Cooperative Localization using Angle of Arrival Measurements in Non-line-of-sight Environments*

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ABSTRACT

We investigate localization of a transmitting node using angle of arrival (AoA) measurements made at a geographically dispersed network of cooperating receivers with known locations. A low-complexity sequential algorithm for updating the source location estimates under line-of-sight (LOS) environments is developed. This serves as a building block for an algorithm that suppresses *outliers* arising due to multipath scattering and reflection in non-line-of-sight (NLOS) scenarios. Maximal likelihood (ML) location estimation requires exhaustive testing of estimates from all possible subsets of measurements. We avoid this by utilizing a randomized algorithm that approaches the ML performance at a complexity that is only quadratic in the number of measurements. The localization error is proportional in the AoA error variance and coverage area, and can be reduced by an increase in the number of estimates with a strong LOS component.

Categories and Subject Descriptors

C.3 [**Special-Purpose and Application-based Systems**]: Signal processing systems; C.2.1 [**Computer-communication networks**]: Network Architecture and Design—*wireless communication, distributed networks*

General Terms

Algorithms, Performance, Measurement

1. INTRODUCTION

Localization of a node in a wireless ad hoc or sensor network has drawn considerable interest in a number of applications, where either the location is auxiliary to the data collected by the network, or location (and hence tracking) itself is the primary goal, such as inventory management using RFID tags [1, 2], WLAN node positioning using access points, surveillance using camera networks and bird habitat monitoring, to name only a few. Even with the dropping costs of GPS receivers, equipping nodes with GPS is expensive and more importantly, GPS is unavailable in most indoor applications. A popular localization solution in indoor environments is to utilize a small collection of static nodes with known positions, such as wireless access points, to infer the locations of the remaining nodes, similar to that used in cellular systems [18]. Moreover, one of the key obstacles to localization in indoor environments is the presence of significant multipath scattering due to reflections from objects such as walls and furniture.

In this paper, we address collaborative localization of a transmitting node using a network of geographically dispersed receivers using angle-of-arrival (AoA) measurements. We envision a paradigm, in which the nodes transmit without any prior coordination with the receivers or for the specific purpose of localization, and the network of receivers is responsible for cooperatively locating the transmitting node. We consider only AoA measurements here, as they require the receivers to be equipped only with a calibrated antenna array, have known locations and capability to associate measurements corresponding to a particular transmitting node. While the localization geometry under line-of-sight (LOS) propagation using AoA measurements is straightforward, our goal here is to develop efficient algorithms for non-line-of-sight (NLOS) (or multipath) environments. Since our work here is pertinent in a larger class of source localization applications as well, we use the term "source" to refer to the transmitting node.

In many applications, other measurements such as time-difference-of-arrival (TDOA) and received signal strength (RSS) have been used for localization. However, TDOA measurements require tight synchronization within the network of receivers to obtain adequate localization resolutions, and RSS measurements are extremely unreliable and need extensive calibration before such measurements can be effectively used. On the other hand, AoA measurements have the potential to provide sufficient localization resolution without the need for extensive coordination between receivers and minimal calibration. We, therefore, study the localization algorithms utilizing AoA measurements alone with the understanding that other forms of location information, if available, can be used to further improve resolution.

Our main contributions are as follows: In order to focus on the localization problem, we abstract away details of the

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receiver algorithms used for AoA estimation (one such algorithm is presented in our prior work [4]) and replace them with models that capture the combined effects of the receiver algorithm and propagation environment on the AoA measurements. We first develop a sequential algorithm to combine the AoA estimates produced by the receivers to estimate the node location under LOS propagation that has linear complexity in the number of measurements and is approximately maximum likelihood. However, NLOS multipaths often results in AoA estimates that are "far" from the true node bearing and act as *outliers* in the localization.

We then use the sequential algorithm as a building block in an outlier suppression algorithm that localizes the node, while minimizing the effect of these outliers. This randomized algorithm has a complexity of $O(MN^2)$, where M is the number of randomizations and N is the number of measurements. Importantly, M, which is chosen to meet a desired probability of failure requirement, does not grow with N. We numerically verify the performance of these algorithms and show that they perform close to the Cramer-Rao lower bound (CRLB). It must noted, however, due to space constraints, we aim to only outline the basic details of these algorithms and provide some key numerical results.

