axis in the left panel contains the absolute values and hence oppositely propagating and damping modes are also included. The modes near growth rate $\sigma t_{\text{cool}} \sim 1$ (also shown in the inset) are the isobaric overstable buoyancy (see section 3.2 and APPENDIX A in Choudhury & Sharma 2016) since the values are below the cyan line signifying maximum Brunt-Väisälä frequency. Both the growth rates and oscillation frequencies weakly depend on l for the fastest mode (confined at small radii). The properties of the fastest mode depends on the background environment, which varies radially. For sufficiently small l , this variation (gradients of density, pressure, temperature, etc) must impact the mode.

The eigenvalues correspond to azimuthally small and large scale modes depending on the value of m . For each eigenvalue (given l), there are $2l+1$ eigenmodes. For small m , the mode is long wavelength in ϕ direction. In Figure 3, the left upper panel (with background in

gray) shows a mode with $l = 10, m = 2$. It is a spiral mode. The lower panel (again with gray background) shows this mode integrated along a range of line-of-sights (LOS). The mode survives in each LOS. In fact, along \hat{z} , the spirals become prominent (third plot in lower panel). These overstable g-modes (buoyancy oscillations) are ideal candidates that can form large scale spiralling cold fronts seen in the core of Perseus core. In practice, the large spirals will emerge in presence of a moderately large scale perturber, e.g., sub-structure passage or any stirring event like a nearly isotropic, gentle AGN feedback as is often expected in relaxed cluster cores. Since these modes are growing in density, the mass loss due to any mixing at the interface with hot medium can be replenished. However, magnetic field along cold fronts is usually believed to be preventing mixing. In the next section, we explore if presence of aligned field destroys the instability itself.

3.2 Effects of weak magnetization

We now explore whether magnetic field supports long wavelength along $\hat{\phi}$. We take a range of plasma β and l, m to obtain the spectra of magnetized modes.

In MHD, both l and m are important parameters entering into the coupled equations. Thus, $k_\theta \sim \frac{l}{r}$ and $k_\phi \sim \frac{2\pi}{2\pi r/m} \sim \frac{m}{r}$. For $m = 2, 5$ at 10 kpc, $k_\phi = 0.2, 0.5$. This also implies the length scales of fluctuations we pick up are $l_\theta \sim 6$ kpc and $l_\phi \sim 12$ kpc (at $k_\phi = 0.5$) or larger in the $r - \phi$ plane (while keeping $l = 10$). Thus these parameters may lead to sufficiently long wavelength along $\hat{\phi}$,

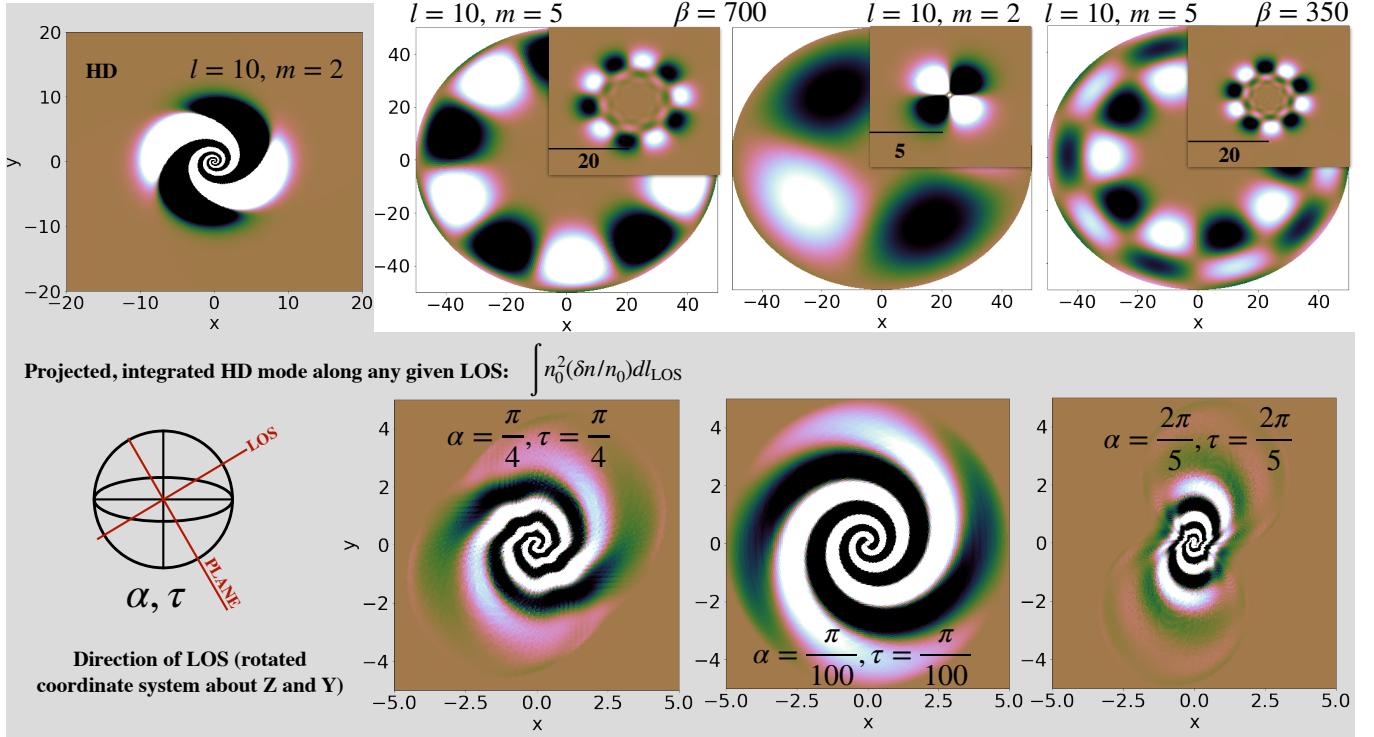


Figure 3. The eigenmodes (in global density perturbation) for hydrodynamic (grey) and MHD cases. In the former case, we show the integrated perturbations in surface brightness (assuming it is dominated by density) in the lower row. The MHD cases (for same l) are presented in three panels with multiple m and β . Both the *domain* modes (inset) and the *reflected* mode at the boundary are shown, the former being identified as the slow compressive mode. There are no real buoyancy spiral/breathing modes at these parameters but see Figure 5 and 6 for lower l .

