An Attitude Estimate Approach using MEMS Sensors for Small UAVs

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***Abstract* — For the small UAVs (Unmanned Aerial Vehicle) using MEMS sensors, this article puts forward a Kalman Filter model to get attitude estimate without long term drift and showing relatively smaller error. Firstly, strapdown inertial attitude algorithm and bi-vector attitude algorithm are presented, which are widely used in small UAV autopilot systems now. However, there is a problem of long term drift with the former and heavy noise with the latter. Due to these shortcomings, accurate attitude control has not been achieved yet in small UAVs. In order to solve these problems, this paper gives out a Kalman filter model which fuses the two types of data into an optimal estimate of real attitude, and overcomes the shortages of both algorithms mentioned above. Simulation results show that this filter can be used to gain fairly good data for more accurate attitude control. Besides, compared with the filters already developed, this Kalman filter has a relatively low order and a loose architecture, which could be more easily adopted in an existed embedded computer system of small UAV.**

I. INTRODUCTION

In recent years, small UAVs are used in many cases for its merits of easy-taking, easy-operating, and easy-maintaining. A small UAV usually refers to an unmanned air vehicle whose wingspan is less than 2 meters and suit for a single person toting. The payloads of small UAVs are usually within several hundred grams, which puts restricts on the autopilot system's volume, weight and power consumption.

Since MEMS sensors have virtues of tiny, light and low power consumption, they are widely used in small UAVs as the data source of attitude control system. But MEMS sensors have some inherent shortcomings such as heavy noises and great drifts, so most of the small UAVs equipped with MEMS sensors only achieve stability control. The theory of stability control adopts angle rate feedback of MEMS gyros to stabilize the aircraft, and calculate the aircraft's attitude through measurements of gravity projections on two horizontal axes of the aircraft by MEMS accelerometer. Obviously, the precision of the attitudes acquired in this method is significantly influenced by vibration of the aircraft. Position navigation and guidance tasks are fulfilled by micro GPS receiver. This method has a simple and reliable structure, and little calculating amount involved. For the static stabilized aircrafts, this method can control them very well in the whole cruise process.

However, a stability control system is always coupled with the small UAV itself. Too many flight experiments are needed for ascertaining the control parameters. Noise and delay of the attitudes acquired by gravity projections method are comparatively remarkable, so it is hard to make a precise control, for those unstable aircrafts, or those tasks asking for accurate attitudes, for example, optic camera platform control.

Next, this article will provide an attitude estimate approach to acquire much more accurate data, and present some examples to prove its feasibility and performance.

II. FRAMEWORK OF THE ATTITUDE ESTIMATE SYSTEM

For the manned aircrafts, there are two ways to get the carrier's attitudes. The first method is to use a vertical gyro, which provides the pitch and roll angles, and use a compass to get the heading data. The other means is to use three high accuracy angular rate gyros to set up a strapdown inertial system. But common vertical gyros are too big and too heavy to fit in small UAVs. On the other hand, when MEMS gyros apply, a strapdown inertial system could have a compact structure and fulfill the demands of payloads' volume and weight of a small UAV. Moreover, the MEMS accelerometer can also give out the attitude by observing the gravity vector, which could be used as a reference. The heading angle can be obtained from the compass system composed of magnetoresistive sensors.

One of the problems comes with the method above is that the error of the strapdown attitude will accumulates due to MEMS gyros' drift. This error enlarges quickly because of the MEMS sensors' great drift level. Even if temperature compensation is carried out, it is not acceptable within tens of minutes. So it prevents MEMS strapdown attitude system to be used in long endurance small UAVs. Another problem is that, although MEMS accelerometers and magnetoresistive sensors have no long term drift accumulation, noise of MEMS sensors is too great that attitudes from observing the gravity vector and geomagnetism vector, namely bi-vector attitude acquisition method, are unlikely to be straightly used in precise attitude control.

Many studies show attitude estimate methods of fusing merits of the strapdown and bi-vector methods, and overcoming their disadvantages at the same time. For instants, a 15 order Kalman filter model and several 6 order Kalman filter models are developed to solve this problem [1][2][3]. The point of these filters is that the model is a whole one, and several attitude acquisition methods are coupled together. This paper introduces another attitude estimate method based on a Kalman filter to get attitudes without long term drift and

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showing relatively smaller error, particularly for the embedded computer system in small UAVs.

