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Multi-period efficiency and Malmquist productivity index in two-stage production systems

Chiang Kao [a,*], Shiuh-Nan Hwang [b]

[a] Department of Industrial and Information Management, National Cheng Kung University, Tainan, Taiwan, ROC
[b] Department of Business Administration, Min Chuan University, Taipei, Taiwan, ROC

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ABSTRACT

Conventional two-stage data envelopment analysis (DEA) models measure the overall performance of a production system composed of two stages (processes) in a specified period of time, where variations in different periods are ignored. This paper takes the operations of individual periods into account to develop a multi-period two-stage DEA model, which is able to measure the overall and period efficiencies at the same time, with the former expressed as a weighted average of the latter. Since the efficiency of a two-stage system in a period is the product of the two process efficiencies, the overall efficiency of a decision making unit (DMU) in the specified period of time can be decomposed into the process efficiency of each period. Based on this decomposition, the sources of inefficiency in a DMU can be identified. The efficiencies measured from the model can also be used to calculate a common-weight global Malmquist productivity index (MPI) between two periods, in that the overall MPI is the product of the two process MPIs. The non-life insurance industry in Taiwan is used to verify the proposed model, and to explain why some companies performed unsatisfactorily in the specified period of time.

1. Introduction

Two-stage production systems are often encountered in the real world where the system has two stages (processes) connected in series, in which the inputs are supplied from outside to the first process, which then produces some intermediate products for the second process to produce the final outputs. Many applications of this type of system have been reported, for example, commercial banks in Bangladesh (Akther et al., 2013), the sustainable design performance of automobile manufacturers in the US (Chen et al., 2012), the development of China's coastal areas (Chiu et al., 2011), airline companies in the US (Lu et al., 2012), and mutual fund families in the US (Premachandra et al., 2012). Cook et al. (2010) and Kao and Hwang (2010) present reviews of the related works.

Fig. 1 shows the structure of a two-stage production system, where X_i, $i = 1, \ldots, m$, Z_f, $f = 1, \ldots, g$, and Y_r, $r = 1, \ldots, s$, are the inputs, intermediate products, and outputs, respectively. If the internal processes are ignored, that is, one merely looks at the inputs used by the system, X_i, and the outputs produced by the system, Y_r, then the data envelopment analysis (DEA) technique proposed by Charnes et al. (1978) can be used to measure the relative efficiency of a set of n units, referred to as decision making units (DMUs), with the same type of production. The model is:

$$
\begin{aligned}
E_k^{CCR} = \max \quad & \sum_{r=1}^{s} u_r Y_{rk} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i X_{ik} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0, \quad j = 1, \ldots, n \\
& u_r, v_i \geqslant \varepsilon, \quad r = 1, \ldots, s, \; i = 1, \ldots, m,
\end{aligned}
\tag{1}
$$

where u_r and v_i are virtual multipliers and ε is a small non-Archimedean number used to avoid ignoring any factor in calculating efficiency (Charnes and Cooper, 1984). Since the efficiency is measured under the most favorable conditions of the DMU being evaluated, the results are persuasive and acceptable by all DMUs.

Without taking the operations of the internal processes into account, an unreasonable result may occur in which the system is efficient while the two processes are not. For this reason, several models that take the operations of the processes into consideration when measuring the system efficiency have been developed, with the network model of Färe and Grosskopf (2000) being the most widely applied. Kao (2009a) classified such models into independent, connected, and relational ones. Recent studies focus on relational models, because they are able to measure the system and

* Corresponding author.
E-mail addresses: ckao@mail.ncku.edu.tw (C. Kao), snhwang@mail.mcu.edu.tw (S.-N. Hwang).

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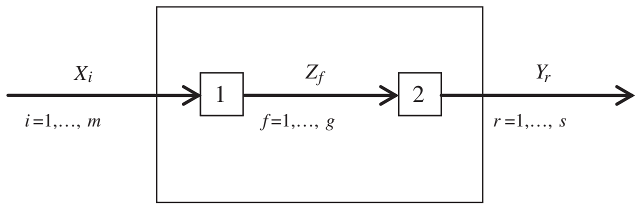

Fig. 1. Two-stage production system.

process efficiencies at the same time, and there is a mathematical relationship between them (Chen et al., 2009; Kao and Hwang, 2008; Tone and Tsutsui, 2009). No matter which model is used, the objective is usually to measure the efficiency of a DMU in a specified period of time.

When the period of time is composed of clearly defined units, such as years, then the data for the inputs, intermediate products, and outputs over the specified period of time can be aggregated in the analysis. For example, Kao and Hwang (2008) used two-year totals to evaluate the efficiency of non-life insurance companies in Taiwan. More often, the average data is used, and one example is Portela et al. (2012), which uses the average results of basic education exams in 2005 and 2006 to measure the performance of Portuguese secondary schools. Since DEA has a unit-invariant property (Lovell and Pastor, 1995), the efficiencies measured from these two types of data, total and average, are the same. If the aggregate data is used, then the resulting efficiency is an overall measure of the performance of the specified period of time. Similar to the case that ignoring the operations of individual processes will result in misleading results, ignoring the operations of individual periods may also produce unreasonable ones. For example, the results may suggest that a DMU is efficient in an overall sense, although every period is inefficient. While one can use the existing two-stage models to calculate the efficiency of each period independently, and then aggregate them to represent the overall performance, there is still a problem with regard how to aggregate the efficiency of individual periods. This paper thus proposes a multi-period model which is able to measure the overall and period efficiencies at the same time, and derive a mathematical relationship between them. The resulting relationship indicates that a DMU in the specified period of time is efficient only if it is efficient in all individual periods.

The proposed model is also able to calculate a common-weight global Malmquist productivity index (MPI) for measuring performance changes between two periods. Since this MPI uses the same frontier facet to calculate efficiencies, the results are more comparable among different DMUs as compared to those calculated from the conventional MPI.

This paper is organized as follows. In the next section, a model for measuring the overall and period efficiencies at the same time is developed. This model may produce multiple solutions, and an approach for determining a solution is described in Section 3. After that, a common-weight global MPI is proposed in Section 4 to measure changes in performance between two periods. An example of the non-life insurance industry in Taiwan is then used to illustrate this approach in Section 5. Finally, in Section 6, some conclusions are drawn from the discussion.

2. Efficiency measurement

Suppose the time span for measuring the efficiency of a set of n DMUs, that have a two-stage structure, covers q periods. Let $X_{ij}^{(p)}$, $Z_{fj}^{(p)}$, and $Y_{rj}^{(p)}$ denote the inputs, intermediate products, and outputs of DMU j in period p, respectively, with totals of $X_{ij}=\sum_{p=1}^{q}X_{ij}^{(p)}$, $Z_{fj}=\sum_{p=1}^{q}Z_{fj}^{(p)}$, and $Y_{rj}=\sum_{p=1}^{q}Y_{rj}^{(p)}$. Fig. 2 is a graphical representation of such a system, and is referred to as a multi-period two-stage production system.

