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Syntax and semantics of the weak consistency model specification language cat

Jade Alglave

Microsoft Research Cambridge
University College London
jaalglav@microsoft.com, j.alglave@ucl.ac.uk

Patrick Cousot

New York University
emer. École Normale Supérieure, PSL Research University
pcousot@cims.nyu.edu, cousot@ens.fr

Luc Maranget

INRIA
Luc.Maranget@inria.fr

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Abstract

We provide the syntax and semantics of the cat language, a domain specific language to describe consistency properties of parallel/distributed programs. The language is implemented in the herd7 tool Alglave and Maranget (2015).

1 Introduction

The cat language Alglave et al. (2015b) is a domain specific language to describe consistency properties succinctly by constraining an abstraction of parallel program executions into a candidate execution and possibly extending this candidate execution with additional constraints on the execution environment. The *analytic semantics* of a program is defined by its *anarchic semantics* that is a set of executions describing computations and a cat *specification cat* describing a weak memory model. An example of anarchic semantics semantics for LISA is given in Alglave and Cousot (2016). An anarchic semantics is a truly parallel semantics, with no global time, describing all possible computations with all possible communications. The cat language operates on abstractions of the anarchic executions called *candidate executions*. The cat specification *cat* checks a candidate execution for the consistency specification (including,

maybe, by defining constraints on the program execution environments, such as the the final writes or the coherence order).

The abstraction of an anarchic execution into a candidate execution is overview in Section 2 while the cat language is introduced is Section 3. Its formal semantics is defined in Section 4. Examples can be found in Alglave [2015].

2 Abstraction to candidate executions

The anarchic semantics is a set of executions. Each execution is abstracted to a candidate execution $\langle evts, \texttt{po}, \texttt{rf}, \texttt{IW}, \texttt{sr} \rangle$ providing

- *events evts*, giving a semantics to instructions; for example in LISA Alglave and Cousot (2016), a write instruction `w[] x v` yields a write event of variable `x` with value `v`. Events can be (for brevity this is not an exhaustive list):
  - *writes*, gathered in the set `W`, including the the set `IW` of *initial writes* coming from the prelude of the program;
  - *reads*, gathered in the set `R`;
  - *branch* events, gathered in the set `B`;
  - *fences*, gathered in the set `F`.
- the program order `po`, relating accesses written in program order in the original LISA program;
- the read-from `rf` describing a communication between a write and a read event;
- the scope relation `sr` relating events that come from threads which reside within the same scope;

A cat specification *cat* may add other components to the candidate execution (*e.g.* to specify constraints on the execution environment) and then checks that this extended candidate execution satisfies the consistency specification, that is, essentially, that the communication relation `rf` satisfies the consistency specification (under hypotheses on the execution environment).

3 The cat language

A weak consistency specification written in the cat language defines constraints to be satisfied by the communication relation `rf` of any candidate execution. A typical cat specification defines new objects depending on the sets and relations of the candidate execution (*e.g.* the program order `po` or the initial writes `IW`) and then imposes constraints on these objects that ultimately restrict the allowed communications `rf` .

3.1 Objects and expressions

3.1.1 Types.

The objects defined in a cat specification may be of the following types (see Appendix 4.9 and Figure 5 for the formal details): evt (event), tag (tag), rel (relation between events), set (set), tuple (tuple), enum (enumeration of tags), fun (unary function type), proc (unary procedure type).

3.1.2 Definitions in binding statements

(see Appendix 4.12.5 and Figure 17 for their formal semantics) can bind an expression to a name, which can be used in place of that expression. For example

```
let rfe = rf & ext
```

defines the relation `rfe` as the restriction of the communications `rf` to events coming from different processes. Formally, `rfe` is built as the intersection (denoted by `&` in `cat`) of the read-from relation `rf` and the predefined relation `ext` which links events coming from different processes (Figure 11).

A set, relation, function or procedure can be given a name by binding (see Figure 7). Bindings (see Appendix 4.12.5 and Figure 17) can be (mutually) recursive (using `let rec ... and ...`).

3.1.3 Functions

(see Appendix 4.12.3 and Figure 15 for their formal semantics) define an object as a function of a unique formal parameter (which may be an empty tuple `()` in absence of parameter or a non-empty tuple for multiple parameters). For example

```
let extof r = r & ext
let rfe = extof rf
```

defines a function `extof` of a parameter `r` which intersects the relation `r` with the relation `ext` between events belonging to different processes. We then define the relation `rfe` as the function `extof` applied to the read-from relation `rf`.

We note that our definition of `extof` above is an abbreviation for the binding of an anonymous function

```
let extof = fun r -> r & ext
```

Functions can be recursive (using `let rec`) and get their actual parameters in a call by tuple-matching their actual argument.

3.1.4 Events.

All events come out of the candidate execution and there is no way in `cat` to generate any other event.

3.1.5 Sets

(see Appendix 4.12.4 and Figure 16 for their formal semantics) are either empty `{}` or a homogeneous set `{`o_1`, ..., `o_n`, ...}`. We do not allow sets of functions or procedures. Predefined sets of events are denoted by the following identifiers (see Appendix 4.11.1 and Figure 10 for their formal semantics):

- the set of all write events `W`, including the initial writes `IW`;
- the set of all read events `R`;
- the set of all branch events `B`;

- the set of all fence events `F`;
- the universe containing all events of the candidate execution, which is denoted "`_`".

New sets can be defined from existing ones using the following operations (see Appendix 4.12.6, Figures 18, 19 and 20 for the formal semantics of these operations):

- the `~`S is the complement of a set S;
- the union of two sets S_1 and S_2 is S_1 `|` S_2;
- the intersection of two sets S_1 and S_2 is S_1 `&` S_2;
- the difference of two sets S_1 and S_2 is $S_1 \setminus S_2$;
- the addition of an element `e` to a set S is `e++`S;

Matching over sets (see Appendix 4.12.4 and Figure 16 for the formal semantics) can be used for (recursive) set definitions. Match is against the empty set `{}` or, for a non-empty set, a partition `e ++ es` into a singleton `{e}` and the rest of the set `es`. For example, given a function f, a set $S = \{e_1, e_2, \ldots, e_n\}$ and an element y, the call `fold` f`(`S`,` y`)` returns the value $f(e_{i_1}, f(e_{i_2}, \ldots f(e_{i_n}, y)))$, where $i_1, i_2, \ldots, i_n$ is some permutation of $1, 2, \ldots, n$:

```
let fold f =
  let rec fold_rec (es,y) = match es with
  || {} -> y
  || e ++ es -> fold_rec (es, f(e,y))
  end
in fold_rec
```

3.1.6 Relations between events

(see Appendix 4.11.1 for their formal semantics) can be the empty relation `0`, the identity relation `id`, or the relations defined from the candidate execution:

- the program order `po`;
- the read-from `rf`,

or predefined relations on events (see Figure 11):

- the relation `loc` between events accessing the same memory location;
- the relation `ext` between events coming from different threads.

New relations (see Appendix 4.12.6) can be defined from sets of events (see Figure 20):

- the cartesian product of two sets of events S_1 and S_2 is S_1`*` S_2

or using unary operators on relations (see Figure 19):

- the identity closure of a relation `r` is `r?`
- its reflexive-transitive closure is `r*`

- its transitive closure is `r+`
- its complement is `~r`
- its inverse is `r^-1`

or using binary operators on relations (see Figure 20):

- the union of two relations `r1` and `r2` is `r1 | r2`
- the intersection of two relations `r1` and `r2` is `r1 & r2`
- the difference of two relations `r1` and `r2` is `r1 \ r2`
- the sequence of two relations `r1` and `r2` is `r1;r2` (*i.e.* the set of pairs (x, y) such that there exists an intervening z, such that $(x, z) \in$ `r1` and $(z, y) \in$ `r2`).

Moreover the following primitives can be used to manipulate sets and relations over events (see Figure 12 for their formal semantics):

- `classes` takes a relation r; if r is an equivalence relation, then we return the equivalence classes of r, otherwise an error is raised;
- `linearisations` takes a set S and a relation r and returns a set of relations; *viz*., if the relation r is acyclic, we return all the possible linearisations (topological sorts in the finite case) of r over S, otherwise we return the empty set.

3.1.7 Tuples

(see Appendix 4.12.4 and Figure 16 for their formal semantics) include the empty tuple `()`, and constructed tuples `(`o_1`, ..., `o_n`)`. Tuples can be heterogeneous. Tuples are essentially used to pass parameters to functions and procedures. Tuples can be destructured by pattern matching; for example in the example of `fold` above, we match the argument of `fold_rec` into the pair `(es,y)`.

3.1.8 Tags.

Events can be tagged (using the annotations on the program instruction generating this event) and these tags can be used to build relations. The tags must be declared (see Appendix 4.12.1 and Figure 14) using the `enum` construct. For example

```
enum memory-order = 'rlx || 'acq || 'rel
```

defines an enumeration type `memory-order`, which contains three tags: `'rlx` (relaxed), `'acq` (acquire), `'rel` (release).

LISA instructions can be annotated with such tags. In cat, tags have a quote `'` to not be confused with identiers. This confusion is impossible in LISA so quotes `'` are omitted. The tags that can be worn by instructions must be declared (see Figure 14 in Appendix 4.12.1), as follows:

```
instructions W[{'rlx,'rel}]
instructions R[{'rlx,'acq}]
```

Events generated by an annotated LISA instruction will bear the same tags as the instruction. The set of events bearing a given tag t is provided by `tag2events (`t`)` (see Figure 13 in the Appendix 4.12.1). For example

```
let Release = tag2events('rel)
let Acquire = tag2events('acq)
```

define the set `Release` (resp. `Acquire`) of events bearing the tag `'rel` (resp. `'acq`).

Tags can be matched against their names as defined in an `enum`, and with the wildcard `_` (see Figure 13); examples are provided in the next section.

3.1.9 Scopes.

The organisation of a parallel system is not always flat. Often, threads (and physical processors or cores alike) are organised in a hierarchical fashion, threads being members of a hierarchy of nested levels, or *scopes*. Examples include: the eponymous scope notion in GPU models (*e.g.* Cooperative Thread Array, or `cta`, in Nvidia PTX), or the notion of shareability domain in ARM (*e.g.* `ish` in ARMv8).

Scopes (see Appendix 4.12.2 for their formal semantics) are special tags which must be declared with the reserved identifier `scopes`:

```
enum scopes = 'cta || 'gpu || 'system
```

The hierarchy of scopes is described in a `cat` file by the functions `narrower` and `wider` (which are reserved identifiers but user-defined, as `scopes` is). In the most simple and frequent case, levels are totally ordered. Then, the `wider` function takes a scope tag as argument and returns the immediately wider scope tag, while the `narrower` function returns the immediately narrower scope tag:[1]

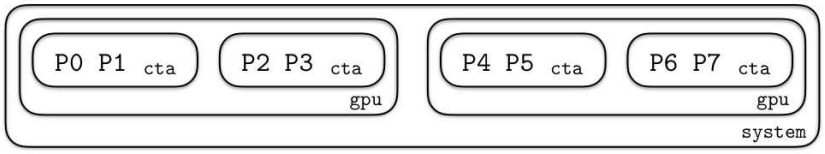
```
let wider(s) = match s with 'gpu -> 'system || 'cta -> 'gpu end
let narrower(s) = match s with 'system -> 'gpu || 'gpu -> 'cta end
```

The above definitions specify that scopes are ordered from narrowest to widest as: `'cta` $<$ `'gpu` $<$ `'system`. In other words, a system contains one or more GPUs, and each GPU contains one or more CTAs.

All LISA litmus tests specify how many threads `P0`, `P1`, *etc.* are involved. Additionally a scoped litmus test specifies how threads are distributed along the scope hierarchy, by means of a *scope tree* such as

```
scopes: (system (gpu (cta P0 P1) (cta P2 P3)) (gpu (cta P4 P5) (cta P6 P7)))
```

which describes the scope hierarchy



[1] One may also consider heterogeneous systems such as coupled CPUs and GPUs. In that case, the hierarchical is no longer total and the function `narrower` returns a set of tags.

The herd7 tool checks that the `wider` function does define a hierarchy, in the sense that each scope has an unique immediately wider scope except one, the root of the hierarchy, which has none. It also checks the compatibility of the `narrower` function and of scope trees with the defined hierarchy.

The events in a given scope s are gathered as an equivalence relation `tag2scope (`s`)` (see Figure 13 in the Appendix 4.12.2). More precisely two events are related by `tag2scope (`s`)` when they are generated by threads that are contained in the same scope instance of level s.

Consider for instance the hierarchy depicted above, and two events e_0, and e_2, generated by `P0` and `P2` respectively. Then e_0 and e_2 are related by `tag2scope ('gpu)` and unrelated by `tag2scope ('cta)` since `P0` and `P2` belong to the same GPU but to different CTAs.

3.2 Constraint statements

After defining sets and relations depending on the candidate execution, we can impose constraints on them (see Figure 22 for the formal semantics).

3.2.1 Checks

(see Appendix 4.13.1 and Figure 22 for their formal semantics) can have the following syntax: [~][`acyclic` | `irreflexive` | `empty`] x.

The checks [~][`acyclic` | `irreflexive` |] r check if the relation r on events is be acyclic or irreflexive. The check or [~][`empty`] S checks if the set S is empty. The check `acyclic`r is a shorthand for `irreflexive`r`+`. The symbol `~` denotes the negation. Failed checks reject the candidate execution which is therefore `forbidden`. For example

```
acyclic po | rf
```

checks whether the union of `po` and `rf` is acyclic in all the candidate executions of a given program.

Users have the option to not enforce the checks, but rather to use them to report properties of the candidate execution. To do so, users must prefix the check they are interested in with the keyword `flag`, and name the flagged test with an identifier *name* (by using the postfix qualifier `as` *name*). A failed flagged check has no consequence over the acceptance or rejection of the candidate execution. It is simply reported (*viz.*, flagged with the name *name*) for the user's information. For example

```
flag ~(acyclic po | rf) as cycle-found
```

will flag, using the name `cycle-found`, all the candidate executions in which there is a cycle in the union of `po` and `rf`.

