Comparative Study of Joint TOA/DOA Estimation Techniques for Mobile Positioning Applications

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***Abstract*—In this paper, we present a comparative study of the performance of the joint Time of Arrival/Direction of Arrival (TOA/DOA) estimation techniques - Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT), Joint Angle and Delay Estimation (JADE) and Space-Alternating Generalised Expectation maximisation (SAGE) - in the context of a strong multipaths model. We focus on a number of performance issues that are of particular interest to the mobile positioning application, such as the estimation accuracy of the Line of Sight (LOS) component within a strong multipaths environment and their robustness to channel model errors. In addition, we also analyse the computational complexity of the algorithms with respect to the number of paths in the channel and the number of antenna elements. Through extensive simulations in MATLAB, we have shown that: 1) SAGE outperforms the other two algorithms in terms of TOA/DOA estimation variance in a strong multipaths environment, thus can give much more accurate position estimate provided the LOS component is relatively strong. 2) SAGE is more robust to channel modeling errors. 3) SAGE's computation complexity exhibits a linear relationship with the number of channel paths (while ESPRIT and JADE are independent) while being unaffected by the number of antenna elements.**

I. Introduction

Network-aided mobile-positioning techniques have attracted great interest in recent years because of their use in supporting highly-lucrative Location Based Services (LBS). Examples of LBS include the automatic dispatch of an emergency service to a mobile terminal's location, vehicle navigation, network optimisation, resource management and automated billing [1] as well as other cases such as indoor and urban environments, for which a GPS receiver is impractical. In addition, embedding GPS receivers into mobile devices leads to increased cost, size, and battery consumption. It also requires the replacement of millions of handsets that are already in the market [2]. It has been widely accepted that mobile-positioning techniques based on the estimation of the DOA and/or TOA of the signal from a mobile terminal at several cell towers can achieve higher positional accuracy than methods based on received signal strength [3].

Recently, various high-resolution joint channel parameter estimation methods have been studied and have potential as candidate algorithms for use in the mobile terminal location technology. Jointly estimating channel parameters, particularly joint TOA/DOA estimation, has a number of advantages [4]. Firstly, the relative estimates of TOA measured at two or more synchronised base stations can be used in conjunction with DOA information measured at each of them to enhance positional accuracy. Secondly, joint estimation can resolve rays having identical directions or times of arrival. Finally, it is possible to locate a mobile device using the joint TOA/DOA estimate method with only one sychronised base station.

The TOA/DOA joint estimation algorithms can be classified into two broad groups, based on their development and fundamental philosophy:

a) Subspace based: These methods rely on each parameter being estimated from a certain eigenvalue problem, where all eigenvalue problems share the same eigenvectors. This allows the problem to be posed as a joint diagonalisation of a collection of data matrices. Such methods include the Joint Angle and Delay Estimation (JADE) technique [5], and the Multi-Dimensional Estimation of Signal Parameters via Rotational Invariance Technique (MD-ESPRIT) [6].

b) Maximum likelihood: The maximum likelihood (ML) method, with its associated optimal properties, shows superior performance at low SNR, when the number of samples is small or the sources are correlated. Several computationally efficient algorithms based on the ML approach have been developed. These include the Expectation Maximisation (EM) [7] and the Space-Alternating Generalised Expectation maximisation (SAGE) [8][9].

While most of these algorithms have been applied with some success either to measured data or realistic channel models, no single comparative study of their relative performance has been reported. The most recent comprehensive surveys are reported in [10] [11], which cover the classical techniques for parameter estimation, such as Beamforming, Weighted Subspace Fitting (WSF), and MUltiple SIgnal Classification (MUSIC). In addition, these literatures mainly cover single parameter estimation, and not the joint case. A comparison between the SAGE and ESPRIT algorithms for 3D channel sounding has been undertaken by Tschudin *et al.* [12] in a study which concentrated on resolvability and multipath identification issues. However, this is not particularly relevant for mobile positioning where knowledge of the accuracy of TOA and DOA estimation is the priority.