Related Work: There exists a rich history of related work in the source localization literature using a combination of TDOA, AoA and signal strength measurements. RSS measurements have been used to locate wireless devices [9, 20] and in acoustic sources [6, 15]. Positioning using TDOA and its derivatives have been used for cellular phone localization [18], in the global positioning system [10] and radar systems [21]. Localization via combining of AoA estimates is presented in [5], and using both AoA and TDOA estimates in [3]. A major drawback of all of the preceding algorithms is that they assume LOS channels between the source and the receivers with no multipath, and model errors due to measurement noise alone.

Our approach to localization in NLOS environments draws upon the significant literature on handling outliers [13]. One approach is to identify and remove outliers *before* location estimation, and such an approach has been adopted in [7, 12, 23]. However, they algorithms require prior statistical knowledge of the NLOS characteristics. In this paper, we adopt the alternative approach of robust location estimation while simultaneously limiting the effect of outliers, thus eliminating outliers "on the go." While we are unaware of prior work using this approach for AoA measurements, similar work for range/TOF measurements includes [11, 8]. Chen [11] proposes an iteratively weighted least-squares localization using range/TOF measurements, which is of exponential complexity in the number of measurements. Casas *et al* [8] solve the multilateration by a least median squares algorithm on TOF estimates, but utilize only three "best" TOF measurements to simultaneously eliminates outliers and estimates the emitter location. In contrast, we propose a linear complexity robust estimation-based approach that tries to estimate the source location using a maximal set of measurements that are not outliers.

The rest of the paper is organized as follows. AoA measurement models under different propagation environments are described in Section 2. The sequential algorithm for localization in LOS scenarios are presented in Section 3. We derive an extension to the sequential localization to suppress NLOS AoA estimates in Section 4, which is motivated by the structure of the ML estimate. The proposed algorithms are numerically investigated in Section 5. Section 6 contains concluding remarks.

2. MODELS OF AOA ESTIMATES

In this section, we develop models that characterize the AoA measurements under LOS and NLOS propagation at each collector and are used to design source localization algorithms in Sections 3 and 4. A number of models for sources with local scattering and methods of AoA estimation under those models have been proposed [16, 22], in which the propagation is characterized by a mean arrival angle (corresponding to the true bearing) and a spatial spreading parameter (quantifying the spatial extent of the multipath effects). The resulting AoA at the antenna array have been represented by Gaussian and Laplacian distributions. On the basis of these ideas, we propose the following models:

LOS Model: We characterize the spatial spreading in LOS propagation scenarios by zero-mean symmetric finite variance "noise" models such as the Gaussian and Laplacian. The AoA estimation with additive white Gaussian noise under LOS propagation results in zero-mean Gaussian errors [17]. Scattering in the vicinity of the source also leads to zero-mean symmetric distributions, although with larger AoA deviations. Therefore, a wide variety of scenarios with strong LOS components, with and without local scattering, can be characterized by such models by varying the spatial spreading parameter. For a source at location $\underline{X}$ along a true bearing $\theta(\underline{X})$, the Gaussian LOS model with local scattering is

$$p_{\text{Gaus}}(\hat{\theta}/\underline{X}) = \frac{\exp\left(-\frac{(\hat{\theta}-\theta(\underline{X}))^2}{2\sigma^2}\right)}{\sqrt{2\pi}\sigma(1-2Q(\frac{\pi}{2\sigma}))} \quad \hat{\theta} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \tag{1}$$

where σ^2 represents the spatial extent of scattering, $Q(t) = \int_t^\infty \exp(-t^2/2)dt/\sqrt{2\pi}$ is the normal tail distribution and all angles are measured with respect to the antenna broadside.

