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A Hybrid SS-ToA Wireless NLoS Geolocation Based on Path Attenuation: Mobile Position Estimation

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***Abstract*—The mobile position estimation using time-of-arrival (ToA) is considered for the wireless NLoS geolocation exploring a signal strength (SS) based on path loss. As exploited the path attenuation, a hybrid SS-ToA approach indicates a performance improvement compared with the usual ToA method. To realize this prospect, it calls for an estimator to determine the mobile position from the time delay. In this paper, we show that the use of line-of-sight (LoS) time delay provides the same performance as that given by using both the LoS and non-LoS (NLoS) time delays. We then propose least squares (LS), weighted least squares (WLS) and maximum likelihood (ML) to estimate the mobile position. Theoretical performance of the LS and WLS is analyzed. It reveals that for different LoS time delay error variances, the LS error variance is larger than the WLS error variance, which is equal to the Cramér-Rao bound (CRB). Numerical results illustrate that the time delay performance analysis is accurate when the time delay estimate is close to its true value, i.e. for small time delay error variance. For high SNR and large effective bandwidth, the LS cannot provide the performance compared with the CRB, whereas the WLS and ML are statistically efficient.**

***Index Terms*—Parameter estimation, non-line-of-sight propagation, path loss.**

I. Introduction

Recently, the Cramér-Rao bound (CRB) has been analyzed in [1] for several geolocation schemes in the presence of non-line-of-sight (NLoS). It is reported that the Fisher information matrix (FIM) of a hybrid scheme using signal strength (SS) and time-difference of arrival (TDoA) can be acquired by the superposition of the FIMs from both schemes. Hybrid schemes outperform those using only one feature in the aspects of estimation accuracy [2] and reliability [3]. It is fruitful to note that the frameworks in [1] and [2] are composed of two separate techniques, which require two different measurements, such as baseband received signal and mean signal strength. Unfortunately, this kind of combination inevitably makes the parameter estimation cumbersome. In [4], a path loss is incorporated into path gain. From wireless geolocation point of view, the composite model enables the receiver to observe only the received signal as well as allows a more realistic propagation investigation. In [5], the inherent accuracy of a mobile station (MS) position estimation is considered by means of the CRB for a handset-based mutilateral geolocation system using time-of-arrival (ToA). Although theoretical performance reveals that the hybrid SS/ToA method outperforms the usual ToA method, there exists a demand to realize the efficiency in practice.

In this paper, we fill in this gap by designing an estimator to achieve the expected theoretical error variance. We first show that the LoS time delays contributes the same mobile position error variance as that of using both LoS and NLoS. Contribution of this paper is that we propose least squares (LS), weighted least squares (WLS) and maximum likelihood (ML) to estimate the mobile position based on the time delay estimates. We then derive the error performance of the LS and the WLS. It is shown that in general, the WLS provides less error variance than the LS. When the time delay error variances are identical, the WLS performance is as same as that of the LS. Compared with the ML, the WLS provides the same error performance, which attains the CRB. Numerical examples are conducted to illustrate the statistical performance of three estimators. It can be summarized that the LS in general cannot achieve the CRB, while the WLS and the ML are statistically efficient.

II. System Model

Let us consider an MS transmitting a radio signal through a wireless channel to a number of base stations (BSs). Let B be the number of all BSs, whose locations, $\mathbf{p}_b = \begin{bmatrix} x_b & y_b \end{bmatrix}^{\mathrm{T}}$; $b \in \{1, 2, \ldots, B\}$, are known. We assume that there is no loss of energy for the transmitted signal when radio waves propagate in a media. There is, however, attenuation by the channel. At each base station, the received energy at the b-th BS can be expressed by (see e.g. [6, p. 46] and [7, p. 38])

$$E_b = \kappa \frac{d_0^{\gamma_b}}{d_b^{\gamma_b}} E_{\mathrm{s}}, \tag{1}$$

where d_0 is the close-in reference in the far field region, d_b is the distance between the MS and the b-th BS, γ_b is the path loss exponent at the b-th BS, $E_{\mathrm{s}} = \int_{-\infty}^{\infty} |s(t)|^2 \mathrm{d}t$ is the energy of a transmitted signal $s(t)$, and κ is the unitless constant depending on antenna characteristics and average channel attenuation given by

$$\kappa = \frac{c^2}{16\pi^2 f_0^2 d_0^2}, \tag{2}$$

with the center frequency f_0 and the speed of light c. Assume that the discrimination between LoS and NLoS has been conducted (see e.g. [8]–[11] and references therein). Let $M < B$ be the number of BSs that receive a set $\{1, 2, \ldots, M\}$ of NLoS signals. The received signal amplitudes $\{a_b\}_{b=1}^{B}$ and the positive delay distances $\{l_m\}_{m=1}^{M}$ are assumed to be unknown, whereas the position of mobile station, $\mathbf{p} = \begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}}$, is the parameter of interest. Let τ_b be the time delay of received signal at the b-th BS:

$$\tau_b(x, y, l_b) = \frac{1}{c}\left(\sqrt{\tilde{x}_b^2 + \tilde{y}_b^2} + l_b\right), \tag{3}$$

where $\tilde{x}_b = x - x_b$, and $\tilde{y}_b = y - y_b$, $l_b = 0$; for $b \in \{M+1, M+2, \ldots, B\}$. As $d_b = c\tau_b$, the loseless energy based on (1) can be rewritten as

$$E_b = \kappa \frac{1}{\left(\sqrt{\left(\frac{\tilde{x}_b}{d_0}\right)^2 + \left(\frac{\tilde{y}_b}{d_0}\right)^2} + \frac{l_b}{d_0}\right)^{\gamma_b}} E_{\mathrm{s}}. \tag{4}$$

Since (1) and (4) are valid only in the far field, it is assumed that d_0 is less than $\sqrt{\tilde{x}_b^2 + \tilde{y}_b^2}$. This means that within a circle of radius d_0 there are no BSs. The received baseband signal can be written as [1]

$$r_b(t) = a_b s(t - \tau_b) + n_b(t), \tag{5}$$

where $s(t)$ is the known waveform, a_b and τ_b are the amplitude and time delay of propagation to the b-th BS, and $n_b(t)$ is an additive noise at the b-th BS and assumed to be a complex-valued white Gaussian process with zero mean and variance σ_{n}^2. Since $E_b = a_b^2 E_{\mathrm{s}}$, the unitless amplitude is given by

$$a_b = \sqrt{\kappa} \frac{1}{\left(\sqrt{\left(\frac{\tilde{x}_b}{d_0}\right)^2 + \left(\frac{\tilde{y}_b}{d_0}\right)^2} + \frac{l_b}{d_0}\right)^{\frac{1}{2}\gamma_b}}. \tag{6}$$

Assume that the transmitted signal is nonzero over the interval $[0, T_{\mathrm{s}}]$, where T_{s} is the signal period.