relative to that in the radial direction. We also carry out searches with smaller l, m for longer wavelength modes at high β . The latter is of type I while the former should be of type II. In what follows, we describe our exploration step-by-step.

Firstly, we find that a wider range of modes are overstable when β is a few hundreds (e.g., $\beta = 350, 700$), but at lower growth rates than t_{isb}^{-1} by a factor of $\sim 0.05 - 0.1$ at $l = 10, m = 5$ (see panels in Figure 2 without gray backgrounds). From our description in section 2, we know that modes with significant velocity perpendicular to the $r - \phi$ plane are not captured in this set-up. Thus transverse waves along $\hat{\theta}$, if unstable, are unavailable in this global analysis. In a 3D realistic simulation, a larger range of modes can be triggered due to this reason and the saturation properties may depend on this factor.

With smaller β , the growth rate tends to decrease slightly (expected from Figure 1 as the growth reduces for stronger field). In Figure 3, we show the two fastest modes for each case (indicated by the green and yellow arrows in the upper right panel in Figure 2) in the upper row. There is a mode close to the outer boundary (marked by green arrow in Figure 2) that is probably a reflection rather than a physical mode. Physically, the mode inside the domain is a robust overstable mode. Note that the physical location may vary for this mode (unlike the one closer to outer boundary). We define this to be *domain* mode in our system. In Figure 4 (left panel), we show a higher order slowly growing *domain* mode (marked by a blue arrow in upper right panel in Figure 2). This is a volume-filling mode but short azimuthal length scales. We understand two characteristics of these new overstable modes so far: the growth rates are typically small, and it may decrease with β (as indicated by the slow wave overstability

in section 2.2). Further, these modes appear to be smaller in length scales than spirals (similar to type II). This can naively indicate support towards small-scale growth and hence turbulence. We carry out a more comprehensive analysis in what follows.

If we keep increasing β , the *domain* modes do not disappear. However, at $\beta = 2 \times 10^5$, we find that a fraction of the hydrodynamic overstable buoyancy modes reappear (see Figure A1). The latter branch is identifiable by the portion approaching N_{BV} (cyan horizontal line). A branch of the overstable *domain* modes still persists at higher oscillation rates. If we take the fastest growing *domain* mode we find that the radial location is closer to the center (last panel Figure A2). This means that the physical scales of the fluctuations are smaller. Basically, the fastest growth happens at smaller scales at high $\beta >> 1$. From the local analysis, the growth rate of type II (discussed after eq. 20) is $\Gamma \sim \frac{k^2 H_{\text{pc}, \delta_{\text{pB}}}}{\beta^{3/2}}$ where δ_{pB} is the ratio of relative pressure fluctuation to relative magnetic field amplitude fluctuation at the radial location. Hence $\Gamma_1^{1/2} \beta_1^{3/4} \delta_{\text{pB},1}^{1/2} / \Gamma_2^{1/2} \beta_2^{3/4} \delta_{\text{pB},2}^{1/2} \propto k_1/k_2 \propto r_2/r_1$ where r denotes radial location. If increasing numbers denote low to high β , the radial location for higher β is expected to be deeper inside the core or in other words, the fastest modes have the length scale conducive to growth. The radial location for this case is indeed at ~ 10 times smaller radius (first and third panel of Figure A2) which matches if $\Gamma_1 \approx \Gamma_2$. But at lower radii, another possible boosting factor for growth is also the higher N_{BV} in the global atmosphere (see Figure 1). This effect of N_{BV} vanishes when we compute the above growth rate (also from eq. 20) for strictly type II modes while in reality faster buoyancy oscillations clearly enhance the growth of the slow mode if we solve the complete dispersion relation. Physi-

cally, stronger buoyancy oscillations (high N_{BV}) also lead to some parallel (to field) motions and hence may cause type II. Thus there is difference in growth rates in the two cases, namely, at high β the fastest growth rate increases. As a result the type II still appears at a sufficiently large radius visible within our domain. On the other hand, the reappearing buoyancy mode transits to the spiralling hydrodynamic eigenmode (right panel of Figure 4). The latter is a test for the transition to hydrodynamic case.

We further use smaller values of l and m at $\beta = 700, 350$ and a large l at $\beta = 150$ in Figure 5. The purpose of the former exploration is to find if global spirals exist at the largest scales in weakly magnetized medium. For the first two cases, we find such azimuthal structures (Figure 6) while for the third case we mostly find all high-frequency unstable slow modes. In section 2.2, we discuss the two broad types of modes and the above exploration clearly extracts type I at large β . For a combination of large $[l, m, \beta]$, retrieving the isobaric mode is difficult. We cannot conclusively determine strongly magnetized cases since our background equilibrium is hydrodynamic (gravity and pressure gradient are the strongest forces) and the effect of magnetic fields may interfere with the consistency of background equilibrium. All we can conclude is that at a few hundred β , the spirals/symmetric-spherical modes (type I) may exist along with the type II. We present examples in Figures 5 (top and bottom in the left panel) and Figure 6 of the largest scales obtained for the aforementioned l, m combinations, noting that these have higher growth rates (approaching isobaric rates).

