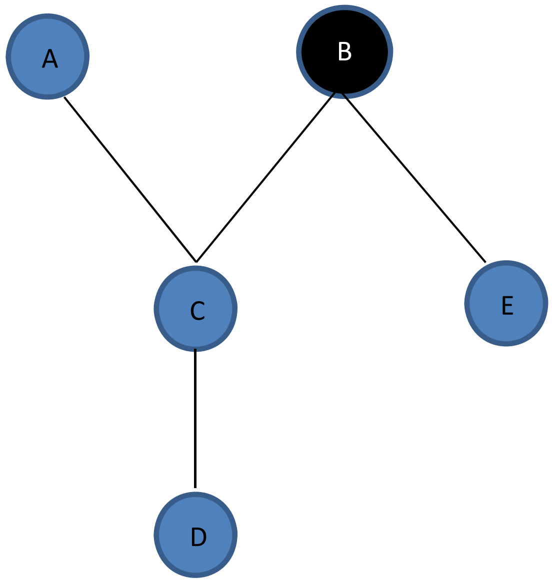
ECSE 6810 Fall, 2020

Assignment 8

MRF and Factor Graph

Due 12/11 at 4pm

1. [20 points] Given the binary MRF and it's parameters below



Each node is a binary variable $x_i = \{-1,1\}$ and $i = A, B, C, D, E$. Model parameters are shown below:

Unary potential:

$$\phi_A(x_A) = [\exp(-2),\ \exp(1.2)]$$
$$\phi_B(x_B) = [\exp(0.8),\ \exp(-0.6)]$$
$$\phi_C(x_C) = [\exp(-0.3),\ \exp(2)]$$
$$\phi_D(x_D) = [\exp(1.3),\ \exp(-0.8)]$$
$$\phi_E(x_E) = [\exp(0.2),\ \exp(-0.2)]$$

Pairwise potential:

$$\psi_{AC}(x_A, x_C) = \begin{bmatrix} \exp(2) & \exp(-1) \\ \exp(-1) & \exp(2) \end{bmatrix}$$

$$\psi_{BC}(x_B, x_C) = \begin{bmatrix} \exp(-0.3) & \exp(1.2) \\ \exp(1.2) & \exp(-0.3) \end{bmatrix}$$

$$\psi_{DC}(x_D, x_C) = \begin{bmatrix} \exp(0.5) & \exp(-1.2) \\ \exp(-1.2) & \exp(0.5) \end{bmatrix}$$

$$\psi_{BE}(x_B, x_E) = \begin{bmatrix} \exp(-1) & \exp(0.7) \\ \exp(0.7) & \exp(-1) \end{bmatrix}$$

Given the evidence $x_B = 1$, perform sum-product and max-product belief propagation to compute

1) $p(x_i|x_B = 1)$ for $i = A, C, D, E$
2) $x_A^*, x_C^*, x_D^*, x_E^* = \arg \max_{x_A, x_C, x_D, x_E} p(x_A, x_C, x_D, x_E|x_B = 1)$

2. [30 points] Given the MRF in problem 1, perform the following tasks
   1) Convert it to the corresponding factor graph, explicitly give the variable nodes and factor functions. Draw the factor graph.

   2) Perform max-sum and max-product belief propagations within the factor graph to compute
      - $p(x_i|x_B = 1)$ for $i = A, C, D, E$
      - $x_A^*, x_C^*, x_D^*, x_E^* = \arg \max_{x_A, x_C, x_D, x_E} p(x_A, x_C, x_D, x_E|x_B = 1)$

3. [20 points] Given a MRF with three nodes A,B,C, and their joint probability can be represented by the following Gibbs distribution

$$p(a, b, c) = \frac{1}{Z} \exp(-w_{ab}E(a, b) - w_{ac}E(a, c) - w_{bc}E(b, c) - w_a E(a) - w_b E(b) - w_c E(c))$$

   The necessary and sufficient condition for the absence of a link between nodes A and B is

$$A \perp\!\!\!\perp B|C$$

   Through the joint distribution and the independence condition above, derive the necessary and sufficient condition in terms of the values of the pairwise and unary parameters for the absence of a link between nodes A and B.