A Note on the Trackability of Dynamic Sensor Networks

[Extended Abstract]

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ABSTRACT

Since the last decade, Semidefinite programming (SDP) has found its important application in locating the ad hoc wireless sensor networks. By choosing proper decomposition and computation schemes, SDP has been shown very efficient to handle the localization problem. Previous research also has shown that the SDP locates the sensor networks in $\mathbb{R}^d$ correctly provided the underlying framework is strong uniquely localizable. In this paper, we consider the localization problem in a more general and practical scenario, that is, the sensors are in movement following a certain trajectory. We show that given the initial position of each sensor and the instantaneous distance data, the dynamic sensor networks can be can be tracked correctly in the near future when the underlying framework is infinitesimal rigid and the trajectories of the sensors are subject to mild conditions. Our result also provides a way to approximate the sensor trajectories using Taylor series based on the distance data.

Categories and Subject Descriptors

G.1 [**Numerical Analysis**]: Optimization

General Terms

Theory

Keywords

Semidefinite Programming, Sensor Network Localization, Uniquely Localizable, Infinitesimal Rigidity

1. INTRODUCTION

A wireless sensor network typically consists of hundreds of sensors, which are spread over a vast physical area and captures local environmental parameters. The sensors can share data with its neighbor sensors so their pairwise distances can be observed. Some sensors have known and fixed locations, which are called anchors to be different from other sensors. The network sensor localization problem is to find the true location of each sensor based on the pairwise distance information from sensors and anchors.

The network sensor localization problem can be considered as a *Graph Realization* problem. Consider the n sensors and m anchors as the vertices of the graph G, and the vertices are connected if and only if the pairwise distances d_{ij} have been detected. Usually, the sensors can communicate with other sensors and anchors only within a certain radio rage. The static sensor network localization problem is to find out a realization locations in $\mathbb{R}^d$, namely $X = (x_i)_{1\le i\le n}$, such that

$$\begin{array}{ll} \|x_i - x_j\| = d_{ij} & \forall (i,j) \in N_x \\ \|x_i - a_{j+n}\| = d_{i,j+n} & \forall (i,j+n) \in N_a, \end{array} \tag{1.1}$$

where a_{j+n} denotes the known locations of anchor j, $E = (N_x, N_a)$ is a partition for the set of edges, depending on whether the edge is adjacent to an anchor node or not.

This problem, even in the case $d = 2$, has been shown NP-complete [1]. The problem raised interest of optimizers after the investigation of applying the semidefinite programming (SDP) to solve the locations [8, 2]. A SDP relaxation for the problem [7, 2] is

$$\begin{array}{ll} \text{minimize} & \mathbf{0} \bullet Z \\ \text{s.t.} & (\mathbf{0}; e_i - e_j)(\mathbf{0}; e_i - e_j)^T \bullet Z = d_{ij}^2, \\ & \quad \forall (i,j) \in N_x \\ & (-a_k; e_i)(-a_k; e_i)^T \bullet Z = d_{i,k+n}^2, \\ & \quad \forall (i,k+n) \in N_a \\ & Z = \begin{pmatrix} I & Z_{21}^T \\ Z_{21} & Z_{22} \end{pmatrix} \succeq 0 \end{array} \tag{1.1}$$

The SDP approach for the network localization problem has been well studied and improved since the last decade. Recent work includes sub-SDP relaxation [9], clique reduction and cone projection [4], and region partition [3]. The current SDP methods can solve thousands of sensors within several minutes [9]. This is cheaper than to compute the locations of sensors from a Global Positioning System (GPS).

The scenario we concern in this paper is somewhat different from those previous works on sensor network localization. Consider in the real life, the sensors usually do not stay still but move along certain trajectories, e.g., motions of molecules, mobiles. Then we expect to attain a sequence of distance data which updates according to the movement of the sensors. Compared with the static network localization model, the dynamic networks certainly provide more information. In [8], it has been studied that under what conditions the SDP can locate network sensors correctly (without considering noisy effect and computational error). It

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turns out the strong realizability of the underlying graph G is indispensable to guarantee the correctness of the SDP solution. In this paper, we will discuss the corresponding conditions for the dynamic sensor tracking problem. It can be expected that the conditions can be weakened because more distance data are available in the dynamic setting. Our results will be of particular interest when people know the historical locations of the sensors and when the movement of the sensors follows a certain kind of natures.