The relational model in Kao and Hwang (2008) for measuring the system (overall) and process efficiencies of the kth DMU for two-stage systems is to take the operations of the two processes into consideration in addition to that of the system defined in the conventional DEA Model (1), where the operation is described by the constraint of requiring the aggregate output to be less than or equal to the aggregate input:

$$
\begin{aligned}
\widehat{E}_k^S = \max \ & \sum_{r=1}^{s} u_r Y_{rk} \\
\text{s.t.} \ & \sum_{i=1}^{m} v_i X_{ik} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0, \quad j=1,\ldots,n \\
& \sum_{f=1}^{g} w_f Z_{fj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0, \quad j=1,\ldots,n \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{f=1}^{g} w_f Z_{fj} \leqslant 0, \quad j=1,\ldots,n \\
& u_r, v_i, w_f \geqslant \varepsilon, \quad r=1,\ldots,s, \quad i=1,\ldots,m, \quad f=1,\ldots,g
\end{aligned}
\tag{2}
$$

Since the sum of the third and fourth constraints is equal to the second constraint for each DMU, the second set of constraints are redundant, and can be deleted. After an optimal solution $\left(u_r^*, v_i^*, w_f^*\right)$ is obtained, the system efficiency, $\widehat{E}_k^S$, and two process efficiencies, $\widehat{E}_k^I$ and $\widehat{E}_k^{II}$, are calculated as:

$$
\begin{aligned}
\widehat{E}_k^S &= \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
\widehat{E}_k^I &= \sum_{f=1}^{g} w_f^* Z_{fk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
\widehat{E}_k^{II} &= \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk}
\end{aligned}
\tag{3}
$$

Clearly, the system efficiency is the product of the two process efficiencies, $\widehat{E}_k^S = \widehat{E}_k^I \times \widehat{E}_k^{II}$.

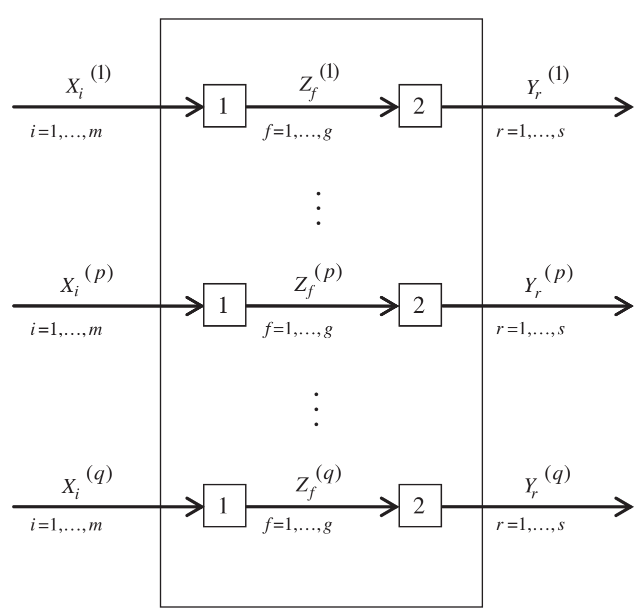

Fig. 2. Multi-period two-stage production system.

Since $\widehat{E}_k^S$, $\widehat{E}_k^I$, and $\widehat{E}_k^{II}$ are calculated from the totals of the inputs, intermediate products, and outputs in all q periods, they represent the overall performance of that period of time. Intuitively, they are the aggregation of the efficiencies of individual periods, and it would be desirable to know the mechanism of the aggregation, so that the period that has the most impact on the overall performance can be identified. For this purpose, the operations of each period must be taken into account in measuring the overall efficiency.

The operations of each period are described by $\sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leqslant 0, \sum_{f=1}^{g} w_f Z_{fj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leqslant 0$, and $\sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{f=1}^{g} w_f Z_{fj}^{(p)} \leqslant 0$. Adding these three sets of constraints to Model (2) yields the following:

$$
\begin{aligned}
E_k^S = \max\ & \sum_{r=1}^{s} u_r Y_{rk} \\
\text{s.t.}\ & \sum_{i=1}^{m} v_i X_{ik} = 1 \\
& u_r, v_i, w_f \geqslant \varepsilon,\ r=1,\ldots,s,\ \ i=1,\ldots,m,\ f=1,\ldots,g \\
& A.\ \text{System constraints}: \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0,\ \ j=1,\ldots,n \\
& \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leqslant 0,\ \ p=1,\ldots,q,\ \ j=1,\ldots,n \\
& B.\ \text{Process 1 constraints}: \\
& \sum_{f=1}^{g} w_f Z_{fj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0,\ \ j=1,\ldots,n \\
& \sum_{f=1}^{g} w_f Z_{fj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leqslant 0,\ \ p=1,\ldots,q,\ \ j=1,\ldots,n \\
& C.\ \text{Process 2 constraints}: \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{f=1}^{g} w_f Z_{fj} \leqslant 0,\ \ j=1,\ldots,n \\
& \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{f=1}^{g} w_f Z_{fj}^{(p)} \leqslant 0,\ \ p=1,\ldots,q,\ \ j=1,\ldots,n
\end{aligned}
\tag{4}
$$

Note that one of the three groups of constraints, A, B, and C, is redundant; moreover, the first set of constraints in each group is also redundant, and can be deleted. At optimality, the overall efficiencies of q periods for the system $\left(E_k^S\right)$ and two processes (E_k^I and E_k^{II}), and the efficiencies of period p $\left(E_k^{S(p)}\right)$ and its two processes ($E_k^{I(p)}$ and $E_k^{II(p)}$), based on Model (4), are:

$$
\begin{aligned}
E_k^S &= \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
E_k^I &= \sum_{f=1}^{g} w_f^* Z_{fk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
E_k^{II} &= \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk} \\
E_k^{S(p)} &= \sum_{r=1}^{s} u_r^* Y_{rk}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}^{(p)} \\
E_k^{I(p)} &= \sum_{f=1}^{g} w_f^* Z_{fk}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}^{(p)} \\
E_k^{II(p)} &= \sum_{r=1}^{s} u_r^* Y_{rk}^{(p)} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk}^{(p)}
\end{aligned}
\tag{5}
$$

Similar to the case of Eq. (3), here the overall efficiency of q periods for the system is also the product of those of its two processes, $E_k^S = E_k^I \times E_k^{II}$; moreover, the efficiency of each period is the product of those of its two processes, $E_k^{S(p)} = E_k^{I(p)} \times E_k^{II(p)}$. Since more constraints have been included, E_k^S, E_k^I, and E_k^{II} measured from the multi-period Model (4) are smaller than those measured from the aggregate Model (2).

A closer examination of Fig. 2 reveals that the multi-period two-stage system has a parallel structure of q subsystems, where each subsystem corresponds to one period and is composed of two processes connected in series. According to Kao (2009b), the parallel structure has a property that the system efficiency is a weighted average of the subsystem efficiencies, where the weight is the proportion of the aggregate input of the corresponding subsystem in that of all q subsystems. Applying this property to the system efficiency E_k^S and process efficiencies E_k^I and E_k^{II} obtains the following relationships:

$$
\begin{aligned}
E_k^S &= \sum_{p=1}^{q} \omega^{(p)} E_k^{S(p)}, \quad \omega^{(p)} = \sum_{i=1}^{m} v_i^* X_{ik}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
E_k^I &= \sum_{p=1}^{q} \omega^{(p)} E_k^{I(p)}, \quad \omega^{(p)} = \sum_{i=1}^{m} v_i^* X_{ik}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
E_k^{II} &= \sum_{p=1}^{q} \varpi^{(p)} E_k^{II(p)}, \quad \varpi^{(p)} = \sum_{f=1}^{g} w_f^* Z_{fk}^{(p)} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk}
\end{aligned}
\tag{6}
$$

