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Simple Localization with Sensors on Grid

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***Abstract*—This paper proposes a novel and simple indoor localization system based on sensors on a grid and on radio map approach. We analyze the impact of different system variables on the localization error: the number of beacons composing the grid, the training points considered during the construction of the radio map, the radio channel fluctuations, the presence of multiple objects to localize simultaneously.**

We are able to give a mathematical expression for the localization error as a function of the training points used in the radio map. In case of multiple objects we specify their proper coverage range that can maximize the throughput and minimize the latency of the localization procedure for a given localization accuracy.

I. Introduction

Context-aware applications are one the most studied field of next generation communication systems [1] [2] and localization of mobile terminals is a fundamental step to support context-aware services. Furthermore, in recent years, wireless sensor networks have become an evolving technology that has a wide range of potential use localization applications [3]. The aim of this work consists in the localization of mobile objects inside a room through the exploitation of sensors on a grid. In particular as a case study we have considered the localization of shopping carts inside a shopping center. The particular scenario chosen as a case study is an example of scenario in which in fact there is the need to build a localization system simple, efficient and cheap. The main idea is to build a grid of beacons on the ceil of the room, placed at distance l meters one from the other. This can help in exploiting some pre-existent grids structures placed over the room ceil inside the shopping centers and reducing therefore the total architectural cost. Each beacon is a radio receiver that is able to estimate the power received by a transmitter embedded in each shopping cart. Information concerning the received power is directly related to the distance between the transmitter and the receiver taken into consideration. For the sake of simplicity and without loss of generality we have considered the shopping center area as a parallelepiped of dimension $X \times Y \times Z$. All the results presented in this paper refer to a room of height $Z = 7$ meters, and each cart having the transmitter located at height $z = 1$ meter.

The localization mechanisms is based on a radio map approach. In general this is realized thanks to the use of N_b beacons, that are radio receivers with known positions, and it consists of two phases: a training phase and the actual localization phase. During the training phase a transmitter is moved in a set of known positions (training positions) around the total area and the power levels received by a set of N_b beacons present in the area are stored in a map in correspondence of each of the training positions; so for each training position a N_b-tuple of values, representing power levels received by the N_b beacons, is stored. The second phase consists in comparing all the stored N_b-tuples related to all the training positions, with the N_b-tuple containing the levels of power received from the beacons by the object in the unknown position: the position of the object is estimated as the one of the training point which has a tuple in the stored map that is the most similar to the tuple measured in the unknown position during the localization phase. Typically the metric used to identify the most similar tuple consists in evaluating the minimum difference between two tuples of values. The training phase is realized by considering points in a virtual grid; the distance between two consequent training points is equal to Δ meters. The carts should transmit a small packet containing their identifier. A central entity should store the tuples, perform the comparisons and make a decision.

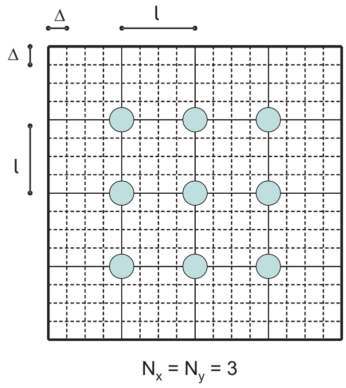

Fig. 1. Example of grid localization scenario. The total area is a square with $N_x = N_y = 3$ beacons per side, with reciprocal horizontal and vertical distance l. The total amount of beacons is therefore $N_b = N_x N_y = 9$. The training phase is realized considering points at distance Δ.

Suppose to have a square area of side $X = Y$ and a number $N_x = N_y$ of beacons per side, so a total amount of beacons available equal to $N_b = N_x N_y$, placed as shown in Figure 1. Localization operation works as follows. Call $T_c = [Pc_1, Pc_2, \cdots, Pc_{N_b}]$ the tuple of received power levels measured at all the beacons after the transmission of the signal by the cart to be localized. Call $T_i = [Pi_1, Pi_2, \cdots, Pi_{N_b}]$ the tuple of received power levels at the beacons during the training phase, referred to the $i-th$ training point. There

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are $Tot_{tr} = (\frac{X}{\Delta}+1)(\frac{Y}{\Delta}+1)$ training points. The following metric is evaluated, for every training point $i = 1, \cdots, Tot_{tr}$: $m_P = \sum_{j=1}^{N_b} |T_c(j) - T_i(j)|$. The point selected as estimated position for the cart is the point i of the training map for which the metric m_P is the minimum one.