The paper is organized as follows. Section 2 will provide some background on the graph realization and discuss the conditions under which SDP locates the sensor network correctly in the static scenario; Section 3 will discuss our main concerns that how the aggregated information in dynamic settings helps us to locate sensors and predict their trajectories. Section 4 is a brief conclusion of our results.

2. STATIC SENSOR NETWORKS AND LOCALIZABILITY

In this section, we will give some basic definitions related with the graph rigidity theory. The graph rigidity theory is a key tool to analyze the localizability of the sensor networks [8]. Even in the dynamic case to be discussed in later sections, our results are based on the graph rigidity theory.

For a sensor network with n sensors and m anchors, we first define its underlying graph $G(V, E)$. Let $V = \{1, 2, \ldots, n+m\}$ denote the set of vertices, which can represent a sensor or an anchor; the edge set E represents the set of pair of sensors (or a sensor and an anchor), whose pairwise distance is known. Let $X = (X_i)_{i=1,\ldots,n+m}$ denote the locations of each sensor or anchor in $\mathbb{R}^2$, namely, $X_i = x_i$ for $i = 1, \ldots, n$ and $X_{i+n} = a_i$ for $i = 1, \ldots, m$. The pair (G, X) is called a *framework*. Assume now the framework is parameterized with a time parameter t, such that the instantaneous locations $X(t)$ are continuously differentiable. Then an assignment of velocities $v(t) = X'(t)$ is called an infinitesimal motion of the framework is

$$(x_i - x_j)^T (v_i - v_j) = 0 \qquad \forall (i, j) \in E. \tag{2.2}$$

If the assigned velocities are a transition or rotation, we call the infinitesimal motion is trivial. If the framework has only trivial velocity assignment, the framework is called infinitesimally rigid; otherwise, the framework is called infinitesimally flexible. In the sensor network setting, the framework is called to have a fixing infinitesimally motion, if it has a nonzero infinitesimal motion with $v_i = 0$ for any anchor nodes $i = n+1, n+2, \ldots, n+m$.

We call a graph uniquely localizable in $\mathbb{R}^d$ if it has a unique realization in $\mathbb{R}^d$ and it has no other realization whose affine space had dimensions larger than d. It has been shown in [6] that the infinitesimal rigidity, 3-connectivity and redundant rigidity are sufficient for unique realization in $\mathbb{R}^2$. Based on this result, the following theorem is proved in [8].

THEOREM 2.1. *The sensor network is uniquely localizable in $\mathbb{R}^2$ if and only if the associated graph G' is uniquely realizable in $\mathbb{R}^2$, where G' is the graph including all the anchor-anchor edges into G.*

The unique realization of G' in $\mathbb{R}^2$ can be checked efficiently, see [5]. If the sensor network is uniquely localizable in $\mathbb{R}^2$, then any feasible solution to the SDP must be the unique locations for the original problem; however, if G' is not uniquely realizable, Theorem 2.1 implies that we can not locate the sensors solely with the distance information.

Unique realization is unstable under small perturbations, which may cause serious problems when we try to use SDP to solve the problem. So the solvability of the network sensor problem by an SDP method requires an even stronger condition, namely, *strongly localizable*. A sensor network is called strongly localizable if and only if the dual of (1.1) has an optimal dual slack matrix with rank n. Based on the concept of strong localizable network, the following theorem ([8]) completes the discussion on the solvability of the sensor network problem with an SDP method.

THEOREM 2.2. *If the sensor network contains a sub-network that is strongly localizable, then the SDP computes a solution that localizes all possibly localizable unknown points.*

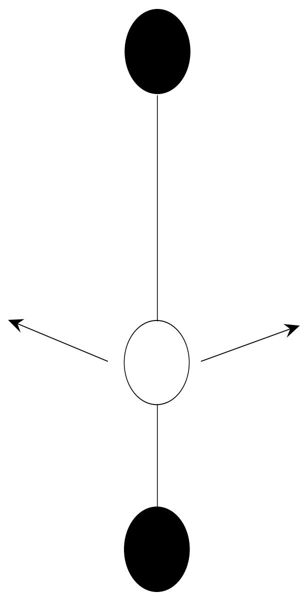
3. DYNAMIC SENSOR NETWORK AND TRACKABILITY

In this section, we will discuss the problem mentioned forehead for tracking the dynamic sensor networks. Assume we known the *exact* location of sensors at a certain time point, namely, $t = 0$. And there arrives the updating pairwise distance data $D(t) = (d_{ij})$ within a time period $[0, \tau]$, we want to compute the new locations $X(t)$ that is consistent with the given data information. It is clear that if there are possibly more than one solution consistent with the given information, then we are unable to tell which location is the true one. So we say the sensors are *trackable* if and only if there is a unique $X(t)$ consistent with the historical location data $X(0)$ and the instantaneous distance data $D(t), 0 \le t \le \tau$. It is not trivial to generalize the localizability results in the static case to the trackability issue in the dynamic case. Figure 3 is an example that even when the graph is uniquely localizable at $t = 0$, the dynamic sensor network is not trackable after $t > 0$ because the sensor in middle can move either towards east or west with completely the same distance information $D(t)$ due to the symmetry.