LOS blockage model: In an extreme case of a NLOS propagation environment, the LOS path to a receiver might be blocked, for instance by structures such as walls or trees. As a result, AoA measurements in such scenarios are fairly uncorrelated with the true bearing of the source and hence, the AoA estimate is drawn uniformly from the feasible set:

$$p_{\text{block}}(\hat{\theta}/\underline{X}) = \frac{1}{\pi}, \quad \hat{\theta} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]. \tag{2}$$

When there are significant contributions from both the LOS path and the NLOS multipath, the model depends on whether the receiver is capable of resolving these contributions spatially. This resolving capability is also dependent on the bandwidth of the source signal.

Narrowband multipath model: With narrowband source transmissions and receivers with small numbers of antennas, the receiver can spatially resolve the AoA of the strongest arriving path (or strongest superposition of paths) only and hence, each receiver produces a single AoA estimate. We model this as one of the least favorable multipath scenarios, where with a probability of α the LOS path to a receiver is blocked resulting in the AoA being drawn according to the LOS blockage model (2). Otherwise, the AoA measurement is obtained from one of LOS models, for instance (1). Thus, we arrive at the following narrowband model:

$$p_{\text{narrow}}(\hat{\theta}/\underline{X}) = \alpha\, p_{\text{block}}(\hat{\theta}/\underline{X}) + (1-\alpha)\, p_{\text{Gaus}}(\hat{\theta}/\underline{X}). \tag{3}$$

Note, that a possibly worse scenario results when all the AoA estimates have correlated errors resulting in an erroneous but mutually consistent location estimate such as when all receivers only receive multipath reflected from a wall. In such cases, additional information about possible sensor locations have to be used to eliminate false virtual sensors from the list of possible locations produced by the algorithms proposed here.

Wideband multipath model: On the other hand, with wideband source transmissions, the receiver has an additional degree of freedom and can resolve paths in time as well. Akin to the narrowband model, the multiple AoA estimates produced by the receiver are modeled as follows: one LOS estimate is represented by the LOS models, and the remaining estimates are drawn uniformly from the feasible set, like the LOS blockage model, as they correspond to multipath with no LOS components (equivalent to LOS blockage). However, it is not known a priori, which one of the multiple estimates corresponds to the LOS path.

3. LOCALIZATION IN LOS SCENARIOS

We present an algorithm for sequential aggregation of the available AoA estimates (generated using LOS model (1)) to produce the source location where source-receiver propagation has strong LOS components and minimal local scattering. Before we describe the sequential algorithm in Section 3.2, we develop, in Section 3.1, the linear minimum mean-squared error (LMMSE) updates at each receiver.

3.1 LMMSE Updates

The LMMSE update only requires knowledge of the second order statistics of the source location estimate. Therefore, the *priors* exchanged between the receivers consist of the latest location error covariance and the source location estimates. The LMMSE updates in polar coordinates (AoA geometry makes this a convenient choice) are optimal under Gaussian measurement error models such as the LOS model in Section 2. Note, the AoA θ is always measured from the x-axis of the global cartesian system (see Figure 1).

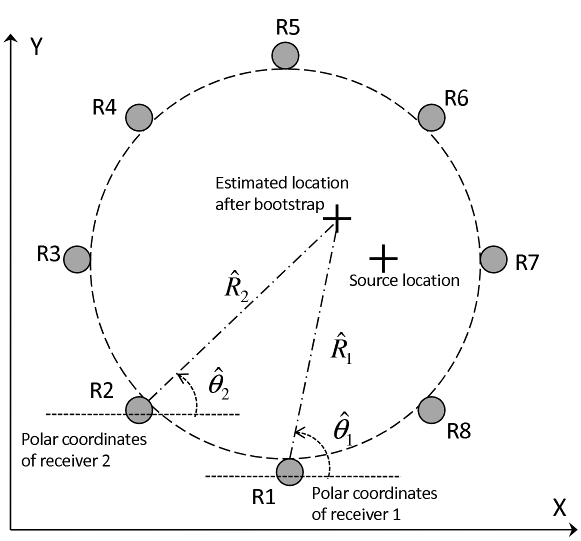

Figure 1: A field with 8 receivers used to illustrate the "bootstrap" geometry using AoA estimates of receivers R1 and R2.