In this model, we can see that $r_b(t)$ is a random signal due to the randomness of $n_b(t)$. Since the position $\mathbf{p}$ and the nuisance parameter l_b are unknown and deterministic, their reparameterizations a_b and τ_b are as well. All unknown parameters can be aggregated into $\boldsymbol{\theta} \in \mathbb{R}^{(M+2)\times 1}$ as

$$\boldsymbol{\theta} = \begin{bmatrix} \mathbf{p}^{\mathrm{T}} & \mathbf{l}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \tag{7}$$

where $(\cdot)^{\mathrm{T}}$ is the transpose and $\mathbf{l} \in \mathbb{R}^{M\times 1}$ is given by

$$\mathbf{l} = \begin{bmatrix} l_1 & l_2 & \cdots & l_M \end{bmatrix}^{\mathrm{T}}. \tag{8}$$

Let the solution of the homogeneous Fredholm integral equation

$$\lambda_{b,k} f_{b,k}(t) = \int_0^T \varphi_b(t, \acute{t}) f_{b,k}(\acute{t}) \mathrm{d}\acute{t} \quad ; k \in \{1, 2 \ldots, K\}, \tag{9}$$

be the eigenvalue $\lambda_{b,k}$ and the orthonormal function $f_{b,k}(t)$, where K is the number of basis functions and the kernel $\varphi_b(t, \acute{t})$ is the eigenfunction, which is equal to the noise autocovariance function. From the Karhunen-Loève (KL) expansion (see e.g. [12, p. 37], [13, p. 279], and [14, p. 298]), the signal can be sampled from $f_{b,k}(t)$ according to

$$r_b(t) = \lim_{K\to\infty} \sum_{k=1}^{K} r_{b,k} f_{b,k}(t), \tag{10}$$

where the received signal sample is given by $r_{b,k} = \int_0^T f_{b,k}(t) r_b(t) \mathrm{d}t$. From (5), the received signal sample can be expressed by

$$r_{b,k} = a_b s_{b,k} + n_{b,k}, \tag{11}$$

where the signal and noise samples are given by $s_{b,k} = \int_0^T f_{b,k}(t) s(t - \tau_b) \mathrm{d}t$ and $n_{b,k} = \int_0^T f_{b,k}(t) n_b(t) \mathrm{d}t$. Assume that the basis function $f_{b,k}(t)$ is chosen such that the noise samples $\{n_{b,k}\}_{k=1}^{K}$ are identically and independently distributed. The probability density function (PDF) of the complex Gaussian multivariate $\{r_{b,k}\}_{k=1}^{K}$ can be written as

$$p(r_{b,1}, \ldots, r_{b,K} | \tau_b) = \frac{1}{(\pi\sigma_{\mathrm{n}}^2)^K} \mathrm{e}^{-\frac{1}{\sigma_{\mathrm{n}}^2} \sum_{k=1}^{K} |r_{b,k} - a_b s_{b,k}|^2}. \tag{12}$$

Given the continuous signal $r_b(t)$; $t \in (0, T]$, the likelihood of τ_b can be written in logarithm scale as

$$\begin{aligned} \ell(\tau_b | r_b(t); t \in (0, T]) &= \lim_{K\to\infty} \ln\left(p(r_{b,1}, \ldots, r_{b,K} | \tau_b)\right) \\ &\doteq -\frac{1}{\sigma_{\mathrm{n}}^2} \int_0^T |r_b(t) - a_b s(t - \tau_b)|^2 \mathrm{d}t, \end{aligned} \tag{13}$$

where $\ln(\cdot)$ is the natural logarithm function and $\doteq$ is the equivalence due to neglecting an irrelevant term. Given the received signal of all base stations

$$\mathbf{r}(t) = \begin{bmatrix} r_1(t) & r_2(t) & \cdots & r_B(t) \end{bmatrix}^{\mathrm{T}}, \tag{14}$$

the log-likelihood function can be derived from

$$\begin{aligned} \ell(\boldsymbol{\tau} | \mathbf{r}(t); t \in (0, T]) &= \lim_{K\to\infty} \ln\left(p(\mathbf{r}[1], \ldots, \mathbf{r}[K] | \boldsymbol{\tau})\right) \\ &\doteq -\frac{1}{\sigma_{\mathrm{n}}^2} \sum_{b=1}^{B} \int_0^T |r_b(t) - a_b s(t - \tau_b)|^2 \mathrm{d}t, \end{aligned} \tag{15}$$

where $\mathbf{r}[k] \in \mathbb{C}^{B\times 1}$ and $\boldsymbol{\tau} \in \mathbb{R}^{B\times 1}$ are defined by

$$\mathbf{r}[k] = \begin{bmatrix} r_{1,k} & r_{2,k} & \cdots & r_{B,k} \end{bmatrix}^{\mathrm{T}}, \tag{16a}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_1 & \tau_2 & \cdots & \tau_B \end{bmatrix}^{\mathrm{T}}. \tag{16b}$$

III. Mobile Position Estimation

The ML estimate of the time delay τ_b can be given by

$$\hat{\tau}_b = \arg\min_{\tau} a_b^2(\tau) E_{\mathrm{s}} - 2a_b(\tau) \int_0^T \Re\left(r_b^*(t) s(t - \tau)\right) \mathrm{d}t, \tag{17}$$

where $a_b(\tau) = \sqrt{\kappa}\left(\frac{d_c}{c\tau}\right)^{\frac{1}{2}\gamma_b}$ is the function of distance, and $\Re(\cdot)$ is the real part of $\cdot$. The above solution is Gaussian distributed, unbiased and provides the estimation error variance as follows [4]

$$\mathrm{E}_{n_b(t)}\{\hat{\tau}_b - \tau_b\} = 0, \tag{18a}$$

$$\mathrm{E}_{n_b(t)}\{(\hat{\tau}_b - \tau_b)^2\} = \frac{1}{8\pi^2 \bar{\beta}^2 \frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} a_b^2 \left(1 + \frac{1}{16\pi^2 \bar{\beta}^2 \tau_b^2} \gamma_b^2\right)}, \tag{18b}$$