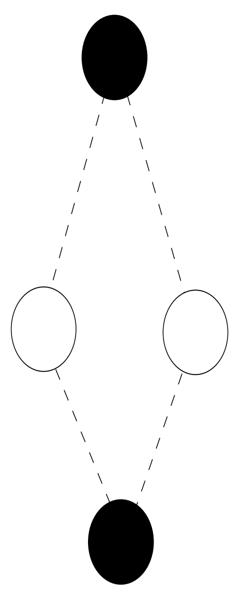
Determining whether the dynamic sensor network is trackable also closely depends on the rigidity of the underlying framework $(G(t), X(t))$, that includes all the sensor and anchor nodes. Here the underlying graph $G(t)$ is no longer fixed because the availability of the pairwise distance changes with the motion of the sensors, e.g., two sensors may lose contact after they departed too far away. To make things simple, let us first consider the case that the edge set E is invariant within a short period $t \in [0, \tau]$. Later, we will generalize the results to the long run by switching the time windows.

THEOREM 3.1. *Suppose the underlying graph $G(t) = (V, E(t))$ is invariant within a short period $t \in [0, \tau]$, and each sensor i moves according to a polynomial of time parameter t with finite degrees in the period $[0, \tau]$, i.e., $X_i(t) = (p_{i1}(t), p_{i2}(t))$ with p_{i1} p_{i2} being polynomials. Also, assume the initial location $\bar{X}$ and the pairwise distance partial matrix $D(t), t \in [0, \tau]$ has been observed correctly. Then the sensors are trackable if the underlying framework $(G, X(0))$ does not admit a fixing infinitesimal motion; or equivalently, the associated framework $(G', X(0))$ is infinitesimally rigid.*

Proof. Suppose both $X(t), \hat{X}(t)$ are consistent with the historical locations $\bar{X}$ and the updated distance data



(a) Initial sensor location at $t = 0$



(b) Two possible sensor locations at $t > 0$ that feedback the same distance data

Figure 1: An Example of Uniquely Realizable Graph Not Trackable (Black-Anchor, White-Sensor)

$D(t)$, $\forall t \in [0, \tau]$. For simplicity, let $X', \hat{X}'$ denote their derivatives of t at time $t = 0$, and let $X^{(k)}, \hat{X}^{(t)}$ denote their k-th derivative of t at time $t = 0$. Then for any $(i, j) \in E$,

$$\begin{aligned} d'_{ij}(0) &= 2(X_i(0) - X_j(0))^T (X'_i - X'_j) \\ &= 2(\hat{X}_i(0) - \hat{X}_j(0))^T (\hat{X}'_i - \hat{X}'_j) \end{aligned} \qquad (3.3)$$

To be consistent with the initial location, we know $X(0) = \hat{X}(0) = \hat{X}$. Denote $v_i^k := X_i^{(k)} - \hat{X}_i^{(k)}$ in subsequent text. Then (3.3) implies $\forall (i, j) \in E$,

$$0 = 2(X_i(0) - \hat{X}_j(0))^T ((X'_i - \hat{X}'_i) - (X'_j - \hat{X}'_j)) \qquad (3.4)$$

The new locations $X(t), \hat{X}(t)$ should be consistent with the anchors, which has fixed locations. So for any anchor node $i = n+1, \ldots, n+m$, $X_i(t) = \hat{X}_i(t) \equiv a_{i-n}$, that is, $v_i^k = 0$ for all anchor nodes $i = n+1, \ldots, n+m$ and any order of derivatives k. Therefore, no fixing infinitesimal motion (2.2) exactly implies $v_i^1 = (X_i)' - (\hat{X}_i)' = 0$ for all $i = 1, 2, \ldots, n$. Now we use induction to finish the proof. Suppose $v_i^s = X_i^{(s)} - \hat{X}_i^{(s)} = 0$ for any $s \le k$, and compute the $k+1$-th derivative of d_{ij} for all $(i, j) \in E$ by the Leibnitz formula:

$$\begin{aligned} d_{ij}^{(k+1)}(0) &= \textstyle\sum_{s=0}^{k+1} C_{k+1}^s ((X_i)^{(s)} - (X_j)^{(s)})^T \\ &\quad ((X_i)^{(k+1-s)} - (X_j)^{(k+1-s)}) \\ &= \textstyle\sum_{s=0}^{k+1} C_{k+1}^s ((\hat{X}_i)^{(s)} - (\hat{X}_j)^{(s)})^T \\ &\quad ((\hat{X}_i)^{(k+1-s)} - (\hat{X}_j)^{(k+1-s)}), \end{aligned} \qquad (3.5)$$

which implies

$$\begin{aligned} 0 &= \textstyle\sum_{s=0}^{k+1} C_{k+1}^s ((((X_i)^{(s)} - (X_j)^{(s)})^T \\ &\quad ((X_i)^{(k+1-s)} - (X_j)^{(k+1-s)}) \\ &\quad -((\hat{X}_i)^{(s)} - (\hat{X}_j)^{(s)})^T \\ &\quad ((\hat{X}_i)^{(k+1-s)} - (\hat{X}_j)^{(k+1-s)})). \end{aligned} \qquad (3.6)$$

By induction assumption, $X^{(s)} = \hat{X}^{(s)}$ for all $s \le k$. So the above equation only left with

$$0 = 2(\bar{X}_i - \bar{X}_j)^T (v_i^{k+1} - v_j^{k+1}), \qquad (3.7)$$

Then infinitesimally rigidity (2.2) implies that $v_i^{k+1} = 0$ for any $i \in V$. Therefore, we have shown that $X(t) - \hat{X}(t)$ has zero derivatives of any order at $t = 0$. Because we assume that $X(t) - \hat{X}(t)$ is a polynomial of t on $[0, \tau]$, it follows that $X(t) \equiv \hat{X}(t)$ within period $t \in [0, \tau]$, or say, there is a unique trajectory $X(t)$ consistent with the given data. ■

Remark 3.1. *The assumption that the motion $X(t)$ is a polynomial function about t can be extended to the case that $X(t)$ has convergent Taylor expansion over $[0, \tau]$. Actually, the proof of Theorem 3.1 provided the approach to calculate the high order derivatives of $X^{(k)}(0)$. And then the sensor trajectories in the near future can be approximated by the Taylor polynomials $X(t) = \sum_{k=0}^{N} \frac{X^{(k)}(0)}{k!} t^k$ for large enough N.*

At last, we state Theorem 3.1 in the long run version.

Theorem 3.2. *Suppose during the period $t \in [0, \tau]$, the underlying graph $G(t)$ changes at time point $t_1, t_2, \ldots, t_k$. Assume each sensor i moves according to a polynomial of time parameter t with finite degrees within each time interval $[t_i, t_{i+1}]$. Also, assume the initial location $X(0)$ and the pairwise distance partial matrix $D(t), t \in [0, \tau]$ has been observed* *correctly. Then the sensors are trackable if at time slot* $t = 0, t_1, t_2, \ldots, t_k$*, the underlying framework* $(G(t), X(t))$ *does not admit a fixing infinitesimal motion; or equivalently, the associated framework* $(G'(t), X(t))$ *is infinitesimally rigid.*

So the trackability of a dynamic sensor network only depends on the framework at each time slot t_i, at which a change occurs to the set E. Here the infinitesimal rigidity of the associated framework G' is a weaker condition than the condition of strong localizability. The reason is, the initial given position $X(0)$ renders us to rule out other solutions to SDP that leave too far away the initial positions. In fact, if the initial locations $X(0)$ are unknown, then according to Theorem 3.1, each possible initial location will corresponds to a sequence of possible new locations $X(t)$. Then the sensors are trackable only if the framework $(G(\tau), X(\tau))$ becomes strongly localizable at some later time $\tau > 0$, the conditions in Theorem 3.2 are satisfied when $t > \tau$.

4. CONCLUSIONS

In this paper, we introduced a new problem of tracking the moving sensors based on their initial positions and the distance data which are updating with time t. We found the infinitesimal rigidity is a sufficient condition for the trackability of a dynamic sensor network under mild assumptions on the trajectories of the sensors. This is actually not surprising because the known historical locations provide some 'global' information. The proof to Theorem 3.1 also provides a method to approximate the trajectories of the sensors with Taylor expansion according to the distance data received in a short period.

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