In order to understand all the mentioned characteristics of *domain* modes that we see in global cluster atmosphere, we investigate the local properties of waves and instabilities now. This helps us to confirm the nature of the overstability and its local dependence on scales (wave number in the local dispersion relation). While local and global behavior may not match precisely (due to absence and presence of gradients of background atmosphere), we claim that the identification of the relevant mode is valid. Further, we will assess from this analysis the effect of the overstabilities on the disruption of the spiral.

3.2.1 Interpretation of the global linear overstable modes using local analysis

In order to interpret our *domain* modes, we take the approximate location of the fastest *domain* mode in the case with $\beta = 700$ from global analysis (Figure 2) which is ~ 10 kpc. We retrieve the physical parameters (e.g., t_{cool} , N_{BV} , v_A , c_s) at the radial location, plug those in the dispersion relation described in section 2.2 (eq. 23). Thus we are able to see the consequence of coupling between magnetoacoustic waves and buoyancy in presence of thermal instability. Figure 7 shows the growth rate (upper panel) and frequencies of oscillations (lower panel). The modes are ordered by the highest to lowest frequencies from left to right (we do not show the second fast mode, identical but oppositely directed to the first). The y -axis is normalized as Figure 2 and the $k_y \in [0, k]$. The slow modes show growth (middle panels) and differ in characteristics at sufficiently large scales (small k_y) depending on the direction of the wave along (positive frequencies) or against (negative frequencies) the background field. In the second slow mode, we have extended the range of k_y to test the behavior at larger values. The other mode in the slow pair behaves similarly at

high k_y . The growth rate is the highest at $k_y \sqrt{\frac{N_{BV}^2}{k^2} + v_A^2 t_{\text{cool}}} = 2\pi$ or in other words at length scale covered by slow mode in a cooling time. This is intuitively clear as the mode can grow only if it doesn't propagate fast. Using the local dispersion, we tested the behavior

of these modes (at the same radial location of the global mode) by increasing v_A by several factors while keeping other parameters constant. We note that the slow mode growth gradually must disappear for stronger magnetic field. The peak growth shifts to smaller k and lower growth rate by a few factors at $\sim 40v_A$, and at $\sim 1000v_A$. The fifth mode is the only dominant mode with small growth rate at large v_A . Note that we cannot test such strongly magnetized regime in global context since we need to modify the background global equilibrium to test low β regimes.

In order to interpret the global modes, the local analysis provides our guiding principles. The high-frequency overstability that we find in the global analysis (*domain* modes), must be the overstable slow modes that oscillate at frequencies $\gtrsim N_{BV}$ (as is evident from the local dispersion relation and global analysis). In Figure 7, there is evidence that this compressive overstability has a strong wavenumber dependence. In the third column we show a larger range for k_y . The oscillation frequencies in second and third are identical except the direction of the wave. The growth rate varies for the slow modes in opposite directions (see discussion in section 2.2 about wave number dependence in local analysis). In the global analysis, overstable modes arise in cases with several combinations of $[l, m]$. A large-scale spiral only occurs when the oscillation frequency is below the maximum N_{BV} (and also below the local values of N_{BV}). To be precise, above N_{BV} the *domain* modes are overstable slow modes (type II) and below N_{BV} these are quite similar to isobaric buoyancy modes (type I). From local and global analysis, it is clear that large-scale buoyancy modes may not be abundant in magnetized medium. Since the ICM is weakly magnetized, we claim that type I azimuthal/spiral modes form over a cooling timescale as expected, along with shorter azimuthal wavelength type II modes depending on the spatial distribution of β . A global picture is conceptualized with the summary image in Figure 8.

The overstable slow modes (*domain* modes) are growing only in a narrow range of scales locally (the length scale a slow mode crosses in t_{cool}). Further, the peak of type II is prominent if β is high (as tested in the local analysis; but type I also reappears at large growth rates as shown in Figure A1). Figure 5 (right panel) also shows $\beta = 150$ case with all overstable slow modes above N_{BV} and one reflected mode at the lowest frequency. This implies at small radial scales and large azimuthal scales, the slow mode grows slowly. Thus regions, with ordered strong and weak background magnetic fields side by side, are predicted to develop fast growing spirals and slowly growing smaller azimuthal scales. On the other hand, at same β (high), depending on frequencies and length scales, both spirals and smaller azimuthal scales can coexist.

4 DISCUSSIONS AND CONCLUSIONS

In this work, we propose that a wide range (in azimuthal scales, growth rates, radial location, etc) of thermally unstable modes in unmagnetized and weakly magnetized plasma can simultaneously explain the presence of large scale spiral cold fronts (type I) and small scale sub-structures (type II) in and around these. This idea is closely aligned with asteroseismology (e.g., Brown & Gilliland 1994) in which an inverted problem is considered from solar/stellar normal modes to estimate the physical conditions in the core of such objects. Multiphase and possibly filamentary plasma is present in the solar atmosphere analogous to the cluster cores (for example, discussed in Choudhury 2023). Here we are assessing if eigen modes may exist in the ICM at sufficiently large scales. Perseus is an example of galaxy cluster in which such a mode analysis can be explored using future