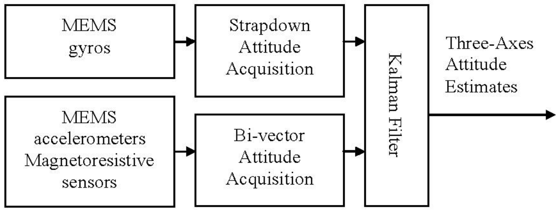

Fig. 1. The framework of attitude estimate system

As shown in figure 1, the attitude estimate architecture discussed in this paper is a loose one. It calculates two different types of attitude respectively in the first step, and uses a Kalman Filter to indicate an optimal estimate in the next step. The second step could be flexibly added to an exist system or created as a new system.

III. METHODS OF ATTITUDE ACQUISITION

A. Strapdown Inertial Attitude Algorithm Based on Quaternion and Rotation Vector

In the strapdown inertial attitude heading reference system (SIAHRS), a quaternion and rotation vector based algorithm is adopted. A rotation vector is defined as,

$$(L - I)\Phi = 0 \tag{1}$$

where $\Phi = [\phi_x, \phi_y, \phi_z]^T$ represents the rotation vector, I is the identity matrix, and L denotes the direction cosine matrix. The direction cosine matrix L_{bg} from local level frame S_g to body frame S_b is shown below (in this article it treat local level frame as inertial frame for analyzing convenience),

$$L_{bg} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} \tag{2}$$

YEON FUH JIANG had deduced a rotation vector differential equation as below [4],

$$\dot{\Phi} = \left\{ I + \frac{1}{2}[\Phi\times] + \frac{1}{\phi^2}\left(1 - \frac{\phi\sin\phi}{2(1-\cos\phi)}\right)[\Phi\times]^2 \right\}\omega \tag{3}$$

where

$$[\Phi\times] = \begin{bmatrix} 0 & -\phi_z & \phi_y \\ \phi_z & 0 & -\phi_x \\ -\phi_y & \phi_x & 0 \end{bmatrix} \tag{4}$$

denotes the skew symmetric matrix has the same efficacy as a vector cross-product operation. And

$$\omega = [\omega_x, \omega_y, \omega_z]^T \tag{5}$$

represents the observed angular rate vector in the body frame. And ϕ is defined as

$$\phi = (\Phi^T\Phi)^{1/2} \tag{6}$$

The last two components in (3) are noncommutativity ones, and also the source of noncommutativity error. When calculating the increment of rotation vector $\Delta\Phi$, the higher order infinitely small component is always ignored. So it could get another equation from (3).

$$\Delta\Phi \approx \int_t^{t+\Delta t} \omega(t)dt + \frac{1}{2}\int_t^{t+\Delta t} \theta(t,\tau)\times\omega(\tau)d\tau \tag{7}$$

where $\theta(t,\tau)$ is the angular increment from t to τ, which is the approximate substitute of $\Phi(\tau)$, according to Ignagni's paper [5]. It can be gotten by the formula below,

$$\theta(t,\tau) = \int_t^{\tau} \omega(t)dt \tag{8}$$

The formula to solve differential equation (7) is presented below. It captures two samples in a calculating cycle, and uses one sample of the previous cycle [6].

$$\Delta\Phi = \Delta\theta + \frac{32}{45}\Delta\theta_1 \times \Delta\theta_2 + \frac{57}{80}\Delta\theta_0 \times (\Delta\theta_1 - \Delta\theta_2) \tag{9}$$

Defining a quaternion as the combination of a scalar component and a vector component ($Q = q_0 + \vec{q} = q_0 + q_1\vec{i} + q_2\vec{j} + q_3\vec{k}$), the relationship between a quaternion and a rotation vector as below could be deduced.

$$\Delta Q = \cos(\Delta\phi/2) + \frac{\Delta\Phi}{\Delta\phi}\sin(\Delta\phi/2) \tag{10}$$

where ΔQ is the quaternion increment of a calculating cycle. So the refreshed quaternion could be gotten,

$$Q_{k+1} = Q_k \otimes \Delta Q \tag{11}$$

Operator $\otimes$ is the quaternion multiplication operator, which is defined as $P \otimes Q = (p_0 q_0 - \vec{p}\cdot\vec{q}) + (p_0\vec{q} + q_0\vec{p} + \vec{p}\times\vec{q})$.

There are four steps in a strapdown attitude calculating cycle.