We often use `flag` in models that involve *data races*, *e.g.* C++ or HSA. In such models, executions that have data races are typically deemed undefined. We handle this in `cat` by flagging candidate executions that exhibit data races with the name `undefined`.

3.2.2 Procedures. (see Appendix 4.13.2 and Figure 23 for their formal semantics)

Definitions of sets and relations and their checks in constraint statements can be gathered and parameterised using procedures and checked by procedure calls. Procedures are not recursive

and return no result. They have one formal parameter (but that can be a tuple, including the empty one). Their body is a non-empty list of statements.

For example the following procedure `sc` implements Sequential Consistency Lamport (1979), given the relation `com` as parameter:

```
procedure sc(com) =
   let sc-order = (po | com)+
   acyclic sc-order
end
```

The procedure may have local definitions (like `sc-order`). The scope of the formal parameter and local definitions is limited to the procedure body. Global definitions (like the relation `po`) can be used in the procedure body.

A call (*e.g.* `call sc(rf)`) passes the actual parameter (a tuple, here `rf`, matching the formal parameter `com`) and the procedure body is evaluated with the actual parameter.

3.2.3 Iteration.

A universally quantified check (*i.e.* a finite conjunction of checks) can be done for all values `e` chosen in a set `S` by a `forall` iterator (see Appendix 4.13.3 and Figure 24 for the formal semantics), as follows:

```
forall e in S do
   call check_contraint(e)
end
```

3.2.4 Candidate execution extension (`with ... from ...`)

The construct `with` o `from` S *requirements* introduces an additional constituant o of the semantics, not already part of the candidate execution. This constituant o of the semantics is introduced in the `cat` file rather than in the anarchic semantics because it only depends on the program execution events (*e.g.* the coherence order).

The `with` construct enumerates all possible objects o in S and checks the *requirements*. Typically S is a set of relations on events and o a relation between events which must satisfy the *requirements* appearing in the remainder of the `cat` file.

For example total orders over certain accesses can be built using `with`:

- the coherence order between writes to a given memory location ;
- SC accesses in C++ or HSA (we use this in our modelisation of HSA, see HSA Foundation (2015)).

3.3 Evaluation of a cat file on a candidate execution

When evaluated on a candidate execution, a `cat` file returns an `error` if the `cat` file is syntactically incorrect. Otherwise the binding definitions, constraint statements, and `with` requirements are evaluated in sequence (see Figures 3 and 25). If some (unflagged *i.e.* mandatory) constraint fails, we return `forbidden` to stipulate that the candidate execution does not satisfy the weak consistency model specified by the `cat` file. Otherwise the candidate execution is accepted, *i.e.*

we return `allowed`. In both cases we return a possibly empty set of flags for conditions that are not enforceable (see Appendix 4.7), as well as the objects introduced by `with` constructs.

4 Syntax and formal semantics of the cat language

4.1 Analytic semantics

The analytic semantics $\mathcal{S}[\![\mathtt{P}]\!]$ of a parallel program P with a given cat consistency specification (or weak consistency model) *cat* is a set of execution behaviors π conforming to this consistency specification. Each such execution behavior $\pi = \langle \Xi, \mathtt{rf}, \Gamma \rangle$ is described in two parts, the computations $\langle \Xi, \mathtt{rf} \rangle$ and the communications $\langle \mathtt{rf}, \Gamma \rangle$ where the *read-from relation* rf is their common interface.

- The possible *computations* $\langle \Xi, \mathtt{rf} \rangle$ are described by the anarchic semantics $\mathcal{S}_a[\![\mathtt{P}]\!]$ of the program P. The read-from relation rf records the correspondance between the reads, the matching writes, and the communicated values on the computation Ξ.

  The anarchic semantics $\mathcal{S}_a[\![\mathtt{P}]\!]$ places only the following restrictions on the communications of P so all possible computations with all possible read-from relations rf are considered.

  - *Satisfaction*: a read event has at least one corresponding communication in rf;
  - *Singleness*: a read event must have at most one corresponding communication in rf;
  - *Match*: if a read reads from a write, then the variables read and written and communicated value must be the same;
  - *Inception*: no communication is possible without the occurrence of both the read and (maybe initial) write it involves (this does not prevent a read to read from a future write).

  Otherwise stated the consistency specification/weak consistency model is not taken into account at all by the anarchic semantics $\mathcal{S}_a[\![\mathtt{P}]\!]$.

- The possible *communications* are described by communications $\langle \mathtt{rf}, \Gamma \rangle$ between communication *i.e.* read and/or write events.

  The cat file *cat* generates all possible communication relations $c \in \Gamma$ (using the with construct). The communication relations $c \in \Gamma$ include the coherence order co, *etc.* More generally, they specify requirements on the execution environment of the program P.

The cat file semantics sorts out the executions $\pi = \langle \Xi, \mathtt{rf}, \Gamma \rangle$ that are feasible for weak consistency model, one by one.

4.2 Consistent semantics specification by cat files

A cat file $cat \in Cat$ defines a check that an execution $\pi = \langle \Xi, \mathtt{rf}, \Gamma \rangle$ satisfies a consistency specification.

- First the computation $\langle \Xi, \mathtt{rf} \rangle$ of the anarchic semantics is abstracted to a candidate execution $X = \alpha_\Xi(\langle \Xi, \mathtt{rf} \rangle) = \langle evts, \mathtt{po}, \mathtt{rf}, \mathtt{IW}, \mathtt{sr} \rangle$ (collecting read, write, branch, fence and rmw events in *evts*, the program order po, the read-from relation rf, initial IW writes, and the program scope tree sr) but where *e.g.* events on local registers or communicated values are abstracted away.

- The `cat` file *cat* is then evaluated on X. Thanks to `with` c_i `from` C_i constructs, the `cat` file generates all necessary communication relations $\Gamma = c_1, \ldots, c_n$ between communication events (including `co` $\in$ `allCo`, *etc.*) which are necessary to express the consistency specification.
- In absence of error in *cat*, the final result
  - can be $\langle$`allowed`, f, $\Gamma\rangle$ meaning that the computation $\langle\Xi,$ `rf`$\rangle$ (with abstraction $X = \alpha_{\Xi}(\langle\Xi,$ `rf`$\rangle)$) together with the communication specification Γ satisfies the consistency specification, or
  - can also be $\langle$`forbidden`, f, $\Gamma\rangle$ meaning that the $\pi = \langle\Xi,$ `rf`, $\Gamma\rangle$ does not satisfy the consistency specification.

  In both cases $f \in \mathcal{F}$ is the set of flagged constraints in *cat* satisfied by the execution $\pi = \langle\Xi,$ `rf`, $\Gamma\rangle$ (without any influence on the `allowed`/`forbidden` result).

4.3 Analytic semantics specified by an anarchic semantics and a cat specification

We define below, in Figure 2, the semantics $\text{😺}[\![cat]\!]\,X$ of a candidate execution X which returns a set of answers of the form $\langle j, f, \Gamma\rangle$ where $j = \{$`allowed`, `forbidden`$\}$, f is the set of flags that have been set up on X and Γ, and Γ defines the communication relation for the execution to be `allowed`/`forbidden`.

The analytic semantics of a program `P` with consistency specification *cat* is therefore

$$\begin{array}{rcl} \mathcal{S}[\![\texttt{P}, cat]\!] & \triangleq & \{\langle\Xi,\ \texttt{rf},\ \Gamma\rangle \mid \langle\Xi,\ \texttt{rf}\rangle \in \mathcal{S}_a[\![\texttt{P}]\!] \wedge \\ & & \qquad \exists f \in \mathcal{F}\ .\ \langle\texttt{allowed},\ f,\ \Gamma\rangle \in \text{😺}[\![cat]\!]\,(\alpha_{\Xi}(\langle\Xi,\ \texttt{rf}\rangle))\} \end{array}$$

This analytic semantics $\mathcal{S}[\![\texttt{P}]\!]$ of a program `P` for a `cat` specification *cat* is the composition $\mathcal{S}[\![\texttt{P}]\!] = \alpha_{\text{😺}}[\![cat]\!] \circ \alpha_{\Xi}(\mathcal{S}_a[\![\texttt{P}]\!])$ of two abstractions of the anarchic semantics *viz.*

$$\begin{array}{rcl} \alpha_{\Xi}(\mathcal{S}) & \triangleq & \{\langle\langle\Xi,\ \texttt{rf}\rangle,\ \alpha_{\Xi}(\langle\Xi,\ \texttt{rf}\rangle)\rangle \mid \langle\Xi,\ \texttt{rf}\rangle \in \mathcal{S}\} \\ \alpha_{\text{😺}}[\![cat]\!](\mathcal{C}) & \triangleq & \{\langle\Xi,\ \texttt{rf},\ \Gamma\rangle \mid \langle\langle\Xi,\ \texttt{rf}\rangle,\ X\rangle \in \mathcal{C} \wedge \\ & & \qquad \exists f \in \mathcal{F}\ .\ \langle\texttt{allowed},\ f,\ \Gamma\rangle \in \text{😺}[\![cat]\!]\,X\} \end{array}$$

4.4 Candidate executions

Candidate executions are tuples:

$$\begin{array}{rcl} X & = & \langle evts,\ \texttt{po},\ \texttt{rf},\ \texttt{IW},\ \texttt{sr}\rangle \quad \in \quad \textit{Candidate} \\ & \triangleq & \textit{Evts} \times \textit{Program-order} \times \textit{Read-from} \times \textit{Writes} \times \textit{Scope-rel} \end{array}$$

which gather the events, the program order `po` on each thread, the read-from relation `rf`, modeling who reads from where, the initial writes `IW`, and a scope relation `sr`.

4.4.1 Events

$e \in \mathit{Evt}$ are abstractions of the events generated by a program execution. Events $e \in \mathit{evts}$ carry the unique program instruction (and its unique program label) which execution generated this event e. However, this information is not directly available to `cat`. Auxiliaries to extract components of an event e are as follows:

$$\mathsf{loc\text{-}of}(e) \triangleq \text{location of } e \qquad \mathsf{kind\text{-}of}(e) \triangleq \text{kind of } e$$
$$\mathsf{pid\text{-}of}(e) \triangleq \text{process identifier of } e \qquad \mathsf{annot\text{-}of}(e) \triangleq \text{annotations of } e$$
$$\mathsf{from\text{-}to\text{-}of}(e) \triangleq \text{events separated by fence event } e$$

The set *evts* of events belong to $\mathit{Evts} \triangleq \wp(\mathit{Evt})$.

- The process identifier $\mathsf{pid\text{-}of}(e)$ refers to the identifier of the unique process at the origin of the event e;
- The location $\mathsf{loc\text{-}of}(e)$ can be a memory location or a register;
- $\mathsf{kind\text{-}of}(e)$ is the kind of event e: write (`W`), read (`R`), branch (`B`), fence (`F`), begin or end of a `rmw`. We define the following sets of events by kind:

$$W(X) \triangleq \{e \in \mathsf{evts\text{-}of}(X) \mid \mathsf{kind\text{-}of}(e) = \texttt{W}\} \qquad R(X) \triangleq \{e \in \mathsf{evts\text{-}of}(X) \mid \mathsf{kind\text{-}of}(e) = \texttt{R}\}$$
$$F(X) \triangleq \{e \in \mathsf{evts\text{-}of}(X) \mid \mathsf{kind\text{-}of}(e) = \texttt{F}\} \qquad B(X) \triangleq \{e \in \mathsf{evts\text{-}of}(X) \mid \mathsf{kind\text{-}of}(e) = \texttt{B}\}$$

- the annotations $\mathsf{annot\text{-}of}(e)$ of e is a possibly empty set of tags and scopes carried by the action at the origin of e;
- let $e_F \in F(X)$ be a fence event generated by a localised fence instruction `f[`ts`] {`$\mathtt{L}_1$`,...,`$\mathtt{L}_n$`} {`$\mathtt{L}'_1$`,...,`$\mathtt{L}'_m$`}` where this instruction and all $\mathtt{L}_1$, ..., $\mathtt{L}_n$, $\mathtt{L}'_1$, ..., $\mathtt{L}'_m$ belong to the same process of $\mathsf{pid\text{-}of}(e_F)$.

  Then $\mathsf{from\text{-}to\text{-}of}(e_F)$ is the set of pairs $\langle e_f,\ e_t\rangle$ such that e_f is an event generated by the execution of a program instruction labelled $\mathtt{L}_i$, $i \in [1, n]$ and e_t is an event generated by the execution of a program instruction labelled $\mathtt{L}'_j$, $j \in [1, m]$. Additionally, we require $\langle e_f,\ e_F\rangle \in \mathsf{po\text{-}of}(X)$ and $\langle e_F,\ e_t\rangle \in \mathsf{po\text{-}of}(X)$, *viz.* the fence does separate the two events e_f and e_t. If the fence carries an empty set of labels this is $\mathsf{from\text{-}to\text{-}of}(e_F) \triangleq \emptyset$. If the fence carries no sets of labels, we set $\mathsf{from\text{-}to\text{-}of}(e_F) \triangleq \{\langle e_f,\ e_t\rangle \mid \langle e_f,\ e_F\rangle \in \mathsf{po\text{-}of}(X) \wedge \langle e_F,\ e_t\rangle \in \mathsf{po\text{-}of}(X)\}$.

4.4.2 Program order,

abbreviated `po` $\in$ *Program-order*, abstracts the order of the events of a process in the execution hence lifts the order in which instructions have been executed to the level of events. For each candidate execution, it is a total order over events within the same thread, hence irreflexive and transitive, and cannot relate events from different threads.

4.4.3 Read-from,

abbreviated `rf` $\in$ *Read-from* $\triangleq \wp(\mathit{Write} \times \mathit{Read})$, relates a read event of a certain shared variable `x` to a unique write event of the same variable. The read-from relation essentially indicates which events read from where.

4.4.4 Initial writes

are gathered in the set $\texttt{IW} \in \mathit{Writes} \triangleq \wp(\mathit{Write})$. The initial writes `IW` simply are the writes in the prelude of the program.