We are interested in evaluating the localization error due to this grid localization method in different working conditions. The localization error is defined as the Euclidean distance between the point in which the cart to be localized is actually located and the point resulting from the localization operation. This work is intended to study different aspects that affect the localization error, such as: the impact of the number N_b of beacons composing the grid, the impact of the training points taken into account during the training phase, the impact of the radio channel fluctuations, the impact of the presence of multiple carts simultaneously in the same area. This paper presents results related to the aforementioned aspects and obtained after a deep simulation campaign.

Literature offers other studies of localization systems in the field of wireless sensor networks, based on grid topologies [4], [5], [6]. Nevertheless to the author knowledge these regards grids composed by both sensors and anchor nodes (nodes with known position) and the target is to define optimal positions inside the grid for the anchor nodes in order to improve the other sensors localization. So these previous works differentiate with the approach presented in this work.

The paper is organized as follows: Section II presents performances analysis of a scenario with a single cart, both in an ideal radio map situation II-A and by considering wireless channel fluctuations II-B; Section III studies the performances in the presence of more carts in the same area. Finally Section IV concludes the paper.

II. Results with a Single Cart

In this section we consider to have only one cart in the total area, so we do not consider here potential interference among different carts and collisions among their transmitted signals.

A. Ideal Radio Map

We consider ideal radio map the case in which the power levels measures are not affected by possible errors and received power levels fluctuations typical of a wireless channel. This optimal and theoretical case is necessary in order to understand the potential bound obtainable as a performance of this localization approach.

In this ideal case, if the cart to be localized is located exactly in a training point, the power level measured during the localization phase and the one measured during the training phase in that particular point coincide.

This means that in this ideal scenario the localization error should be influenced only by two factors: the training parameter Δ and the availability of a sufficient number of reference points in the tuple used for the localization, the parameter N_b. The last condition influences the localization error since an insufficient number of beacons could potentially

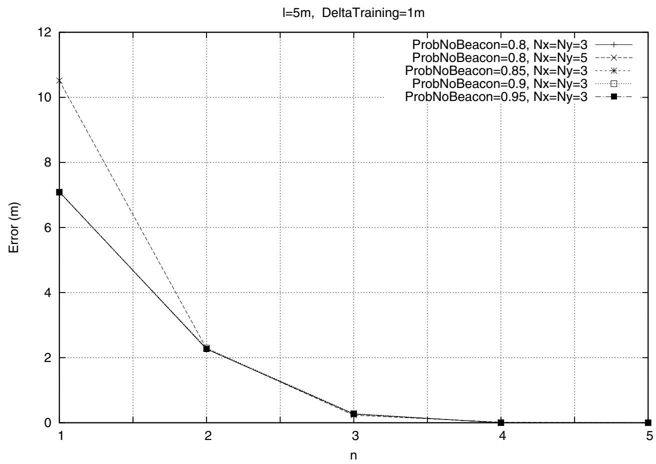

Fig. 2. Localization error as a function of the total number of beacons (n) present in the area. The localization process has considered a training map with a training step $\Delta = 1$ m and an inter-beacons distance $l = 5$ m.

lead to have different points with the same value of the metric used to discriminate the possible resulting points.

Figure 2 confirm this statement. The n beacons used for the localization phase and considered in the x axes of the graphic are obtained by building an initial grid composed by $N_b = N_x \times N_y$ beacons and by removing each beacon with a uniform distributed probability $Prob\_No\_Beacon$. This means that the curve obtained is the result of a an average of cases in which the n beacons used are located in different positions inside the grid. The probability of having n beacons available for the localization is expressed by the following Equation: $P(n) = \binom{N_b}{n}(1-\beta)^n\beta^{(N_b-n)}$. So the mean number of beacons available for the localization is $n = N_b(1-\beta)$. We have also verified that a scenario composed by N_b beacons in the grid and a $Prob\_No\_Beacon = \beta$ and a scenario with a grid composed by n beacons give the same results in terms of mean localization error.

The results plotted in this Figure have been obtained with a set of simulations where the cart has been placed exactly in a position coincident with a position considered during the training phase, therefore a possible localization error could only be due to the presence of positions that have with the same value of the metric; in this case in fact the localization algorithm simply stops by choosing one among these possible points. We can observe that when n is greater than four this effect is vanishing. The entity of the error depends on the area considered, so it depends on the parameters l or N_x, N_y if l is fixed. It also depends on the cart position, this is why the Figure has been obtained by averaging a lot of cases in which the cart position has been randomly chosen among all the possible training positions.