- Calculate equivalent rotation vector of a cycle by (9)
- Calculate quaternion increment $\Delta\Phi$ and refreshed quaternion Q_{k+1} by (10) and (11)
- Quaternion representing a rotation must yield to the equation ${q_0}^2 + {q_1}^2 + {q_2}^2 + {q_3}^2 = 1$, so adjust the refreshed quaternion as below,

$$\overline{Q}_{k+1} = \frac{Q_{k+1}}{\sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}} \tag{12}$$

- Finally make use of the new refreshed quaternion; it get pitch, roll, and heading angles.

$$\left.\begin{aligned} \sin\theta &= -2(q_3 q_1 - q_0 q_2) \\ \tan\phi &= [2(q_2 q_3 + q_0 q_1)]/[1-2(q_1^2 + q_2^2)] \\ \tan\psi &= [2(q_1 q_2 + q_0 q_3)]/[1-2(q_2^2 + q_3^2)] \end{aligned}\right\} \tag{13}$$

B. Attitude Algorithm Based on Gravity Vector and Geomagnetism Vector (bi-Vector Method)

In the local level frame, gravity vector could be considered as a constant $g_g = [0 \quad 0 \quad g_0]^T$. By obtaining the observed gravity vector $g_b = [g_{bx} \quad g_{by} \quad g_{bz}]^T$ in body frame, the pitch and roll angle can be calculated as below,

$$\left.\begin{aligned} g_{bx} &= g_0 \sin\theta \\ g_{by} &= g_0 \sin\phi \end{aligned}\right\} \tag{14}$$

In the same time, Wang Song gives out a formula which solves the heading angle from outputs of magnetoresistive sensors [7].

$$\begin{aligned} \psi &= \mathrm{arctg}\frac{\mathrm{H}_{gY}}{\mathrm{H}_{gX}} \\ &= \mathrm{arctg}\frac{-\mathrm{H}_{bY}\cos\phi + \mathrm{H}_{bZ}\sin\phi}{\mathrm{H}_{bX}\cos\theta + \mathrm{H}_{bY}\sin\theta\sin\phi + \mathrm{H}_{bZ}\sin\theta\cos\phi} \end{aligned} \tag{15}$$

where $H_b = [H_{bX} \quad H_{bY} \quad H_{bZ}]^T$ denotes the magnetic field measured from the magnetoresistive sensors with three axes that are fixed on the carrier. θ and ϕ represent pitch angle and roll angle respectively. They could be the results of (13) or (14), and also could be the optimal estimate by the Kalman Filter discussed below.

IV. ATTITUDE ESTIMATE APPROACH

A. Kalman Filter System Model

Considering the virtual platform frame (virtual inertial frame) in the strapdown computer system has a small error relative to inertial space, the error could be described by a quaternion Q_e. Since the error is small, it could be assumed that $Q_e \approx (1, \vec{q}_e)$. Define the strapdown platform error as $\varepsilon^i = \vec{q}_e = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z]^T$ (measured in inertial frame), and the random drift of MEMS gyros as $\delta\omega_{ib}^b = [\delta\omega_x \quad \delta\omega_y \quad \delta\omega_z]^T$ (measured in body frame) [2]. Because of observing noises and gyros bias, there will be an error on the direction cosine matrix from local level frame to body frame. It is marked as δL_{ib}, and

$$\widetilde{L}_{ib} = L_{ib} + \delta L_{ib} = (I + E^i) L_{ib} \tag{16}$$

where $\widetilde{L}_{ib}$ is the calculating result, and E^i is the skew symmetric matrix corresponding to strapdown platform error ε^i,

$$E^i = \begin{bmatrix} 0 & -\varepsilon_z & \varepsilon_y \\ \varepsilon_z & 0 & -\varepsilon_x \\ -\varepsilon_y & \varepsilon_x & 0 \end{bmatrix} \tag{17}$$

The strapdown platform error is brought by the observing error of gyros. There is a relationship between them as below [1],

$$\dot{\varepsilon}^i = L_{ib}\delta\omega_{ib}^b \tag{18}$$

The gyro bias consists of null drift, temperature drift, random drift and white noise. Null drift and temperature drift can be compensated by specific algorithms. So this paper only considers random drift and white noise. There is an important assume of the random drift, which yields to the first order Markov process.

$$\left.\begin{aligned} \delta\dot{\omega}_{ib}^b &= A_{IMU}\delta\omega_{ib}^b + w_r \\ A_{IMU} &= diag[-\frac{1}{t_{rx}} \quad -\frac{1}{t_{ry}} \quad -\frac{1}{t_{rz}}] \end{aligned}\right\} \tag{19}$$

where t_{rx}, t_{ry}, and t_{rz} are the autocorrelation parameters.