4.4.5 Scope relation,

abbreviated $\texttt{sr} \in \mathit{Scope\text{-}rel}$, relates events that come from threads which reside within the same *scope*; this is a notion that is mostly used for scoped models such as GPUs (see *e.g.*, Alglave et al. (2015a) and Sections 3.1.9).

Auxiliaries to extract components of a candidate execution $X \triangleq \langle \mathit{evts}, \texttt{po}, \texttt{rf}, \texttt{IW}, \texttt{sr} \rangle$ are as follows:

$$\mathsf{evts\text{-}of}(X) \triangleq \mathit{evts} \qquad \mathsf{po\text{-}of}(X) \triangleq \texttt{po} \qquad \mathsf{rf\text{-}of}(X) \triangleq \texttt{rf}$$
$$\mathsf{sr\text{-}of}(X) \triangleq \texttt{sr} \qquad \mathsf{init\text{-}of}(X) \triangleq \texttt{IW}$$

4.5 Program scope relation defined by a scope tree and cat scope hierarchy

A program scope tree specifies a scope relation. The syntax of program scope trees and their semantics, that is the scope relation that they define are defined in Figure 1. Program scope trees must match the scope hierarchy defined by the `cat` file through a scope tag declaration (see Figure 14) and the user specified functions with reserved names `narrower` and `wider`, as checked in Figure 13.

4.6 Values

The `cat` language is much inspired by OCaml Leroy et al. (2014), featuring for example types, immutable bindings, first-class functions and pattern matching. However, `cat` is a domain specific language, with important differences from OCaml:

- base values are specialised; they are: sets of events, relations over events, first class functions; there are also tags, including scope tags, akin to C enumerations or OCaml constant constructors. There are two structured values: sets of values and tuples of values, see Figure 5.
- there is a distinction between expressions in Figure 7 that evaluate to some value, statements in Figure 3, which introduce new definitions or constraints, and requirements in Figure 25 which introduce new communication relations on the execution environment and constraints on them.

We use the following notations: square brackets $[\ldots]$ denote optional components, parentheses $(\ldots)$ denote grouping, $(\ldots)^*$ (resp. $(\ldots)^+$) denotes zero, one or several (resp. one or several) repetitions of the enclosed components.

Scope trees —

$$
\begin{array}{lcll}
st & ::= & (\textit{s-tag}\ \mathtt{P}_0\ \ldots\ \mathtt{P}_n) & \text{program scope trees} \\
 & \mid & (\textit{s-tag}\ st_0\ \ldots\ st_n) &
\end{array}
$$

where $\{\mathtt{P}_0, \ldots, \mathtt{P}_n\} \subseteq \{\mathsf{pid\text{-}of}(e) \mid e \in \mathsf{evts\text{-}of}(X)\}$.

Given a scope tree st, a set E of events and a scope-tag st, define $\mathsf{srel}(st)\ E\ \textit{s-tag}$ to be the relation between different events that come from threads which reside in the scope $\textit{s-tag}$, as follows

$$
\begin{array}{rcl}
\mathsf{srel}((\textit{s-tag}\ \mathtt{P}_0\ \ldots\ \mathtt{P}_n))\ E\ \textit{s-tag}' & \triangleq & \emptyset \qquad (\text{when } \textit{s-tag} \neq \textit{s-tag}') \\
\mathsf{srel}((\textit{s-tag}\ \mathtt{P}_0\ \ldots\ \mathtt{P}_n))\ E\ \textit{s-tag} & \triangleq & \{\langle e,\ e'\rangle \mid e, e' \in E \wedge \exists i, j \in [0, n]\ .\ \mathsf{pid\text{-}of}(e) = \mathtt{P}_i \wedge \mathsf{pid\text{-}of}(e') = \mathtt{P}_j\} \\
\mathsf{srel}((\textit{s-tag}\ st_0\ \ldots\ st_n))\ E\ \textit{s-tag}' & \triangleq & \bigcup_{i=0}^{n} \mathsf{srel}(st_i)\ E\ \textit{s-tag}' \qquad (\text{when } \textit{s-tag} \neq \textit{s-tag}') \\
\mathsf{srel}((\textit{s-tag}\ st_0\ \ldots\ st_n))\ E\ \textit{s-tag} & \triangleq & \{\langle e,\ e'\rangle \mid e, e' \in E \wedge \\
 & & \quad \exists \mathtt{P}_i, \mathtt{P}_j \in \bigcup_{i=0}^{n} \mathsf{processes}(st_0\ \ldots\ st_n)\ .\ \mathsf{pid\text{-}of}(e) = \mathtt{P}_i \wedge \mathsf{pid\text{-}of}(e') = \mathtt{P}_j\} \\
\mathsf{processes}((\textit{s-tag}\ \mathtt{P}_0\ \ldots\ \mathtt{P}_n)) & \triangleq & \{\mathtt{P}_0, \ldots, \mathtt{P}_n\} \\
\mathsf{processes}((\textit{s-tag}\ st_0\ \ldots\ st_n)) & \triangleq & \bigcup_{i=0}^{n} \mathsf{processes}(st_i) \\
\mathsf{tags\text{-}of}((\textit{s-tag}\ \mathtt{P}_0\ \ldots\ \mathtt{P}_n)) & \triangleq & \{\textit{s-tag}\} \\
\mathsf{tags\text{-}of}((\textit{s-tag}\ st_0\ \ldots\ st_n)) & \triangleq & \{\textit{s-tag}\} \cup \bigcup_{i=0}^{n} \mathsf{tags\text{-}of}(st_i)
\end{array}
$$

If the program has a scope-tree st then the candidate execution X must have its scope relation component $\mathtt{sr} = \mathsf{sr\text{-}of}(X)$ be such that for all $\textit{s-tag} \in \mathsf{tags\text{-}of}(st)$, $\mathtt{sr}(\textit{s-tag}) = \mathsf{srel}\ (st)\ (\mathsf{evts\text{-}of}(X))\ \textit{s-tag}$.

Figure 1: Semantics of program scope trees

4.7 Consistency specifications

Consistency specifications (or cat files/specifications) *cat* filter candidate executions and extend them with communication relations. In other words, the semantics $\text{(=^..^=)}[\![cat]\!]\, X$ of a cat specification *cat* is defined with respect to a candidate execution X and its result extend it to specify requirements on the execution environment.

4.7.1 Evaluating a *cat* specification

means allowing or forbidding that candidate execution. More precisely, evaluating a cat file makes a result object $\langle j,\ f,\ \rho,\ \omega\rangle$ evolve, where:

- *Judgements* $j \in \mathcal{J} \triangleq \{\texttt{allowed}, \texttt{forbidden}\}$ can be of two kinds: `allowed` when a candidate execution passes all the checks imposed by the cat specification, or `forbidden` when a candidate execution fails on one of the checks of the cat specification *cat*.
- *Flagged checks* $f \in \mathcal{F} \triangleq \wp(\mathit{Identifier})$ collect identifiers of checks that have been flagged and are recorded to signal certain executions (*e.g.*, the ones with data races).
- *Environments* $\rho \in \mathcal{E}$ associate identifiers (which belong to the set *Identifier*) to typed values; more precisely environments are partial functions from identifiers to values:

  $$\mathcal{E} \quad \triangleq \quad \mathit{Identifier} \nrightarrow \mathcal{V}\ .$$

  During the evaluation of the cat file *cat*, the environment $\rho \in \mathcal{E}$ gets augmented with new definitions as evaluation progresses. It evolves also locally when evaluating functions and procedures, according to the static scoping or block-structured visibility rule.
- *Sets of communication relation identifiers* ω record the identifiers of communication relations introduced by a `with` requirement.

  $$\omega \quad \in \quad \mathcal{W} \quad \triangleq \quad \wp(\mathit{Communication\text{-}relation\text{-}identifier})$$

  During the evaluation of the cat file *cat*, the set $\omega \in \mathcal{W}$ of communication relation identifiers gets augmented with new identifiers introduced by `with` *id* `from` ... requirements, see Figure 25. The relation $\rho(id)$ which is the value of such communication relation identifiers *id* is found in the environment ρ. The final verdict in Figure 2 collects this information in the final result of the *cat* evaluation.
- *Results* collect judgements, flagged checks, environments, and communication relation identifiers or raise error if needed.

  $$r \quad = \quad \langle j,\ f,\ \rho,\ \omega\rangle \quad \in \quad \mathcal{R} \quad \triangleq \quad (\mathcal{J} \times \mathcal{F} \times \mathcal{E} \times \mathcal{W}) \cup \{\mathsf{error}\}$$

  A result may be undefined *e.g.* when an implementation might not terminate, for example, when evaluating a non-terminating function. The result can also be error when the cat file is incorrect. The difference is that an implementation of cat is assumed to signal error but is not required to report undefined results. The final result is collected in the final verdict $\langle j,\ f,\ \prod_{id \in \omega} \rho(id)\rangle$, see Figure 2.

Initially, the judgement is `allowed`, the set of flags is empty, predefined identifiers are implicitly bound to event sets and relations over events as described in Section 4.11.1 and Figures 10 and 11, and the set of communication relation identifiers is empty, see Figure 2.

4.7.2 Specifications

(or cat files) are lists of *requirements* preceded by an identifier, used for documentation purposes. We give the syntax and semantics of specifications in Figure 2.

The requirements constitutive of the specification are evaluated in sequence, until one requirement raises error or `forbidden`, or until the end of the requirement list. In that latter case, the specification accepts the candidate execution, hence raises `allowed`.

4.7.3 The final verdict

in Figure 2 is given at the top-level, gets rid of the environment, and returns the communication relations in S obtained by finding the value of the communication relation identifiers id in the environment. If S is empty, we return `forbidden` (with unmodified flags). If S contains error, the error is returned.

$$
\begin{array}{lcl}
\mathit{cat} & \in & \mathit{Cat} \\
\mathit{cat} & ::= & \mathit{identifier} \\
 & \mid & \mathit{identifier\ requirements}
\end{array}
$$

$$
\text{😺}[\![\mathit{identifier}]\!]\ X \triangleq \{\langle \texttt{allowed}, \emptyset, \emptyset \rangle\}
$$

$$
\text{😺}[\![\mathit{identifier\ requirements}]\!]\ X \triangleq \mathsf{verdict}(\text{😺}[\![\mathit{requirements}]\!]\ X\ \langle \texttt{allowed}, \emptyset, \emptyset, \emptyset \rangle)
$$

$$
\begin{array}{lcl}
\text{😺}[\![\mathit{cat}]\!] & \triangleq & \{\langle X, \Gamma \rangle \mid X \in \mathit{Candidate} \wedge \Gamma \in \mathit{Communication\text{-}relation} \wedge \\
 & & \qquad \exists f \in \mathcal{F}\ .\ \langle \texttt{allowed}, f, \Gamma \rangle \in \text{😺}[\![\mathit{cat}]\!]\ X\}
\end{array}
$$

$$
\begin{array}{lcll}
\mathsf{verdict}\ \emptyset & \triangleq & \{\langle \texttt{forbidden}, \emptyset, \emptyset \rangle\} & \\
\mathsf{verdict}\ S & \triangleq & \mathsf{error} & \text{when } \mathsf{error} \in S \\
\mathsf{verdict}\ \{\langle j_i, f_i, \rho_i, \omega_i \rangle \mid i \in \Delta\} & \triangleq & \{\langle j_i, f_i, \prod_{id \in \omega_i} \rho_i(id) \rangle \mid i \in \Delta\} & \text{otherwise}
\end{array}
$$

Figure 2: Semantics of specifications

4.8 Statements

Requirements can be *statements* introducing new binding definitions and checking constraints, or the `with` id `from` S requirement introducing a new communication relation identified by id.

Statements are evaluated for their effect: adding new *definitions* or checking *constraints*. We give their syntax and semantics in Figure 3. Note that once an error has been raised, we stay in that state. Moreover statements have no `with` requirement so cannot introduce new

communication identifiers. Therefore the set of communication relation identifiers is unchanged by the evaluation of a statement, $\omega' = \omega$ in Figure 3.

$$
\begin{array}{rcl}
\mathit{statements} & \in & \mathit{Statements} \\
\mathit{statements} & ::= & \{\mathit{statement}\}^+ \\
\mathit{statement} & \in & \mathit{Statement} \\
\mathit{statement} & ::= & \mathit{definition} \\
 & | & \mathit{constraint}
\end{array}
$$

$$
\begin{array}{l}
\text{😸}[\![\mathit{statements}]\!] \quad \in \quad \mathit{Candidate} \to \mathcal{R} \to \mathcal{R} \\
\text{😸}[\![\mathit{statement}\ \ \mathit{statements}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle \quad \triangleq \\
\quad \mathsf{let}\ \langle j',\ f',\ \rho',\ \omega'\rangle = \text{😸}[\![\mathit{statement}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \ \mathsf{in} \\
\quad\quad \mathsf{if}\ \ j' = \texttt{allowed}\ \ \mathsf{then} \\
\quad\quad\quad \text{😸}[\![\mathit{statements}]\!]\ X\ \langle j',\ f',\ \rho',\ \omega'\rangle \\
\quad\quad \mathsf{else}\ \ \langle j',\ f',\ \rho',\ \omega\rangle
\end{array}
$$

$$
\text{😸}[\![\mathit{statement}]\!]\ X\ \mathsf{error} \quad \triangleq \quad \mathsf{error}
$$

Figure 3: Semantics of statements

4.9 Typed values and semantic domains

Typed values, (gathered in the set $\mathcal{V}$) are given in Figure 5. Events (of type evt) belong to the set *Evt*. There are no operation on events so the type evt can only be used to type elements of relations or sets. Typed values include (see Figure 5):

- the error symbol;
- tags (of type tag), which belong to *Tag*;
- relations over events (of type rel), which belong to $\wp(\mathit{Evt} \times \mathit{Evt})$;
- sets (of type set) of values, which belong to $\wp(\mathcal{V})$; sets have to be homogeneous, and cannot be sets of functions or procedures, as reflected by the predicate well-formed;
- tuples (of type tuple) of values, which belong to $\bigcup_{n\in\mathbb{N}} \prod_{i=1}^{n} \mathcal{V}$;
- enumerations of tags (of type enum), which belong to $\wp(\mathit{Tag})$;
- functions (of type fun);
- non-recursive procedures (of type proc).