Figure 3 shows the localization error as a function of the number of beacons present per side in a square area ($N_x = N_y$) and for different value of the distance inter-beacon: $l = 5, 10, 20, 30$ m. The training phase has been realized with $\Delta = 1$ m. We can see that the error is independent from the parameter l while it depends on the number of available

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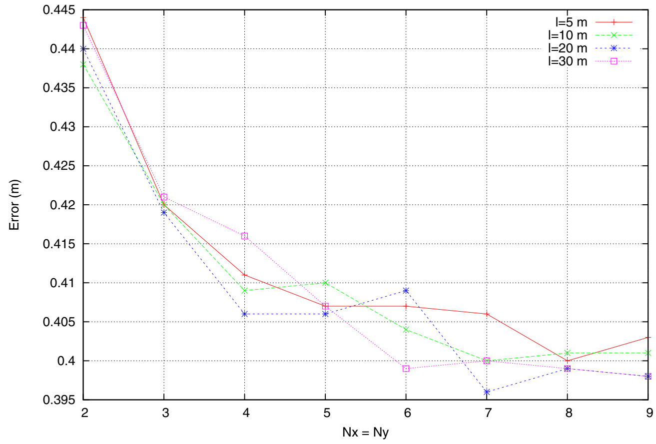

Fig. 3. Localization error as a function of the number of beacons per side ($N_x = N_y$) present in the area and for different values of the inter-beacon distance $l = 5, 10, 20, 30$ m. The localization process has considered a training map with a training step $\Delta = 1$ m.

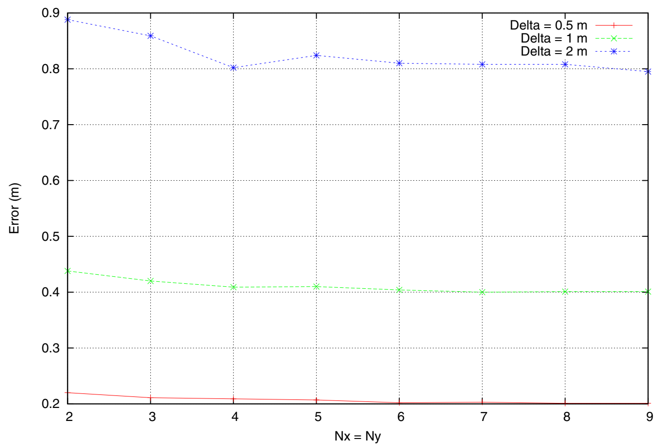

Fig. 4. Localization error as a function of the number of beacons per side ($N_x = N_y$) present in the area and for different values of the inter-beacon distance $l = 5, 10, 20, 30$ m. The localization process has considered a training map with a training step $\Delta = 1$ m.

beacons and it shows a floor as the number of beacons tends to increase. The same trend is confirmed also by the Figure 4 plot the results obtained with training step values of 0.5m, 1m and 2m. The difference between these three cases in represented by the value of the floor, that results therefore a function of the parameter Δ. These considerations suggest that we can try to evaluate the floor value when border effects can be neglected.

To this purpose the following model can be adopted.

During the simulation we have placed the cart by following a random uniform distribution inside the total area. We focus the attention on a single little square of side Δ, delimited by the four corners coincident with four points of the training phase, as shown in Figure 5. We can say that the cart location is uniformly distributed inside this square area. We can neglect the variable z since it does not modify the results, being the always same for the cart and the beacons. So we focus the attention on the coordinates x and y that are both variables described with a uniform pdf: $f_X(x) = 1/\Delta$ if $0 \le x \le \Delta$ and 0 otherwise; $f_Y(y) = 1/\Delta$ if $0 \le y \le \Delta$ and 0 otherwise. The variable x and y are independent, so the joint pdf describing the cart position is $f_{XY}(x, y) = 1/\Delta^2$ if $0 \le x \le \Delta, 0 \le y \le \Delta$ and 0 otherwise. Let (x, y) be the actual position of the cart to localize. If the number of beacons used for the localization is enough the localization algorithm is able to give as a result the training point that is closer to the actual cart position; therefore in this case the localization error in only influenced by the Δ parameter. In this case the total space can be thought to be divided into square Voronoi regions, as specified in Figure 5, each composed by the points of the area that are closer to a training point.