Consider a receiver $\mathbf{R}$ that receives the following prior information: source location estimate $\hat{\mu}_{\text{old}} = [\hat{R}_{\text{old}} \quad \hat{\theta}_{\text{old}}]^T$ and error covariance $\hat{\mathbf{\Sigma}}_{\text{old}}$, where

$$\hat{\mathbf{\Sigma}}_{\text{old}} = \begin{pmatrix} \Sigma_{RR}^{(\text{o})} & \Sigma_{R\theta}^{(\text{o})} \\ \Sigma_{R\theta}^{(\text{o})} & \Sigma_{\theta\theta}^{(\text{o})} \end{pmatrix}.$$

The AoA estimate at receiver $\mathbf{R}$ is $\hat{\theta}$ with spatial spread $\Sigma_{\theta\theta}^{(\text{n})}$. After some manipulation (cf. chapter 8, [19]), the updated location estimate at receiver $\mathbf{R}$ is obtained as

$$\hat{R}_{\text{new}} = \hat{R}_{\text{old}} + \frac{\Sigma_{R\theta}^{(\text{o})}(\hat{\theta} - \hat{\theta}_{\text{old}})}{\Sigma_{\theta\theta}^{(\text{o})} + \Sigma_{\theta\theta}^{(\text{n})}} \; ; \; \hat{\theta}_{\text{new}} = \frac{\Sigma_{\theta\theta}^{(\text{o})}\ \hat{\theta} + \Sigma_{\theta\theta}^{(\text{n})}\ \hat{\theta}_{\text{old}}}{\Sigma_{\theta\theta}^{(\text{o})} + \Sigma_{\theta\theta}^{(\text{n})}}. \tag{4}$$

The update for the covariance matrix is

$$\hat{\mathbf{\Sigma}}_{\text{new}}^{-1} = \hat{\mathbf{\Sigma}}_{\text{old}}^{-1} + \begin{pmatrix} 0 & 0 \\ 0 & (\Sigma_{\theta\theta}^{(\text{n})})^{-1} \end{pmatrix}. \tag{5}$$

When the observations $\hat{\theta}_{\text{new}}$ have Gaussian errors as in the LOS model, the LMMSE updates in (4) are also the optimal minimum mean squared error updates. Under other non-Gaussian error models, the LMMSE updates are the optimal linear updates in the mean squared error sense.

In this manner, given priors on the source location and new AoA estimates, an updated source location estimate $\hat{\mu}_{\text{new}}$ and error covariance $\hat{\mathbf{\Sigma}}_{\text{new}}$ can be produced in a polar coordinate system centered at the receiver. This location estimate and error covariance is then transformed to the global cartesian system (a common frame of reference) to provide the information in a form accessible to the next receiver. These transformations between polar and cartesian systems are easily derived and are omitted here for brevity.

3.2 Sequential Localization Algorithm

We now describe the steps involved in sequentially aggregating receiver AoA estimates to produce a location estimate. Let N receivers be located at $\underline{X}_k^c = [x_k \ y_k]^T$ indexed by k, and let the source be at $\underline{X}_s = [x_s \ y_s]^T$. Receiver k's AoA estimate is $\hat{\theta}_k$. Further, the polar coordinates with receiver k at its origin is designated $\mathcal{P}_k$. For ease of exposition, we assume that the receivers are indexed in the order in which their estimates are combined.

The Bootstrap procedure: The algorithm is initialized by considering the first two receivers in the combining order, which are receivers 1 and 2 by convention. Their AoA estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ are used to obtain an initial estimate of the source location and error covariance to "bootstrap" the Bayesian algorithm (see Figure 1). However, care must be taken to ensure that the initial estimate is within the space of possible source locations. When the direction of arrival to two receivers are almost parallel, small errors in AoA estimates can lead to extremely large location errors and hence, such initializations are avoided. The sequential algorithm for source localization has the following steps:

Step 1 (Bootstrap): Estimate initial location $\underline{\hat{\mu}}^{(1)}$ and error covariance $\hat{\mathbf{\Sigma}}_{\text{pol}}^{(1)}$ in $\mathcal{P}_1$. Transform the location and error covariance into the global cartesian coordinates as $\underline{\hat{X}}_s^{(1)}$ and $\hat{\mathbf{\Sigma}}_{\text{car}}^{(1)}$. Pass *prior* $[\underline{\hat{X}}_s^{(1)}, \hat{\mathbf{\Sigma}}_{\text{car}}^{(1)}]$ to the next receiver.