The value of functions and procedures are closures memorising their parameter (which belongs to *Pat*), their body (which in the case of functions belongs to *Expr*, and in the case of procedures can be a list of elements of *Statement*), and declaration *environment* (which belongs to $\mathcal{E}$). On a call, the actual parameters are evaluated in the calling environment and the body in the declaration environment enriched by the value of the formal parameters and the local bindings. After the call, evaluation goes on in the calling environment. This is therefore static scoping.

type	$\in$	*Type*	
type	::=		
		evt	events
	|	tag	tag
	|	rel	relation between events
	|	set	set
	|	tuple	tuple
	|	enum	enumeration
	|	fun	unary function type
	|	proc	unary procedure type

Figure 4: Typed values

4.10 Auxiliaries

To define the semantics of operators over sets and relations in particular we need to define a certain number of auxiliaries (summarised in Figure 6).

4.11 Expressions

Expressions let the user build new sets or relations over tags and events. Figure 7 summarises the syntax of expressions.

Several constructs are non-deterministic: the set matching of Section 4.12.4, the iteration over sets of Section 4.13.3. In the semantics, only one result is nondeterministically picked out of all possible ones. This is different from the `with` requirement of Section 4.14.2 where all possibilities for choosing the communication relation are enumerated.

The semantics of an expression is error whenever the semantics of any one of its subexpressions is error. To leave this check implicit, we assume that the mathematical construct let $\mathsf{type}_i : v_i = \text{😺}[\![expr_i]\!] X \rho, \quad i \in [1, \ell]$ in ... equals error whenever there exists i in $[1, \ell]$ such that $\text{😺}[\![expr_i]\!] =$ error.

$$\mathsf{well\text{-}formed}(S) \triangleq \forall\, \mathsf{type}\colon v \in S \,.\, \mathsf{type} \notin \{\mathsf{fun}, \mathsf{proc}\} \wedge \forall\, \mathsf{type}'\colon v' \in S \,.\, \mathsf{type} = \mathsf{type}'$$

check sets

Semantic domains

$$\begin{array}{rcll}
\mathcal{V} & \triangleq & & \text{typed values} \\
 & & \{\mathsf{error}\} & \\
 & & \cup\, \mathsf{tag}\colon \mathit{Tag} & \\
 & & \cup\, \mathsf{rel}\colon \wp(\mathit{Evt} \times \mathit{Evt}) & \\
 & & \cup\, \mathsf{set}\colon \{S \in \wp(\mathcal{V}) \mid \mathsf{well\text{-}formed}(S)\} & \\
 & & \cup\, \mathsf{tuple}\colon (\{()\} \cup \bigcup_{n \in \mathbb{N}, n>1} \prod_{i=2}^{n} \mathcal{V}) & \\
 & & \cup\, \mathsf{enum}\colon \wp(\mathit{Tag}) & \\
 & & \cup\, \mathsf{fun}\colon ((\mathit{Pat} \to \mathit{Expr}) \times \mathcal{E}) & \\
 & & \cup\, \mathsf{proc}\colon ((\mathit{Pat} \to \{\mathit{Statement}\}^+) \times \mathcal{E}) & \\
\rho \in \mathcal{E} & \triangleq & \mathit{Identifier} \nrightarrow \mathcal{V} & \text{environments} \\
j \in \mathcal{J} & \triangleq & \{\texttt{allowed}, \texttt{forbidden}\} & \text{judgements} \\
f \in \mathcal{F} & \triangleq & \wp(\mathit{Identifier}) & \text{flagged checks} \\
\omega \in \mathcal{W} & \triangleq & \wp(\mathit{Communication\text{-}relation\text{-}identifier}) & \text{set of com. identifiers} \\
r \in \mathcal{R} & \triangleq & (\mathcal{J} \times \mathcal{F} \times \mathcal{E} \times \mathcal{W}) \cup \{\mathsf{error}\} & \text{results}
\end{array}$$

Figure 5: Semantic domains

$$\begin{array}{lll}
\mathbb{i}_{\mathcal{X}} \triangleq \{\langle e,\, e\rangle \mid e \in \mathcal{X}\} & & \text{identity relation on set } \mathcal{X} \\
r \fatsemi r' \triangleq \{\langle e,\, e'\rangle \mid \exists e'' \,.\, \langle e,\, e''\rangle \in r \wedge \langle e'',\, e'\rangle \in r'\} & & \text{sequence of relations} \\
\mathsf{dom}(r) \triangleq \{x \mid \exists y \,.\, \langle x,\, y\rangle \in r\} & & \text{domain of relation } r \\
\mathsf{range}(r) \triangleq \{y \mid \exists x \,.\, \langle x,\, y\rangle \in r\} & & \text{range of relation } r \\
\mathsf{fld}(r) \triangleq \mathsf{dom}(r) \cup \mathsf{range}(r) & & \text{field of relation } r \\
\mathbf{lfp}^{\subseteq} F = \bigcap \{X \in \wp(S) \mid F(X) \subseteq X\} & & \text{the least fixpoint of the } \subseteq\text{-increasing operator } F \text{ on the powerset } \wp(S) \text{ Tarski (1955)}
\end{array}$$

Figure 6: Auxiliaries for defining operators' semantics

4.11.1 Identifiers

are either predefined or defined by the user through *definition* statements. We list the reserved identifiers in Figure 8. User-defined identifiers cannot be reserved identifiers and are bound in the environment ρ (see Figure 9).

simple	$\in$	*Simples*	
simple	::=		
	$\mid$	*id*	identifiers
	$\mid$	*tag*	tags
	$\mid$	*function*	anonymous functions
	$\mid$	*procedure*	procedures
	$\mid$	*set*	sets
	$\mid$	*tuple*	tuples

clause	$\in$	*Clauses*	
clause	::=		
		[`||`] *tag* `->` *expr* {`||` *tag* `->` *expr* }* [`___` `->` *expr*]	
	$\mid$	[`||`] `{}` `->` *expr* `||` *id* `++` *id* `->` *expr*	

expr	$\in$	*Expr*	
expr	::=		
	$\mid$	*simple*	simples
	$\mid$	*expr expr*	function application
	$\mid$	(*expr*) $\mid$ `begin` *expr* `end`	grouping
	$\mid$	`let` [`rec`] *binding* {`and` *binding*}* `in` *expr*	binding expressions
	$\mid$	`match` *expr* `with` *clause* `end`	matching
	$\mid$	*op*	operators on sets and relations

definition	$\in$	*Definition*	
definition	::=	*decl*	
	$\mid$	`let` [`rec`] *binding* {`and` *binding*}*	

Figure 7: Simple expressions, expressions and definitions

Predefined identifiers denoting sets of events appear in Figure 10. We have: the universal sets, the set of all write, read, memory, branch and fences events, as well as the set of initial writes. The semantics of these identifiers, given in Figure 10 is straightforward; they denote the eponymous sets of events.

Predefined identifiers denoting relations on events appear in Figure 11. We have: the empty and identity relations, the relation over events accessing the same memory location, the relation over events with different pids, the program order, and the read-from relation.

Keywords $\triangleq$

{`acyclic`, `and`, `as`, `begin`, `call`, `do`, `empty`, `end`, `enum`, `flag`, `forall`, `from`, `fun`, `in`, `instructions`, `irreflexive`, `let`, `match`, `procedure`, `rec`, `scopes`, `with` }

Primitives $\triangleq$

{`classes`, `fromto`, `linearisations`, `tag2events`, `tag2scopes`}

Names $\triangleq$

{`___`, `0`, `B`, `ext`, `F`, `id`, `IW`, `loc`, `M`, `narrower`, `po`, `R`, `rf`, `rmw`, `W`, `wider` }

Reserved $\triangleq$ Keywords $\cup$ Primitives $\cup$ Names

Figure 8: List of reserved identifiers

$id \in$ *Identifier*

$id \in$ *Communication-relation-identifier* $\triangleq$ *Identifier* $\setminus$ Reserved

$$\text{😺}[\![id]\!]\ X\ \rho \triangleq \text{if } id \in \text{dom}(\rho) \text{ then} \quad \text{(when } id \notin \text{Names)}$$
$$\text{let type: } v = \rho(id) \text{ in}$$
$$\text{if type } = \text{enum then set: } v \text{ else type: } v$$
$$\text{else error}$$

(for $id \in$ Names, see Figures 10 or 11)

Figure 9: Semantics of identifiers

aevt	::=		annotable events
	|	`W`	write events
	|	`R`	read events
	|	`B`	branch events
	|	`F`	fence events

predefined-events	::=		
	|	`___`	all events
	|	`IW`	initial writes
	|	`M`	memory events, $\texttt{M} = \texttt{W} \cup \texttt{R}$
	|	*aevt*	annotable events

$\llbracket \texttt{\_\_} \rrbracket\ X\ \rho$	$\triangleq$	$\mathsf{set}\{\mathsf{evt}: e \mid e \in \mathsf{evts\text{-}of}(X)\}$	events
$\llbracket \texttt{W} \rrbracket\ X\ \rho$	$\triangleq$	$\mathsf{set}\{\mathsf{evt}: e \mid e \in \mathsf{evts\text{-}of}(X) \cap W(X)\}$	write events
$\llbracket \texttt{R} \rrbracket\ X\ \rho$	$\triangleq$	$\mathsf{set}\{\mathsf{evt}: e \mid e \in \mathsf{evts\text{-}of}(X) \cap R(X)\}$	read events
$\llbracket \texttt{B} \rrbracket\ X\ \rho$	$\triangleq$	$\mathsf{set}\{\mathsf{evt}: e \mid e \in \mathsf{evts\text{-}of}(X) \cap B(X)\}$	branch events
$\llbracket \texttt{F} \rrbracket\ X\ \rho$	$\triangleq$	$\mathsf{set}\{\mathsf{evt}: e \mid e \in \mathsf{evts\text{-}of}(X) \cap F(X)\}$	fence events
$\llbracket \texttt{M} \rrbracket\ X\ \rho$	$\triangleq$	$\mathsf{set}\{\mathsf{evt}: e \mid e \in \mathsf{evts\text{-}of}(X) \cap (R(X) \cup W(X))\}$	memory events
$\llbracket \texttt{IW} \rrbracket\ X\ \rho$	$\triangleq$	$\mathsf{set}\{\mathsf{evt}: e \mid e \in \mathsf{init\text{-}of}(X)\}$	initial write events

where $\llbracket \texttt{IW} \rrbracket\ X\ \rho \subseteq \llbracket \texttt{W} \rrbracket\ X\ \rho$

Figure 10: Predefined sets and their semantics

The semantics of these predefined identifiers, given in Figure 11 is relatively straightforward again: `0` is the empty relation, `id` is the identity relation, `loc` the relation between events accessing the same variable, and `ext` the relation between events from different threads. It is the eponymous relation for `po` and `rf`.

Predefined relations over events —

predefined-relations	::=	`0`	empty relation
	|	`id`	identity
	|	`loc`	same location
	|	`ext`	external (different pids)
	|	`po`	program order
	|	`rf`	read-from
	|	`rmw`	read-modify-write

Semantics of predefined relations —

$$
\begin{aligned}
&\text{😸}[\![\texttt{0}]\!]\ X\ \rho \triangleq \mathsf{rel}\colon \emptyset \\
&\text{😸}[\![\texttt{id}]\!]\ X\ \rho \triangleq \mathsf{rel}\colon \mathbb{i}_{\mathsf{evts\text{-}of}(X)} \triangleq \mathsf{rel}\colon \{\langle e,\ e\rangle \mid e \in \mathsf{evts\text{-}of}(X)\} \\
&\text{😸}[\![\texttt{loc}]\!]\ X\ \rho \triangleq \mathsf{rel}\colon \{\langle e,\ e'\rangle \in \mathsf{evts\text{-}of}(X) \times \mathsf{evts\text{-}of}(X) \mid \mathsf{loc\text{-}of}(e) = \mathsf{loc\text{-}of}(e')\} \\
&\text{😸}[\![\texttt{ext}]\!]\ X\ \rho \triangleq \mathsf{rel}\colon \{\langle e,\ e'\rangle \in \mathsf{evts\text{-}of}(X) \times \mathsf{evts\text{-}of}(X) \mid \wedge \mathsf{pid\text{-}of}(e) \neq \mathsf{pid\text{-}of}(e')\} \\
&\text{😸}[\![\texttt{po}]\!]\ X\ \rho \triangleq \mathsf{rel}\colon \mathsf{po\text{-}of}(X) \\
&\text{😸}[\![\texttt{rf}]\!]\ X\ \rho \triangleq \mathsf{rel}\colon \mathsf{rf\text{-}of}(X) \\
&\text{😸}[\![\texttt{rmw}]\!]\ X\ \rho \triangleq \mathsf{let}\ RMW\ =\ \{\langle r,\ w\rangle \mid \exists e_b, e_e \in \mathsf{evts\text{-}of}(X)\ .\ \\
&\qquad \mathsf{kind\text{-}of}(e_b) = \texttt{beginrmw} \wedge \mathsf{kind\text{-}of}(e_e) = \texttt{endrmw} \wedge \\
&\qquad \langle e_b,\ r\rangle \in \mathsf{po\text{-}of}(X) \wedge (\nexists e \in \mathsf{evts\text{-}of}(X)\ .\ \langle e_b,\ e\rangle \in \mathsf{po\text{-}of}(X) \wedge \\
&\qquad \langle e,\ r\rangle \in \mathsf{po\text{-}of}(X)) \wedge \langle r,\ w\rangle \in \mathsf{po\text{-}of}(X) \wedge \langle w,\ e_e\rangle \in \mathsf{po\text{-}of}(X) \wedge \\
&\qquad (\nexists e \in \mathsf{evts\text{-}of}(X)\ .\ \langle w,\ e\rangle \in \mathsf{po\text{-}of}(X) \wedge \langle e,\ e_e\rangle \in \mathsf{po\text{-}of}(X)) \wedge \\
&\qquad (\nexists e \in \mathsf{evts\text{-}of}(X)\ .\ (\langle e_b,\ e\rangle \in \mathsf{po\text{-}of}(X) \wedge \langle e,\ e_e\rangle \in \mathsf{po\text{-}of}(X)) \wedge \\
&\qquad\quad \mathsf{kind\text{-}of}(e) \in \{\texttt{beginrmw}, \texttt{endrmw}\})\} \\
&\quad \mathsf{in} \\
&\quad\ \mathsf{rel}\colon RMW
\end{aligned}
$$

Figure 11: Predefined relations over events and their semantics

4.12 Primitives to manipulate sets and relations over events

appear in Figure 12. We have five primitives ($\mathsf{Primitives} \triangleq \{$`classes`, `fromto`, `linearisations`, `tag2events`, `tag2scope`$\}$). We will detail the primitives `tag2events` and `tag2scope` in Section 4.12.1. For the other three primitives:

- `classes` takes as argument an expression *expr*, which should evaluate as a relation r; if r is an equivalence relation, then we return the equivalence classes of r, otherwise we raise an error;
- `linearisations` takes as argument a pair of two expressions $expr_1$, which should evaluate to a set S, and $expr_2$, which should evaluate to a relation r; if this relation is acyclic, then we return all the possible linearisations (topological sorts) of r over S, otherwise we return the empty set.
- `fromto` takes as argument a expression *expr*, which should evaluate to a set S of tags, the events tagged with these tags should be fence events, and the result is the union of all their sets of pairs of events separated by these fence events.

