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# Comparison between the unscented Kalman filter and the extended Kalman filter for the position estimation module of an integrated navigation information system

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## Abstract

*An integrated navigation information system must know continuously the current position with a good precision. The required performance of the positioning module is achieved by using a cluster of heterogeneous sensors whose measurements are fused. The most popular data fusion method for positioning problems is the extended Kalman filter. The extended Kalman filter is a variation of the Kalman filter used to solve non-linear problems. Recently, an improvement to the extended Kalman filter has been proposed, the unscented Kalman filter. This paper describes an empirical analysis evaluating the performances of the unscented Kalman filter and comparing them with the extended Kalman filter's performances.*

## 1. Introduction

An integrated navigation information system is an embedded system installed in a car which provides useful functionalities to the driver like trip planning, guidance, digital map and points of interest directory. The guidance module uses a planned trip to indicate the driver which road to take. To avoid giving bad turn direction, the system must know the localization of the vehicle precisely and continuously.

GPS receiver can measure the position of the vehicle to the required precision. However, a GPS solution could be unavailable for several seconds because of the occlusions of satellite signals by high buildings or heavy foliages. Thus, a cluster of sensors including a GPS receiver is usually used. Two other popular sensors are differential odometer and inertial measurement unit (IMU). A differential odometer measures the distance traveled by a vehicle and the current azimuth. An IMU measures the acceleration and the angular velocity along the axis of a Cartesian coordinate system. With these two sensors, the position of the vehicle is reckoned by applying basic kinematics' equations and and using an initial position obtained from another information source. The estimated position will eventually drift from the real position because of the accumulation of errors. So the position must be reset periodically with an absolute position like the position estimated by the GPS.

A more complex fusion method than the reckon/reset positioning system described above is usually used to improve the precision of the estimation. These methods fuse continuously the available measurements in some optimal sense. The most popular method is the Kalman filter. This method uses the a priori information on the sensor noises, the vehicle dynamic and the kinematics' equations to compute recursively an optimal position which minimizes the mean square error.

### 1.1 Previous Work

The Kalman filter is an optimal linear estimator introduced in 1960 [1]. The filter is optimal when the process noise and the measurement noise can be modelized by white Gaussian noise. Non-linear problems can be solved with the extended Kalman filter (EKF) [2, 3]. This filter is based upon the principle of linearizing the state transition matrix and the observation matrix with Taylor series expansions. The linearization can lead to poor performance and divergence of the filter for highly non-linear problems. A recent improvement to the EKF is the unscented Kalman filter (UKF) [4]. The UKF approximates the probability density resulting from the non-linear transformation of a random variable instead of approximating the nonlinear functions with a Taylor series expansion. The approximation is done by evaluating the nonlinear function with a minimal set of carefully chosen sample points. The posterior mean and covariance estimated from the sample points are accurate to the second order for any nonlinearity [5]. If the priori random variable is Gaussian, the posterior mean and covariance are accurate to the third order for any nonlinearity [6].

The architecture of a positioning system can be decentralized or centralized. In the centralized architecture, all the sensor measurements are fused by one fusion method only. So it is easy to compare the performance of two different fusion methods when the cluster of sensors is the same. This is the architecture that has been chosen for the empirical analysis presented here. The equations of a centralized extended Kalman filter for land navigation positioning system are described in [7].

The extended Kalman filter has been very popular for land navigation system [8, 9, 10]. The extended Kalman filter can be replaced by better algorithms like the unscented Kalman filter. The first use of an unscented Kalman filter for land navigation positioning system is described in [11]. To our knowledge, only one paper has been recently written on the use of the unscented Kalman filter as the fusion method in an integrated navigation information system [12]. An unscented Kalman filter has also been used for GPS positioning [13]. The goal of the present paper is to analyse the performance improvement of the unscented Kalman filter over the extended Kalman filter for an integrated navigation information system.

## 2 Methodology

### 2.1 Simulation

A simulator has been built to evaluate the performances of the unscented Kalman filter and the extended Kalman filter. The simulator has two parts. The first part computes the kinematics, the position and the attitude of a car travelling a route. The road segments are from the city of Calgary. The kinematics was computed as a function of the acceleration of a real vehicle measured while travelling on the route used in the simulator. The second part emulates the sensor measurements. The models of the different sensors are based on automotive grade real sensors. The measurements are then fused as depicted in figure 1.