Step 2 (Transformation): Let the index of the current receiver be k. Transform the priors $[\underline{\hat{X}}_s^{(k-1)}, \hat{\mathbf{\Sigma}}_{\text{car}}^{(k-1)}]$ to $[\underline{\hat{\mu}}^{(k-1)}, \hat{\mathbf{\Sigma}}_{\text{pol}}^{(k-1)}]$ in the local polar coordinates, $\mathcal{P}_k$.

Step 3 (Aggregation): Update the prior estimates $[\underline{\hat{\mu}}^{(k-1)}, \hat{\mathbf{\Sigma}}_{\text{pol}}^{(k-1)}]$ with the AoA estimate of the kth receiver, $\hat{\theta}_k$, using (4) and (5). Transform updated estimates $[\underline{\hat{\mu}}^{(k)}, \hat{\mathbf{\Sigma}}_{\text{pol}}^{(k)}]$ into the global cartesian coordinates as $[\underline{\hat{X}}_s^{(k)}, \hat{\mathbf{\Sigma}}_{\text{car}}^{(k)}]$.

Step 4 (Termination): If unaggregated AoA estimates exists, pass priors on to the next unaggregated receiver and go to *Step 2*, otherwise output the location estimate and Stop.

Since only a mean and covariance need to be exchanged, this algorithm is amenable to a completely distributed implementation, with each receiver needing to know only its own location and orientation. Moreover, this sequential Bayesian algorithm is quite general, and can, for example, incorporate probabilistic information on the source range obtained from signal strength measurements. Further, the complexity of this algorithm grows only linearly in the number of receivers. This scalability is required to realize performance improvements with the number of receivers.

4. NLOS LOCALIZATION SCENARIOS

We now address the narrowband multipath scenario, described in Section 2, in which some receivers experience LOS blockage and thus produce outlying AoA measurements. In Section 4.2, we develop a randomized algorithm that uses robust estimation techniques to estimate the source location, while mitigating the effects of the *outlying* NLOS measurements. This outlier suppression algorithm uses the sequential algorithm in Section 3.2 as a building block and is motivated by the ML algorithm presented in Section 4.1. Although we focus on the narrowband scenario here due to its simplicity, we outline how the source can be located in the wideband scenario using only the "good" LOS AoA estimates, where each receiver produces a list of source bearings.

4.1 ML Localization

Under the narrowband mixture model described in (3), the ML location estimate $\underline{\hat{X}}$ can be approximated as

$$\underline{\hat{X}} \approx \arg\min \sum_{k=1}^{N} \min\left[(\hat{\theta}_k - \theta_k(\underline{X}))^2, \Theta_{\max}^2\right], \qquad (6)$$

where $\hat{\theta}_k$ are the AoA estimates and $\Theta_{\max}$ is a function of σ^2 and α. The ML algorithm in (6) is the standard LS minimization, where only those angular errors below the threshold $\Theta_{\max}$ are minimized. This cost function ensures that there is no incentive to reducing 'large' angular errors (larger than the threshold) and therefore, outliers or NLOS estimates only have a limited effect on the location estimate. This ML method provides one possible modification of the LS metric to suppress outliers, however, other nonlinear functions of the error [13] can be used, but are not considered here. As desired, the ML estimator in (6) reduces to the ML method for the LOS scenario in the absence of outliers ($\alpha \to 0$), and the threshold $\Theta_{\max} \to \infty$. To extend the sequential algorithm to the NLOS scenario, we impose this constraint, $\Theta_{\max}$, on the largest observed angular error at each step in the algorithm and describe this modified algorithm in the following section.