where $\mathrm{E}_{n_b(t)}\{\cdot\}$ is the expectation with respect to $n_b(t)$ and $\bar{\beta}$ is the effective (root-mean-square) bandwidth defined by $\bar{\beta} = \sqrt{\frac{\int_{-\infty}^{\infty} f^2 |S(f)|^2 \mathrm{d}f}{\int_{-\infty}^{\infty} |S(f)|^2 \mathrm{d}f}}$, with $S(f)$ being the Fouriér transform of $s(t)$. Let us introduce $\bar{\mathbf{\Phi}} = \frac{\partial}{\partial \mathbf{p}} \bar{\mathbf{d}}^{\mathrm{T}}(\mathbf{p}) \in \mathbb{R}^{2\times(B-M)}$, where $\bar{\mathbf{d}} = c\bar{\boldsymbol{\tau}}$ with

$$\bar{\boldsymbol{\tau}} = \begin{bmatrix} \tau_{M+1} & \tau_{M+2} & \cdots & \tau_B \end{bmatrix}^{\mathrm{T}}. \tag{19}$$

It can be shown that

$$\bar{\mathbf{\Phi}} = \begin{bmatrix} \cos(\phi_{M+1}) & \cos(\phi_{M+2}) & \cdots & \cos(\phi_B) \\ \sin(\phi_{M+1}) & \sin(\phi_{M+2}) & \cdots & \sin(\phi_B) \end{bmatrix}, \tag{20}$$

where ϕ_b is defined by

$$\phi_b = \arctan\left(\frac{y_b - y}{x_b - x}\right). \tag{21}$$

Let us introduce

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_B \end{bmatrix}^{\mathrm{T}}, \tag{22}$$

$$\bar{\boldsymbol{\sigma}}^2 = \begin{bmatrix} \sigma_{M+1}^2 & \sigma_{M+2}^2 & \cdots & \sigma_B^2 \end{bmatrix}^{\mathrm{T}}, \tag{23}$$

where $\sigma_b^2 = \mathrm{E}_{n_b(t)}\{(\hat{\tau}_b - \tau_b)^2\}$ is given by (18b).

Lemma 1: Using either $\hat{\bar{\boldsymbol{\tau}}}$ or $\hat{\boldsymbol{\tau}}$, the position estimates $\hat{\mathbf{p}}_{\bar{\boldsymbol{\tau}}}$ and $\hat{\mathbf{p}}_{\boldsymbol{\tau}}$ provide zero mean and the same variance, i.e.

$$\begin{aligned} &\mathrm{E}_{\mathbf{n}(t)}\left\{(\hat{\mathbf{p}}_{\boldsymbol{\tau}} - \mathbf{p})(\hat{\mathbf{p}}_{\boldsymbol{\tau}} - \mathbf{p})^{\mathrm{T}}\right\} \\ &= \mathrm{E}_{\bar{\mathbf{n}}(t)}\left\{(\hat{\mathbf{p}}_{\bar{\boldsymbol{\tau}}} - \mathbf{p})(\hat{\mathbf{p}}_{\bar{\boldsymbol{\tau}}} - \mathbf{p})^{\mathrm{T}}\right\} \\ &\simeq c^2(\bar{\mathbf{\Phi}}\bar{\mathbf{\Phi}}^{\mathrm{T}})^{-1}\bar{\mathbf{\Phi}}\mathbf{D}(\bar{\boldsymbol{\sigma}}^2)\bar{\mathbf{\Phi}}^{\mathrm{T}}(\bar{\mathbf{\Phi}}\bar{\mathbf{\Phi}}^{\mathrm{T}})^{-1}, \end{aligned} \tag{24}$$

where $\mathbf{D}(\cdot)$ is the diagonal matrix whose diagonal vector is $\cdot$, $\simeq$ is the equality, which neglects $o(\|\hat{\mathbf{p}} - \mathbf{p}\|_{\mathrm{E}}^2)$ and the little oh of $u(\hat{\mathbf{p}} - \mathbf{p}) = o(v(\hat{\mathbf{p}} - \mathbf{p}))$ stands for $\lim_{\hat{\mathbf{p}}\to\mathbf{p}} \frac{u(\hat{\mathbf{p}}-\mathbf{p})}{v(\hat{\mathbf{p}}-\mathbf{p})} = 0$

Proof: Taking the first-order Taylor series around the true value $\bar{\boldsymbol{\tau}}$, we have

$$\begin{aligned} \hat{\bar{\boldsymbol{\tau}}} &= \bar{\boldsymbol{\tau}}(\mathbf{p}) + \left(\frac{\partial}{\partial \mathbf{p}}\bar{\boldsymbol{\tau}}^{\mathrm{T}}(\mathbf{p})\right)^{\mathrm{T}}(\hat{\mathbf{p}} - \mathbf{p}) + o(\|\hat{\mathbf{p}} - \mathbf{p}\|_{\mathrm{E}}^2) \\ &\simeq \bar{\boldsymbol{\tau}}(\mathbf{p}) + \frac{1}{c}\bar{\mathbf{\Phi}}^{\mathrm{T}}(\hat{\mathbf{p}} - \mathbf{p}). \end{aligned} \tag{25}$$

Then, we consider the analysis to $\boldsymbol{\tau}(\mathbf{p})$ in such a way that

$$\begin{aligned} \hat{\boldsymbol{\tau}} &= \boldsymbol{\tau}(\mathbf{p}) + \left(\frac{\partial}{\partial \mathbf{p}}\boldsymbol{\tau}^{\mathrm{T}}(\mathbf{p})\right)^{\mathrm{T}}(\hat{\mathbf{p}} - \mathbf{p}) + o(\|\hat{\mathbf{p}} - \mathbf{p}\|_{\mathrm{E}}^2) \\ &\simeq \boldsymbol{\tau}(\mathbf{p}) + \boldsymbol{\nabla}_{\mathbf{p}\boldsymbol{\tau}}^{\mathrm{T}}(\hat{\mathbf{p}} - \mathbf{p}), \end{aligned} \tag{26}$$

where the Jacobian matrix $\boldsymbol{\nabla}_{\mathbf{p}\boldsymbol{\tau}} = \frac{\partial}{\partial \mathbf{p}}\boldsymbol{\tau} \in \mathbb{R}^{2\times B}$ is given by

$$\boldsymbol{\nabla}_{\mathbf{p}\boldsymbol{\tau}} = \frac{1}{c}\begin{bmatrix} \tilde{\mathbf{\Phi}} & \bar{\mathbf{\Phi}} \end{bmatrix}, \tag{27}$$

with $\tilde{\mathbf{\Phi}} \in \mathbb{R}^{2\times M}$ given by

$$\tilde{\mathbf{\Phi}} = \begin{bmatrix} \cos(\phi_1) & \cos(\phi_2) & \cdots & \cos(\phi_M) \\ \sin(\phi_1) & \sin(\phi_2) & \cdots & \sin(\phi_M) \end{bmatrix}. \tag{28}$$

Using algebraic manipulation, the result in (24) can be obtained. ■

Note that the error variance in (24) depends on only the LoS.