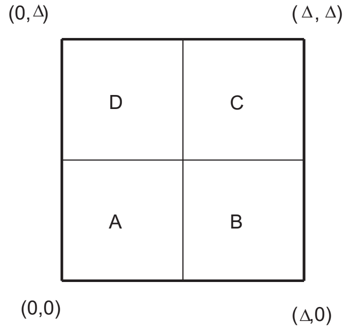

Fig. 5. Scenario utilized for the model.

The error can be defined in a proper manner by focusing the attention on a single Voronoi region and simply multiply by a factor 4 that considers all the four regions. By considering for example the region called A in Figure 5, we can say that the localization algorithm will assign the cart the training position $(0, 0)$, so the error is $\sqrt{x^2 + y^2}$. So the error can be calculated as:

$$\begin{aligned} \epsilon(\Delta) &= \frac{4}{\Delta^2} \int_0^{\Delta/2} \int_0^{\Delta/2} \sqrt{x^2 + y^2}\, dx dy \\ &= (\sqrt{2} + ArcSinh(1))/6 \end{aligned} \quad (1)$$

The error function $\epsilon(\Delta)$ is a straight line with angular coefficient $(\sqrt{2} + ArcSinh(1))/6 \cong 0.3826$. Numerical results from this model are the following: $\epsilon(0.5) = 0.19$, $\epsilon(1) = 0.38$, $\epsilon(2) = 0.76$. While the numerical results for the error floor obtained by simulations are: $\epsilon(0.5) \cong 0.2$, $\epsilon(1) \cong 0.4$, $\epsilon(2) \cong 0.8$.

B. Radio Map and Wireless Channel Fluctuations

The hypothesis of ideal radio channel is actually never realized in practical cases. In general the conditions under which the localization is realized differ from the original ones that have characterized the training phase. Differences influence the power received levels at the beacons and the causes may be due for example to the presence of people or obstacles that were not present during the training phase. As a result the localization is affected by these elements and typically the localization error tends to increase since, as a

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consequence of these power levels fluctuations, an object is estimated closer or farther from its actual position.

We have chosen to model the wireless channel perturbations affecting the localization phase with a gaussian distributed random variable that is added to the original power level stored into the map.

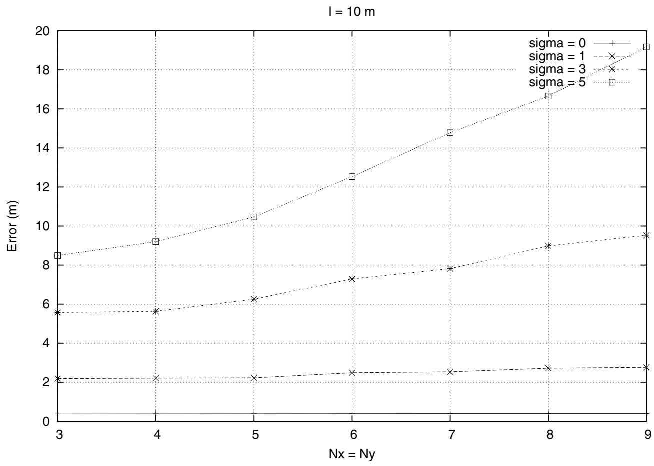

Fig. 6. Localization error as a function of the number of beacons per side ($N_x = N_y$) present in the area and for different values of the parameter $\sigma = 0, 1, 3, 5$ dBm. $\Delta = 1m$.

Figure 6 shows the effects of the channel fluctuations on the localization error. These results have been obtained by simulating square ares composed by $N_b = 9, 16, 25, 36, 49, 64, 81$ beacons with inter-beacons distance of 10 meters. Different curves are related to the simulation of different channel conditions, characterized by the parameter $\sigma = 1, 3, 5$ dBm compared to the ideal case ($\sigma = 0$). The increasing trend of he error with the increasing of the variable N_x, N_y is due to the fact that, on equal terms of l, the considered area increases. This trend is more evident for high values of σ while the error tends to be independent from the area for small values of σ. The effect of the channel variations and of the environment variations can be mitigated, as suggested in many papers, by increasing the complexity of the radio map construction, by storing for each training point the average of more power levels, measured in different conditions (angular position, people present in the room, obstacles in the path); nevertheless also in this case the training phase and the localization phase happen in different actual condition, so the approach of modeling the channel with a gaussian variable can nevertheless be adopted to characterize any possible change of the initial condition and to estimate its consequence on the localization accuracy.