These relationships can be verified easily:

$$
\sum_{p=1}^{q} \omega^{(p)} E_k^{S(p)} = \sum_{p=1}^{q} \left(\frac{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}} \right) \left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}} \right) = \sum_{p=1}^{q} \left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}} \right) = E_k^S
$$

$$
\sum_{p=1}^{q} \omega^{(p)} E_k^{I(p)} = \sum_{p=1}^{q} \left(\frac{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}} \right) \left(\frac{\sum_{f=1}^{g} w_f^* Z_{fk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}} \right) = \sum_{p=1}^{q} \left(\frac{\sum_{f=1}^{g} w_f^* Z_{fk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}} \right) = E_k^I
$$

$$
\sum_{p=1}^{q} \varpi^{(p)} E_k^{II(p)} = \sum_{p=1}^{q} \left(\frac{\sum_{f=1}^{g} w_f^* Z_{fk}^{(p)}}{\sum_{f=1}^{g} w_f^* Z_{fk}} \right) \left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{f=1}^{g} w_f^* Z_{fk}^{(p)}} \right) = \sum_{p=1}^{q} \left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{f=1}^{g} w_f^* Z_{fk}} \right) = E_k^{II}
$$

We thus conclude that the overall efficiency in q periods is a weighted average of the efficiencies of the q individual periods, and each DMU being evaluated selects the most favorable weights to form the overall efficiency from period efficiencies.

Altogether, we have two ways to decompose the system efficiency into the process efficiencies of each period:

$$
\begin{aligned}
E_k^S &= E_k^I \times E_k^{II} = \left(\sum_{p=1}^{q} \omega^{(p)} E_k^{I(p)} \right) \left(\sum_{p=1}^{q} \varpi^{(p)} E_k^{II(p)} \right) \\
E_k^S &= \sum_{p=1}^{q} \omega^{(p)} E_k^{S(p)} = \sum_{p=1}^{q} \omega^{(p)} \left(E_k^{I(p)} \times E_k^{II(p)} \right)
\end{aligned}
\tag{7}
$$

This decomposition indicates that a multi-period two-stage system is efficient if and only if all its period-specific processes are. As a result, it identifies the processes that cause the inefficiency of the system, and thus provides a direction for making improvements to it.

3. Resolution for multiple solutions

One attractive aspect of efficiency decomposition is that it identifies the source of inefficiency in a system. It also shows the relative performance of a period (or process) of a DMU compared with that of other DMUs. However, due to multiple solutions, the decomposition may not be unique, which makes the efficiencies of different DMUs not comparable. A common basis for measuring the efficiencies is thus necessary.

In order to make the process efficiencies of different DMUs comparable, Kao and Hwang (2008) suggested maximizing the efficiency of the process to be compared, while maintaining the system efficiency at the value obtained previously. Following this idea, in comparing the efficiency of period t, one should find the maximum efficiency of this period for each DMU, while maintaining the system efficiency at the level of E_k^S calculated from Model (4). That is:

$$\begin{aligned} E_k^{S(t)} = \max\ & \sum_{r=1}^{s} u_r Y_{rk}^{(t)} \\ \text{s.t.}\ & \sum_{i=1}^{m} v_i X_{ik}^{(t)} = 1 \\ & \sum_{r=1}^{s} u_r Y_{rk} = E_k^S \sum_{i=1}^{m} v_i X_{ik} \\ & \sum_{f=1}^{g} w_f Z_{fj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leqslant 0,\ p=1,\ldots,q,\ j=1,\ldots,n \\ & \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{f=1}^{g} w_f Z_{fj}^{(p)} \leqslant 0,\ p=1,\ldots,q,\ j=1,\ldots,n \\ & u_r, v_i, w_f \geqslant \varepsilon,\ r=1,\ldots,s,\ i=1,\ldots,m,\ f=1,\ldots,g \end{aligned} \tag{8}$$

Note that in this formulation all redundant constraints have been deleted. Since every DMU is given the first priority to determine the efficiency of period t that, together with other period efficiencies, form the fixed overall efficiency E_k^S, $E_j^{S(t)}$, $j = 1, \ldots, n$, have a common basis for comparison. If period h is of second concern, then one simply finds the maximum efficiency of this period, while maintaining the overall efficiency at E_k^S and the efficiency of period t at $E_k^{S(t)}$. This process can be continued to find the efficiencies of the periods of lower-priority concerns.

In each period there are two processes. To make the process-specific efficiency of a period comparable among all DMUs, the same idea can be applied. Suppose decision makers are more concerned with process one than process two. Then one simply maximizes the efficiency of process one, while requiring the overall and period t efficiencies to be E_k^S and $E_k^{S(t)}$, respectively. The corresponding model is:

$$\begin{aligned} E_k^{I(t)} = \max\ & \sum_{f=1}^{g} w_f Z_{fk}^{(t)} \\ \text{s.t.}\ & \sum_{i=1}^{m} v_i X_{ik}^{(t)} = 1 \\ & \sum_{r=1}^{s} u_r Y_{rk} = E_k^S \sum_{i=1}^{m} v_i X_{ik} \\ & \sum_{r=1}^{s} u_r Y_{rk}^{(t)} = E_k^{S(t)} \sum_{i=1}^{m} v_i X_{ik}^{(t)} \\ & \sum_{f=1}^{g} w_f Z_{fj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leqslant 0,\ p=1,\ldots,q,\ j=1,\ldots,n \\ & \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{f=1}^{g} w_f Z_{fj}^{(p)} \leqslant 0,\ p=1,\ldots,q,\ j=1,\ldots,n \\ & u_r, v_i, w_f \geqslant \varepsilon, r=1,\ldots,s, i=1,\ldots,m,\ f=1,\ldots,g \end{aligned} \tag{9}$$

After all of the periods and processes for which the corresponding efficiencies are to be compared have been enumerated via the above process, an optimal solution $\left(u_r^*,\ v_i^*,\ w_f^*\right)$ is determined from the multiple solutions to calculate different types of efficiencies via Eq. (5) and the weights via Eq. (6).

4. Performance changes

Top managers are usually interested in performance changes between two periods for multi-period problems, because the results are able to provide directions for achieving better performance. A measure that has been widely used for this purpose is the Malmquist productivity index (MPI).

Different forms of the MPI have been proposed in the literature. Suppose the efficiencies of periods t and h are to be compared, where h is later than t. The idea of Caves et al. (1982) is to use the technology of period t to measure the efficiencies of periods t and h, and take the ratio of the latter to the former as the MPI. A value greater than, equal to, or less than 1 indicates that the performance has improved, remained the same, or worsened between periods t and h, respectively. Since period h can also be used as

Table 1
Three-year totals of inputs (X), intermediate products (Z), and outputs (Y) of 21 non-life insurance companies in Taiwan.