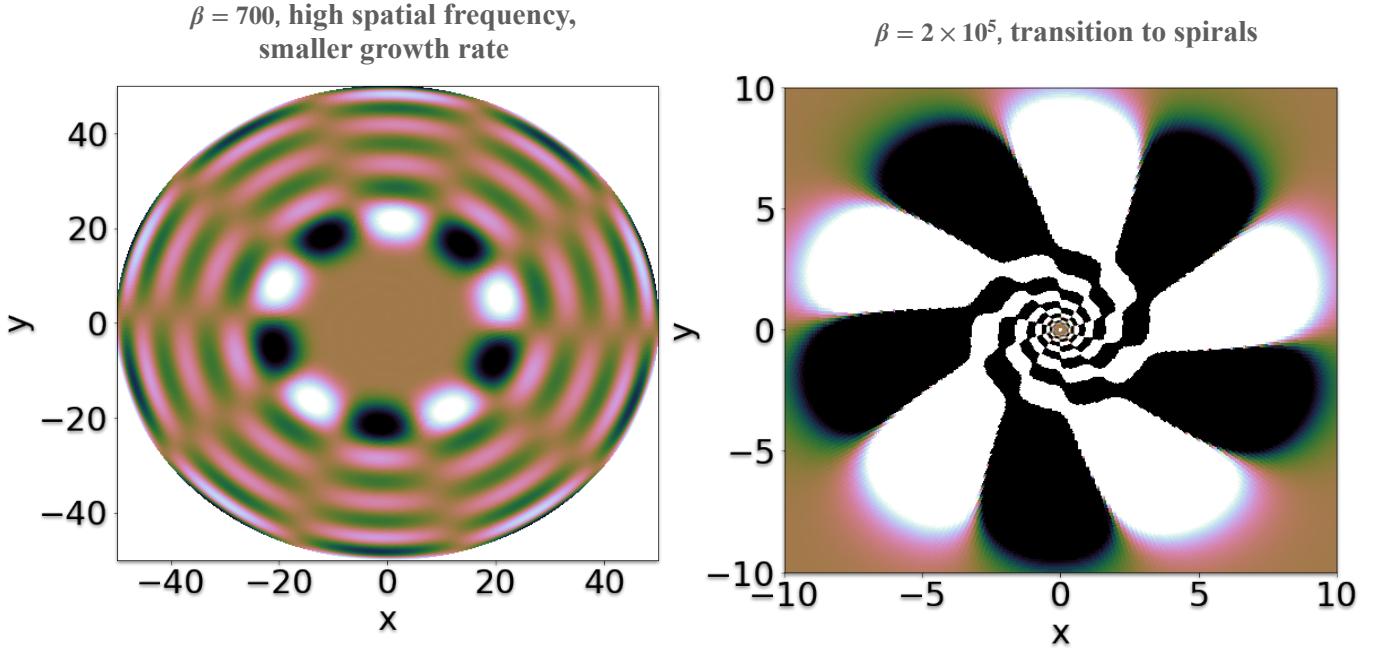


Figure 4. In the left panel, a high frequency *domain* mode is shown and it is volume-filling. The frequency is marked by a blue arrow in Figure 2 in the upper rightmost panel. The corresponding $l = 10$ as is used in the hydrodynamic case. On the right panel, the transition to spiral at extremely large β is demonstrated and this is a buoyancy mode with higher growth rate (see Figure A1 for the parameter space of growth rate and oscillation rate).

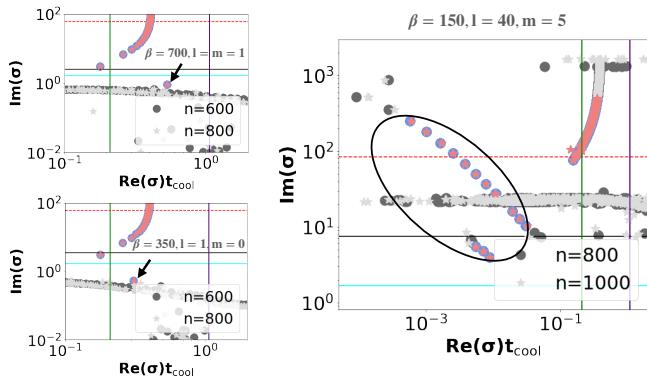


Figure 5. The growth (damping) rates and oscillation rates of more global overstable modes for lower values of l (left) and large value of l (right). The former cases show emergence of azimuthal long wavelength spiral/breathing modes. At large l , no such modes are easily found in our exploration.

X-ray mission at high spatial resolution like AXIS (Russell et al. 2024). A tentative picture of a relaxed cluster core filled up with eigen modes is given in Figure 8. Further, these modes are possibly relevant for the circumgalactic medium (CGM), which is basically scaled down in mass, size, and other properties compared to the ICM (e.g., Figure 4 and Figure 8 in Choudhury et al. 2019a). Magnetic field strengths in the CGM is unknown except some upper limits ($\lesssim \mu\text{G}$) from Faraday Rotation measures (e.g., Lan & Prochaska 2020) which translates to high plasma β .

We find that large scale spiral modes and breathing modes are expected in the galaxy cluster core in unmagnetized and weakly magnetized cases, although in latter such modes are rarer. At timescales longer than inverse Brunt Väisälä frequency (N_{BV}^{-1}), such modes form in weakly magnetized plasma (high β). These are also thermally unstable within comparable timescales ($t_{\text{cool}}^{-1} \sim N_{\text{BV}}$). At higher frequencies than N_{BV} , we find that the slow wave is overstable in a local approximation. In the global atmosphere, depending on the length scales and frequencies, both slow wave and buoyancy wave overstabilities may coexist.