A. Estimation of position parameters

Since the time delay $\boldsymbol{\tau}$ contains a sufficient relation to $\mathbf{p}$ and $\mathbf{l}$, the time delay estimate $\hat{\boldsymbol{\tau}}$ will be transformed into the mobile position $\mathbf{p}$ by an estimator.

1) Least squares (LS):

Proposition 1 (LS estimate of mobile position): The mobile position can be calculated from

$$\hat{\mathbf{p}}_{\mathrm{LS}} = \arg\min_{x,y} \sum_{b=M+1}^{B} \left(\hat{\tau}_b - \frac{1}{c}\sqrt{(x_b - x)^2 + (y_b - y)^2}\right)^2. \tag{29}$$

Note that the concentrated LS estimate of the mobile position appears dependent on only the LoS time delays.

2) Weighted least squares (WLS):

Proposition 2 (WLS estimate of mobile position): The WLS estimate of $\mathbf{p}$ is given by

$$\hat{\mathbf{p}}_{\mathrm{WLS}} = \arg\min_{x,y} \sum_{b=M+1}^{B} \frac{1}{\sigma_b^2(x,y)}\left(\hat{\tau}_b - \frac{1}{c}\sqrt{(x_b - x)^2 + (y_b - y)^2}\right)^2. \tag{30}$$

3) Maximum Likelihood:

Proposition 3 (ML estimate): The ML estimate of $\mathbf{p}$ is calculated by

$$\hat{\mathbf{p}}_{\mathrm{ML}} = \arg\min_{x,y} \sum_{b=M+1}^{B} \ln\left(\sigma_b^2(x,y)\right) + \frac{1}{\sigma_b^2(x,y)}(\hat{\tau}_b - \tau_b(x,y))^2. \tag{31}$$

B. Error variance of the position estimate using least squares

Lemma 2: For estimation error, both LS and WLS criteria provide zero mean and the variances from

$$\mathrm{E}_{\mathbf{n}(t)}\left\{(\hat{\mathbf{p}}_{\mathrm{LS}} - \mathbf{p})(\hat{\mathbf{p}}_{\mathrm{LS}} - \mathbf{p})^{\mathrm{T}}\right\} = c^2(\bar{\mathbf{\Phi}}\bar{\mathbf{\Phi}}^{\mathrm{T}})^{-1}\bar{\mathbf{\Phi}}\mathbf{D}(\bar{\boldsymbol{\sigma}}^2)\bar{\mathbf{\Phi}}^{\mathrm{T}}(\bar{\mathbf{\Phi}}\bar{\mathbf{\Phi}}^{\mathrm{T}})^{-1}, \tag{32}$$

$$\mathrm{E}_{\mathbf{n}(t)}\left\{(\hat{\mathbf{p}}_{\mathrm{WLS}} - \mathbf{p})(\hat{\mathbf{p}}_{\mathrm{WLS}} - \mathbf{p})^{\mathrm{T}}\right\} = c^2(\bar{\mathbf{\Phi}}\mathbf{D}^{-1}(\bar{\boldsymbol{\sigma}}^2)\bar{\mathbf{\Phi}}^{\mathrm{T}})^{-1}. \tag{33}$$

It results in $\mathrm{E}_{\mathbf{n}(t)}\left\{(\hat{\mathbf{p}}_{\mathrm{LS}} - \mathbf{p})(\hat{\mathbf{p}}_{\mathrm{LS}} - \mathbf{p})^{\mathrm{T}}\right\} \succeq \mathrm{E}_{\mathbf{n}(t)}\left\{(\hat{\mathbf{p}}_{\mathrm{WLS}} - \mathbf{p})(\hat{\mathbf{p}}_{\mathrm{WLS}} - \mathbf{p})^{\mathrm{T}}\right\}$, where the equality holds when the variances of the time delay errors are identical.

Proof: Let a generalized LS (GLS) be

$$f_{\mathrm{GLS}}(\mathbf{p}) = \|\hat{\bar{\boldsymbol{\tau}}} - \bar{\boldsymbol{\tau}}\|_{\bar{\mathbf{W}}}^2, \tag{34}$$

where $\|\mathbf{e}\|_{\mathbf{W}}^2 = \mathbf{e}^{\mathrm{H}}\mathbf{W}\mathbf{e}$ is the weighted Euclidean norm with the Hermitian transpose $(\cdot)^{\mathrm{H}}$. The first derivative of the GLS is given by

$$\frac{\partial}{\partial \mathbf{p}} f_{\mathrm{GLS}}(\mathbf{p}) = -\frac{1}{c}2\bar{\mathbf{\Phi}}\bar{\mathbf{W}}(\hat{\bar{\boldsymbol{\tau}}} - \bar{\boldsymbol{\tau}}), \tag{35}$$

Let the asymptotic Hessian matrix of $f_{\mathrm{GLS}}(\mathbf{p})$ be

$$\begin{aligned} \overline{\mathbf{H}}_{\mathbf{pp}} &= \mathrm{E}_{\mathbf{n}(t)}\left\{\frac{\partial^2}{\partial \mathbf{p}\partial \mathbf{p}^{\mathrm{T}}} f_{\mathrm{GLS}}(\mathbf{p})\right\} \\ &= \frac{1}{c^2}2\bar{\mathbf{\Phi}}\bar{\mathbf{W}}\bar{\mathbf{\Phi}}^{\mathrm{T}}. \end{aligned} \tag{36}$$