Semantics of primitive functions —

$$
\begin{array}{l}
\text{😺}[\![\texttt{classes}\ expr]\!]\ X\ \rho \ \triangleq\ \mathsf{let\ type}\colon r\ =\ \text{😺}[\![expr]\!]\ X\ \rho\ \mathsf{in} \\
\quad \mathsf{if}\ (\mathsf{type}\ =\ \mathsf{rel}\ ) \wedge (\mathbb{i}_{\mathsf{fld}(r)} \subseteq r \wedge (r)^{-1} \subseteq r \wedge r \fatsemi r \subseteq r)\ \mathsf{then} \\
\quad\quad \mathsf{set}\colon \{\mathsf{set}\colon \{\mathsf{evt}\colon e \in \mathsf{fld}(r) \mid \langle e,\ e' \rangle \in r\} \mid e' \in \mathsf{fld}(r)\} \\
\quad \mathsf{else\ error} \\
\\
\text{😺}[\![\texttt{linearisations}\ expr]\!]\ X\ \rho \ \triangleq\ \mathsf{let\ type}\colon v\ =\ \text{😺}[\![expr]\!]\ X\ \rho\ \mathsf{in} \\
\quad \mathsf{if}\ \ (\mathsf{type}\colon v = \mathsf{tuple}\colon \langle \mathsf{set}\colon s,\ \mathsf{rel}\colon r \rangle) \wedge (\forall\, \mathsf{type}_v \colon v \in s\ .\ \mathsf{type}_v\ =\mathsf{evt}\ )\ \mathsf{then} \\
\quad\quad \mathsf{if}\ r^+ \cap \mathbb{i}_s = \emptyset\ \mathsf{then} \\
\quad\quad\quad \mathsf{set}\colon \{\mathsf{rel}\colon r' \in \wp(s \times s) \mid r \cap (s \times s) \subseteq r' \wedge r' \fatsemi r' \subseteq r' \wedge \\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (\forall e \neq e' \in s : \langle e,\ e' \rangle \in r' \vee \langle e',\ e \rangle \in r')\} \\
\quad\quad \mathsf{else\ set}\colon \emptyset \\
\quad \mathsf{else\ error} \\
\\
\text{😺}[\![\texttt{fromto}\ expr]\!]\ X\ \rho \ \triangleq\ \mathsf{let\ type}\colon S\ =\ \text{😺}[\![expr]\!]\ X\ \rho\ \mathsf{in} \\
\quad \mathsf{if}\ \ (\mathsf{type}\ =\mathsf{set}\ ) \wedge (\forall\, \mathsf{type}_e \colon e \in S\ .\ \mathsf{type}_e\ =\mathsf{evt}\ \wedge e \in F(X))\ \mathsf{then} \\
\quad\quad \mathsf{rel}\colon \bigcup_{e \in S} \mathsf{from\text{-}to\text{-}of}\,(e) \\
\quad \mathsf{else\ error}
\end{array}
$$

Figure 12: Semantics of primitives

4.12.1 Tags

Tags essentially are identifiers preceded by a quote `'` (to distinguish them form identifiers in bindings); and we gather them in sets, as shown in Figure 13. We first define an auxiliary over a tag *tag*:

- is-tag-declared checks that *tag* has been defined in an environment ρ, i.e. belongs to an enumeration *tag-set* in ρ.

Now, the value of a tag `'`*id* is the corresponding typed value if the tag has been declared in the environment ρ, or an error if not.

Finally, the primitive `tag2events` gathers all events bearing the tag *tag*, provided that the tag *tag* is declared in the environment ρ.

Declarations. One can declare enumerations of tags named by an identifier with the construct `enum`. One can use these tags to *annotate* LISA instructions, using the eponymous `instructions` construct.

Declarations (see Figure 14) augment the environment. The effect of an `enum` declaration is to extend the environment with the corresponding set of tags. In other terms, the semantics of `enum` *id* = [||] tag_1 ... ||tag_n is to augment the environment ρ with the set of typed tags $tag_1, \ldots tag_n$, under the name *id*.

The semantics of an `instruction` declaration is as follows: if there is a tag not in the environment, we raise an error; and if there is an event whose i^{th} tag is not in the i^{th} tag set, we raise an error.

4.12.2 Scopes.

Semantically, we distinguish *scope tags* *s-tag* from other tags, as shown in Figure 13. Thus for `enum` declarations, the identifier `scopes` is reserved to declare scopes. If an `enum scopes` declaration is provided then two functions `narrower` and `wider` must be declared on scope tags, to define the set of all possible scope hierarchies. Finally, the primitive `tag2scope` builds the relation between events coming from instructions that belong to the same scope (*viz.*, the scope instances of that scope) — relatively to a scope tree appearing in the original program. We give its semantics in Figure 13.

Matching over tags is as follows:

```
match expr with
  ||  tag_1 -> expr_1
  ||  ...
  ||  tag_n -> expr_n
  || ___ -> expr_d
end
```

The value of the `match` expression is computed as follow: first evaluate *expr* to some value v, which must be a tag `t`. Then v is compared with the tags tag_1, ..., tag_n, in that order. If some tag pattern tag_i equals `t`, then the value of the `match` is the value of the corresponding expression $expr_i$. Otherwise, the value of the `match` is the value of the default expression $expr_d$. As the default clause `___->` $expr_d$ is optional, the `match` construct may fail in error. We give the semantics of matching over tags in Figure 13.

Tags —

$$\begin{array}{rcll} tag & \in & Tag & \\ tag & ::= & \texttt{'}id & \\ s\text{-}tag & ::= & tag & \text{scope tags} \end{array}$$

Auxiliaries over tags $\texttt{'}id$ —

$$\mathsf{is\text{-}tag\text{-}declared}[\![\texttt{'}id]\!]\,\rho \triangleq \exists\ tag\text{-}set \in \mathsf{dom}(\rho)\ .\ \rho(tag\text{-}set) = \mathsf{enum}{:}\,T \wedge \mathsf{tag}{:}\,\texttt{'}id \in T$$

$$\mathsf{tag\text{-}set\text{-}of}[\![\texttt{'}id]\!]\,\rho \triangleq \{\texttt{'}id' \mid \exists\ tag\text{-}set \in \mathsf{dom}(\rho)\ .\ \rho(tag\text{-}set) = \mathsf{enum}{:}\,T \wedge \mathsf{tag}{:}\,\texttt{'}id \in T \wedge \mathsf{tag}{:}\,\texttt{'}id' \in T\}$$

Value of a tag $\texttt{'}id$ —

$$\text{😺}[\![\texttt{'}id\,]\!]\,X\,\rho \triangleq \mathsf{if}\ \mathsf{is\text{-}tag\text{-}declared}[\![\texttt{'}id\,]\!]\,\rho\ \mathsf{then}\ \mathsf{tag}{:}\,\texttt{'}id\ \mathsf{else}\ \mathsf{error}$$

Gathering all events bearing the same tag $\texttt{'}id$ —

$$\text{😺}[\![\texttt{tag2events('}id\texttt{)}]\!]\,X\,\rho \triangleq \mathsf{if}\ \mathsf{is\text{-}tag\text{-}declared}[\![\texttt{'}id]\!]\,\rho\ \mathsf{then}\ \{e \in \mathsf{evts\text{-}of}(X) \mid \texttt{'}id \in \mathsf{annot\text{-}of}(e)\}\ \mathsf{else}\ \mathsf{error}$$

Building the scope instance of level $s\text{-}tag$ —

$$\begin{array}{l} \text{😺}[\![\texttt{tag2scope(}s\text{-}tag\texttt{)}]\!]\,X\,\rho \triangleq \\ \quad \mathsf{let}\ \mathsf{type}_w{:}\,t = \text{😺}[\![\texttt{wider}\ s\text{-}tag]\!]\,X\,\rho \\ \quad \mathsf{and}\ \mathsf{type}_n{:}\,n = \text{😺}[\![\texttt{narrower}\ s\text{-}tag]\!]\,X\,\rho\ \mathsf{in} \\ \qquad \mathsf{if}\ \mathsf{type}_w = \mathsf{tag}{:} \wedge \mathsf{type}_n \in \{\mathsf{tag}\,,\mathsf{set}\} \wedge \mathsf{is\text{-}tag\text{-}declared}[\![s\text{-}tag]\!]\,\rho \wedge \forall\texttt{'}id \in \mathsf{tag\text{-}set\text{-}of}[\![s\text{-}tag]\!]\,\rho\ . \\ \qquad\quad t = \texttt{'}id \Rightarrow \mathsf{sr\text{-}of}(X)\ s\text{-}tag \subseteq \mathsf{sr\text{-}of}(X)\ \texttt{'}id \wedge \\ \qquad\quad \mathsf{type}_n = \mathsf{tag} \wedge n = \mathsf{tag}{:}\,\texttt{'}id \Rightarrow \mathsf{sr\text{-}of}(X)\ \texttt{'}id \subseteq \mathsf{sr\text{-}of}(X)\ s\text{-}tag \wedge \\ \qquad\quad \mathsf{type}_n = \mathsf{set} \wedge \mathsf{tag}{:}\,\texttt{'}id \in n \Rightarrow \mathsf{sr\text{-}of}(X)\ \texttt{'}id \subseteq \mathsf{sr\text{-}of}(X)\ s\text{-}tag \\ \qquad \mathsf{then} \\ \qquad\quad \mathsf{sr\text{-}of}(X)\ s\text{-}tag \\ \qquad \mathsf{else} \\ \qquad\quad \mathsf{error} \end{array}$$

Tag matching —

$$\begin{array}{l} \text{😺}[\![\texttt{match}\ expr_0\ \texttt{with}\ [\texttt{||}]\ tag_1\ \texttt{->}\ expr_1\ \ldots\ \texttt{||}\ tag_n\ \texttt{->}\ expr_n\ otherwise\ \texttt{end}]\!]\,X\,\rho \triangleq \\ \quad \mathsf{let}\ \mathsf{type}{:}\,t = \text{😺}[\![expr_0]\!]\,X\,\rho\ \mathsf{in} \\ \qquad \mathsf{if}\ \mathsf{type} \neq \mathsf{tag}\ \ \mathsf{then}\ \mathsf{error}\ \mathsf{else}\ \mathsf{let}\ i = \min\{k \mid t = tag_k\}\ \mathsf{in} \\ \qquad\quad \mathsf{if}\ \ i \neq +\infty\ \ \mathsf{then}\ \text{😺}[\![expr_i]\!]\,X\,\rho \\ \qquad\quad \mathsf{else}\ \ \mathsf{if}\ \ otherwise = \texttt{||\_\_\_}\ \texttt{->}\ \ expr_{n+1}\ \ \mathsf{then}\ \ \text{😺}[\![expr_{n+1}]\!]\,X\,\rho \\ \qquad\quad \mathsf{else}\ \ \mathsf{error} \end{array}$$

Figure 13: Tags and their semantics

Declarations —

annotation	$\in$	*Annotation*	
annotation	::=	*set*	(annotations are sets of tags)
decl	$\in$	*Decl*	
decl	::=	`enum` id = `[`[`||`]tag {`||` tag}*`]`	($when id \neq$ `scopes`)
	|	`enum scopes` = `[`[`||`]$s\text{-}tag$ {`||` $s\text{-}tag$}*`]`	
	|	`instructions` $ainstr$ `[`{$annotation$}$^+$`]`	

Semantics of `enum` —

$$\text{🐱}[\![\texttt{enum}\ id\ =\ [\texttt{||}]\ tag_1\ \dots\ \texttt{||}\,tag_\ell]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle \quad \triangleq \qquad \text{(including } id = \texttt{scopes}\text{)}$$
$$\langle j,\ f,\ \rho[id := \textsf{enum}\text{: }\{\textsf{tag}\text{: } tag_1, \dots, \textsf{tag}\text{: } tag_\ell\}],\ \omega\rangle$$

Semantics of `instructions` —

$$\text{🐱}[\![\texttt{instructions}\ aevt\ \texttt{[}annotation_1\texttt{,}\ \dots\texttt{,}annotation_n\texttt{]}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle \quad \triangleq$$
$$\textsf{let type}_i\text{: } T_i = \text{🐱}[\![annotation_i]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle, \quad i \in [1,n]\ \textsf{in}$$
$$\textsf{if}\ \ \forall i \in [1,n]\ .\ \textsf{type}_i = \textsf{set} \land \forall\, \textsf{type}'\text{: } t \in T_i\ .\ \textsf{type}' = \textsf{tag}\ \land$$
$$\forall e \in \textsf{evts-of}(X)\ .\ (\textsf{kind-of}(e) = aevt) \Rightarrow (\forall i \in [1,n]\ .\ \textsf{annot-of}(e)_i \in T_i)$$
$$\textsf{then}\ \ \langle j,\ f,\ \rho,\ \omega\rangle\ \textsf{else}\ \ \textsf{error}$$

Figure 14: Declarations

4.12.3 Functions

Functions are first class values, as reflected by the anonymous function construct `fun` *pat* `->` *expr*. We call the expression *expr* the *body* of the function. A function takes one argument *pat* only. When this argument is a tuple, it may be destructured by a tuple pattern `(`id_1`, ..., `id_n`)`.