There is a large uncertainty about the morphology of magnetic field in the cluster core. A combination of multiple techniques like Faraday Rotation (e.g., see Di Gennaro et al. 2021 for a discussion on how and when cluster magnetic field reached the current amplitude of a few μG) and Synchrotron intensity gradient (e.g., Hu et al. 2024 for a recent discussion) have suggested that coherent magnetic field may span a sufficiently large range of scales (5 – 500 kpc) compared to the cluster core length scale (100 – 200 kpc). In fact, the above work supports the case for elongated magnetic field lines along the direction of merger axis for merging clusters; this hints that any global perturber can produce coherent field in a given direction. Moreover, a kinetic jet from the central AGN activity may also produce misaligned density and temperature gradients in the diffuse medium ($\nabla p \times \nabla \rho$) and hence a coherent magnetic field in the plane perpendicular to jet axis. Radio images of galactic atmospheres often reveal relatively large scale coherent filamentary magnetic field produced by possibly jet activities or unknown processes (e. g., discussed in Kale 2021, Yusef-Zadeh et al. 2022 for Milky Way halo, Rudnick et al. 2022 for jet-cluster interaction, Rajpurohit et al. 2022

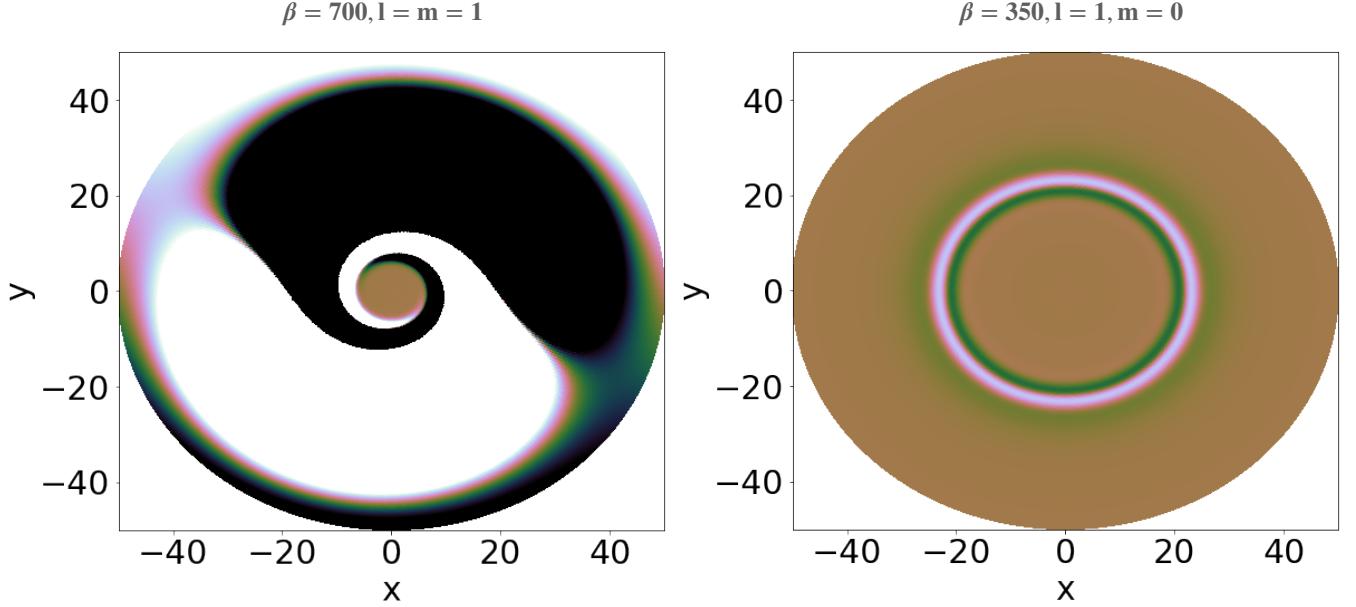


Figure 6. Two large scale (azimuthal) modes obtained for weakly magnetized cases at frequencies below N_{BV} , the characteristic buoyancy oscillation frequency. The left panel shows large scale spiral while the right panel is a symmetric-spherical mode. Availability of these modes at large growth rates in the weakly magnetized cases confirm that spiralling is a persistent characteristic for global buoyancy modes irrespective of plasma β . While these are rarer in presence of magnetic field, these are expected to be robust growing features in weakly magnetized ICM core at sufficiently long timescales.

for filaments in merging clusters and so on). Recently, Omoruyi et al. 2024 show closely aligned multiphase gas in merging galaxies in a cluster environment in which diffuse radio, H α , and molecular phases have been revealed along a 25 kpc arc. This X-ray deficient region is different from cold front that we envisage in this work, but such a region can be an end-state of saturated instabilities if plasma β is sufficiently large. It is essential to assess the non-linear stage of our instabilities to conclude on the latter scenario.

In a conservative sense, there is no concrete evidence of fully azimuthal field that we use in the ICM core. Thus we use a simple, idealized model for the field morphology ($\hat{\phi}$) which may not be the most realistic consideration for a global atmosphere. The purpose of the field is to demonstrate that if there is a coherent magnetic field along the azimuthal direction, thermally unstable spirals or arcs are easily expected. Realistically, smaller scale field structures (as opposed to a completely coherent field along $\hat{\phi}$) can be more common and lead to the development of type II modes at shorter timescale and type I at longer timescale. Our upcoming ideal MHD simulations can robustly confirm if azimuthal long wavelength modes are sustained in the core under such circumstances.

The other missing aspect of the global linear theory is the absence of the perpendicular plane (3D) and the fifth slowly growing mode over large range of wave numbers along the background field in our local analysis (see last panel in Figure 7). If plasma β reduces locally, this may grow faster. Since this grows at distinctly different β , this may not disrupt any pre-existing spiral structure. However, in order to assess this effect, a simulation of the cluster core is ideal

(to be pursued in an upcoming manuscript but Appendix B shows an example of preliminary ideal MHD simulations).