Company	Operating expenses (X_1)	Insurance expenses (X_2)	Direct written premiums (Z_1)	Reinsurance premiums (Z_2)	Underwriting profit (Y_1)	Investment profit (Y_2)
1. Taiwan Fire	1827.432	995.522	10666.753	1304.158	1493.047	925.639
2. Chung Kuo	2116.181	2057.884	14511.415	2796.008	2217.009	1576.967
3. Fubon	9220.278	5238.588	53920.821	2867.737	11493.844	6819.887
4. Zurich	3907.807	1295.927	14245.301	2075.088	3028.077	737.136
5. Taian	2912.386	2092.616	15618.798	1210.307	3489.606	439.039
6. Ming Tai	5647.574	2680.438	25313.691	1723.19	5578.269	943.65
7. Central	2389.384	1702.415	16602.115	1024.872	1369.658	330.516
8. The First	1931.069	1869.863	11740.238	800.403	2493.718	762.183
9. Kuo Hua	2863.916	1050.302	11115.813	1072.358	2196.43	39.004
10. Union	3728.748	911.181	13469.749	1605.048	2564.809	1459.153
11. Shingkong	3773.255	1965.278	19787.019	1243.916	5566.127	365.543
12. South China	2101.314	1527.049	10938.948	1019.356	2434.294	332.283
13. Cathay Century	3223.054	931.553	14970.463	1215.814	5193.637	572.255
14. Allianz President	1789.888	606.467	7740.185	670.221	1292.8	257.637
15. Newa	2062.182	1671.825	11372.429	523.649	4467.499	384.563
16. AIU	1041.406	720.632	4738.118	1315.635	986.789	218.418
17. North America	216.249	247.189	1429.957	648.171	599.95	79.25
18. Federal	229.236	76.477	539.762	205.785	428.929	40.525
19. Asia	23.88	16.49	79.534	21.52	84.217	6.797
20. AXA	81.64	37.074	334.571	86.418	2.511	30.142
21. Mitsui Sumitomo	219.611	306.772	554.859	644.816	294.812	22.082

the basis for measuring efficiency, and the results may not be the same as those obtained from using period t as the basis, Färe et al. (1994) proposed using both periods, but separately, to calculate two Cave's MPIs, and use their geometric average to be the MPI. This MPI can be decomposed into two parts to show efficiency and technological changes.

Both Cave's and Färe's MPIs use the technology of one period to measure efficiency. Pastor and Lovell (2005) proposed a global MPI, which uses the global technology of all q periods to measure efficiency. The MPI between two periods is the ratio of the efficiencies of these two periods. Since the observations of all q periods are used to construct the efficiency frontier for measuring efficiency, the MPI possesses an attractive property of circularity (Pastor and Lovell, 2007). That is, the MPI between periods t and h, denoted as $MPI^{t,h}$, is the product of $MPI^{t,d}$ and $MPI^{d,h}$, where d is any period between t and h. This property ensures a consistent measure of performance changes between periods t and h based on any period.

For multi-period two-stage systems the global technology is defined by:

$$\left\{\begin{array}{l}\sum_{r=1}^{s}u_rY_{rj}-\sum_{i=1}^{m}v_iX_{ij}\leqslant 0,\ \forall j;\ \sum_{f=1}^{g}w_fZ_{fj}-\sum_{i=1}^{m}v_iX_{ij}\leqslant 0,\ \forall j;\\ \sum_{r=1}^{s}u_rY_{rj}-\sum_{f=1}^{g}w_fZ_{fj}\leqslant 0,\ \forall j;\ \sum_{r=1}^{s}u_rY_{rj}^{(p)}-\sum_{i=1}^{m}v_iX_{ij}^{(p)}\leqslant 0,\ \forall p,j;\\ \sum_{f=1}^{g}w_fZ_{fj}^{(p)}-\sum_{i=1}^{m}v_iX_{ij}^{(p)}\leqslant 0,\ \forall p,j;\ \sum_{r=1}^{s}u_rY_{rj}^{(p)}-\sum_{f=1}^{g}w_fZ_{fj}^{(p)}\leqslant 0,\ \forall p,j\end{array}\right\}.$$

By deleting redundant constraints, it reduces to:

$$\left\{\sum_{f=1}^{g}w_fZ_{fj}^{(p)}-\sum_{i=1}^{m}v_iX_{ij}^{(p)}\leqslant 0,\ \forall p,j;\ \sum_{r=1}^{s}u_rY_{rj}^{(p)}-\sum_{f=1}^{g}w_fZ_{fj}^{(p)}\leqslant 0,\ \forall p,j\right\}.$$

Suppose the MPI between periods t and h are desired. The efficiency of period t is calculated via the following DEA model:

$$\begin{aligned}\widetilde{E}_k^{S(t)}=\max\ &\sum_{r=1}^{s}u_rY_{rk}^{(t)}\\ \text{s.t.}\ &\sum_{i=1}^{m}v_iX_{ik}^{(t)}=1\\ &\sum_{f=1}^{g}w_fZ_{fj}^{(p)}-\sum_{i=1}^{m}v_iX_{ij}^{(p)}\leqslant 0,\ \ p=1,\ldots,q,\ \ j=1,\ldots,n\\ &\sum_{r=1}^{s}u_rY_{rj}^{(p)}-\sum_{f=1}^{g}w_fZ_{fj}^{(p)}\leqslant 0,\ p=1,\ldots,q,\ \ j=1,\ldots,n\\ &u_r,v_i,w_f\geqslant\varepsilon,\ r=1,\ldots,s,\ i=1,\ldots,m,\ f=1,\ldots,g\end{aligned}\tag{10}$$

The efficiency of period h, $\widetilde{E}_k^{S(h)}$, can be calculated similarly. The global MPI is $M\widetilde{P}I^{S(t,h)}=\widetilde{E}_k^{S(h)}/\widetilde{E}_k^{S(t)}$.

Comparing Model (4) (with the redundant constraints deleted) with Model (10), it is noted that they have the same constraints, which implies that the same technology is defined. The only difference is that the former measures the overall efficiency E_k^S, and uses Eq. (5) to obtain period efficiencies $E_k^{S(t)}$ and $E_k^{S(h)}$, whereas the latter measures $\widetilde{E}_k^{S(t)}$ and $\widetilde{E}_k^{S(h)}$ directly and independently. To measure $\widetilde{E}_k^{S(t)}$ and $\widetilde{E}_k^{S(h)}$ independently implies that different sets of multipliers, or different frontier facets, have been used to measure efficiency. The efficiencies $E_k^{S(t)}$ and $E_k^{S(h)}$, in contrast, are measured from the same set of multipliers, indicating that they are measured based on the same frontier facet. In other words, the MPIs calculated from the efficiencies obtained from Model (4) are common-weight global MPIs. Many scholars believe that efficiencies calculated from different frontier facets are not comparable, and thus propose different approaches to generate common-weight measures (Doyle and Green, 1994; Friedman and Sinuany-Stern, 1997; Kao, 2010; Kao and Hung, 2005; Roll et al., 1991; Sinuany-Stern and Friedman, 1998; Sueyoshi, 2001). From this point of view, the common-weight global MPI calculated from the ratio of $E_k^{S(h)}$ to $E_k^{S(t)}$, i.e., $MPI_k^{S(t,h)}=E_k^{S(h)}/E_k^{S(t)}$, is a more appropriate measure of performance changes than that calculated from the ratio of $\widetilde{E}_k^{S(h)}$ to $\widetilde{E}_k^{S(t)}$. Note that the period efficiency calculated from Model (4) will never exceed that calculated from Model (10), because the former is more restrictive in selecting frontier facets to measure efficiency.