This paper therefore presents a comparative study on the

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performance of some of the recent joint TOA/DOA estimation techniques for mobile positioning applications. We implemented the 2D Unitary ESPRIT [6], Shift Invariance-JADE (SI-JADE) [13] and SAGE algorithms in MATLAB, and carried out Monte Carlo simulations. We focus a number of issues which are of particular interest to mobile positioning applications, such as the estimation accuracy of the LOS component within a strong multipaths environment, as well as the robustness of these algorithms when the order of the channel model is not accurately estimated. In addition, we also analyse the computational complexity of the algorithms with respect to the number of paths in the channel and the number of antenna elements.

II. System Model

A typical wireless multipath propagation channel is illustrated in Figure 1, where data symbols from a mobile device are modulated by a known pulse shape and transmitted through the spatial multipath channel. The transmitted signal then undergoes a series of scattering processes (namely diffraction, reflection and refraction) before arriving at the M elements of a Uniform Linear Array (ULA) at the receiver of a base station. There are assumed to be L different paths; the ℓ^{th} path is received from a particular DOA θ_ℓ, has a TOA τ_ℓ, and a complex path attenuation β_ℓ. The spatial multipath channel may then be modelled as the M-element impulse response vector

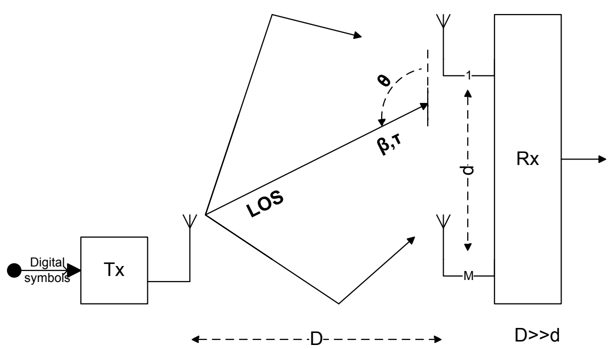


Fig. 1. Illustration of a wireless multipath environment. Each path is parameterised by its TOA τ_ℓ, DOA θ_ℓ, and complex path attenuation β_ℓ

$$\mathbf{h}(t) = \sum_{\ell=1}^{L} \mathbf{a}(\theta_\ell)\beta_\ell g(t-\tau_\ell) \tag{1}$$

where $\mathbf{a}(\theta_\ell)$ is an M-element vector describing the response of the array to a path from direction θ_ℓ, and $g(t)$ is the known modulation pulse shape.

The transmitted pulse shape $g(t)$ employed throughout the experiment is the Raised Cosine filter of excess bandwidth 0.35. The length of the pulse shape is $8T$, where T is the symbol period. While the oversampling factor P for each symbol period is 4.

III. A Brief Review of JADE and SAGE

This section provides a very brief overview of the JADE and SAGE algorithms. A detailed mathematical analysis of the algorithms will not be included due to the lack of space. Interested readers may refer to the references within. Since the ESPRIT algorithm is a very well-known parameter estimation technique, its details will be omitted completely.

A. JADE

The JADE [5] algorithm is a method that exploits the stationarity of the angles and delays, as well as the independence of fading over many time-slots in a time slotted mobile system, by combining multiple estimates of the channel impulse response over many time slots.

The array manifold vector $\mathbf{a}(\theta_\ell)$ and the delay manifold vector $\mathbf{g}(\tau_\ell)$ can be combined and re-defined as the *space-time response vectors*:

$$\mathbf{u}(\theta_\ell,\tau_\ell) = \mathbf{g}(\tau_\ell) \otimes \mathbf{a}(\theta_\ell) \tag{2}$$

where $\otimes$ denotes the Kronecker product.

Analogous to the traditional steering vector, the *space-time response vector* $\mathbf{u}(\theta_\ell,\tau_\ell)$ is the response of the array to a channel with a single path with direction θ_ℓ and delay τ_ℓ and includes the pulse-shape function. The corresponding space-time response matrix for the L paths is

$$\mathbf{U}(\boldsymbol{\theta},\boldsymbol{\tau}) = \mathbf{G}(\boldsymbol{\tau}) \circ \mathbf{A}(\boldsymbol{\theta}) = [\mathbf{u}(\theta_1,\tau_1),\ldots,\mathbf{u}(\theta_L,\tau_L)] \tag{3}$$

where $\circ$ denotes the Khatri-Rao product.