Using the Taylor series $\frac{\partial}{\partial \mathbf{p}} f_{\mathrm{GLS}}(\mathbf{p}) = \frac{\partial}{\partial \mathbf{p}} f_{\mathrm{GLS}}(\mathbf{p})\Big|_{\mathbf{p}=\hat{\mathbf{p}}_{\mathrm{GLS}}} + \frac{\partial^2}{\partial \mathbf{p} \partial \mathbf{p}^{\mathrm{T}}} f_{\mathrm{GLS}}(\mathbf{p})\Big|_{\mathbf{p}=\check{\mathbf{p}}} (\hat{\mathbf{p}}_{\mathrm{GLS}} - \mathbf{p})$ with $\check{\mathbf{p}}$ lying between the estimated value $\hat{\mathbf{p}}_{\mathrm{GLS}}$ and the true value $\mathbf{p}$, we have $\hat{\mathbf{p}}_{\mathrm{GLS}} - \mathbf{p} \simeq -\overline{\mathbf{H}}_{\mathbf{pp}}^{-1} \frac{\partial}{\partial \mathbf{p}} f_{\mathrm{GLS}}(\mathbf{p})$, i.e.

$$\hat{\mathbf{p}}_{\mathrm{GLS}} - \mathbf{p} \simeq c(\bar{\boldsymbol{\Phi}}\bar{\mathbf{W}}\bar{\boldsymbol{\Phi}}^{\mathrm{T}})^{-1}\bar{\boldsymbol{\Phi}}\bar{\mathbf{W}}(\hat{\bar{\boldsymbol{\tau}}} - \bar{\boldsymbol{\tau}}). \tag{37}$$

Finally, $\bar{\mathbf{W}} = \mathbf{I}$ for the LS and $\bar{\mathbf{W}} = \mathbf{D}^{-1}(\bar{\boldsymbol{\sigma}}^2)$ for the WLS are invoked to obtain (32) and (33), respectively. ■

It can be seen that

- the WLS and ML achieve the same variance of estimation error, which is lower than that by the LS,
- the use of LoS signal is sufficient to achieve an optimal performance using the time delay,
- the variance of the estimation error depends only on the LoS portion,
- as $\bar{\boldsymbol{\Phi}}\bar{\boldsymbol{\Phi}}^{\mathrm{T}} = \sum_{b=M+1}^{B} \begin{bmatrix}\cos(\phi_b)\\ \sin(\phi_b)\end{bmatrix} \begin{bmatrix}\cos(\phi_b) & \sin(\phi_b)\end{bmatrix}$ and $\bar{\boldsymbol{\Phi}}\mathbf{D}^{-1}(\bar{\boldsymbol{\sigma}}^2)\bar{\boldsymbol{\Phi}}^{\mathrm{T}} = \sum_{b=M+1}^{B} \frac{1}{\sigma_b^2} \begin{bmatrix}\cos(\phi_b)\\ \sin(\phi_b)\end{bmatrix} \begin{bmatrix}\cos(\phi_b) & \sin(\phi_b)\end{bmatrix}$, the inverses $(\bar{\boldsymbol{\Phi}}\bar{\boldsymbol{\Phi}}^{\mathrm{T}})^{-1}$ and $(\bar{\boldsymbol{\Phi}}\mathbf{D}^{-1}(\bar{\boldsymbol{\sigma}}^2)\bar{\boldsymbol{\Phi}}^{\mathrm{T}})^{-1}$ exist when their ranks are full. In other words, for different ϕ_b, $B - M \geq 2$, i.e. $B \geq M + 2$.

IV. Cramér-Rao Lower Bound

Lemma 3: When the path loss exponent $\boldsymbol{\gamma}$ is known, the FIM is given by

$$\begin{aligned}\mathbf{J}_{\boldsymbol{\tau\tau}} &= -\mathrm{E}_{\mathbf{n}(t)}\left\{\frac{\partial^2}{\partial \boldsymbol{\tau} \partial \boldsymbol{\tau}^{\mathrm{T}}} \ln\left(p(\mathbf{r}(t); t \in (0,T] | \boldsymbol{\theta})\right)\right\} \\ &= \frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}\mathbf{D}^2(\mathbf{a})\left(8\pi^2\bar{\beta}^2\mathbf{I} + \frac{1}{2}\mathbf{D}^2(\boldsymbol{\gamma})\mathbf{D}^{-2}(\boldsymbol{\tau})\right),\end{aligned} \tag{38}$$

where $\mathbf{a} \in \mathbb{R}^{B\times 1}$ is defined by

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \cdots & a_B \end{bmatrix}^{\mathrm{T}}. \tag{39}$$

Proof: Let the desired signal vector $\mathbf{s}_a(t;\boldsymbol{\tau})$ be

$$\mathbf{s}_a(t;\boldsymbol{\tau}) = \mathbf{a} \odot \mathbf{s}(t;\boldsymbol{\tau}), \tag{40}$$

where $\odot$ is the Schur-Hadamard or element-wise product. The FIM can be written as

$$\mathbf{J}_{\boldsymbol{\tau\tau}} = 2\frac{1}{\sigma_{\mathrm{n}}^2}\int_0^T \Re\left(\left(\frac{\partial}{\partial \boldsymbol{\tau}}\mathbf{s}_a^{\mathrm{T}}(t;\boldsymbol{\tau})\right)\left(\frac{\partial}{\partial \boldsymbol{\tau}}\mathbf{s}_a^{\mathrm{T}}(t;\boldsymbol{\tau})\right)^{\mathrm{H}}\right)\mathrm{d}t. \tag{41}$$

Consider the derivative

$$\begin{aligned}\frac{\partial}{\partial \boldsymbol{\tau}}\mathbf{s}_a^{\mathrm{T}}(t;\boldsymbol{\tau}) &= \nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}(\mathbf{a}) + \left(\frac{\partial}{\partial \boldsymbol{\tau}}\mathbf{a}^{\mathrm{T}}\right)\mathbf{D}\left(\mathbf{s}(t;\boldsymbol{\tau})\right) \\ &= \nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}(\mathbf{a}) - \frac{1}{2}\mathbf{D}\left(\mathbf{a}\odot\boldsymbol{\gamma}\odot\mathbf{s}(t;\boldsymbol{\tau})\right)\mathbf{D}^{-1}(\boldsymbol{\tau}),\end{aligned} \tag{42}$$

where $\nabla_{\boldsymbol{\tau}\mathbf{s}} \in \mathbb{C}^{B\times B}$ is the Jacobian matrix defined by $\nabla_{\boldsymbol{\tau}\mathbf{s}} = \frac{\partial}{\partial \boldsymbol{\tau}}\mathbf{s}^{\mathrm{T}}(t;\boldsymbol{\tau})$. Using $\int_0^T \nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}\left(\mathbf{s}^*(t;\boldsymbol{\tau})\right) = \mathbf{O}$, the FIM yields