By the same token, changes in performance for process one (and two) between periods t and h can also be measured by a common-weight global MPI calculated from the ratio of $E_k^{I(h)}$ (or $E_k^{II(h)}$) to $E_k^{I(t)}$ (or $E_k^{II(t)}$), i.e., $MPI_k^{I(t,h)}=E_k^{I(h)}/E_k^{I(t)}$ (or $MPI_k^{II(t,h)}=E_k^{II(h)}/E_k^{II(t)}$). This leads to the following relationship between period and process MPIs:

$$MPI_k^{S(t,h)}=\frac{E_k^{S(h)}}{E_k^{S(t)}}=\frac{E_k^{I(h)}\times E_k^{II(h)}}{E_k^{I(t)}\times E_k^{II(t)}}=MPI_k^{I(t,h)}\times MPI_k^{II(t,h)}\tag{11}$$

That is, the period MPI between two periods is the product of the two process MPIs between the same periods. Changes in period performance can thus be attributed to changes in process performances. Lee and Johnson (2011) proposed the same idea for decomposing the period MPI into the process MPIs. However, they overlooked the phenomenon of multiple solutions, and the decomposition thus does not show the correct relationship.

5. Non-life insurance companies in Taiwan

Kao and Hwang (2008) study the performance of the non-life insurance industry in Taiwan, with the whole production process divided into two stages, premium acquisition and profit generation. The first stage is characterized by marketing of the insurance, where clients are attracted to pay direct written premiums and reinsurance premiums are received from other insurance companies. The second stage is characterized by investment, where premiums are invested in a portfolio to earn profit. For this two-stage production system, the inputs X_i, intermediate products Z_f, and outputs Y_r used by Kao and Hwang (2008) are:

Operating expenses (X_1): salaries of the employees and various types of costs incurred in daily operations.

Insurance expenses (X_2): expenses paid to agencies, brokers, and solicitors, and other expenses associated with marketing the service of insurance.

Direct written premiums (Z_1): premiums received from insured clients.

Reinsurance premiums (Z_2): premiums received from ceding companies.

Underwriting profit (Y_1): profit earned from the insurance business.

Investment profit (Y_2): profit earned from the investment portfolio.

Twenty-four companies are evaluated in Kao and Hwang (2008), with the data taken from 2001 and 2002.

In order to have a more general discussion, data from one more year, 2000, is included in the current study. Since some observations of one company in 2000 (Tai Ping) and two companies in 2002 (China Mariners and Royal & Sun Alliance) are missing, they are excluded to leave twenty-one companies for this study. Table 1 shows the total inputs, intermediate products, and outputs of the twenty-one companies in the three years 2000–2002. All values are in 1,000 Taiwan dollars (1 USD ≅ 30 NTD). The complete data set for all three years is shown in the Appendix.

By using the aggregate data contained in Table 1, the system (overall) and two process efficiencies of the twenty-one companies are measured via Model (2), with the results shown in the left part of Table 2 under the heading "Aggregate Model." As expected, the overall efficiency, $\widehat{E}_k^S$, is the product of the two process efficiencies, $\widehat{E}_k^I$ and $\widehat{E}_k^{II}$. None of the twenty-one companies are efficient, even though there are three companies which are efficient in the first process and two in the second. In an overall sense, the best three companies are Fubon (No. 3), Chung Kuo (No. 2), and Union (No. 10), whereas the worst three are Central (No. 7), Mitsui Sumitomo (No. 21), and Kuo Hua (No. 9).

If the variations in individual periods are considered, the multi-period Model (4) can be applied to measure different kinds of efficiencies. Since there are three periods, and multiple solutions may occur, we maximize the efficiency of year 2000 first via Model (8), and year 2001 second to provide a common basis for these three years to be comparable among different companies. The right part of Table 2 under the heading "Multi-period Model" shows the system (overall) and two process efficiencies in the three-year period. The efficiencies measured from this model are smaller than those measured from the aggregate model for each company, as more constraints are involved. However, the rankings of the twenty-one companies measured from the two models are quite similar. For example, the best three and worst three companies measured by this model are the same as those measured from the aggregate model, although the rankings are a little different. A Spearman rank correlation test (Daniel, 1978) shows that the rankings obtained from the two models based on the overall efficiencies are highly correlated, with an r_s statistic equal to 0.9402 and the corresponding p value less than 0.001. For the two processes, process one in general has a better performance than process two, as indicated by the average efficiencies shown in the last row of Table 2, as noted in Kao and Hwang (2008). It is also noted that companies with a high process two efficiency, e.g., Chung Kuo (No. 2), Fubon (No. 3), Union (No. 10), and Asia (No. 19), also have good overall performance.

Table 3 shows the detailed results obtained from the multi-period Model (4). As indicated in Eq. (6), the efficiencies of the overall and two processes of the three-year period are weighted averages of those of the three years. Using Taiwan Fire (No. 1) to illustrate this, the weighted average of the three period efficiencies is $0.3354 \times 0.3357 + 0.3177 \times 0.4259 + 0.3468 \times 0.4887$, or 0.4174, which is exactly the overall efficiency of the three-year period shown in the first entry of the third-to-last column of Table 2. This relationship also applies to the two processes, in that $0.3354 \times 0.8305 + 0.3177 \times 0.8887 + 0.3468 \times 0.9816$ is equal to 0.9014 for process one and $0.3090 \times 0.4043 + 0.3133 \times 0.4792 + 0.3777 \times 0.4978$ is equal to 0.4631 for process two.

Based on Eq. (5), the period efficiency $\left(E_k^{S(p)}\right)$ is the product of the two process efficiencies ($E_k^{I(p)}$ and $E_k^{II(p)}$), which corresponds to $0.3357 = 0.8305 \times 0.4043$, $0.4259 = 0.8889 \times 0.4792$, and $0.4887 = 0.9816 \times 0.4978$ for the three periods for Taiwan Fire. From Eq. (11), the MPI of a company between two periods is the product of those of the two processes between the same periods. For Taiwan Fire, we have $1.2685 = 1.0701 \times 1.1854$, $1.1474 = 1.1045 \times 1.0388$, and $1.4555 = 1.1820 \times 1.2314$ during 2000–2001, 2001–2002, and 2000–2002, respectively. Moreover, the circular property shows that the MPI during 2000–2002 is the product of those during 2000–2001 and 2001–2002: $1.4555 = 1.2685 \times 1.1474$ for the company, $1.1820 = 1.0701 \times 1.1045$ for process one, and $1.2314 = 1.1854 \times 1.0388$ for process two.

The numbers, in bold, in the last three rows of Table 3 are the averages of the twenty-one companies of the corresponding entries of the three years. The three average period efficiencies, 0.2555 for 2000, 0.3065 for 2001, and 0.2334 for 2002, indicate that year 2001 has the best performance. The average MPI during 2000–2002, 1.4215, indicates that the general performance of the twenty-one companies from 2000 to 2002 has improved. The average MPIs for 2000–2001, 2.9742, and 2001–2002, 0.8567, further indicate that this improvement is due to the large improvement in 2000–2001, followed by a small decline in 2001–2002.

The efficiencies in Table 3 also show the sources of inefficiency for each company in the three-year period. For example, Chung Kuo (No. 2) has an overall efficiency of 0.5199, as shown in Table 2. Table 3 indicates that the inefficient part (1-0.5199) is caused by the relatively unsatisfactory performances in 2001 (with an efficiency score of 0.4943) and 2002 (with an efficiency score of

Table 2
Three-year efficiencies for 21 non-life insurance companies in Taiwan.