Since the angle/delay parameters are quasi-stationary, then the space-time response matrix $\mathbf{U}(\boldsymbol{\theta},\boldsymbol{\tau})$ can be assumed to be time invariant over the observation interval. The goal is then to estimate $(\boldsymbol{\theta},\boldsymbol{\tau})$ from the estimated channel $\hat{\mathbf{H}}$:

$$\hat{\mathbf{H}} = \mathbf{U}(\boldsymbol{\theta},\boldsymbol{\tau})\boldsymbol{\beta} + \mathbf{V} \tag{4}$$

Many of the well-known methods such as ML, subspace fitting, MUSIC and ESPRIT that have been developed for DOA estimation are applicable to the JADE problem [5].

B. SAGE

The SAGE algorithm is a two-fold extension of the EM algorithm, which replaces the high-dimensional optimisation process in the EM algorithm by several separate low-dimensional maximisation steps.

Each step of the SAGE algorithm consists of estimating a subset of $\hat{\boldsymbol{\Omega}}_\ell = [\tau_\ell,\theta_\ell,\beta_\ell]$, while keeping the estimates of the other components fixed. The coordinate-wise update procedure used to obtain the new estimate $\hat{\boldsymbol{\Omega}}''$ of the wave ℓ given the current estimate $\hat{\boldsymbol{\Omega}}'$ is

$$\begin{aligned}
\hat{\tau}_\ell'' &= \arg\max_{\tau} \left\{ \left| z\left(\tau, \hat{\theta}_\ell'; \hat{x}_\ell(t;\hat{\boldsymbol{\Omega}}')\right) \right| \right\} \\
\hat{\theta}_\ell'' &= \arg\max_{\theta} \left\{ \left| z\left(\hat{\tau}_\ell', \theta; \hat{x}_\ell(t;\hat{\boldsymbol{\Omega}}')\right) \right| \right\} \\
\hat{\beta}_\ell'' &= \frac{1}{MTP} z\left(\hat{\tau}_\ell', \hat{\theta}_\ell'; \hat{x}_\ell(t;\hat{\boldsymbol{\Omega}}')\right)
\end{aligned} \tag{5}$$

where

$$z\Big(\hat{\tau}'_\ell, \hat{\theta}'_\ell; \hat{x}_\ell(t; \hat{\mathbf{\Omega}}')\Big) \triangleq \int_{D_o} g^*(t' - \tau) a^H(\theta) x_\ell(t'; \hat{\mathbf{\Omega}})\ dt' \quad (6)$$

where D_o is the observation interval of the received data $y(t)$, then

$$\hat{x}_\ell(t; \hat{\mathbf{\Omega}}) = y(t) - \sum_{\ell'=1, \ell' \neq \ell}^{L} s(t; \hat{\mathbf{\Omega}}'_\ell) \quad (7)$$

Carrying out this updating procedure for all L components defines one iteration cycle of the SAGE algorithm. Equations (7) and (5) are the E-step and M-step respectively of the SAGE algorithm. The E-step calculates an estimate $\hat{x}_\ell(t; \hat{\mathbf{\Omega}})$ of the noise corrupted version of $s(t, \mathbf{\Omega}_\ell)$ by subtracting the estimated contribution of all waves, except the ℓ^{th} one, from the received signal. This process is known as parallel interference cancellation (PIC).

IV. Key Aspects and Discussions

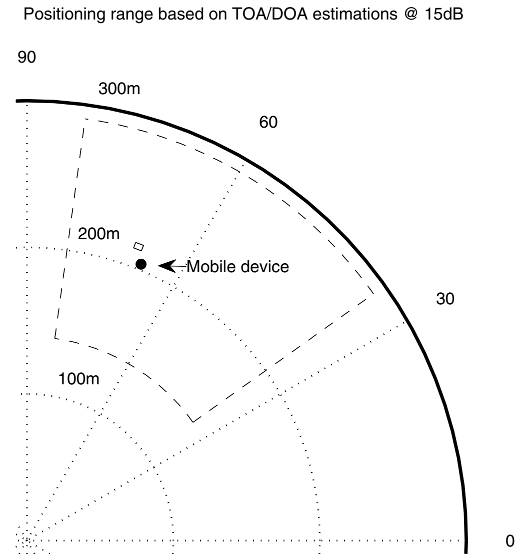
In this section, the performance of the 2-D Unitary ESPRIT, SI-JADE and SAGE algorithms are compared in a strong multipaths environment. The main goal of this study is to compare the positioning accuracies that can be obtained using the above mentioned joint TOA/DOA estimation algorithms. We will first compare their ability to identify the multipaths before discussing the positioning accuracy attainable from their estimates of the LOS component. In addition, key aspects of these algorithms, such as their robustness to model errors, in particular, the effects of the under-estimation of the number of paths as well as computational complexity will be addressed.