4.2 Sequential Aggregation with Outlier Suppression

In NLOS scenarios, the localization algorithm must identify the subset of receivers with LOS channels and use these AoA measurements to estimate the source location. Since the problem of searching for the largest subset of mutually consistent receivers is prohibitive computationally, we resort to a randomization of the algorithm in Section 3.2, in which we randomize the choice of the first two receivers used in the bootstrap phase. Thereafter, each subsequent receiver's AoA estimate is combined ensuring that angular errors in all the aggregated receivers remain below the threshold $\Theta_{\max}$. This procedure is terminated if no more AoA estimates can be aggregated without causing some angular errors to exceed the threshold. Thus, by bootstrapping with different pairs of receivers, this randomized algorithm produces source position estimates corresponding to different subsets of receivers that mutually agree, leading to a list of possible explanations for the observed $\underline{\hat{\theta}}$.

Sequential aggregation with outlier suppression involves repetitions of the following basic steps with multiple random bootstraps:

Step 0 (Initialization): Set the list of receivers already aggregated $\mathcal{A} = \emptyset$ and list of receivers yet to be combined, $\mathcal{C} = \{1, \ldots, N\}$.

Step 1 (Bootstrap): Select a new pair of receivers at random, say $\{i, j\}$. Compute the initial estimate $\underline{\hat{X}}_s$ and error covariance $\hat{\mathbf{\Sigma}}$ using the bootstrap procedure. Add $\{i, j\}$ to the list of aggregated (inlying) receivers, $\mathcal{A} = \mathcal{A} \cup \{i, j\}$, and remove it from the list of remaining receivers $\mathcal{C} = \mathcal{C} - \{i, j\}$.

Step 2 (Angular Error Computation): Compute angular errors $e(\hat{\theta}, \underline{\hat{X}}_s)$ over the remaining receivers $\mathcal{C}$, i.e.,

$$e(\hat{\theta}_i, \underline{\hat{X}}_s) = |\hat{\theta}_i - \theta_i(\underline{\hat{X}}_s)| \quad \forall\ i \in \mathcal{C}.$$

Step 3 (Candidate Selection and Aggregation): Find the receiver k with the smallest error $e(\hat{\theta}_k, \underline{\hat{X}}_s)$. Combine $\underline{\hat{X}}_s$ with $\hat{\theta}_k$ to obtain the new *candidate* estimate $\underline{\hat{X}}'_s$ and covariance $\hat{\mathbf{\Sigma}}'$.

Step 4 (Threshold Verification): If the angular error using the *candidate* location $\underline{\hat{X}}'_s$ is below the threshold for all the aggregated receivers, i.e.,

$$e(\hat{\theta}_l, \underline{\hat{X}}'_s) < \Theta_{\max},\ \forall\ l \in \mathcal{A} \cup \{k\},$$

then retain the *candidate* location and covariance, $\underline{\hat{X}}_s = \underline{\hat{X}}'_s$, and $\hat{\mathbf{\Sigma}} = \hat{\mathbf{\Sigma}}'$. Also, $\mathcal{A} = \mathcal{A} \cup \{k\}$.

Step 5 (Termination): Remove receiver k from further consideration $\mathcal{C} = \mathcal{C} - \{k\}$. If there are no remaining receivers ($\mathcal{C} = \emptyset$) then Stop else goto *Step 2*.

The above algorithm is repeated M times with different random bootstraps (each run is of $O(N^2)$) in order to detect the source with a high probability, and each run produces a likely source location and a confidence in that estimate. Of these M solutions, *the ML cost function* in Section 4.1 is used to select the most likely estimate. If in a wideband system, a receiver resolves two arriving paths, the above algorithm can still be used by introducing a second *virtual* receiver at the same location as the original receiver and assigning to it the second arriving path, while ensuring that both AoA estimates are not part of the same location estimate as that would be physically impossible.