The main conclusions from this paper are:

- **Spirals in hydrodynamic cluster - cold fronts:** In the hydrodynamic cluster, thermally unstable isobaric buoyancy modes produce perfect spirals of varying azimuthal length scales. Theoretically, the modes are modelled as spherical harmonics with radial dependence characterized on a Chebyshev basis. A large range of l, m associated with spherical harmonics gives rise to spirals in any given radial eigenmode. These spirals, if formed, must appear in high resolution X-ray imaging of cluster cores as “cold fronts”.

- **Spirals and shorter azimuthal modes in ideal MHD:** In a weakly magnetized medium, there are two modes that may grow in density. We study an idealized case in which the magnetic field is perpendicular to the gravity and is along $\hat{\phi}$. Note that there is no observational evidence of magnetic field morphology to be globally along $\hat{\phi}$ for any cluster. Such fields are conceivable in a plane perpendicular to AGN jet axis due to misaligned density and pressure gradients. Even sub-structure passage may form large-scale coherent fields perpendicular to the direction of motion. The latter may cause dragging of fields via stripping of interstellar (ISM) or circumgalactic medium (CGM). We find that large scale spirals may still form similar to buoyancy oscillations in hydrodynamics. In addition, an overstable slow wave, which propagates faster than buoyancy modes, can also grow at rates ~ 10 times smaller than the spirals. Although the latter is predicted to be prevalent at wide range of length scales, the growth is insignificant at most of these wavelengths. Thus slow

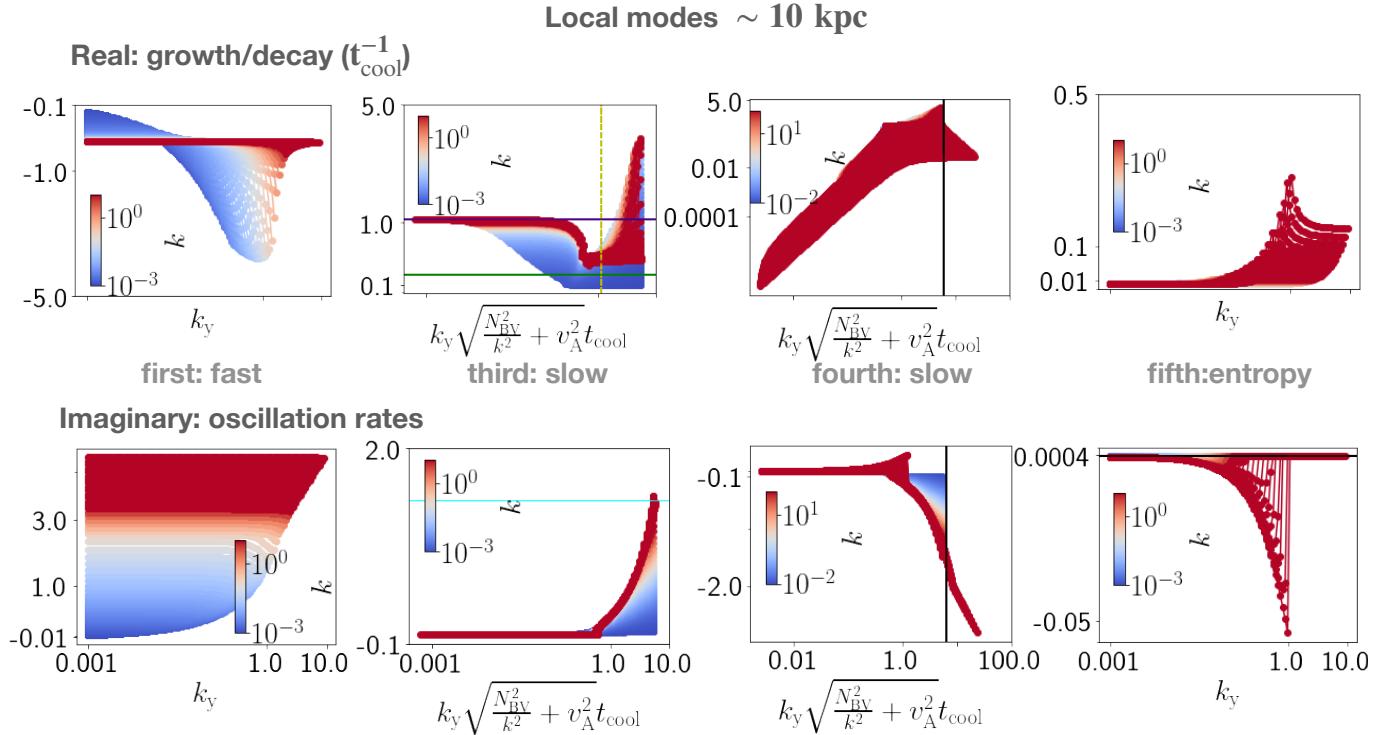


Figure 7. The analytic local modes of a simple model of 2D plane defined by the direction of gravity and magnetic field. The physical parameters of density, pressure, cooling time, etc are extracted for the location of the overstable *domain* mode in the global analysis with $\beta = 700$, $l = 10$, $m = 5$. Five modes are present in this system including two pairs of fast and slow compressive modes and a purely growing mode. The upper panel consists of the four growth rates and the lower panel shows the oscillation rates of the modes (excluding one of the fast magnetosonic modes which is identical to the first but oppositely directed). The second and the third panels in both rows show the slow compressive overstable modes. The yellow dashed line in second panel and top row represents $k_y \sim \sqrt{k_\phi^2 + k_\theta^2} = 1.12$ (see section 3.2). The purple and green horizontal lines show local t_{isb}^{-1} and t_{isc}^{-1} which implies the global growth rate for type II is smaller than the local estimate (in red) by a factor of $\gtrsim 2$ and expected to be well below t_{isb}^{-1} . If type I modes appear, these will always dominate the growth for a large range of k_y . The black vertical lines in third column denote $k_y \sqrt{\frac{N_{\text{BV}}^2}{k^2} + v_A^2 t_{\text{cool}}} = 2\pi$.

modes cannot probably destroy a pre-existing spiral or a spiral that forms in long timescale (without or with a moderate magnetic field).