Then the system state could be defined as $X = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \delta\omega_x \quad \delta\omega_y \quad \delta\omega_z]^T$. The state equation is,

$$\dot{X} = A_{6\times6}X + G_{6\times6}W \tag{20}$$

where $W = [w_{gx} \quad w_{gy} \quad w_{gz} \quad w_{rx} \quad w_{ry} \quad w_{rz}]$ denotes the process noise vector. w_g is the white noise of gyro. w_r is the first order Markov process noise of gyro.

Matrix A and G are respectively defined as,

$$A_{6\times6} = \begin{bmatrix} 0 & L_{ib} \\ 0 & A_{IMU} \end{bmatrix} \quad G_{6\times6} = \begin{bmatrix} L_{ib} & 0 \\ 0 & I_{3\times3} \end{bmatrix} \tag{21}$$

Take the difference between strapdown attitude and bi-vector attitude as measurement. Firstly calculate $L_{ib}^S(\theta+\delta\theta, \phi+\delta\phi, \psi+\delta\psi)$ by (2) from strapdown attitude and $L_{ib}^{BV}(\theta,\phi,\psi)$ from bi-vector attitude. In the next, from (16) strapdown platform error measurement can be represented as,

$$\widetilde{E}^i = (L_{ib}^S - L_{ib}^{BV})L_{bi} = (L_{ib}^S - L_{ib}^{BV})L_{ib}^{-1} \tag{22}$$

where $\widetilde{\varepsilon}^i$ can be gotten according to (17). Defining measurement vector $Z = \widetilde{\varepsilon}^i$, measurement equation is,

$$Z = H_{3\times6}X + V \tag{23}$$

where $H_{3\times6} = [I_{3\times3} \quad 0]^T$ is the measurement matrix. $V = [v_{a1} \quad v_{a2} \quad v_{a3}]^T$ denotes the measurement noise. Then derive discrete-time linear stochastic difference equations from (20) and (23),

$$\left.\begin{aligned} X_k &= \Phi_{k,k-1}X_{k-1} + \Gamma_{k,k-1}W \\ Z_k &= HX_k + V \end{aligned}\right\} \tag{24}$$

After every cycle of Kalman Filter calculation, there will be a new strapdown platform error estimate $\hat{\varepsilon}^i$. From $\hat{\varepsilon}^i$, the skew symmetric matrix $\hat{E}^i$ could be deduced, and the estimate of direction cosine matrix is,

$$\hat{L}_{ib}^k = L_{ib}^S - \hat{E}^i \hat{L}_{ib}^{k-1} \tag{25}$$

Using equation (2) and the fact that $\hat{L}_{ib}^k = \hat{L}_{bi}^{k\ -1} = \hat{L}_{bg}^{k\ -1}$ (note the assume of treating local level frame as inertial frame in this paper), the optimal attitude estimation could be solved out,

$$\left.\begin{aligned} \hat{\theta} &= \arcsin(-\hat{L}_{13}) \\ \hat{\phi} &= \arctan(\hat{L}_{23} / \hat{L}_{33}) \\ \hat{\psi} &= \arctan(\hat{L}_{12} / \hat{L}_{11}) \end{aligned}\right\} \tag{26}$$

From equations above, it could be seen that strapdown, bi-vector and Kalman Filter calculations are divided into three parts in this optimal attitude estimate system. For the embedded computer system of small UAVs always has no backup and limited computing capability. This low order loose architecture brings great convenience and flexibility to the embedded computer system. If organize the system in this architecture, it does not lead to the whole system failure even when a subsystem breaks down.

B. Attitude Estimate Examples

In this section some simulation results are revealed to demonstrate performances of the aforementioned Kalman Filter. The example consists of a static carrier simulation and an attitude tracking of a sine wave carrier motion. It is assumed that the 1-sigma measurement noise of bi-vector attitude method is 2.5 deg, which corresponds the noise level of the MEMS accelerometer and magnetoresistive sensor widely using now. The white noise and Markov process noise of MEMS gyros is 1.0 deg/s and 0.8×10^{-3} rad/s^2 respectively. The simulation uses fixed step method, with step of 0.02s. Within this example, there is an assumption that attitude errors relative to best estimates are small, because it is the presupposition of formula (18).

Firstly, the Kalman Filter is put into a static carrier. Figure2 shows typical attitude error of only bi-vector method. Figure3 indicates the same error result through Kalman Filter. It is seen that this Kalman Filter outputs fairly good estimates of real attitude, with errors less than 1.5 deg.