Choice of M and $\Theta_{\max}$: Assuming that the AoA spread at each receiver and fraction of receivers with strong NLOS are known, the threshold $\Theta_{\max}$ from the ML estimator can be used. In practice, only the largest expected fraction of NLOS receivers, $\alpha_{\max}$, can be set for the specific deployment scenario, which results in a conservative threshold for lower levels of multipath scattering as $\Theta_{\max}$ is monotonically decreasing with α. We have observed numerically that the algorithm almost always converges to the vicinity of the true source location, if two receivers with LOS channels are chosen in the bootstrap phase (our current focus is on providing corresponding theoretical guarantees). Hence, we choose the number of randomizations M to ensure that probability of choosing two receivers with NLOS channels (or bootstrap failure) is below a desired level. We can derive a bounding relationship to choose M:

$$M \leq \frac{\log(P(\text{bootstrap failure}))}{\log(1-(1-\alpha)^2)}. \tag{7}$$

We observe that the probability of bootstrap failure is independent of N. This ensures that the outlier suppression algorithm, which has complexity $O(MN^2)$, is only quadratic in the number of measurements.

5. NUMERICAL RESULTS

We study the performance of the proposed algorithms via Monte-Carlo simulations. We consider a circular field of unit radius with N equally spaced receivers along the perimeter. As the localization error grows linearly with distance from the receiver, by selecting a field of unit radius, we obtain scale-invariant measures of performance. Each receiver produces AoA estimates that are generated according to the models described in Section 2. We assume throughout that all the receivers have the same AoA spread and that only a single source transmits at any given time. We use the CRLB and ML estimator as a performance benchmarks.

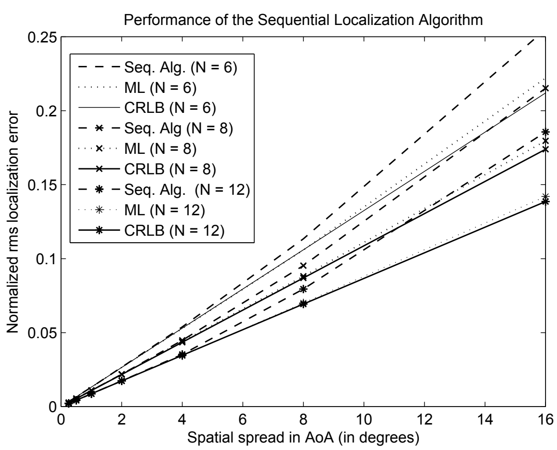

Figure 2: Localization performance of the sequential estimation algorithm (in dashed lines) in the LOS scenario for 6,8 and 12 receivers. The performance is compared against the CRLB (in solid lines) and the ML estimator (in dotted lines)

5.1 Performance under LOS scenarios

The performance of the sequential algorithm in Section 3.2 for different numbers of receivers is shown in Figure 2. The algorithm achieves the CRLB for small angular estimation errors (for $N \geq 6$ receivers), but the performance deteriorates for large spreads in AoA. The sequential algorithm, which is an instance of a stochastic approximation algorithm [14], has a tendency to get stuck in local minima for large AoA spreads, i.e., when there are multiple smaller subsets of mutually consistent receivers. This problem can be addressed by multiple runs of the sequential estimation using different pairs of receivers to bootstrap, and thereafter, by selecting the most likely location (i.e., has the *highest ML cost*), like that employed in the outlier suppression algorithm in Section 4.2. Note, that the simulation used random combining orders for the receivers, which appears to lead to insignificant loss in performance against the CRLB. In reality, the nonlinear coordinate transformations at each step in the sequential algorithm are location dependent making the final estimate weakly dependent on the specific order.