The value of a function, given in Figure 15, is its *closure*; we write $\langle \boldsymbol{\lambda}\, id \cdot body,\ \rho' \rangle$ for a closure with parameter *id*, body *body* and declaration environment ρ' (this closure can also be understood as a triple $\langle id,\ body,\ \rho' \rangle$).

Function calls are written $expr_1\ expr_2$. That is, functions are of arity one and the application operator is left implicit. Notice that function application binds tighter than all binary operators (see Section 4.12.6) and looser that postfix operators (see Section 4.12.6). Furthermore the implicit application operator is left-associative.

The `cat` language has call-by-value semantics. That is, the effective parameter $expr_2$ is evaluated before being bound to the function formal parameter, see Figure 15.

4.12.4 Sets and tuples.

Sets are written as follows: `{` $expr_1$`,` $expr_2$`, ...,` $expr_n$`}` with n greater than 0. As events are not values, one cannot build a set of events using explicit set expressions. Sets are homogeneous, *i.e.* contain elements of the same type. We give their semantics in Figure 16. The value of `{}` is the empty set, and the value of `{`$expr_1$`, ..., `$expr_n$`}` is the set of values $\{v_1 \dots v_n\}$ where the v_i are the values of $expr_i$.

Matching over sets is as follows:

```
match expr with
  ||  {}  -> expr1
  ||  id1 ++ id2 -> expr2
end
```

We compute the value of the `match` as follow: first evaluate *expr* to some value v, which must be a set. If v is the empty set `{}`, then the value of the `match` is the value of $expr_1$. Otherwise, if v is a non-empty set S, then let e be some element in S and S' be the set S minus the element e. The value of the `match` is the value of $expr_2$ in a context where id_1 is bound to e and id_2 is bound to S'. We give the semantics of matching over sets in Figure 16, where the non-deterministic choice $e \in s$ is arbitrary (and unknown). So the semantics in Figure 16 returns one possible match (as opposed to all possibilities).

Tuples include the empty tuple `()`, and constructed tuples `(`$expr_1$`, `$expr_2$`, ..., `$expr_n$`)`, with n greater than 2. In other words there is no tuple of size one (which avoids ambiguity with grouping between parentheses).

We give their semantics in Figure 16. The value of `()` is the empty tuple $\langle\rangle$, and the value of $\langle expr_1,\ \dots,\ expr_n \rangle$ is the tuple of values $\langle v_1,\ \dots,\ v_n \rangle$ where the v_i are the values of $expr_i$; we do not impose that these values $\langle v_1,\ \dots,\ v_n \rangle$ have the same type.

Patterns —

$$\begin{array}{lcl} \mathit{pat} & \in & \mathit{Pat} \\ \mathit{pat} & ::= & \mathit{id} \mid \texttt{()} \mid \texttt{(}\mathit{id}\ \{\texttt{,}\ \mathit{id}\}^*\texttt{)} \end{array}$$

Anonymous functions —

$$\begin{array}{lcl} \mathit{function} & \in & \mathit{Function} \\ \mathit{function} & ::= & \texttt{fun}\ \mathit{pat}\ \texttt{->}\ \mathit{expr} \end{array}$$

Values of functions —

$$\begin{array}{l} \text{😸}[\![\texttt{fun}\ \mathit{id}\ \texttt{->}\ \mathit{expr}]\!]\ X\ \rho' \triangleq \mathsf{fun}\colon \langle \boldsymbol{\lambda}\, \mathit{id} \bullet \mathit{expr},\ \rho' \rangle \\ \text{😸}[\![\texttt{fun ()}\ \texttt{->}\ \mathit{expr}]\!]\ X\ \rho' \triangleq \mathsf{fun}\colon \langle \boldsymbol{\lambda}\, () \bullet \mathit{expr},\ \rho' \rangle \\ \text{😸}[\![\texttt{fun (}\mathit{id}_1\texttt{,}\ \ldots\texttt{,}\ \mathit{id}_\ell\texttt{)}\ \texttt{->}\ \mathit{expr}]\!]\ X\ \rho' \triangleq \qquad \ell \geqslant 1 \\ \quad \mathsf{fun}\colon \langle \boldsymbol{\lambda}\, \langle \mathit{id}_1,\ \ldots,\ \mathit{id}_\ell \rangle \bullet \mathit{expr},\ \rho' \rangle \end{array}$$

Semantics of function calls —

$$\begin{array}{l} \text{😸}[\![\ \mathit{expr}_1\ \ \mathit{expr}_2\ ]\!]\ X\ \rho \triangleq \qquad \text{(when } \mathit{expr}_1 \notin \mathsf{Primitives}\text{)} \\ \quad \mathsf{let}\ \mathsf{type}_i\colon v_i\ =\ \text{😸}[\![\ \mathit{expr}_i]\!]\ X\ \rho, \quad i \in [1,2]\ \ \mathsf{in} \\ \qquad \mathsf{match}\ \ \mathsf{type}_1\colon v_1\ \ \mathsf{with} \\ \qquad \mid \mathsf{fun}\colon \langle \boldsymbol{\lambda}\, \mathit{id} \bullet \mathit{expr},\ \rho' \rangle \twoheadrightarrow \text{😸}[\![\ \mathit{expr}]\!]\ X\ \rho'[\mathit{id} := \mathsf{type}_2\colon v_2] \\ \qquad \mid \mathsf{fun}\colon \langle \boldsymbol{\lambda}\, () \bullet \mathit{expr},\ \rho' \rangle \twoheadrightarrow \mathsf{if}\ \mathsf{type}_2\colon v_2 = \mathsf{tuple}\colon \langle\rangle\ \ \mathsf{then} \\ \qquad\qquad \text{😸}[\![\ \mathit{expr}]\!]\ X\ \rho' \\ \qquad\quad \mathsf{else}\ \ \mathsf{error} \\ \qquad \mid \mathsf{fun}\colon \langle \boldsymbol{\lambda}\, \langle \mathit{id}_1,\ \ldots,\ \mathit{id}_n \rangle \bullet \mathit{expr},\ \rho' \rangle \twoheadrightarrow \mathsf{if}\ \mathsf{type}_2\colon v_2 = \mathsf{tuple}\colon \langle e_1,\ \ldots,\ e_\ell \rangle \wedge \ell = n\ \mathsf{then} \\ \qquad\qquad \text{😸}[\![\ \mathit{expr}]\!]\ X\ \rho'[\mathit{id}_1 := e_1] \ldots [\mathit{id}_n := e_n] \\ \qquad\quad \mathsf{else}\ \ \mathsf{error} \\ \qquad \mid \_\_\_ \twoheadrightarrow \mathsf{error} \end{array}$$

Figure 15: Semantics of functions

Grouping is straightforward, as shown in Figure 16: the semantics of a parenthesised expression `(`*expr*`)` is the semantics of *expr*, idem for `begin` *expr* `end`.

4.12.5 Bindings

are of the form *pat* `=` *expr* or *id pat* `=` *expr*, where *id pat* `=` *expr* is syntactic sugar for *id* `=` `fun` *pat* `->` *expr*. As shown in Figure 17, bindings simply update the environment ρ. The bindings

Sets and tuples —

$$
\begin{array}{rcll}
\mathit{set} & \in & \mathit{Set} & \\
\mathit{set} & ::= & \texttt{\{\}} \mid \texttt{\{}\mathit{expr}\ \{\texttt{,}\ \mathit{expr}\}^*\texttt{\}} & \text{sets} \\
\mathit{tuple} & \in & \mathit{Tuple} & \\
\mathit{tuple} & ::= & \texttt{()} \mid \texttt{(}\mathit{expr}\texttt{,}\ \mathit{expr}\ \{\texttt{,}\ \mathit{expr}\}^*\texttt{)} & \text{tuples}
\end{array}
$$

Semantics of sets —

$$
\begin{array}{l}
\text{😸}[\![\texttt{\{\}}]\!]\ X\ \rho \quad \triangleq \quad \mathsf{set}\colon \emptyset \\
\text{😸}[\![\texttt{\{}\mathit{expr}_1\texttt{,}\ \ldots\texttt{,}\ \mathit{expr}_n\texttt{\}}]\!]\ X\ \rho \quad \triangleq \qquad\qquad n \geqslant 1 \\
\quad \mathsf{let}\ \mathsf{type}_i\colon v_i \ = \ \text{😸}[\![\mathit{expr}_i]\!]\ X\ \rho, \quad i \in [0, n]\ \ \mathsf{in} \\
\quad\quad \mathsf{let}\ S = \mathsf{set}\colon \{\mathsf{type}_1\colon v_1, \ldots, \mathsf{type}_n\colon v_n\}\ \ \mathsf{in} \\
\quad\quad\quad \mathsf{if}\ \ \mathsf{well\text{-}formed}(S)\ \ \mathsf{then}\ \ S\ \mathsf{else}\ \ \mathsf{error}
\end{array}
$$

Set matching —

$$
\begin{array}{l}
\text{😸}[\![\texttt{match}\ \mathit{expr}_0\ \texttt{with}\ \texttt{[||]}\ \texttt{\{\}}\ \texttt{->}\ \mathit{expr}_1\ \texttt{||}\ \mathit{id}_1\ \texttt{++}\ \mathit{id}_2\ \texttt{->}\ \mathit{expr}_2\ \texttt{end}]\!]\ X\ \rho \quad \triangleq \\
\quad \mathsf{let}\ \mathsf{type}\colon s = \text{😸}[\![\mathit{expr}_0]\!]\ X\ \rho\ \ \mathsf{in} \\
\quad\quad \mathsf{if}\ \mathsf{type} \neq \mathsf{set}\ \ \mathsf{then}\ \ \mathsf{error} \\
\quad\quad \mathsf{else}\ \ \mathsf{if}\ \ s = \emptyset\ \ \mathsf{then}\ \ \text{😸}[\![\mathit{expr}_1]\!]\ X\ \rho \\
\quad\quad \mathsf{else}\ \ \mathsf{let}\ e \in s\ \ \mathsf{in} \\
\quad\quad\quad \text{😸}[\![\mathit{expr}_2]\!]\ X\ \rho[\mathit{id}_2 := \mathsf{set}\colon (s \setminus \{e\})][\mathit{id}_1 := e]
\end{array}
$$

Semantics of tuples —

$$
\begin{array}{l}
\text{😸}[\![\texttt{()}]\!]\ X\ \rho \quad \triangleq \quad \mathsf{tuple}\colon \langle\rangle \\
\text{😸}[\![\texttt{(}\mathit{expr}_1\texttt{,}\ \ldots\texttt{,}\ \mathit{expr}_n\texttt{)}]\!]\ X\ \rho \quad \triangleq \qquad\qquad n \geqslant 2 \\
\quad \mathsf{tuple}\colon \langle \text{😸}[\![\mathit{expr}_1]\!]\ X\ \rho, \ \ldots, \ \text{😸}[\![\mathit{expr}_n]\!]\ X\ \rho\rangle
\end{array}
$$

Semantics of grouping —

$$
\begin{array}{l}
\text{😸}[\![\texttt{(}\mathit{expr}\texttt{)}]\!]\ X\ \rho \quad \triangleq \quad \text{😸}[\![\mathit{expr}]\!]\ X\ \rho \\
\text{😸}[\![\texttt{begin}\ \mathit{expr}\ \texttt{end}]\!]\ X\ \rho \quad \triangleq \quad \text{😸}[\![\mathit{expr}]\!]\ X\ \rho
\end{array}
$$

Figure 16: Semantics of sets, tuples and grouping

for *pat* `=` *expr* are as follows: if *pat* is `()`, then *expr* must evaluate to the empty tuple; if *pat* is *id* or `(`*id*`)`, then *id* is bound to the value of *expr*; if *pat* is a proper tuple pattern `(`id_1, ..., id_n`)` with n greater than 2, then *expr* must evaluate to a tuple value of size n `(` v_1, ..., v_n`)` and the names id_1, ..., id_n are bound to the values v_1, ..., v_n.