III. Results with Multiple Carts

This Section investigates the impact of the presence of multiple carts in the same area. The carts can access the wireless channel to realize the localization by following the aloha approach. Each cart transmits its message at random instants, in order to minimize the possibility of collision. In case of collision the cart localization is not performed: the cart should transmit its identification message again after a random interval time. So the aloha access mechanism only affect the latency of the localization operation: the localization if all the carts is ensured but generally it can involve more than one localization attempt. In order to study the worst possible case of collisions we consider therefore the case in which all the carts taken into consideration are sending their localization packet simultaneously. In this way we can understand the entity of the number of carts that can be present simultaneously in the area in order to obtain a certain level of performances, concerning the localization error and the percentage of carts that are able to realize the localization phase in a single attempt. This is a useful information to achieve, especially if carts mobility is considerably high and localization of each cart has to be realized by a certain time.

In the rest of the work we will use the following parameters. Let R be the coverage range of each cart; we consider all the carts to have the same range. In the previous Section with only one cart we have always considered R to be sufficiently high in order to have all the beacons of the grid in car visibility. Obviously, in a scenario with multiple carts, if all the carts have a range that can cover the whole area the number of carts that can perform the localization simultaneously is null; so we have to limit the radio coverage range of the carts and to study the entity of this parameter in order to maximize the number of carts that can be localized simultaneously with a predefined accuracy. To do that we introduce the parameter b, that represents the mean number of visible beacons for each cart. If the grid is composed by N_b beacons, b can be approximated (a part from the border cases of the area) with the following expression: $b \cong \frac{N_b \pi (R^2 - (Z-z)^2)}{XY}$. The larger is the area, the less the border effects will vary in the practice this parameter. To simulate the reciprocal collisions of signals arriving to the beacons in visibility of more carts we remove from the list of visible beacons of each cart the ones that are receiving signals from more carts and for each cart we realize the localization estimate only with the beacons that are receiving signal only from the cart into exam; these beacons are called beacons available for the localization purpose of a cart.

In paragraph II-A we have verified from Figure 2 that in order to achieve the lowest error as possible, accordingly to the Δ parameter chosen during the training phase, at least four beacons should be used during the localization phase. So let $Loc\_Threshold$ be the minimum number of beacons that have to be available for a cart in order to make the localization process possible. If a cart has a number of beacons available that is lower, it can not perform the localization phase at the moment (it will try another attempt in a second moment).

By setting the parameter $Loc\_Threshold = 4$ we ensure that the cart that we define as localized will be affected by a localization error of magnitude $\simeq 0.3826\Delta$, as defined by the mathematical model described in II-A.

Figure 7 plots the localization error as a function of the number of carts present in the shopping center, for different room dimensions: $N_x = N_y = 5, 10, 20$. Simulations have

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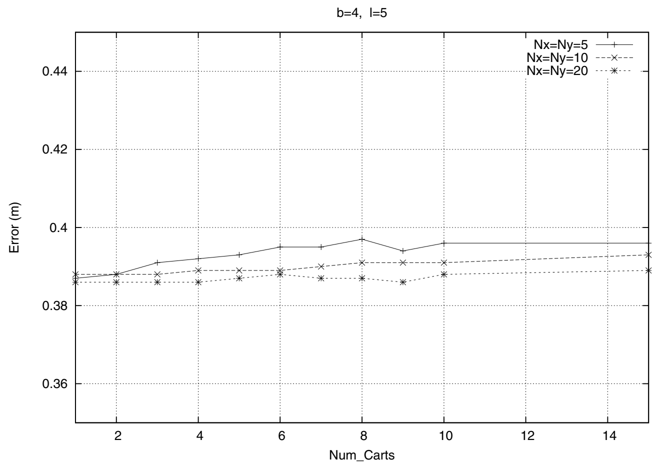

Fig. 7. Localization error as a function of the number of carts present in the shopping center, for different room dimensions: $N_x = N_y = 5, 10, 20$. Simulations have used the following parameters: inter-beacons distance of 5 meters, $b = 4$, $Loc\_Threshold = 4$.