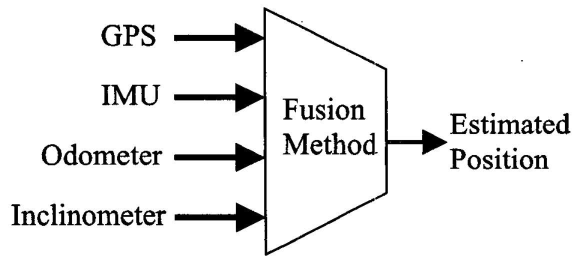


**Figure 1 : Centralized fusion**

The sensors' measurements are distortioned by deterministic and random errors. A source of random errors can be described as a stochastic process. The simulator uses a random number generator to emulate random errors. The estimated position is the posteriori random variable resulting from the mathematical transformation of the stochastic processes which modelize the sensors' imperfect measurements. The fusion method defines the mathematical transformation.

100 Monte Carlo simulations has been run for each fusion method. For each sampling time, the estimated positions from the Monte Carlo simulations form the sampling distribution. There are 26639 measurement vectors for each Monte Carlo simulation. These sampling distributions approximate the truth continuous distributions of the posteriori random variables describing the estimated positions. The first moment of each sampling distribution has been computed and used for the computation of the performance metrics.

### 2.2 Algorithms

The extended Kalman filter predicts the states of the random process with equation (1). The predicted states are updated with the measurements in equation (2).

$$x_{k+1|k} = \Phi_{k+1|k}[x_{k|k}] + w_k \tag{1}$$

$$z_{k+1} = H_{k+1|k}[x_{k+1|k}] + v_{k+1} \tag{2}$$

where $x_{k+1|k}$ is the predicted process state vector, $x_{k|k}$ is the estimated process state vector, $\Phi_{k+1|k}$ is the discrete state transition matrix from k to k+1, $w_k$ is the process noise vector, $z_{k+1}$ is the measurement vector, $H_{k+1|k}$ is the observation matrix and $v_{k+1}$ is the measurement noise vector.

In our study, we have 13 states to describe the random process. A position-velocity-acceleration model is used for each component of the position [14]. The last four states include the slope, the pitch, the azimuth and the yaw velocities. The state transition matrix $\Phi_{k+1|k}$ is linear. Only the observation matrix $H_{k+1|k}$ contains nonlinear equations, the most relevant for horizontal positioning is described by equation 3.

$$\begin{bmatrix} a_R \\ a_P \\ a_Y \end{bmatrix} = \begin{bmatrix} \cos(\Phi_y)*\cos(\Phi_p) & -\sin(\Phi_y) & \cos(\Phi_y)*\sin(\Phi_p) \\ \sin(\Phi_y)*\cos(\Phi_p) & \cos(\Phi_y) & \sin(\Phi_y)*\sin(\Phi_p) \\ -\sin(\Phi_p) & 0 & \cos(\Phi_p) \end{bmatrix} * \begin{bmatrix} a_N \\ a_E \\ a_D \end{bmatrix} \quad (3)$$

where $a_R$, $a_P$, $a_Y$ are the acceleration vector components along the roll, the pitch and the yaw axis respectively, $\Phi_p$ and $\Phi_y$ are the euler angles for the pitch and the yaw axis respectively, $a_N$, $a_E$, $a_D$ are the acceleration along the north axis, the east axis and the down axis respectively.

The extended Kalman filter approximates the non-linear matrix H based on the Taylor series expanded about the estimated state vector with

$$H[\hat{x}_{k+1|k}] \approx H[\hat{x}_{k|k}] + \frac{\partial H[\hat{x}_{k|k}]}{\partial \hat{x}_{k|k}} (\hat{x}_{k+1|k} - \hat{x}_{k|k}) \quad (4)$$

The linear approximation often introduces large errors in the estimated state vector and can lead to the divergence of the filter.

The unscented Kalman filter is based on the unscented transformation, which is a method for reckoning the statistics of a random variable undergoing a non-linear transformation. A set of $2*n_\chi + 1$ weighted samples are deterministically chosen to capture the true mean and variance of the prior random variable.

$$n_\chi = n_x + n_w + n_v \quad (5)$$

where $n_x$ is the number of process states, $n_w$ is the dimension of $w_k$ and $n_v$ is the dimension of $v_k$.