$$\begin{aligned}\mathbf{J}_{\boldsymbol{\tau\tau}} = 2\frac{1}{\sigma_{\mathrm{n}}^2}\int_0^T \Re\Big(&\nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}^2(\mathbf{a})\nabla_{\boldsymbol{\tau}\mathbf{s}}^{\mathrm{H}} \\ &- \frac{1}{2}\nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}\left(\mathbf{s}^*(t;\boldsymbol{\tau})\right)\mathbf{D}^2(\mathbf{a})\mathbf{D}^{-1}(\boldsymbol{\tau})\mathbf{D}(\boldsymbol{\gamma}) \\ &- \frac{1}{2}\mathbf{D}(\boldsymbol{\gamma})\mathbf{D}^2(\mathbf{a})\mathbf{D}^{-1}(\boldsymbol{\tau})\mathbf{D}(\mathbf{s}(t;\boldsymbol{\tau}))\nabla_{\boldsymbol{\tau}\mathbf{s}}^{\mathrm{H}} \\ &+ \frac{1}{4}\mathbf{D}^2\left(\mathbf{a}\odot\boldsymbol{\gamma}\right)\mathbf{D}^{-2}(\boldsymbol{\tau})\mathbf{D}\left(\mathbf{s}(t;\boldsymbol{\tau})\odot\mathbf{s}^*(t;\boldsymbol{\tau})\right)\Big)\mathrm{d}t \\ = 2\frac{1}{\sigma_{\mathrm{n}}^2}\int_0^T &\Re\left(\nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}^2(\mathbf{a})\nabla_{\boldsymbol{\tau}\mathbf{s}}^{\mathrm{H}}\right)\mathrm{d}t + \frac{1}{2}\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}\mathbf{D}^2\left(\mathbf{a}\odot\boldsymbol{\gamma}\right)\mathbf{D}^{-2}(\boldsymbol{\tau}).\end{aligned} \tag{43}$$

Let us consider $\int_0^T \nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}^2(\mathbf{a})\nabla_{\boldsymbol{\tau}\mathbf{s}}^{\mathrm{H}}\mathrm{d}t = \sum_{b=1}^{B} a_b^2 \int_0^T \frac{\partial}{\partial \boldsymbol{\tau}} s(t-\tau_b)\left(\frac{\partial}{\partial \boldsymbol{\tau}} s(t-\tau_b)\right)^{\mathrm{H}}\mathrm{d}t$. The (b_1, b_2)-th element of $\int_0^T \frac{\partial}{\partial \boldsymbol{\tau}} s(t-\tau_b)\left(\frac{\partial}{\partial \boldsymbol{\tau}} s(t-\tau_b)\right)^{\mathrm{H}}\mathrm{d}t$ is given by

$$\begin{aligned}&\left[\int_0^T \frac{\partial}{\partial \boldsymbol{\tau}} s(t-\tau_b)\left(\frac{\partial}{\partial \boldsymbol{\tau}} s(t-\tau_b)\right)^{\mathrm{H}}\mathrm{d}t\right]_{[b_1,b_2]} \\ &= \int_0^T \left|\frac{\partial}{\partial \tau_b} s(t-\tau_b)\right|^2 \mathrm{d}t\,\delta_{b,b_1}\delta_{b_1,b_2} \\ &= \int_{-\tau_b}^{T-\tau_b} \left|\left(\frac{\partial}{\partial t}t'\right)\frac{\partial}{\partial t'} s(t')\right|^2 \mathrm{d}t'\,\delta_{b,b_1}\delta_{b_1,b_2}\ ; t' = t - \tau_b \\ &= \int_{-\infty}^{\infty} |(j2\pi f)S(f)|^2 \mathrm{d}f\,\delta_{b_1,b_2} \\ &= 4\pi^2\bar{\beta}^2 E_{\mathrm{s}}\delta_{b_1,b_2},\end{aligned} \tag{44}$$

where j is the unit imaginary number, and $\delta_{\cdot,\cdot}$ is the Kronecker delta function. Therefore, it provides

$$\int_0^T \nabla_{\boldsymbol{\tau}\mathbf{s}}\mathbf{D}^2(\mathbf{a})\nabla_{\boldsymbol{\tau}\mathbf{s}}^{\mathrm{H}}\mathrm{d}t = 4\pi^2\bar{\beta}^2 E_{\mathrm{s}}\mathbf{D}^2(\mathbf{a}). \tag{45}$$

Substituting (45) into (43), we obtain (38). ■

We can see that each diagonal element of (38) corresponds to (18b). For the case of known $\boldsymbol{\gamma}$, the unbiased CRB of the position is given by

$$\mathbf{B}_{\mathbf{pp}} = \frac{1}{8\pi^2\bar{\beta}^2\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}}c^2\left(\bar{\boldsymbol{\Phi}}\mathbf{D}^2(\bar{\mathbf{a}})\left(\mathbf{I} + \frac{1}{16\pi^2\bar{\beta}^2}\mathbf{D}^2(\bar{\boldsymbol{\gamma}})\mathbf{D}^{-2}(\bar{\boldsymbol{\tau}})\right)\bar{\boldsymbol{\Phi}}^{\mathrm{T}}\right)^{-1}, \tag{46}$$

where $\bar{\mathbf{a}} \in \mathbb{R}^{(B-M)\times 1}$ and $\bar{\boldsymbol{\gamma}} \in \mathbb{R}^{(B-M)\times 1}$ are defined by

$$\bar{\mathbf{a}} = \begin{bmatrix} a_{M+1} & a_{M+2} & \cdots & a_B \end{bmatrix}^{\mathrm{T}}, \tag{47a}$$

$$\bar{\boldsymbol{\gamma}} = \begin{bmatrix} \gamma_{M+1} & \gamma_{M+2} & \cdots & \gamma_B \end{bmatrix}^{\mathrm{T}}. \tag{47b}$$

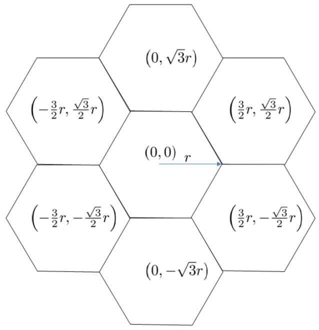

Fig. 1. Cellular system with cell radius r.