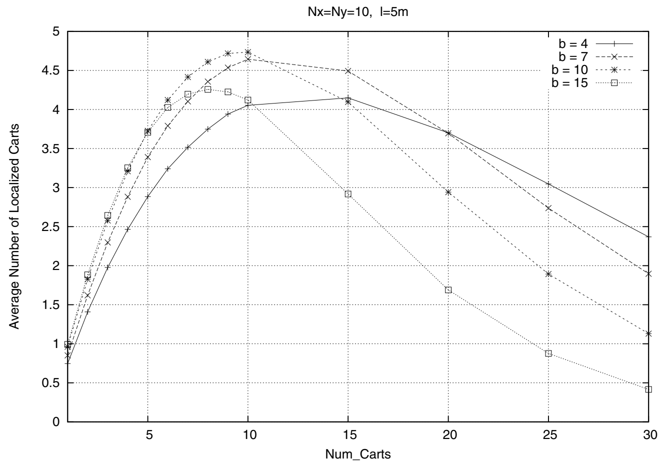

Fig. 8. Average number of localized carts as a function of the number of carts present in the shopping center and for different values of the number of visible beacons for each carts ($b = 4, 7, 10, 15$). The shopping center in this case is a room with $N_x = N_y = 10$ beacons per side, with inter-distance of 5 meters. A $Loc\_Threshold = 4$ has been used.

used the following parameters: inter-beacons distance of 5 meters, $b = 4$, $Loc\_Threshold = 4$. As stated before the error remains around 0.4.

Some simulations tests have been realized in order to understand the impact of the parameters b and $Loc\_Threshold$ on the system performances.

Figure 8 shows the average number of localized carts that we can obtain in a scenario composed by $N_x = N_y = 10$, $l = 5$ meters and $Loc\_Threshold = 4$: the curves are plotted for different values of the parameter $b = 4, 7, 10, 15$. First of all we notice a trend typical of aloha approaches: the curves present a point of maximum and then progressively decreases as the number of users users, so of requested traffic, increases. What's interesting is that until the number of carts is meagre the presence of more beacons in cart's visibility (bigger values of the parameter b) increases the number of carts that can be localized while as the number of carts increases performances get worse as the parameter b increases, since this increment the probability of collisions among the carts. So the availability of more beacons to take into account for the purpose of localization can improve the process only with limited carts density, otherwise the incremented number of collisions lowers substantially the number of carts simultaneously localized with a predefined accuracy. These considerations and the curves plotted in Figure 8 suggest that a choice of $b = 4$ can be a good trade off.

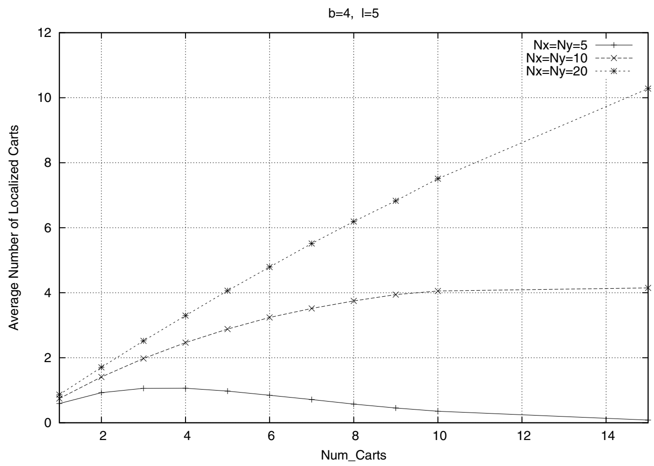

Fig. 9. Average number of localized cart as a function of the number of carts present in the shopping center, for different room dimensions: $N_x = N_y = 5, 10, 20$. Simulations have used the following parameters: inter-beacons distance of 5 meters, $b = 4$, $Loc\_Threshold = 4$.

The performances in terms of number of carts localized simultaneously with a defined accuracy is also dependent on the room dimensions. Figure 9 shows the average number of localized carts as a function of the total carts present in the area for different room dimensions: having fixed the inter-beacons distance at 5 meters the curves are realized for different area so for different number of beacons composing the grid ($N_x = N_y = 5, 10, 20$). Each cart has an average number of visible beacons equal to 4 ($b = 4$). Obviously as the area dimensions increases the probability of collisions among beacons decreases and the performances shows a positive improvement. So the choice of the parameter b should also take into account the area dimensions in order to understand the entity of the performance that can be achieved. Furthermore, the smallest the area, the more some border effects tends to reduce the performances for small values of b: for example with $b = 4$ and $N_x = N_y = 5$ also in case of only one cart the localization process can be unapplicable when the cart is located at the border of the room, for lake of beacons in its visibility (remember that b is a parameter that directly derives from the coverage radio range of a cart).