Company	Aggregate model			Multi-period model		
	Overall [rank] $\widehat{E}_k^S$	Process I $\widehat{E}_k^I$	Process II $\widehat{E}_k^{II}$	Overall [rank] E_k^S	Process I E_k^I	Process II E_k^{II}
1. Taiwan Fire	0.6633 [4]	0.9676	0.6855	0.4174 [4]	0.9014	0.4631
2. Chung Kuo	0.8469 [2]	0.9883	0.8570	0.5199 [3]	0.8478	0.6132
3. Fubon	0.9481 [1]	0.9481	1.0000	0.5959 [1]	0.7889	0.7553
4. Zurich	0.3618 [11]	0.8637	0.4189	0.2313 [11]	0.7738	0.2989
5. Taian	0.2464 [18]	0.7797	0.3160	0.1365 [18]	0.6451	0.2115
6. Ming Tai	0.3009 [14]	0.7710	0.3903	0.1817 [13]	0.6914	0.2629
7. Central	0.1840 [21]	1.0000	0.1840	0.1078 [19]	0.8194	0.1315
8. The First	0.4826 [10]	0.8751	0.5515	0.2897 [9]	0.7402	0.3914
9. Kuo Hua	0.2217 [19]	0.6805	0.3259	0.0539 [21]	0.5067	0.1063
10. Union	0.7897 [3]	0.9199	0.8585	0.5445 [2]	0.9374	0.5809
11. Shingkong	0.3083 [13]	0.5684	0.5423	0.1382 [17]	0.7497	0.1844
12. South China	0.2491 [17]	0.7634	0.3263	0.1387 [16]	0.6370	0.2177
13. Cathay Century	0.6219 [5]	0.9438	0.6589	0.2921 [8]	0.9571	0.3052
14. Allianz President	0.3000 [15]	0.8374	0.3583	0.1818 [12]	0.8253	0.2202
15. Newa	0.3395 [12]	0.7336	0.4628	0.1768 [14]	0.5792	0.3052
16. AIU	0.2894 [16]	0.7948	0.3641	0.1695 [15]	0.7414	0.2286
17. North America	0.5340 [8]	1.0000	0.5340	0.2995 [7]	0.8436	0.3551
18. Federal	0.6109 [6]	1.0000	0.6109	0.2801 [10]	0.6785	0.4128
19. Asia	0.5874 [7]	0.5874	1.0000	0.3093 [6]	0.5388	0.5740
20. AXA	0.5266 [9]	0.7394	0.7122	0.3333 [5]	0.8492	0.3925
21. Mitsui Sumitomo	0.1869 [20]	0.4192	0.4459	0.0934 [20]	0.3493	0.2675
Average	**0.4571**	**0.8182**	**0.5525**	**0.2615**	**0.7334**	**0.3466**

Table 3
Period efficiencies and MPIs for 21 non-life insurance companies in Taiwan.