The unscented Kalman filter approximates the non-linear observation matrix by

$$H[\hat{x}_{k+1|k}] \approx \sum_{i=0}^{2*n_\chi} W_i * H[\chi^x_{i,k+1|k}] + \chi^v_{i,k+1} \quad (6)$$

where Wi are the weights, $\chi^x_{i,k+1|k}$ are the sigma points describing the prior predicted states and $\chi^v_{i,k+1}$ are the sigma points describing the measurement noise.

### 2.3 Results

The two performance metrics are the accuracy/precision of the fusion and the computational time to perform the fusion. The accuracy is evaluated by taking the Euclidian distance between the estimated position and the true position. The mean and the variance of the Euclidian distances for the whole simulation are reckoned. The variance describes the precision of the fusion method. The horizontal position is described by the tangential plane located at the real vehicle position whose coordinates are given by the latitude and the longitude.

**Table 1: Mean Position Error**

| Component | Gain (%) | Ukf (m) | Ekf (m) |
|---|---|---|---|
| Latitude | -40.74 | -1.52 | -1.08 |
| Longitude | 11.25 | 2.92 | 3.29 |
| Horizontal | 3.60 | 7.76 | 8.05 |

**Table 2 : Position Error Variance**

| Component | Gain (%) | Ukf ($m^2$) | Ekf ($m^2$) |
|---|---|---|---|
| Latitude | 12.77 | 40.10 | 45.97 |
| Longitude | -4.82 | 38.08 | 36.33 |
| Horizontal | 2.47 | 28.81 | 29.54 |

The unscented Kalman filter has slightly better results for horizontal positioning than the extended Kalman filter. In table 1, the estimated position is less biased for the unscented Kalman filter than for the extended Kalman filter. The results in table 2 shows that the unscented Kalman filter is more precise than the extended Kalman filter.

For each estimated position the execution time taken by the fusion method was recorded. Table 3 shows the mean computational time.

**Table 3 : Mean Computational Time**

| Ekf (s) | Ukf (s) | Gain (%) |
|---|---|---|
| 0.0028 | 0.0658 | -2250 |

Contrary to the claim in [5, 6], the computational cost of the unscented Kalman filter is significantly greater than the computational cost of the extended Kalman filter. This is shown in table 3. The significant execution time difference is related to the number of times equations 1 and 2 are evaluated for each fusion algorithm. With the unscented Kalman filter, these equations are evaluated 75 times, once for each sigma point. With the extended Kalman filter, the Taylor series expansion of these equations are only evaluated once at each iteration. Furthermore, the Jacobian of the matrix H used in the Taylor series expansion is calculated only once because the observation equations are static. Thus the multiple computations of equations 1 and 2 by the unscented Kalman filter at each iteration is responsible for the larger computational cost.

**Table 4 : Horizontal position error when no GPS solution is available**

| Statistical moment | Gain (%) | Ukf | Ekf |
|---|---|---|---|
| Mean (m) | -2.03 | 21.63 | 21.20 |
| Variance (m^2) | -10.02 | 599.63 | 545.03 |

Surprisingly, the unscented Kalman filter is less performant than the extended Kalman filter when there is no GPS solution available. In that situation, the acceleration of the vehicle measured by the IMU is used to estimate the vehicle's position described by equation 3. This equation represents the nonlinear transformation of the estimated states which are assumed to be Gaussian random variable in order to predict the IMU measurement. The performance of both filters depends on their capacity to estimated the mean of the resulting random variable. An empirical analysis has been made to evaluate this capacity. In this experiment, each state has been modelized by a discrete Gaussian random variable with 100 realizations distributed uniformely in the range of possible value with a 99% probability of realization. Each realization is present a number of times proportional to its probability of realization in the statistical data representing the probability function. Thus, 24060 samples modelized each random variable. The nonlinear function described by equation 3 is then applied to these random variables and the means of the resulting random variables are computed. The same discrete random variables are used with the Taylor series expansion of equation 3. In the extended Kalman filter, the linearization occurs around the states estimated at the previous iteration. The linearized equation is applied to the predicted states at the current time. The linearization error is directly proportional to the difference between the these estimated states and predicted states. For the empirical analysis, the mean variation between the estimated value and the predicted value obtained with the extended Kalman filter for one Monte Carlo simulation has been taken. Table 5 shows the variation between the real mean of the a posteriori probability density and the estimated mean of the a posteriori probability density obtained with the Taylor series expansion and the unscented Kalman filter. As can be seen, the unscented Kalman filter provides no significant improvement over the extended Kalman filter and even brings a deterioration for two acceleration components.