A. Positioning Accuracy

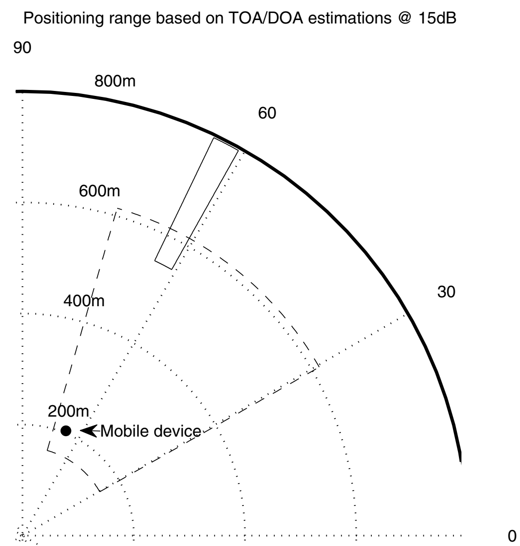
We have studied the algorithms' performance in the multipaths environment where the channel contain 10 paths within a delay spread of $10T$ to represent a relatively strong multipaths scenario. To begin with, we assess the algorithms' ability to identify the multipaths. Then we compare the positioning accuracies attainable from their estimates of the LOS component in both a strong and weak LOS scenario.

Fig. 2 shows the distributions of the path estimates from 50 simulations at a SNR environment of 15dB. We can observe similar performances in terms of identifying each of the multipath, such that the 4 closest paths were easily visible to all three algorithms, while the remaining paths were estimated with varying degrees of success. However, it is clear from Fig. 2(a) and (b) that JADE and ESPRIT's estimates for most of these paths appear to be some what random, in addition, a number of paths has been missed completely. On the other hand, SAGE produced very accurate estimates of the 3 closest paths, and partially correct estimates for the distant paths (e.g. correct delay, wrong angle), which is clearly evident for the 2 furtherest paths in Fig. 2(c). In fact, the large variance in JADE and ESPRIT's estimates can have a significant impact on the positioning accuracies if these data were used in the locationing of a mobile device.

Assuming no prior knowledge of the location of the mobile device, hence choosing the path estimate with the shortest TOA value as the LOS estimate, we were able to show the positioning accuracies for each algorithm based on the mean error and standard deviation values from the 50 simulations. This is shown in Fig. 3(a) for a strong LOS channel, where



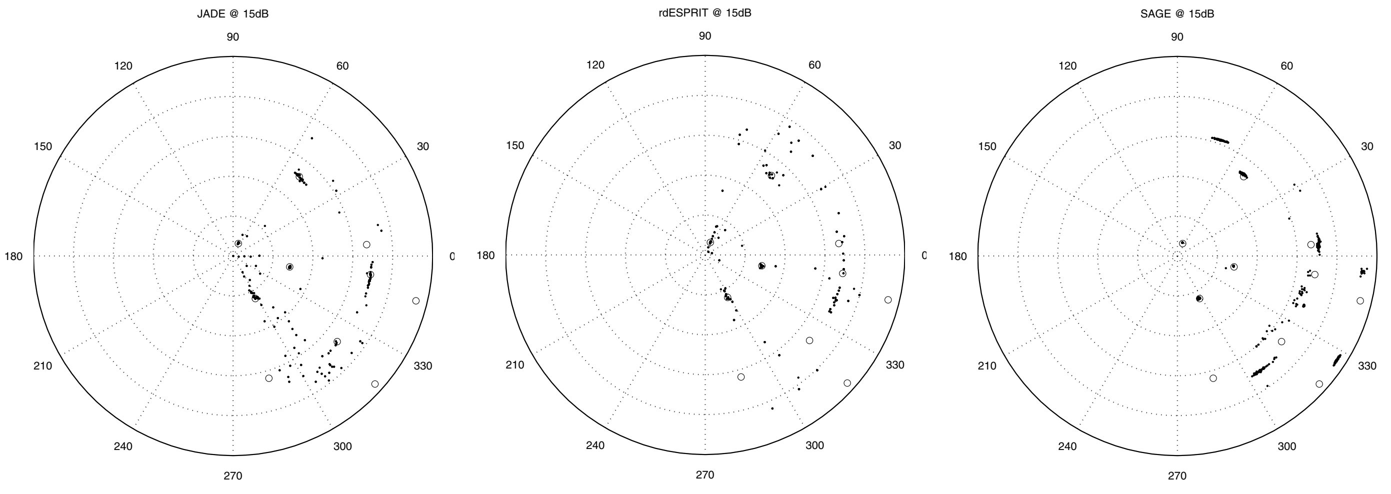
(a) Strong LOS. β_{LOS} is 108% of the second strongest path.