No.	Year	[Period]	Period		Process I			Process II		
		$[t-h]$	Eff. $E_k^{S(p)}$	[MPI] $MPI_k^{S(t,h)}$	Eff. $E_k^{I(p)}$	(weight) $\omega^{(p)}$	[MPI] $MPI_k^{I(t,h)}$	Eff. $E_k^{II(p)}$	(weight) $\varpi^{(p)}$	[MPI] $MPI_k^{II(t,h)}$
1	2000	[00–01]	0.3357	[1.2685]	0.8305	(0.3354)	[1.0701]	0.4043	(0.3090)	[1.1854]
	2001	[01–02]	0.4259	[1.1474]	0.8887	(0.3177)	[1.1045]	0.4792	(0.3133)	[1.0388]
	2002	[00–02]	0.4887	[1.4555]	0.9816	(0.3468)	[1.1820]	0.4978	(0.3777)	[1.2314]
2	2000	[00–01]	0.7038	[0.7024]	0.7586	(0.3470)	[1.1189]	0.9278	(0.3105)	[0.6278]
	2001	[01–02]	0.4943	[0.7031]	0.8487	(0.3319)	[1.1114]	0.5824	(0.3322)	[0.6326]
	2002	[00–02]	0.3475	[0.4938]	0.9433	(0.3211)	[1.2436]	0.3684	(0.3573)	[0.3971]
3	2000	[00–01]	0.7399	[0.8476]	0.7399	(0.3345)	[1.0098]	1.0000	(0.3138)	[0.8394]
	2001	[01–02]	0.6272	[0.6095]	0.7472	(0.3398)	[1.0876]	0.8394	(0.3218)	[0.5604]
	2002	[00–02]	0.3822	[0.5166]	0.8126	(0.3538)	[1.0982]	0.4704	(0.3644)	[0.4704]
4	2000	[00–01]	0.2469	[1.3760]	0.6929	(0.4089)	[1.1958]	0.3563	(0.3661)	[1.1507]
	2001	[01–02]	0.3397	[0.3285]	0.8285	(0.2822)	[1.0027]	0.4100	(0.3022)	[0.3277]
	2002	[00–02]	0.1116	[0.4521]	0.8308	(0.3089)	[1.1990]	0.1343	(0.3317)	[0.3771]
5	2000	[00–01]	0.0464	[5.0932]	0.6802	(0.3175)	[0.8728]	0.0682	(0.3348)	[5.8357]
	2001	[01–02]	0.2362	[0.5203]	0.5936	(0.3341)	[1.1160]	0.3979	(0.3075)	[0.4663]
	2002	[00–02]	0.1229	[2.6502]	0.6625	(0.3484)	[0.9740]	0.1855	(0.3577)	[2.7210]
6	2000	[00–01]	0.1921	[1.0998]	0.7088	(0.3126)	[0.9512]	0.2710	(0.3205)	[1.1562]
	2001	[01–02]	0.2113	[0.6907]	0.6742	(0.3269)	[1.0261]	0.3134	(0.3188)	[0.6731]
	2002	[00–02]	0.1459	[0.7596]	0.6918	(0.3605)	[0.9760]	0.2109	(0.3607)	[0.7782]
7	2000	[00–01]	0.0115	[26.454]	0.6486	(0.4105)	[1.3551]	0.0177	(0.3250)	[19.5219]
	2001	[01–02]	0.3033	[0.1386]	0.8789	(0.2998)	[1.1378]	0.3451	(0.3215)	[0.1218]
	2002	[00–02]	0.0420	[3.6668]	1.0000	(0.2897)	[1.5419]	0.0420	(0.3535)	[2.3782]
8	2000	[00–01]	0.2461	[0.9198]	0.6859	(0.3251)	[1.1033]	0.3588	(0.3012)	[0.8337]
	2001	[01–02]	0.2264	[1.7361]	0.7567	(0.3331)	[1.0251]	0.2991	(0.3405)	[1.6936]
	2002	[00–02]	0.3930	[1.5969]	0.7757	(0.3418)	[1.1310]	0.5066	(0.3582)	[1.4119]
9	2000	[00–01]	0.0514	[0.9016]	0.5511	(0.3385)	[0.8686]	0.0933	(0.3682)	[1.0379]
	2001	[01–02]	0.0464	[1.3603]	0.4787	(0.3165)	[1.0210]	0.0969	(0.2990)	[1.3323]
	2002	[00–02]	0.0631	[1.2264]	0.4888	(0.3450)	[0.8869]	0.1291	(0.3328)	[1.3828]
10	2000	[00–01]	0.7201	[0.8016]	0.9855	(0.2856)	[0.9655]	0.7306	(0.3003)	[0.8303]
	2001	[01–02]	0.5772	[0.6913]	0.9515	(0.3020)	[0.9392]	0.6066	(0.3066)	[0.7360]
	2002	[00–02]	0.3990	[0.5541]	0.8937	(0.4124)	[0.9068]	0.4465	(0.3932)	[0.6111]
11	2000	[00–01]	0.0735	[3.9284]	0.7419	(0.3053)	[1.0034]	0.0991	(0.3021)	[3.9150]
	2001	[01–02]	0.2887	[0.1948]	0.7444	(0.3302)	[1.0223]	0.3878	(0.3279)	[0.1905]
	2002	[00–02]	0.0562	[0.7652]	0.7610	(0.3645)	[1.0258]	0.0739	(0.3700)	[0.7459]
12	2000	[00–01]	0.0482	[4.2745]	0.6528	(0.3471)	[0.9203]	0.0739	(0.3558)	[4.6445]
	2001	[01–02]	0.2061	[0.8153]	0.6008	(0.3220)	[1.0911]	0.3430	(0.3038)	[0.7472]
	2002	[00–02]	0.1680	[3.4850]	0.6555	(0.3308)	[1.0042]	0.2563	(0.3404)	[3.4706]
13	2000	[00–01]	0.1388	[3.6329]	1.0000	(0.3025)	[0.8726]	0.1388	(0.3161)	[4.1634]
	2001	[01–02]	0.5042	[0.4414]	0.8726	(0.3369)	[1.1460]	0.5778	(0.3071)	[0.3851]
	2002	[00–02]	0.2225	[1.6035]	1.0000	(0.3606)	[1.0000]	0.2225	(0.3768)	[1.6035]
14	2000	[00–01]	0.1455	[1.7110]	0.7597	(0.3191)	[1.0414]	0.1915	(0.2937)	[1.6429]
	2001	[01–02]	0.2489	[0.5997]	0.7911	(0.3383)	[1.1632]	0.3146	(0.3243)	[0.5156]
	2002	[00–02]	0.1493	[1.0261]	0.9203	(0.3425)	[1.2114]	0.1622	(0.3819)	[0.8470]
15	2000	[00–01]	0.0711	[3.8843]	0.5625	(0.3347)	[1.0486]	0.1263	(0.3251)	[3.7043]
	2001	[01–02]	0.2760	[0.6953]	0.5899	(0.3010)	[0.9928]	0.4679	(0.3066)	[0.7003]
	2002	[00–02]	0.1919	[2.7007]	0.5856	(0.3643)	[1.0411]	0.3277	(0.3683)	[2.5942]
16	2000	[00–01]	0.1754	[1.0511]	0.7015	(0.2503)	[0.9532]	0.2500	(0.2368)	[1.1027]
	2001	[01–02]	0.1843	[0.8194]	0.6686	(0.3704)	[1.2545]	0.2757	(0.3340)	[0.6532]
	2002	[00–02]	0.1510	[0.8613]	0.8388	(0.3793)	[1.1958]	0.1801	(0.4292)	[0.7203]
17	2000	[00–01]	0.4099	[0.7240]	0.6557	(0.2628)	[1.1986]	0.6252	(0.2042)	[0.6041]
	2001	[01–02]	0.2968	[0.7882]	0.7859	(0.3079)	[1.2725]	0.3777	(0.2869)	[0.6194]
	2002	[00–02]	0.2339	[0.5707]	1.0000	(0.4293)	[1.5252]	0.2339	(0.5089)	[0.3742]
18	2000	[00–01]	0.2366	[1.2383]	0.7976	(0.3315)	[0.7422]	0.2966	(0.3898)	[1.6684]
	2001	[01–02]	0.2930	[1.0593]	0.5920	(0.3359)	[1.0930]	0.4949	(0.2931)	[0.9691]
	2002	[00–02]	0.3103	[1.3117]	0.6470	(0.3326)	[0.8112]	0.4796	(0.3172)	[1.6169]
19	2000	[00–01]	0.2107	[0.9346]	0.5166	(0.3524)	[0.9813]	0.4078	(0.3379)	[0.9525]
	2001	[01–02]	0.1969	[3.1416]	0.5069	(0.3926)	[1.2203]	0.3884	(0.3694)	[2.5744]
	2002	[00–02]	0.6186	[2.9362]	0.6186	(0.2549)	[1.1975]	1.0000	(0.2927)	[2.4520]
20	2000	[00–01]	0.4496	[0.8049]	0.9765	(0.2745)	[0.9144]	0.4604	(0.3157)	[0.8803]
	2001	[01–02]	0.3619	[0.6071]	0.8930	(0.3548)	[0.7984]	0.4053	(0.3731)	[0.7604]
	2002	[00–02]	0.2197	[0.4887]	0.7130	(0.3707)	[0.7301]	0.3082	(0.3112)	[0.6694]
21	2000	[00–01]	0.1132	[0.8091]	0.2360	(0.2564)	[1.5528]	0.4797	(0.1733)	[0.5211]
	2001	[01–02]	0.0916	[0.9037]	0.3665	(0.3218)	[1.2179]	0.2500	(0.3376)	[0.7420]
	2002	[00–02]	0.0828	[0.7312]	0.4464	(0.4218)	[1.8912]	0.1855	(0.5389)	[0.3867]

Table 3 (*continued*)

No.	Year	[Period] [$t-h$]	Period Eff. $E_k^{S(p)}$	Period [MPI] $MPI_k^{S(t,h)}$	Process I Eff. $E_k^{I(p)}$	Process I (weight) $\omega^{(p)}$	Process I [MPI] $MPI_k^{I(t,h)}$	Process II Eff. $E_k^{II(p)}$	Process II (weight) $\varpi^{(p)}$	Process II [MPI] $MPI_k^{II(t,h)}$
	Average		**0.2555**	[**2.9742**]	**0.7087**	(**0.3215**)	[**1.0352**]	**0.3513**	(**0.3095**)	[**2.7056**]
	Average		**0.3065**	[**0.8567**]	**0.7171**	(**0.3284**)	[**1.0878**]	**0.4121**	(**0.3203**)	[**0.7829**]
	Average		**0.2334**	[**1.4215**]	**0.7746**	(**0.3514**)	[**1.1320**]	**0.3058**	(**0.3725**)	[**1.2971**]

Table 4
Complete three-year data of 21 non-life insurance companies in Taiwan.