- **Physical reason for two modes in magnetized medium:** In section 2.2, we attempt to reconcile the results with previous literature on instabilities in presence of magnetized buoyancy. The spiral modes that appear in weakly magnetized medium (type I) is similar to buoyancy waves except that the oscillations happen as a flux tube in response to buoyant perturbations of fluid. The smaller (azimuthal) scale modes (type II) appear due to inhomogeneities along the field (perpendicular to gravity) that generate compression/rarefaction in that direction. In principle, even an arbitrarily small velocity, thus produced along field, can trigger weak growth for sufficiently small-scale modes ($\mathbf{k} \cdot \mathbf{B}_0$) and unbalanced tension and pressure forces. Stronger gravitational force along the propagation of wave can suppress growth (eq. 20). In reality, each type of mode will trigger the other. Whether type I or II dominates depends on timescale and source of perturbation (Figure 8).

- **Future direction:** The key question, *can type II modes (described in previous conclusion point) destroy type I?* is addressed with the following arguments:

- type II propagates fast (global result) and grows weakly,
- type I is isobaric, propagates over a longer timescale (also confined spatially as g-modes) and grows faster,

hence the former cannot destroy the latter. We explore in an upcoming work with 3D MHD simulations (e.g., Appendix B), if this prediction based on 2D model holds in the non-linear 3D model.

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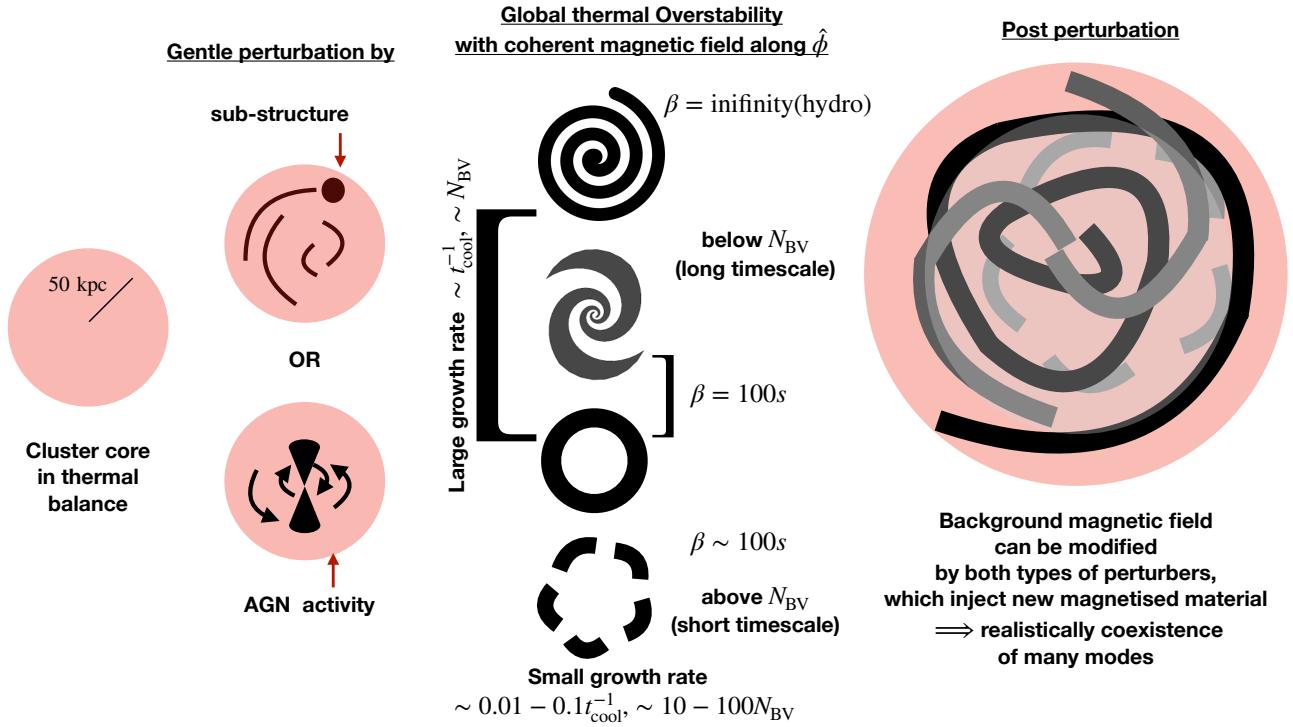


Figure 8. A cartoon demonstrating the physically realisable scenario of the modes in relaxed galaxy cluster cores (e.g., Perseus). Perturbations at sufficiently large spatial scales from either sub-structure passage or gentle AGN feedback can generate multiple modes of a range of frequencies and wavelengths in the core. Out of these modes, the buoyancy modes are spiral and thermally unstable in the hydrodynamic limit ($\beta = \infty$). At $\beta =$ few hundreds, spirals and spherically symmetric modes may form as well but these are rarer and only below the characteristic buoyancy frequency. Above that frequency, all modes are fast propagating, overstable slow compressive modes. The magnetic field (or β) in the background medium can change over time due to injection of fields by the AGN or the satellite gaseous halo. Moreover, modeling the ICM core with a coherent magnetic field in $\hat{\phi}$ is also unrealistic. Consequently, the core is infested with many modes of varying characteristics in reality and not just one of these modes. The robust prediction of all these modes is the filamentary nature of these with lengths aligned along a local coherent magnetic field.