(b) Weak LOS. β_{LOS} is 18% of the strongest path.

Fig. 3. Positioning accuracies of JADE and SAGE based on the mean value and standard deviation in TOA/DOA estimates of the LOS component. Sector enclosed by the dashed and solid lines represents the location estimate for JADE and SAGE respectively. Assuming a symbol period T of 1μs.

the amplitude of the LOS component is 108% of that of the second strongest path. The area enclosed by the dashed lines implies the possible location of the mobile device based on the JADE's TOA/DOA estimates. This is also shown for SAGE as the solid lines. We can see a significant difference in terms of the positioning accuracies for these two algorithms, where the region of uncertainty for SAGE (albeit slightly incorrect), is only a fraction of that JADE. (ESPRIT showed similar



(a) SI-JADE (b) 2D Unitary ESPRIT (c) SAGE

Fig. 2. Distribution of path estimates from 50 simulations at 15dB. The channel consists of 10 paths which are shown in the plot as open circles. Origin in centre represent location of the base station, radius represents distances from base station, and azimuthal scale indicates the DOA to the base station.

performance to JADE hence was omitted to reduce clutter)

Fig. 3(b) shows the same result but for a weak LOS channel, where the amplitude of the LOS component is only 18% of the strongest path. We notice performance degradetion for both algorithms. This is specially the case for SAGE whose position estimate is now signifiantly different from the actual. This is expected due to the sequential nature of which each path is estimated in SAGE. Since the strongest path is normally estimated first, paths that are much weaker suffer from errors acculmated in all previous stages.

Hence, for a real positioning application, where no prior knowledge of the LOS component can be assumed, provided the channel contains a relatively strong LOS component, SAGE's ability to provide TOA/DOA estimates with much lower variances (and hence more accurate position estimate) makes it much better suited.

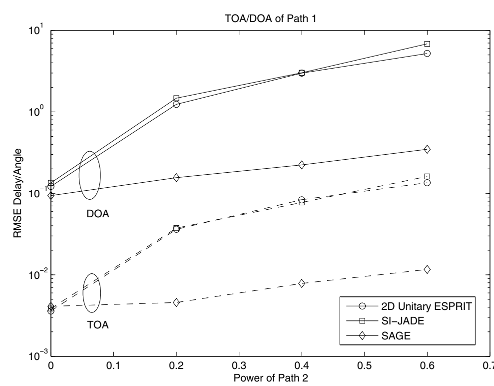


Fig. 4. RMSE in DOA and TOA of the first path when L is underestimated. Solid and dashed lines represents RMSE values of DOA and TOA estimates respectively.

B. Robustness to Model errors

An important aspect of these estimation algorithms is their robustness to model errors and noise. Here, the robustness of the algorithms to the (mild) under-estimation of the number of channel path is investigated.

We assume that there is a four-element ULA receiving signals from a single transmitter via L paths. The idea is to examine the effects on the RMSE of the DOA and TOA estimates of the first path when underestimating the value of L by 1. The true setup parameters are: $L = 3$, with the angle, delay and normalised powers: $\theta = [-32.5; -5; 15]^{\circ}$, $\tau = [1.1; 2.1; 5]T$, $\beta = [1; x; 0.6]$ where $x = 0, \ldots, 0.6$. In the experiment, the power factor β of the first and third paths is held constant, while that of the second varies from 0 to 0.6. When $\beta = 0$, there are really only two paths. In our simulation, we specify the estimated path number to be 2. Hence a model error occurs when the power of the second path becomes non-zero. Fig. 4 shows how the RMSE in the estimated TOA and DOA of the first path varies as the power in the second path increases. Notice that SI-JADE and 2D Unitary ESPRIT are badly affected as soon as the power of the second path increases even slightly, where we can see a ten-fold increase in RMSE when the power factor of the second path reaches 0.2; this is for both angles and delays estimates. However, only a slight degradation is obvious for SAGE.