No.	Year	Operating expenses (X_1)	Insurance expenses (X_2)	Direct written premiums (Z_1)	Reinsurance premiums (Z_2)	Underwriting profit (Y_1)	Investment profit (Y_2)
1	2000	648.688	322.010	3214.996	447.423	508.904	243.952
	2001	564.644	321.646	3386.667	383.885	516.094	298.338
	2002	614.100	351.866	4065.090	472.850	468.049	383.349
2	2000	734.359	705.129	4491.141	983.114	988.507	742.213
	2001	702.273	711.591	4821.776	923.288	681.452	498.089
	2002	679.549	641.164	5198.498	889.606	547.05	336.665
3	2000	3221.215	1706.974	16527.959	1113.943	3642.615	2894.615
	2001	3287.374	1728.368	17414.956	888.637	3490.349	2472.629
	2002	3411.689	1803.246	19977.906	865.157	4360.88	1452.643
4	2000	1280.100	627.564	4497.393	1122.762	1314.479	322.078
	2001	1301.796	304.614	4509.821	523.776	905.664	325.261
	2002	1325.911	363.749	5238.087	428.55	807.934	89.797
5	2000	969.553	649.516	4933.341	566.895	1250.013	0.000
	2001	966.906	701.246	4938.001	297.892	951.187	311.786
	2002	975.927	741.854	5747.456	345.520	1288.406	127.253
6	2000	1858.573	806.908	8046.425	588.590	1678.739	320.782
	2001	1901.036	857.973	8125.085	518.916	1628.500	385.301
	2002	1887.965	1015.557	9142.181	615.684	2271.030	237.567
7	2000	821.638	751.983	5128.953	478.535	325.880	0.000
	2001	839.531	469.232	5478.492	252.408	200.583	330.516
	2002	728.215	481.200	5994.670	293.929	843.195	0.000
8	2000	627.820	571.393	3529.849	295.875	795.777	207.377
	2001	643.203	646.221	4003.584	227.592	631.973	198.834
	2002	660.046	652.249	4206.805	276.936	1065.968	355.972
9	2000	901.468	377.888	3893.435	429.180	710.416	0.000
	2001	965.860	312.894	3453.317	298.354	598.207	39.004
	2002	996.588	359.520	3769.061	344.824	887.807	0.000
10	2000	1135.958	260.229	4035.343	486.559	990.618	549.858
	2001	1192.209	275.161	4040.666	535.189	637.120	477.648
	2002	1400.581	375.791	5393.740	583.300	937.071	431.647
11	2000	1163.314	596.476	5865.555	432.573	1956.891	0.000
	2001	1335.321	621.355	6476.792	413.115	1821.296	365.543
	2002	1274.620	747.447	7444.672	398.228	1787.940	0.000
12	2000	705.312	538.161	3542.552	553.847	1033.094	0.000
	2001	655.503	498.841	3436.948	247.223	573.139	191.110
	2002	740.499	490.047	3959.448	218.286	828.061	141.173
13	2000	1038.110	280.490	4548.166	465.921	1838.440	16.773
	2001	1029.370	315.008	4628.822	359.811	1452.394	426.805
	2002	1155.574	336.055	5793.475	390.082	1902.803	128.677
14	2000	578.172	191.396	2134.172	267.340	438.746	59.690
	2001	613.606	202.728	2475.627	234.925	289.183	136.805
	2002	598.110	212.343	3130.386	167.956	564.871	61.142
15	2000	608.385	586.806	3676.968	181.160	1323.015	12.579
	2001	662.692	489.291	3459.789	175.208	1146.263	232.465
	2002	791.105	595.728	4235.672	167.281	1998.221	139.519
16	2000	283.891	172.635	1106.634	320.015	294.058	54.491
	2001	348.262	279.373	1510.742	478.769	284.062	94.149

(*continued on next page*)

Table 4 (*continued*)

No.	Year	Operating expenses (X_1)	Insurance expenses (X_2)	Direct written premiums (Z_1)	Reinsurance premiums (Z_2)	Underwriting profit (Y_1)	Investment profit (Y_2)
	2002	409.253	268.624	2120.742	516.851	408.669	69.778
17	2000	56.827	64.851	288.007	164.880	80.829	32.393
	2001	66.592	74.837	408.320	201.171	121.784	25.949
	2002	92.830	107.501	733.630	282.120	397.337	20.908
18	2000	83.794	22.959	222.933	73.865	73.305	13.988
	2001	67.685	28.553	152.011	63.434	188.194	12.130
	2002	77.757	24.965	164.818	68.486	167.430	14.407
19	2000	7.887	5.988	27.471	6.946	2.076	2.616
	2001	9.185	6.538	32.200	6.406	1.899	2.738
	2002	6.808	3.964	19.863	8.168	80.242	1.443
20	2000	26.947	8.666	88.661	36.554	0.000	11.162
	2001	28.251	13.394	121.601	34.016	2.511	11.614
	2002	26.442	15.014	124.309	15.848	0.000	7.366
21	2000	56.314	71.678	78.440	253.285	152.442	5.106
	2001	70.673	102.440	183.277	250.178	54.931	8.067
	2002	92.624	132.654	293.142	394.638	87.439	8.909

0.3475), and these are due to the unsatisfactory performances in process two. Based on results like this, a company may design more suitable strategies to improve its performance.

6. Conclusion

The DEA technique has been widely applied to measure the relative efficiency of a set of DMUs in a specified period of time. Conventionally, the aggregate data of the periods involved, either totals or averages, is used in measuring efficiency. This paper investigates the effects of the operations of individual periods on the overall performance of a DMU with a two-stage structure in the specified period of time. The aggregate model developed by Kao and Hwang (2008) is extended to incorporate the variations in individual periods. This model can not only measure different kinds of efficiencies, but also calculate a common-weight global MPI, and is applied to the non-life insurance industry in Taiwan. Several conclusions can be drawn from the previous sections.

First, when the operations of individual periods are taken into account, a DMU is efficient if and only if every period is efficient, and the efficiency scores measured from the multi-period model are smaller than those measured from the conventional aggregate one. However, the results of the empirical study of the twenty-one non-life insurance companies in Taiwan show that the rankings obtained from the two models are similar in a statistical sense. Second, the overall efficiency of a DMU in multiple periods of time is a weighted average of those of individual periods. This relationship also applies to the two processes. The decomposition helps decision makers identify the key sources of inefficiency of a DMU. Third, the MPI of a DMU between two periods is the product of those of the two processes between the same periods. Based on the circular property of the global MPI, the MPI between two periods of longer duration is the product of two (or more) MPIs of non-overlapping periods of shorter duration.

The production of the non-life insurance companies is composed of two stages, premium acquisition and profit generation. The fourth conclusion is that in general the stage of premium acquisition (process one) has a better performance than the stage of profit generation (process two) for non-life insurance companies in Taiwan. This implies that more effort should be devoted to process two in order to achieve better overall performance. Fifth, the performance of the twenty-one companies in general has improved from 2000 to 2002, due to the excellent performance in 2001, although this was followed by a small decline in 2002.

Both the DEA models and the MPI discussed in this paper are developed under the assumption of constant returns to scale (CRS), which prevents the investigation of scale issues. It would be more informative if models under the assumption of variable returns to scale (VRS) can be developed. Kao and Hwang (2011) extended the CRS models of Kao and Hwang (2008), on which this paper is based, to VRS ones. Their models can be adopted for exploring scale issues; however, they are limited to two-stage structures, and cannot be generalized to multi-stage ones. Amatatsu et al. (2012) proposed an additive network DEA model to calculate the industrial efficiency and the corresponding returns to scale governed by local governments in Japan. Although the scale economy of each process can be identified, their relationship with that of the system is not known. For this topic, there are a lot of details which need to be clarified, and are thus a direction for future research.

The multi-period system studied in this paper is a two-stage one. The results are also applicable to systems with more than two stages, i.e., series systems. There are two basic structures for general network systems, series and parallel. The works of Beasley (1995) and Kao (2012) on the teaching and research functions of universities present a typical example of the parallel structure. How to apply the idea developed in this paper to this type of system is a topic for future research. Another more challenging area is to investigate the efficiencies and MPIs for general network systems in multiple periods of time.

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Appendix A

see Table 4.

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