**Table 5 : Difference between the real mean and the estimated mean of the a posteriori density**

| Estimated state | EKF | UKF |
|---|---|---|
| Roll acceleration | 0.0064 % | 0.8070 % |
| Pitch aceleration | 0.0218 % | 1.2876 % |
| Yaw acceleration | 0.2482 % | 0.0754 % |

The superiority of the unscented Kalman filter happens only when the variation between the predicted states and the estimated states is important. With the simulated data, approximatively 98% of the variations encountered are not important enough to generate a significant linearization error. This is due to the low dynamics of the vehicle.

## 3 Conclusion

The unscented Kalman filter has a slightly better performance than the extended Kalman filter when used as a fusion method in a positioning module of an integrated navigation information system. Unfortunatly, there is no gain of performance when there are no GPS solution available. An empirical analysis has demonstrated that the low dynamics of a vehicle make the potential linearization errors of the extended Kalman filter negligible. Futhermore, the computational time of the unscented Kalman filter is much greater than the computational time of the extended Kalman filter. One way to diminish the computational time of the UKF might be to use an decentralized architecture instead of a centralized one. In a decentalized architecture, the local filter for the IMU and the local filter for the differential odometer would be an UKF. The local filter for the GPS would be a standard linear Kalman filter.

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## References

[1] R. E. Kalman, "*A new approach to linear filtering and prediction problems*", Transactions of the ASME, Journal of Basic Engineering, 1960.

[2] B. Anderson and J. Moore, "*Optimal Filtering*". Prentice Hall, Englewood, NJ, 1979.

[3] A. Gelb, "*Applied Optimal Estimation*". The MIT Press, Cambridge, Massachusetts, 1974.

[4] S. J. Julier and J. K. Uhlmann, "*A New Extension of the Kalman Filter to Nonlinear Systems*", In Proc. of AeroSense: The 11th Int. Symp. on Aerospace/Defence Sensing, Simulation and Controls, 1997.

[5] A. D. R. Merwe, N. Freitas, E. Wan, "*The Unscented Particle Filter, Tech report CUED/F-INFENG/TR-380*", Cambridge University Engineering Department, Cambridge, England August 2000.

[6] E. A. Wan and R. v. d. Merwe, "*The Unscented Kalman Filter for Nonlinear Estimation*", Proceedings of Symposium 2000 on Adaptive Systems for Signal Processing, Communication and Control (AS-SPCC), Lake Louise, Alberta, Canada, 2000.

[7] M. S. Grewal, L. R. Weill, and A. P. Andrews, "*Global Positioning Systems, Inertial Navigation, and Integration*". John Wiley & Sons, New York, 2001.

[8] E. Abbott and D. Powell, "*Land-Vehicle Navigation Using GPS*", Proceedings of the IEEE, 1999.

[9] Q. Wu, Z. Gao, and D. Wan, "*An adaptive information fusion method to vehicle*", IEEE Position location and navigation symposium, 2002.

[10] K. A. Redmill, T. Kitajima, and U. Özgüner, "*DGPS/INS Integrated Positioning for Control of Automated Vehicle*", IEEE Intelligent Transportation Systems, Oakland, 2001.

[11] S. Julier, "*Process Models for the Navigation of High-Speed Land Vehicles*", Wadham College, Oxford, 1997, pp. 184.

[12] W. Li and H. Leung, "*Constrained Unscented Kalman Filter Based Fusion of GPS/INS/Digital Map for Vehicle Localization*", IEEE INTERNATIONAL CONFERENCE ON INTELLIGENT TRANSPORTATION SYSTEMS, Shangai, 2003.

[13] X. Mao, M. Wada, H. Hashimoto, M. Saito, and I. Suzuki, "*EM/Unscented Smoothing Based Parameter Learning for Nonlinear Models for GPS Positioning*", 10th World Congress on Intelligent Transport Systems, Madrid, 2003.

[14] R. G. Brown and P. Y. C. Hwang, "*Introduction to random signals and applied kalman filtering with MATLAB exercices and solutions*", 3 ed. John Wiley & Sons1997.