Bindings —

$$
\begin{array}{lcl}
\mathit{binding} & \in & \mathit{Binding} \\
\mathit{binding} & ::= & \mathit{pat}\ \texttt{=}\ \mathit{expr} \mid \mathit{id}\ \mathit{pat}\ \texttt{=}\ \mathit{expr} \\
 & & \text{where } \mathit{id}\ \mathit{pat}\ \texttt{=}\ \mathit{expr} \triangleq \mathit{id}\ \texttt{=}\ \texttt{fun}\ \mathit{pat}\ \texttt{->}\ \mathit{expr}
\end{array}
$$

Value of a binding —

$$
\begin{array}{l}
\text{😸}[\![\ \mathit{pat}\ \texttt{=}\ \mathit{expr}\ ]\!]\ X\ \rho \triangleq \mathsf{match}\ \mathit{pat}\ \mathsf{with} \\
\mid \texttt{()} \twoheadrightarrow \mathsf{tuple}\colon \langle\rangle \\
\mid \mathit{id} \mid \texttt{(}\mathit{id}\texttt{)} \twoheadrightarrow \rho[\mathit{id} := \text{😸}[\![\mathit{expr}]\!]\ X\ \rho] \\
\mid \texttt{(}\mathit{id}_1\texttt{,}\ \ldots\ \texttt{,}\mathit{id}_m\texttt{)} \twoheadrightarrow \\
\quad \mathsf{match}\ (\text{😸}[\![\mathit{expr}]\!]\ X\ \rho)\ \mathsf{with} \\
\qquad \mid \mathsf{tuple}\colon \langle e_1,\ \ldots,\ e_m\rangle \twoheadrightarrow \rho[\mathit{id}_1 := e_1] \ldots [\mathit{id}_m := e_m] \\
\qquad \mid \_\_\_ \twoheadrightarrow \mathsf{error} \\
\text{😸}[\![\ \mathit{id}\ \mathit{pat}\ \texttt{=}\ \mathit{expr}\ ]\!]\ X\ \rho \triangleq \text{😸}[\![\ \mathit{id}\ \texttt{=}\ \texttt{fun}\ \mathit{pat}\ \texttt{->}\ \mathit{expr}\ ]\!]\ X\ \rho
\end{array}
$$

Binding definitions —

$$
\begin{array}{l}
\text{😸}[\![\texttt{let}\ \mathit{binding}_1\ \texttt{and}\ \ldots\ \texttt{and}\ \mathit{binding}_n]\!]\ X\ \rho \triangleq \qquad n \geqslant 1 \\
\quad (\text{😸}[\![\mathit{binding}_n]\!]\ X(\ldots(\text{😸}[\![\mathit{binding}_1]\!]\ X\ \rho)\ldots))
\end{array}
$$

Recursive function binding:

$$
\begin{array}{l}
\text{😸}[\![\texttt{let rec}\ \mathit{id}_1\ \mathit{pat}_1\ \texttt{=}\ \mathit{expr}_1\ \texttt{and}\ \ldots\ \texttt{and}\ \mathit{id}_n\ \mathit{pat}_n\ \texttt{=}\ \mathit{expr}_n]\!]\ X\ \rho \triangleq \\
\quad \mathsf{let}\ \mathit{cl}^\infty = \prod_{j=1}^{n} \text{😸}[\![\texttt{fun}\ \mathit{pat}_i\ \texttt{->}\ \mathit{expr}_i]\!]\ X\ \rho[\mathit{id}_1 := \mathit{cl}_1^\infty] \ldots [\mathit{id}_n := \mathit{cl}_n^\infty]\ \mathsf{in} \\
\qquad \rho[\mathit{id}_1 := \mathit{cl}_1^\infty] \ldots [\mathit{id}_n := \mathit{cl}_n^\infty]
\end{array}
$$

Recursive set/relation binding:

$$
\begin{array}{l}
\text{😸}[\![\texttt{let rec}\ \mathit{id}_1\ \texttt{=}\ \mathit{expr}_1\ \texttt{and}\ \ldots\ \texttt{and}\ \mathit{id}_n\ \texttt{=}\ \mathit{expr}_n]\!]\ X\ \rho \triangleq \\
\quad \mathsf{let}\ F_i(\prod_{j=1}^{n} x_j) \triangleq \mathsf{let}\ \mathsf{type}_i\colon s_i = \text{😸}[\![\mathit{expr}_i]\!]\ X\ \rho[\prod_{j=1}^{n} \mathit{id}_j := \prod_{j=1}^{n} x_j]\ \mathsf{in} \\
\qquad\qquad \mathsf{if}\ (\mathsf{type}_i \in \{\mathsf{set}, \mathsf{rel}\})\ \mathsf{then}\ \mathsf{type}_i\colon s_i\ \mathsf{else}\ \mathsf{error}, \quad i \in [1,n] \\
\qquad \mathsf{in}\ \mathsf{let} \prod_{i=1}^{n} e_i = \mathbf{lfp}^{\subseteq}\ \boldsymbol{\lambda} \cdot \prod_{i=1}^{n} x_i \prod_{i=1}^{n} F_i(\prod_{i=1}^{n} x_i)\ \mathsf{in} \\
\qquad\quad \mathsf{if}\ (\exists i \in [1,n]\ .\ e_i = \mathsf{error})\ \mathsf{then}\ \mathsf{error}\ \mathsf{else}\ \rho[\mathit{id}_1 := e_1] \ldots [\mathit{id}_n := e_n]
\end{array}
$$

Binding expressions —

$$
\begin{array}{l}
\text{😸}[\![\texttt{let}\ \mathit{binding}_1\ \texttt{and}\ \ldots\ \texttt{and}\ \mathit{binding}_n\ \texttt{in}\ \mathit{expr}]\!]\ X\ \rho \triangleq \qquad n > 1 \\
\quad \text{😸}[\![\mathit{expr}]\!]\ X\ (\text{😸}[\![\texttt{let}\ \mathit{binding}_1\ \texttt{and}\ \ldots\ \texttt{and}\ \mathit{binding}_n]\!]\ X\ \rho)
\end{array}
$$

$$
\begin{array}{l}
\text{😸}[\![\texttt{let rec}\ \mathit{id}_1\ \texttt{=}\ \mathit{expr}_1\ \texttt{and}\ \ldots\ \texttt{and}\ \mathit{id}_n\ \texttt{=}\ \mathit{expr}_n\ \texttt{in}\ \mathit{expr}]\!]\ X\ \rho \triangleq \\
\quad \text{😸}[\![\mathit{expr}]\!]\ X\ (\text{😸}[\![\texttt{let rec}\ \mathit{id}_1\ \texttt{=}\ \mathit{expr}_1\ \texttt{and}\ \ldots\ \texttt{and}\ \mathit{id}_n\ \texttt{=}\ \mathit{expr}_n]\!]\ X\ \rho)
\end{array}
$$

Figure 17: Bindings and their semantics

Binding definitions happen through the `let` and `let rec` constructs, which bind value names for the rest of a specification evaluation. We give the semantics of binding definitions in Figure 17.

First, the construct `let` $binding_1$ `and` ... `and` $binding_n$, that is, `let` pat_1 `=` $expr_1$ `and` ... `and` pat_n `=` $expr_n$, evaluates $expr_1$, ..., $expr_n$, and binds the names in the patterns pat_1, ..., pat_n to the resulting values.

Second, for recursive function bindings `let rec` id_1 pat_1 `=` $expr_1$ `and` ... `and` id_e pat_n `=` $expr_n$, we follow Milner and Tofte (1991) where the proof of existence and unicity of the infinite closure cl^{∞} is based on Aczel (1988).

Third, for recursive set or relation bindings `let rec` id_1 `=` $expr_1$ `and` ... `and` id_n `=` $expr_n$, we compute the least solution of the equations id_1 `=` $expr_1$, ..., id_n `=` $expr_n$ on sets or relations using inclusion for ordering. These fixpoint equations must satisfy the $\subseteq$-monotony (increasingness) hypotheses of Tarski (1955) fixpoint theorem or else the result is undefined.

The recursive bindings may be mutually recursive. We suppose these recursive definitions well-formed, *i.e.* terminating. The result of ill-formed definitions is undefined (*i.e.* an implementation might return an error or never terminate).

Binding expressions happen through the construct `let [rec]` *bindings* `in` *expr*, which locally binds the names defined by *bindings* to evaluate *expr*. Both non-recursive and recursive bindings are allowed.

4.12.6 Operators on sets and relations

Operators can be unary or binary. We list them in Figure 18, and detail their semantics below.

Operators on sets and relations —

op	∈	*Operators*	
op	::=	*expr* `++` *expr*	set addition
	|	*expr* `*` *expr*	cartesian product
	|	*expr* `|` *expr*	union
	|	*expr* `&` *expr*	intersection
	|	*expr* `;` *expr*	relation composition
	|	*expr* `+`	transitive closure
	|	*expr* `?`	reflexive closure
	|	*expr* `*`	reflexive and transitive closure
	|	*expr*`^-1`	inverse
	|	*expr* `\` *expr*	subtraction
	|	`~` *expr*	complement

Figure 18: List of both unary and binary operators

Unary operators. Given an expression denoting a relation, we can build its identity closure with the operator `?`, its reflexive-transitive closure with the operator `*`, its transitive closure with `+`, its complement with `~` and its inverse with `^-1`. These operators are postfix, and are defined on relations only, except for the complement, which can apply to sets of events or tags as well. Figure 19 gathers them all. We recall that the value of identifier `0` is the empty relation.

postfixed op	cat-operation-of$[\![\mathsf{op}]\!]\ X\ r$	relation operation
`*`	$\mathbf{lfp}^{\subseteq}\ \boldsymbol{\lambda}\bullet x \mathbin{\mathbb{i}} _{\mathsf{evts\text{-}of}(X)}\ \cup\ x \mathbin{;} r$	reflexive closure
`+`	$\mathbf{lfp}^{\subseteq}\ \boldsymbol{\lambda}\bullet xr\ \cup\ x \mathbin{;} r$	irreflexive closure
`?`	$\mathbb{i}_{\mathsf{evts\text{-}of}(X)} \cup r$	identity closure
`^-1`	$\{\langle e',\ e\rangle \mid \langle e,\ e'\rangle \in r\}$	inverse

Unary operators on relations —

$$
\begin{array}{l}
\text{😺}[\![expr\ \mathsf{op}]\!]\ X\ \rho\ \triangleq \\
\quad \mathsf{let\ type:}\ r\ =\ \text{😺}[\![expr]\!]\ X\ \rho\ \mathsf{in} \\
\quad\quad \mathsf{if\ type}\ =\ \mathsf{rel\ then} \\
\quad\quad\quad \mathsf{rel:\ cat\text{-}operation\text{-}of}[\![\mathsf{op}]\!]\ X\ r \\
\quad\quad \mathsf{else\ \ error}
\end{array}
$$

Complement of a set or relation —

$$
\begin{array}{l}
\text{😺}[\![\mathsf{\sim}\ expr]\!]\ X\ \rho\ \triangleq \\
\quad \mathsf{let\ type:}\ s\ =\ \text{😺}[\![expr]\!]\ X\ \rho\ \mathsf{in} \\
\quad\quad \mathsf{if\ \ type}\ =\mathsf{set}\ \wedge\ \forall\,\mathsf{type:}\ e \in s\ .\ \mathsf{type}\ =\mathsf{evt\ \ then} \\
\quad\quad\quad \mathsf{set:}\ (\{\mathsf{evt:}\ e\mid e \in \mathsf{evts\text{-}of}(X)\} \setminus s) \\
\quad\quad \mathsf{else\ if\ \ type}\ =\mathsf{set}\ \wedge\ \exists\ id \in \mathsf{dom}(\rho)\ .\ \rho(id) = \mathsf{enum:}\,T\ \wedge\ s \subseteq T\ \ \mathsf{then} \\
\quad\quad\quad \mathsf{set:}\ (T \setminus s) \\
\quad\quad \mathsf{else\ \ if\ \ type}\ =\mathsf{rel\ \ then} \\
\quad\quad\quad \mathsf{rel:}\ ((\mathsf{evts\text{-}of}(X) \times \mathsf{evts\text{-}of}(X)) \setminus s) \\
\quad\quad \mathsf{else\ \ error}
\end{array}
$$

Figure 19: Semantics of unary operators

Binary operators. We can build the sequence (or composition in the sense of Figure 6) of two expressions with the operator `;`, defined on relations only. We can add an element to a set: the addition operator $expr_1$ `++` $expr_2$ operates on sets. The value of $expr_2$ must be a set of values S and the operator returns the set S augmented with the value of $expr_1$. We can build

a new relation out of the cartesian product of two sets of events, with the infix operator `*`.

We can build the union, intersection, and difference of sets and relations as summarised in Figure 20. The semantics of $expr_1$ `op` $expr_2$ is the operator `op` applied to the sets (resp. relations) s_1 and s_2, *viz.*, the values of $expr_1$ and $expr_2$.

Sequence of relations —

$$\text{😺}[\![expr_1 \ ; \ expr_2]\!]\ X\ \rho \ \triangleq$$
$$\quad \mathsf{let}\ \mathsf{type}_i\colon r_i \ = \ \text{😺}[\![expr_i]\!]\ X\ \rho, \quad i \in [1,2] \ \ \mathsf{in}$$
$$\quad\quad \mathsf{if}\ \mathsf{type}_1 \ = \mathsf{type}_2 \ = \mathsf{rel}\ \mathsf{then}$$
$$\quad\quad\quad \mathsf{rel}\colon r_1 \fatsemi r_2$$
$$\quad\quad \mathsf{else}\ \ \mathsf{error}$$

Adding an element to a set —

$$\text{😺}[\![expr_1 \ \texttt{++}\ expr_2]\!]\ X\ \rho \ \triangleq$$
$$\quad \mathsf{let}\ \mathsf{type}_i\colon v_i \ = \ \text{😺}[\![expr_i]\!]\ X\ \rho, \quad i \in [1,2] \ \ \mathsf{in}$$
$$\quad\quad \mathsf{let}\ S = \mathsf{set}\colon (\{\mathsf{type}_1\colon v_1\} \cup v_2) \ \ \mathsf{in}$$
$$\quad\quad\quad \mathsf{if}\ \ \mathsf{well\text{-}formed}(S)\ \ \mathsf{then}\ \ S\ \ \mathsf{else}\ \ \mathsf{error}$$

Cartesian product of two sets of events —

$$\text{😺}[\![expr_1 \ \texttt{*}\ expr_2]\!]\ X\ \rho \ \triangleq$$
$$\quad \mathsf{let}\ \mathsf{type}_i\colon s_i \ = \ \text{😺}[\![expr_i]\!]\ X\ \rho, \quad i \in [1,2] \ \ \mathsf{in}$$
$$\quad\quad \mathsf{if}\ \ \mathsf{type}_1 \ = \mathsf{type}_2 \ = \mathsf{set} \wedge \forall\, \mathsf{type}\colon v \in s_1 \cup s_2 \ .\ \mathsf{type} \ = \mathsf{evt} \ \ \mathsf{then}$$
$$\quad\quad\quad \mathsf{rel}\colon s_1 \times s_2$$
$$\quad\quad \mathsf{else}\ \ \mathsf{error}$$

op	cat-operation-of $[\![\mathsf{op}]\!]$	
`|`	$\cup$	union
`&`	$\cap$	intersection
`\`	$\setminus$	difference

Binary operators relative to both sets and relations —

$$\text{😺}[\![expr_1 \ \mathsf{op}\ expr_2]\!]\ X\ \rho \ \triangleq$$
$$\quad \mathsf{let}\ \mathsf{type}_i\colon s_i \ = \ \text{😺}[\![expr_i]\!]\ X\ \rho, \quad i \in [1,2] \ \ \mathsf{in}$$
$$\quad\quad \mathsf{if}\ \ \mathsf{type}_1 \ = \mathsf{type}_2 \ = \mathsf{set} \wedge \mathsf{well\text{-}formed}(s_1 \cup s_2) \ \ \mathsf{then}$$
$$\quad\quad\quad \mathsf{set}\colon s_1\ (\mathsf{cat\text{-}operation\text{-}of}\,[\![\mathsf{op}]\!])\ s_2$$
$$\quad\quad \mathsf{else}\ \ \mathsf{if}\ \mathsf{type}_1 \ = \mathsf{type}_2 \ = \mathsf{rel}\ \mathsf{then}$$
$$\quad\quad\quad \mathsf{rel}\colon s_1\ (\mathsf{cat\text{-}operation\text{-}of}\,[\![\mathsf{op}]\!])\ s_2$$
$$\quad\quad \mathsf{else}\ \ \mathsf{error}$$

Figure 20: Semantics of binary operators

4.13 Constraints

Constraints (see Figure 21) can be checks, procedure calls, or iteration over sets.