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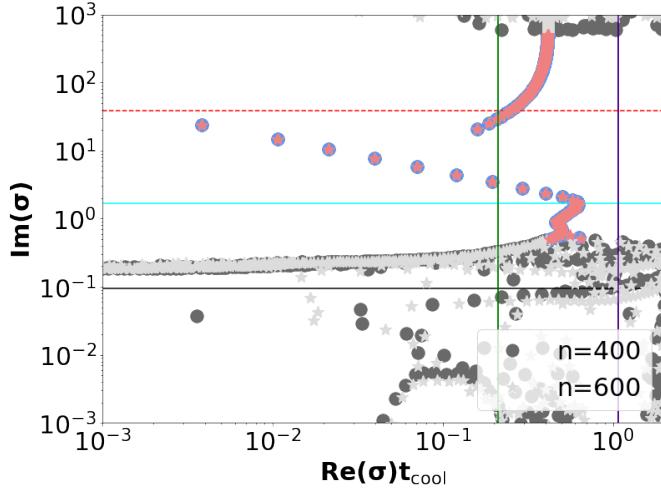


Figure A1. Growth rates and oscillation rates for a case with $\beta = 2 \times 10^5$ which shows that the branch of solution that converges shifts towards the isobaric growth rates (purple vertical line) as expected from previous literature. These are overstable buoyancy modes as evident from the prevalence of points below the horizontal cyan line denoting N_{BV} .

APPENDIX A: LIMITING TO HYDRODYNAMIC CASE

We tested the hydrodynamic limit of zero magnetic field using our pseudospectral code. We find that even an infinitesimal field triggers acoustic overinstability above oscillation rate N_{BV} (cyan line in Figure A1). In this case, overstable buoyancy oscillations appear below N_{BV} at growth rates closer to t_{isb} which also appear in pure hydrodynamic case. When the magnetic tension term, proportional to the wave number of a mode, is relevant in the dynamics (e.g., magnetorotational instability), even for weak magnetic field, the wavenumber of small-scale modes becomes extremely large. Figure A2 shows the one-dimensional radial eigen modes for the fiducial magnetized case and the case for hydrodynamic limit (marked in gray). In the latter case, the overstable buoyancy mode (middle panel) and slow acoustic mode (lowest panel) distinctly emerge.

APPENDIX B: A VIEW OF 3D MHD SIMULATIONS

In this section, we include a snapshot of one of our ongoing MHD simulations that hint at the formation of large scale azimuthal features in the cluster core. It is a 200 kpc box with the cluster in the centre as described in Choudhury & Reynolds 2022. In addition, we include a magnetic field along $\hat{\phi}$ along with a small random perturbation in magnetic field in all directions. Our simulations demonstrate that presence of a wide range of length scales in overinstability in the presence of stratification, local radiative cooling, and weak magnetic field is possible even in non-linear model.

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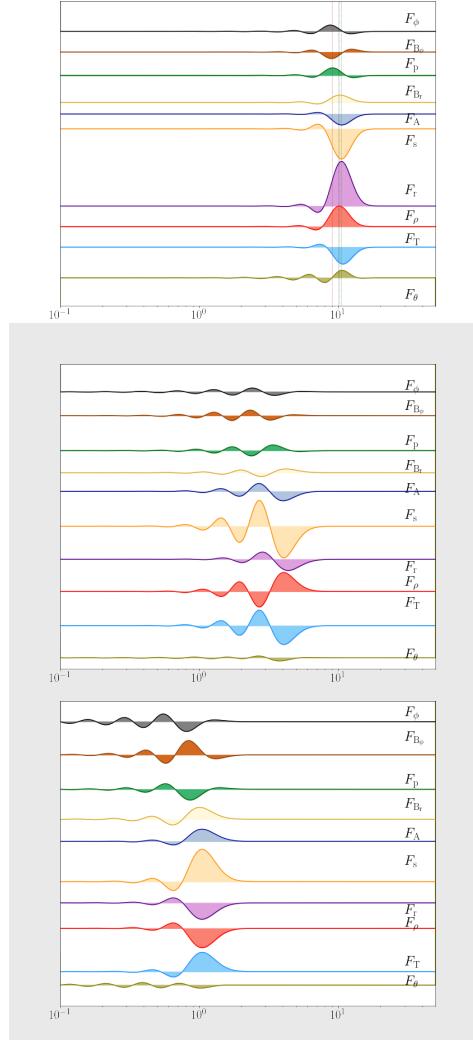


Figure A2. The upper panel shows the fastest global eigen mode in 1D in the fiducial magnetized case of $\beta = 700$. The middle and lowest panels show the overstable buoyancy and acoustic modes for $\beta = 2 \times 10^5$. In all the panels $10F_\theta \ll F_t$ as required by our method.

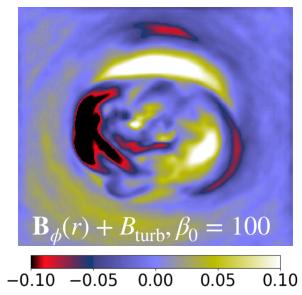


Figure B1. The line-of-sight integrated temperature contrast in a slice of a 3D MHD simulation performed in PLUTO (a conservative hydrodynamic code with constrained transport scheme to maintain divergence-free condition for magnetic field). The magnetic field is similar to the analysis done in this paper but with additional random perturbations. The core is multiphase but closer to isobaric condition with large-scale dense features (black/red). The natural expectation, from what we understand in the current paper for isobaric state, is that large-scale azimuthal modes must easily form.