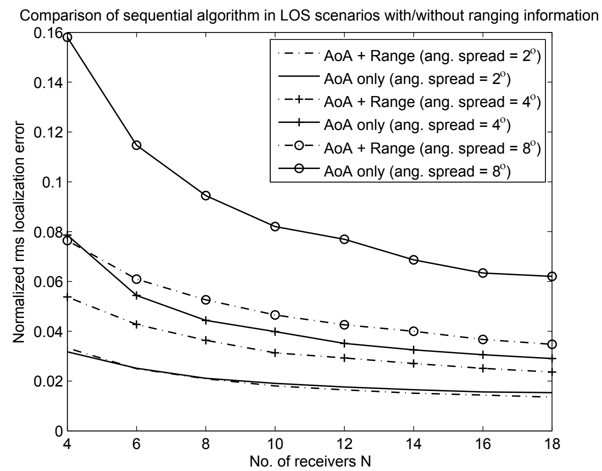

Figure 3: Localization performance in a system with only AoA information and one where all nodes have RSS-based range estimates in addition to AoA measurements for different numbers of receivers.

In order to demonstrate the flexibility of our algorithm to leverage range information as well, we compare our system, utilizing only AoA measurements, to one where all the receivers additionally obtained RSS-based range information. The RSS-based range estimate is modeled as a log-normal variable (independent of the AoA estimate) [18].

Using an extension to the update formulation in Section 3.1 to include range estimates, we show in Figure 3 the localization performance for different AoA spreads under LOS conditions. We observed that when the fractional range estimation errors were at least as large as the fractional AoA measurement errors (middle and lower pair of curves), resolution using both AoA and range modalities is similar to that with AoA alone with moderate increases in N. However, with very good range measurements, very large increases in N are necessary to achieve competing resolution.

5.2 Performance under NLOS scenarios

We present next numerical results for the narrowband multipath model, where each receiver produces a single AoA estimate corresponding to either the LOS path, or the reflected and scattered multipath. In Figure 4, the localization performance of the outlier suppression algorithm proposed in Section 4.2 is shown to be very close to that of the optimal ML estimator in Section 4.1 for a system with 8 receivers. For different fractions of outliers, α, the outlier suppression algorithm is run with the optimal ML threshold $\Theta_{\max}$. The number of random seeds, $M = \{4, 7, 11, 15\}$, was $\alpha = \{0.125, 0.25, 0.375, 0.5\}$ (or $\{1, 2, 3, 4\}$ outliers) using (7) to achieve a probability of bootstrap failure less than 10^{-3}. The ML estimate was obtained by brute force minimization of the same cost function.

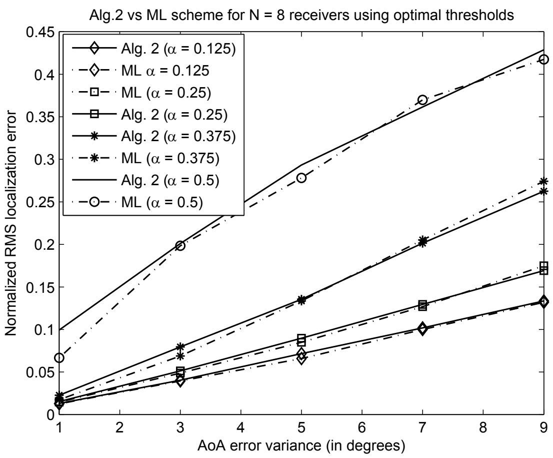

Figure 4: Localization performance with NLOS suppression for different fractions, α, of receivers with NLOS channels with optimal thresholds. The location error variance (solid line) is compared against the ML error variance (dash-dotted line).

6. CONCLUSIONS

In LOS scenarios, sequential algorithm of linear complexity nearly achieves the CRLB. In NLOS multipath scenarios, although the outlier suppression algorithm is approximately ML, its performance is heavily dependent on the specific type of environment and the capability of the receivers to resolve the contributions of the LOS path and NLOS multipath in the received signal. In a narrowband system, where each collector resolves only a superposition of the arriving paths, the algorithm performs effectively given a sufficiently large number number of receivers with strong LOS channels. However, the capacity of a wideband system to estimate AoA from the LOS and multipath individually, is vital for good performance in settings where all the collectors experience NLOS propagation. The proposed algorithms are amenable to distributed implementation and can incorporate RSS-based range measurements as well.

There are however several open issues that need further investigation such as physical layer dependent AoA models, capability to handle multiple simultaneous transmissions, guarantees on convergence, using time of arrival measurements and effects of propagation loss and multipath fading.

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