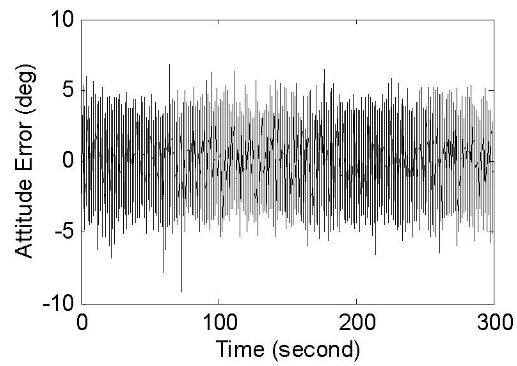

Fig. 2. Attitude error from only bi-vector output

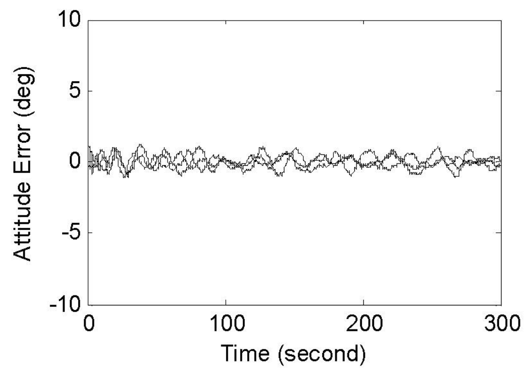

Fig. 3. Attitude error from Kalman Filter output
(Static carrier condition)

Next, a sine wave motion is applied to the carrier. The motion could be described as,

$$\left.\begin{aligned} \theta(t) &= 11^\circ \sin(\frac{2\pi}{11}t) \\ \phi(t) &= 15^\circ \sin(\frac{2\pi}{15}t) \\ \psi(t) &= 17^\circ \sin(\frac{2\pi}{17}t) \end{aligned}\right\} \tag{27}$$

The same type of result with figure 3 is figure 4, which shows that the dynamic performance is similar to that in static condition. But in a real dynamic situation, acceleration of the carrier itself will disturb bi-vector attitude measurement. Some additional procedures must be employed to compensate this interferential acceleration. For example, GPS measurements may be used to estimate carrier acceleration [8].

Figure 5 is the convergence process of the Kalman Filter. The curve shows the change of error quaternion gain, which denotes the weight of residual state in the "correct" procedure of Kalman Filter. It costs about 50 seconds – an acceptable period in applications – to converge at a stable value.

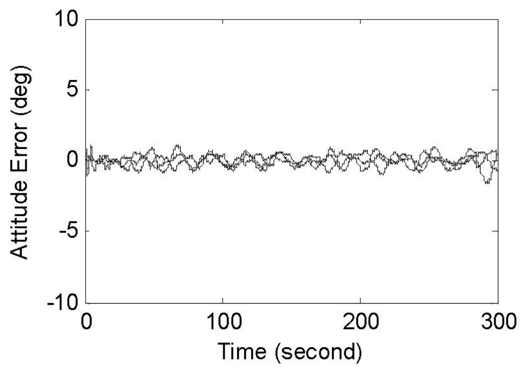

Fig. 4. Attitude error from Kalman Filter output (Dynamic carrier condition)

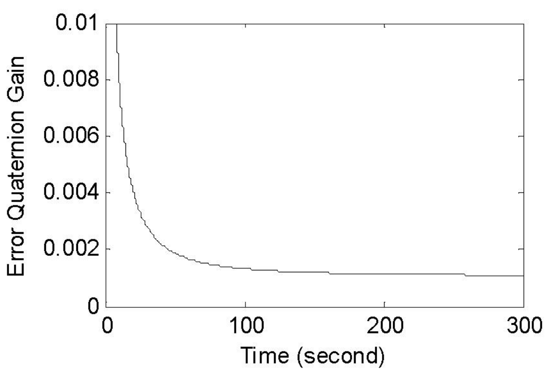

Fig. 5. Typical convergence process of error quaternion gain

V. CONCLUSION

In the flight of small UAVs, attitude control performance is critical to the security and reliability of holistic autopilot system, while attitude acquisition is the pivot of attitude control. Drifts and delays of measurements in attitude acquisition system are vital factors of control divergence. So estimating three-axis attitudes exactly and timely becomes primary problems to be solved in the small UAV autopilot system. The loose Kalman Filter architecture presented in this article has merits of small error and no long term drift, and is based on MEMS sensors on shelf. But temperature drift of MEMS gyros – the primary ingredient of drift – is not taken into account in this article. It needs large numbers of experiments to compensate temperature bias, which lies beyond the scope of this paper. Besides, some problems such as stability of Kalman Filter at special conditions must be further considered in order to put it in real applications.

VI. ACKNOWLEDGMENT

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