The explicit expression of $\mathrm{trace}(\mathbf{B_{pp}})$ from (46) is available in [5]. Since we have $\mathrm{E}_{\mathbf{n}(t)}\left\{(\hat{\mathbf{p}}_{\mathrm{WLS}}-\mathbf{p})(\hat{\mathbf{p}}_{\mathrm{WLS}}-\mathbf{p})^{\mathrm{T}}\right\} = \mathrm{E}_{\mathbf{n}(t)}\left\{(\hat{\mathbf{p}}_{\mathrm{ML}}-\mathbf{p})(\hat{\mathbf{p}}_{\mathrm{ML}}-\mathbf{p})^{\mathrm{T}}\right\} = c^2\left(\bar{\mathbf{\Phi}}\mathbf{D}^2(\bar{\boldsymbol{\sigma}}^2)\bar{\mathbf{\Phi}}^{\mathrm{T}}\right)^{-1}$, the error variance ratio of position estimate is $\sqrt{\frac{\mathrm{trace}\left(\mathrm{E}_{\mathbf{n}(t)}\{(\hat{\mathbf{p}}_{\mathrm{ML}}-\mathbf{p})(\hat{\mathbf{p}}_{\mathrm{ML}}-\mathbf{p})^{\mathrm{T}}\}\right)}{\mathrm{trace}(\mathbf{B_{pp}})}} = 1$, which can be inferred that the WLS and the ML are statistically efficient.

V. Numerical Examples

Consider a certain configuration of a cellular system operating at the center frequency $f_0 = 1.9$ GHz. In seven hexagonal cells, let the origin of the Cartesian coordinate lie at the center of the central cell according to Fig. 1. The BSs are located at the center of each cell with $\mathbf{P} = r\begin{bmatrix} 0 & \frac{3}{2} & 0 & -\frac{3}{2} & -\frac{3}{2} & 0 & \frac{3}{2} \\ 0 & \frac{\sqrt{3}}{2} & \sqrt{3} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & -\sqrt{3} & -\frac{\sqrt{3}}{2}\end{bmatrix}^{\mathrm{T}}$, where r is the cell radius. The mobile station is located at $\mathbf{p} = \frac{1}{2}r\cos\left(\frac{1}{6}\pi\right)\left[\cos\left(\frac{1}{6}\pi\right) \quad \sin\left(\frac{1}{6}\pi\right)\right]^{\mathrm{T}}$ m. The mobile is therefore $\frac{\sqrt{3}}{4}r$ m apart from the center of the central cell. With respect to the MS position, the associated angles of BSs become $\boldsymbol{\phi} = \left[-150^\circ \quad 30^\circ \quad 103.9^\circ \quad 160.9^\circ \quad -150^\circ \quad -100.9^\circ \quad -43.9^\circ\right]^{\mathrm{T}}$. The time delay estimate $\hat{\tau}_b$ is generated from a Gaussian random variable according to the ML error performance in (18). We assume that the first M base stations receive the NLoS signals. From the objective functions in (29), (30) and (31), the mobile position is searched by a simplex method whose initial value is given by the true value perturbed by a zero-mean unit-variance Gaussian random variable. The root mean square error (RMSE) is calculated by the square root of the trace of the error variances from the LS, the WLS, and the ML, i.e. $\bar{\epsilon} = \sqrt{\frac{1}{N_R}\sum_{n_R=1}^{N_R}\|\hat{\mathbf{p}}[n_R]-\mathbf{p}\|_{\mathrm{E}}^2} = \sqrt{\frac{1}{N_R}\sum_{n_R=1}^{N_R}(\hat{x}[n_R]-x)^2+(\hat{y}[n_R]-y)^2}$, where N_R is the number of experimental realizations. From (1), the link budget can be computed from

$$10\log_{10}\left(\frac{E_b}{\sigma_{\mathrm{n}}^2}\right) = 10\log_{10}\left(\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}\right) + 10\log_{10}(\kappa) + 10\gamma_b\log_{10}\left(\frac{d_0}{d_b}\right), \tag{48}$$

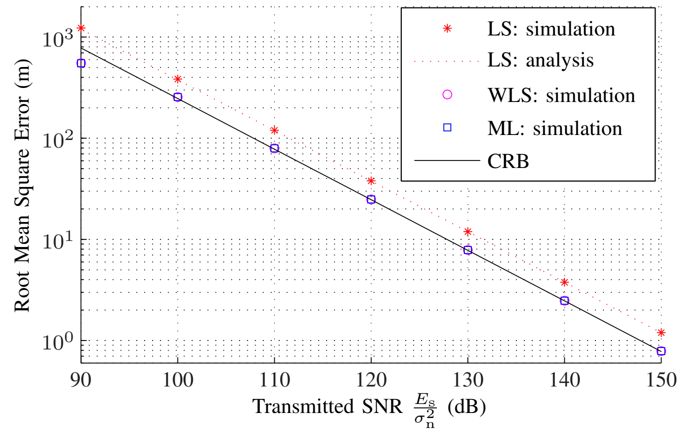

Fig. 2. RMSE of the position estimate as a function of the transmitted SNR for $M = 0$ NLoS BSs, $\bar{\beta} = 10\frac{1}{\sqrt{3}}\pi$ MHz, $r = 2{,}000$ m, $\gamma_b = 4.5425$ and $N_R = 10{,}000$ independent runs.

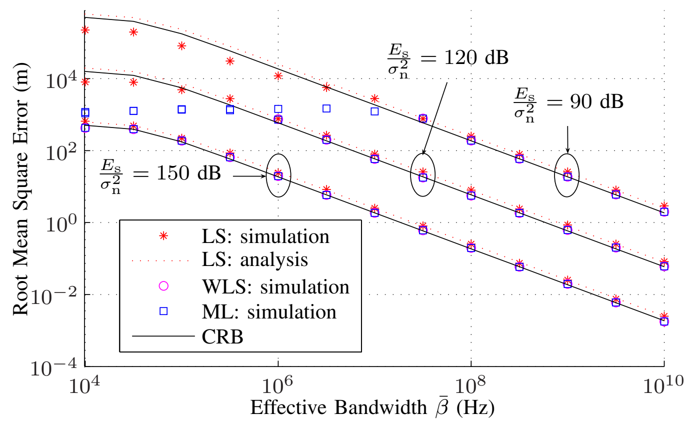

Fig. 3. RMSE of the position estimate as a function of the effective bandwidth of the transmitted signal for $\frac{E_s}{\sigma_n^2} = 90, 120, 150$ dB, $M = 1$ NLoS BSs, $r = 2{,}000$ m, $\gamma_b = 4.5425$ and $N_R = 100$ independent runs.

where $\frac{E_b}{\sigma_{\mathrm{n}}^2}$ is the received signal-to-noise ratio (SNR) at the b-th base station and $\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2}$ is the transmitted SNR. The purpose of the link budget is to point out that the transmitted SNR should be high in order to maintain an acceptable received SNR.