In fact, SAGE detects the specified number of the most significant paths, and simply ignores the rest. This is a desirable behaviour for a real system such that a slight underestimate of the actual number of paths should not prevent the most significant paths from being identified.

Hence, under-estimation of the path number is much worse for SI-JADE and 2D Unitary ESPRIT as it relies on the angle-delay subspace. This in turn depends greatly on accurate knowledge of L. So if there are more paths than the dimensions of the computed angle-delay subspace, the detection of certain multi-path components becomes difficult. This indicates that, for mobile positioning applications which are sensitive to the shortest path delay estimation, SAGE is a better candidate in comparison to SI-JADE and 2D Unitary ESPRIT.

C. Computational complexity and real time processing

In this section, our aim is to analyse each algorithm's complexity with respect to the number of multipaths and the number of antenna elements (i.e. input data size). Here, we measure computational complexity in terms of CPU time per Monte Carlo run of the algorithms. It is important to stress that the magnitude, or even the order, of the CPU execution time of each algorithm is not the emphasis of this section. Our focus is on the relationship of the complexity to the number of paths and number of antenna elements, since the magnitude/order of the complexity for each method is highly dependent on code implementation, which is outside the scope of this study. Fig. 5a shows the CPU time against

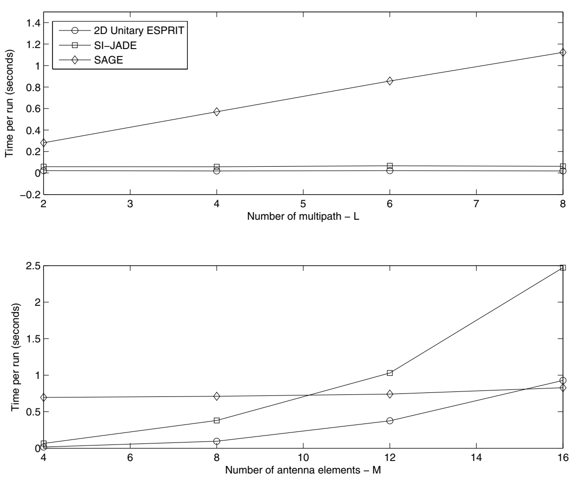


Fig. 5. Computational complexity comparison of the algorithms measured in CPU time per Monte Carlo run. The focus here is on the relationship of the complexity to the number of paths and number of antenna elements, not the magnitude or even order.

the number of multipaths. Unsurprisingly, a linear relationship is shown between the execution time and path number for SAGE, since each SAGE cycle only estimates the parameters of one path. The subspace approaches estimate all the paths through a joint diagonalisation approach, hence increasing the number of paths has no effect on the CPU time. One might expect that increasing the size of the channel matrix will have a significant effect on all the algorithms. However, this is not the case as shown in Fig. 5b, where the CPU time is plotted as a function of antenna elements. This is because the most complicated step of SAGE, e.g. the cost function in (6), involves only matrix-vector multiplications as supposed to matrix-matrix multiplications within that of the other two methods. In addition, the actual optimisation of the cost function (which is independent of the matrix size) far outweighs the numerical calculation of the integral, hence an increase in matrix size contributes little to the total CPU time.

V. Conclusion

We have presented a comparative study of the performance of the joint TOA/DOA techniques, ESPRIT, JADE and SAGE. Through extensive simulations in MATLAB, we have demonstrated that: 1) SAGE outperforms the other two algorithms in terms of TOA/DOA estimation variance in a strong multipaths environment, thus can give much more accurate position estimate provided the LOS component is relatively strong. 2) SAGE is more robust to channel modeling errors. 3) SAGE's computation complexity exhibits a linear relationship with the number of channel paths (while ESPRIT and JADE are independent) while being unaffected by the number of antenna elements.

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