Finally another trade off can be achieved by setting a proper value of the parameter $Loc\_Threshold$. Figure 10 plots the average number of carts simultaneously localized in an area of $Nx = Ny = 10$ beacons with distance $l = 5$ meters and $b = 4$, for different values of $Loc\_Threshold = 1, 2, 3, 4$. We

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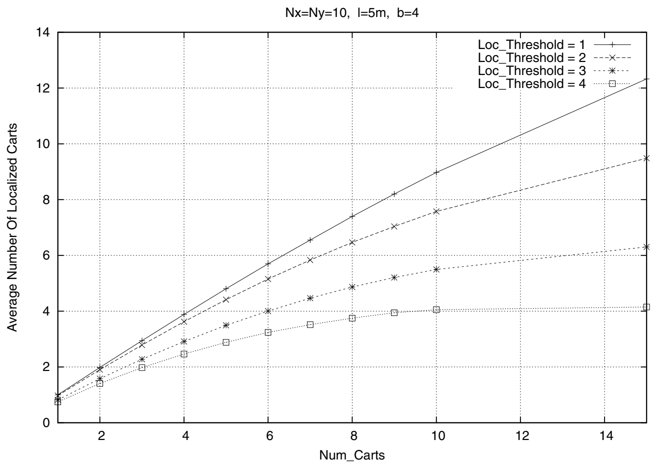

Fig. 10. Average number of localized cart as a function of the number of carts present in the shopping center, for different value of the localization threshold $(1, 2, 3, 4)$. The shopping center in this case is a room with $N_x = N_y = 10$ beacons per side, with inter-distance of 5 meters, and $b = 4$.

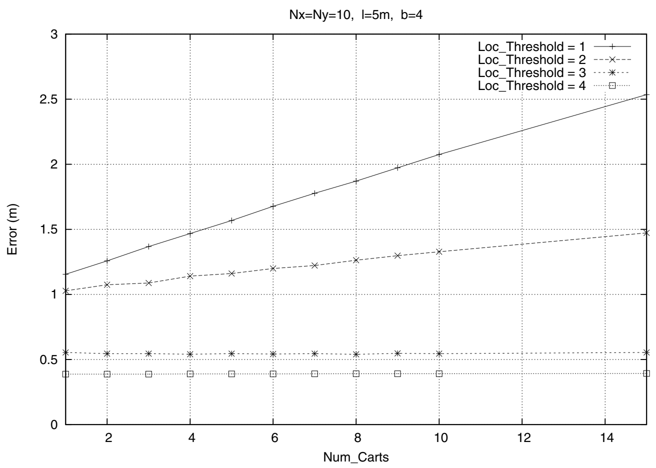

Fig. 11. Localization error as a function of the number of carts present in the shopping center, for different value of the localization threshold $(1, 2, 3, 4)$. The shopping center in this case is a room with $N_x = N_y = 10$ beacons per side, with inter-distance of 5 meters, and $b = 4$.

can see that by decreasing the limit of beacons that have to be available for each cart in order to localize it the number of collisions decreases and the number of localized carts increments. Nevertheless by decreasing the localization threshold the localization error that affects the carts localization can be bigger that the one ensured with $Loc\_Threshold \geq 4$ and set during the training phase with the selected Δ parameter (see Figure 11). Practical use of these results could be the following: the training phase could be realized with a big precision (small value of Δ) since, being realized only once it does not imply too much effort; then the error accuracy that the final user desire to obtain can be chosen dynamically according to the specific application: so in some cases also an error bigger than the one forecast from the Δ parameter used in the training phase can be sufficient and in this case a smaller localization threshold can so be chosen giving a performance improvement in terms of number of simultaneous localized carts (so of localization speed).

IV. Conclusions

The localization system prosed and described in this paper is a very simple approach that can be used for indoor applications.

We have shown how to obtain a final desired accuracy by setting some basic parameters, such as the number of beacons composing the grid and the training points considered during the construction of the radio map. Furthermore we have shown the entity of the error variations due to external and non predictable environment agents (presence of obstacles or people). In case of multiple objects we specify their proper coverage range that can maximize the throughput and minimize the latency of the localization procedure for a given localization accuracy.

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