In Fig. 2, the RMSE is shown as a function of the transmitted SNR from $\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} = 90$ dB to $\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} = 150$ dB. In this case, we have $\frac{E_1}{\sigma_{\mathrm{n}}^2} = \frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} - 120.5873$ dB. It means the central cell actually receives the SNR from $\frac{E_1}{\sigma_{\mathrm{n}}^2} \approx -30$ dB to $\frac{E_1}{\sigma_{\mathrm{n}}^2} \approx 30$ dB. From $\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} = 100$ dB to $\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} = 150$ dB, the LS in Fig. 2 well coincides with its expected error analysis, but cannot attain the CRB, whereas the WLS and the ML do. For $\frac{E_{\mathrm{s}}}{\sigma_{\mathrm{n}}^2} < 100$ dB, the WLS and the ML of the mobile position deviate from the CRB. This is because the ML error performance in (18) holds true for high received SNR. At low received SNR, the time delay error variance σ_b^2 is inaccurate and leads to an erroneous value of $\sigma_b^2(x, y)$ for the WLS in (30) and for the ML in (31).

In Fig. 3, the error variance of the mobile position estimate decreases with the increase in the effective bandwidth. From a

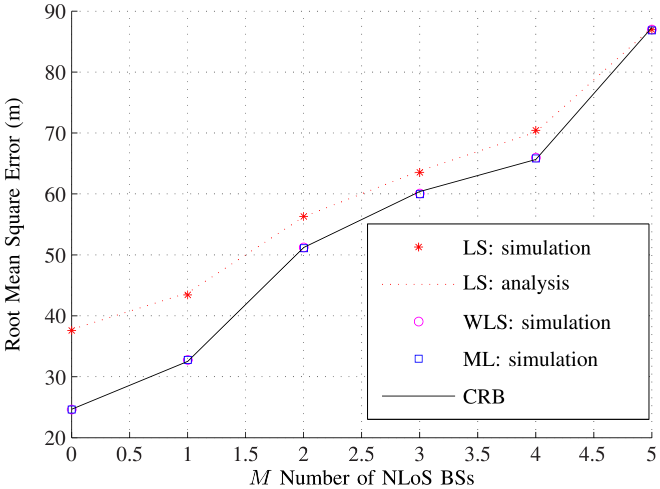

Fig. 4. RMSE of the position estimate as a function of the number of the NLoS BSs for $\frac{E_s}{\sigma_n^2} = 120$ dB, $\bar{\beta} = 10\frac{1}{\sqrt{3}}\pi$ MHz, $r = 2{,}000$ m, $\gamma_b = 4.5425$ and $N_R = 10{,}000$ independent runs.

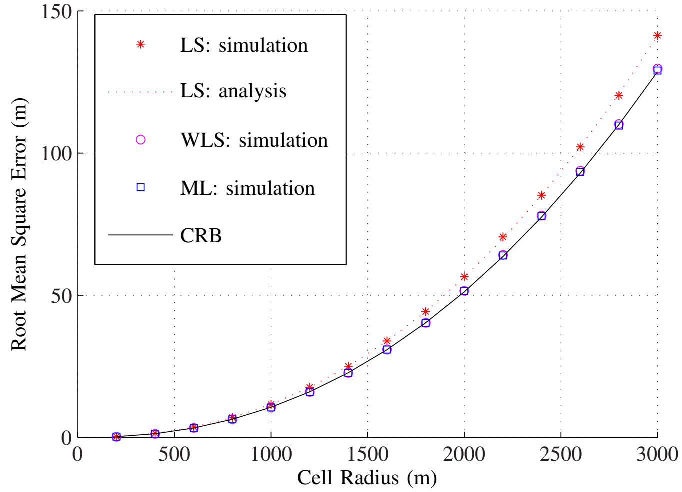

Fig. 5. RMSE of the position estimate as a function of the cell radius for $\frac{E_s}{\sigma_n^2} = 120$ dB, $\bar{\beta} = 10\frac{1}{\sqrt{3}}\pi$ MHz, $M = 2$ NLoS BSs, $\gamma_b = 4.5425$ and $N_R = 10{,}000$ independent runs.

condition of the Taylor expansion, the performance analysis of the time delay estimate $\hat{\tau}_b$ is accurate when the estimate $\hat{\tau}_b$ is close to the true value τ_b, i.e. for small σ_b^2 in (18), which means high SNR and/or large effective bandwidth. Therefore, for low effective bandwidth, all the estimators based on the time delay cannot well correspond to their theoretical analyzes, i.e. LS and the CRB. However, for high SNR, e.g. $\frac{E_s}{\sigma_n^2} = 150$ dB, the predictions by the analyzed LS and the CRB are reliable.

In Fig. 4, the number of all BSs is kept as a constant, i.e. $B = 7$, while the number of the LoS BSs is varied from 0 to 5 according to the condition $B \geq M + 2$. We can see that the statistically efficient estimators, the WLS and the ML, outperform the LS, especially for low M. The reason is that the WLS and the ML employ more information in their criteria, which perform well when they gain more LoS information.

In Fig. 5, the RMSE is shown as a function of the cell radius, which is related to the distance between the MS and the BS. It can be seen that the use of more sophisticated estimators can obtain more accuracy at the expense of more computation.

VI. Conclusion

The estimation of the mobile position from the time delay has been considered for the NLoS geolocation exploiting the path attenuation. The use of LoS time delay provides the same performance as that given by using both the LoS and NLoS time delays. The LS, WLS and ML are proposed to estimate the mobile position. Numerical results illustrate that the time delay performance is accurate when the time delay estimate is close to its true value, i.e. for small time delay error variance. For high SNR and large effective bandwidth, the LS cannot provide the performance compared to the CRB, whereas the WLS and ML are statistically efficient. Even though the time delay is estimated by the ML, the direct least squares fit to obtain the mobile position, in general, is suboptimal. Rather, the weighted least squares and the sophisticated maximum likelihood satisfy the optimal performance.

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