Constraints —

constraint	∈	*Constraint*
constraint	::=	
	|	*flagoption check expr* `as` *id*
	|	`call` *id expr*
	|	`forall` *id* `in` *expr* `do` *statements* `end`
flagoption	::=	[`flag`]

Figure 21: Constraints

4.13.1 Checks

happen through the construct *check expr*, which evaluates *expr* and applies the check *check*. There are six checks: acyclicity (keyword `acyclic`), irreflexivity (keyword `irreflexive`) and emptyness (keyword `empty`); and their negations. If the check succeeds, the candidate execution is allowed so far. Otherwise, the candidate execution is forbidden.

A check can optionally be named *id*, using the keyword `as`. A check can also be flagged, by prefixing it with the `flag` keyword. Flagged checks must be named with the `as` construct. Failed flagged checks do not stop evaluation; instead failed flagged checks are recorded under their name in the component f of the semantics of the `cat` specification, for example to handle flagged candidate executions later within our `herd7` tool. We give the semantics of checks in Figure 22.

Flagged checks are useful for specifications with statements that impact the semantics of an entire program, *e.g.*, in the case of specifications phrased in terms of data races, such as C++ Batty et al. (2016) or HSA HSA Foundation (2015).

4.13.2 Procedures

Procedures have no result and cannot be recursive: the body of a procedure is a list of statements and the procedure will be invoked to apply the constraints within its body. Intended usage of procedures is to define constraints that are checked later. Figure 23 gives the semantics of procedures: just like functions, procedure declarations simply augment the environment ρ with their closure.

Procedure calls are written `call` *id expr*, where *id* is the name of a previously defined procedure. The bindings performed during the call of a procedure are discarded when the procedure returns, all other effects (*e.g.* checks or flags, see Section 4.13.1) performed are retained. Procedures cannot be recursive.

Checks —

$$\begin{array}{rcl} \mathit{check} & ::= & \mathit{checkname} \mid \sim \mathit{checkname} \\ \mathit{checkname} & ::= & \texttt{acyclic} \mid \texttt{irreflexive} \mid \texttt{empty} \end{array}$$

$$\begin{array}{l} \mathsf{check\text{-}condition\text{-}on\text{-}relation}[\![\mathit{check}]\!]\ R \triangleq \\ \quad \mathsf{match}\ \mathit{check}\ \mathsf{with} \\ \quad \mid\ \texttt{acyclic} \to \mathsf{check\text{-}condition\text{-}on\text{-}relation}[\![\texttt{irreflexive}]\!]\ R^+ \\ \quad \mid\ \texttt{irreflexive} \to (\forall e \in \mathsf{fld}(R)\ .\ \langle e,\ e\rangle \notin R) \\ \quad \mid\ \texttt{empty} \to R = \emptyset \\ \quad \mid \sim \mathit{check}' \to \neg(\mathsf{check\text{-}condition\text{-}on\text{-}relation}[\![\mathit{check}']\!]\ R) \end{array}$$

$$\begin{array}{l} \mathsf{cat\text{-}check\text{-}relation}[\![\mathit{flagoption}\ \mathit{check}\ R\ \texttt{as}\ \mathit{id}]\!]\ \langle j,\ f,\ \rho,\ \omega\rangle \triangleq \\ \quad \mathsf{if}\ (\mathsf{check\text{-}condition\text{-}on\text{-}relation}[\![\mathit{check}]\!]\ R)\ \mathsf{then} \\ \quad\quad \langle j,\ f,\ \rho,\ \omega\rangle \\ \quad \mathsf{else\ if}\ \mathit{flagoption} = \texttt{flag}\ \mathsf{then} \\ \quad\quad \langle j,\ f \cup \{\mathit{id}\},\ \rho,\ \omega\rangle \\ \quad \mathsf{else}\ \langle \texttt{forbidden},\ f,\ \rho,\ \omega\rangle \end{array}$$

Value of checks —

$$\begin{array}{l} \text{😺}[\![\mathit{flagoption}\ \mathit{check}\ \mathit{expr}\ \texttt{as}\ \mathit{id}]\!]\ X\ \langle j,\ f,\ ,\ \rho\rangle \triangleq \\ \quad \mathsf{let\ type:}\ R = \text{😺}[\![\mathit{expr}]\!]\ X\ \rho\ \mathsf{in} \\ \quad\quad \mathsf{if}\ (\mathsf{type} = \mathsf{rel}) \vee (\mathsf{type} = \mathsf{set} \wedge \mathit{check} \in \{\texttt{empty}, \sim \texttt{empty}\})\ \mathsf{then} \\ \quad\quad\quad \mathsf{cat\text{-}check\text{-}relation}[\![\mathit{flagoption}\ \mathit{check}\ R\ \texttt{as}\ \mathit{id}]\!]\ \langle j,\ f,\ \rho,\ \omega\rangle \\ \quad\quad \mathsf{else\ error} \end{array}$$

Figure 22: Checks and their semantics

Procedures —

$$
\begin{array}{lcl}
\mathit{procedure} & \in & \mathit{Procedure} \\
\mathit{procedure} & ::= & \texttt{procedure}\ \mathit{id}\ \mathit{pat}\ =\ \{\mathit{statement}\}^+\ \texttt{end}
\end{array}
$$

Declarations of procedures —

$$
\begin{array}{l}
\text{😺}[\![\texttt{procedure}\ \mathit{id}_1\ \mathit{id}_2\ =\ \{\mathit{statement}\}^+\ \texttt{end}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \triangleq \\
\quad \langle j,\ f,\ \rho[\mathit{id}_1 := \mathsf{proc}\colon \langle \boldsymbol{\lambda}\ \mathit{id}_2 \bullet \{\mathit{statement}\}^+,\ \rho\rangle],\ \omega\rangle \\
\text{😺}[\![\texttt{procedure}\ \mathit{id}\ \texttt{()}\ =\ \{\mathit{statement}\}^+\ \texttt{end}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \triangleq \\
\quad \langle j,\ f,\ \rho[\mathit{id} := \mathsf{proc}\colon \langle \boldsymbol{\lambda}\ \texttt{()} \bullet \{\mathit{statement}\}^+,\ \rho\rangle],\ \omega\rangle \\
\text{😺}[\![\texttt{procedure}\ \mathit{id}\ \texttt{(}\mathit{id}_1\texttt{, }\ldots\texttt{, }\mathit{id}_\ell\texttt{)}\ =\ \{\mathit{statement}\}^+\ \texttt{end}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \triangleq \\
\quad \langle j,\ f,\ \rho[\mathit{id} := \mathsf{proc}\colon \langle \boldsymbol{\lambda}\ \langle \mathit{id}_1,\ \ldots,\ \mathit{id}_\ell\rangle \bullet \{\mathit{statement}\}^+,\ \rho\rangle],\ \omega\rangle
\end{array}
$$

Procedure calls —

$$
\begin{array}{l}
\text{😺}[\![\texttt{call}\ \mathit{id}\ \mathit{expr}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \triangleq \\
\quad \mathsf{if}\ \ \mathit{id} \notin \mathsf{dom}(\rho)\ \ \mathsf{then}\ \ \mathsf{error}\ \ \mathsf{else} \\
\quad \mathsf{match}\ \ \rho(\mathit{id})\ \ \mathsf{with} \\
\quad\quad \mid \langle \boldsymbol{\lambda}\ \mathit{id}_1 \bullet \{\mathit{statement}\}^+,\ \rho'\rangle\ \rightarrowtriangle \\
\quad\quad\quad \text{😺}[\![\{\mathit{statement}\}^+]\!]\ X\ \rho'[\mathit{id}_1 := (\text{😺}[\![\mathit{expr}]\!]\ X\ \rho)] \\
\quad\quad \mid \langle \boldsymbol{\lambda}\ \texttt{()} \bullet \{\mathit{statement}\}^+,\ \rho'\rangle\ \rightarrowtriangle \\
\quad\quad\quad \mathsf{let\ type}\colon v\ =\ \text{😺}[\![\mathit{expr}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \ \mathsf{in} \\
\quad\quad\quad\quad \mathsf{if\ type}\colon v = \mathsf{tuple}\colon \langle\rangle\ \ \mathsf{then} \\
\quad\quad\quad\quad\quad \text{😺}[\![\{\mathit{statement}\}^+]\!]\ X\ \langle j,\ f,\ \rho',\ \omega\rangle \\
\quad\quad\quad\quad \mathsf{else\ \ error} \\
\quad\quad \mid \langle \boldsymbol{\lambda}\ \langle \mathit{id}_1,\ \ldots,\ \mathit{id}_n\rangle \bullet \{\mathit{statement}\}^+,\ \rho'\rangle\ \rightarrowtriangle \\
\quad\quad\quad \mathsf{let\ type}\colon v\ =\ \text{😺}[\![\mathit{expr}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \ \mathsf{in} \\
\quad\quad\quad\quad \mathsf{if\ type}\colon v = \mathsf{tuple}\colon \langle e_1,\ \ldots,\ e_\ell\rangle \wedge (n = \ell)\ \mathsf{then} \\
\quad\quad\quad\quad\quad \text{😺}[\![\{\mathit{statement}\}^+]\!]\ X\ \langle j,\ f,\ \rho'[\mathit{id}_1 := e_1] \ldots [\mathit{id}_n := e_n],\ \omega\rangle \\
\quad\quad\quad\quad \mathsf{else\ \ error} \\
\quad\quad \mid \_\_\_\ \rightarrowtriangle\ \mathsf{error}
\end{array}
$$

Figure 23: Semantics of procedures

4.13.3 Iteration over sets

We can iterate checks over sets with the `forall` construct:

```
forall id in expr do
  statements
end
```

The expression *expr* must evaluate to a set S. Then, the list of statements *statements* is evaluated for all bindings of the name *id* to some element e of S. In practice, as failed checks forbid the candidate execution, this amounts to checking the conjunction of the checks within *statements* for all the elements of S. Similarly to procedure calls, the bindings performed during an iteration are discarded when iteration ends, all other cumulated effects (e.g. checks) being retained. We give the semantics of iteration in Figure 24; the iteration is non-deterministic since the choice e in S is arbitrary and unknown.

$$
\begin{array}{l}
\textsf{cat-iterate}\ id\ S\ \{statement\}^+\ X\ \langle j,\ f,\ \rho,\ \omega\rangle \quad \triangleq \\
\quad \textsf{if}\ \ S = \emptyset\ \ \textsf{then}\ \langle j,\ f,\ \rho,\ \omega\rangle \\
\quad \textsf{else}\ \ \textsf{let}\ e \in S\ \ \textsf{in} \\
\qquad \textsf{let}\ r = \text{😺}[\![\{statement\}^+]\!]\ X\ \langle j,\ f,\ \rho[id := e],\ \omega\rangle\ \ \textsf{in} \\
\qquad\quad \textsf{if}\ \ r = \textsf{error}\ \ \textsf{then}\ \ \textsf{error}\ \textsf{else} \\
\qquad\quad \textsf{let}\ \langle j',\ f',\ \rho',\ \omega\rangle = \ r\ \ \textsf{in} \\
\qquad\qquad \textsf{if}\ \ j' = \texttt{allowed}\ \ \textsf{then} \\
\qquad\qquad\quad \textsf{cat-iterate}\ id\ (S \setminus \{e\})\ \{statement\}^+\ X\ \langle j',\ f',\ \rho',\ \omega\rangle \\
\qquad\qquad \textsf{else}\ \ \langle j',\ f',\ \rho',\ \omega\rangle
\end{array}
$$

Iteration over sets —

$$
\begin{array}{l}
\text{😺}[\![\texttt{forall}\ id\ \texttt{in}\ expr\ \texttt{do}\ \{statement\}^+\ \texttt{end}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle \quad \triangleq \\
\quad \textsf{let type:}\ S\ =\ \text{😺}[\![expr]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle\ \ \textsf{in} \\
\qquad \textsf{if}\ \ \textsf{type} \neq \textsf{set}\ \ \textsf{then}\ \ \textsf{error} \\
\qquad \textsf{else}\ \ \textsf{cat-iterate}\ id\ S\ \{statement\}^+\ X\ \langle j,\ f,\ \rho,\ \omega\rangle
\end{array}
$$

Figure 24: Semantics of iteration

4.14 Requirements

Requirements are the constitutive blocks of a `cat` specification. Their evaluation goes as given in Figure 25. Requirements can be statements, or `with` bindings.

4.14.1 Statements

and their semantics have been presented in the sections above. At the level of requirements, we evaluate lists of statements, gather their evaluation $\langle j,\ f,\ \rho,\ \omega \rangle$, and the final verdict forgets the environment ρ to built the result (see Figure 2).

4.14.2 Candidate extension via `with` binding

happens through the construct `with` *id* `from` *expr*. This construct extends the current environment by one binding (see Figure 25). The grammar only allows `with` bindings to occur at the top-level. The expression *expr* is evaluated to a set S. Then the remainder of the specification is evaluated for each choice of element e in S in an environment extended by a binding of the name *id* to e.

The final verdict at top level in Figure 2 gets rid of the environment and returns the communication relations obtained by finding the value of the communication relation identifiers *id* in the environment.

Requirements —

$$
\begin{array}{lcl}
\mathit{requirements} & \in & \mathit{requirements} \\
\mathit{requirements} & ::= & \mathit{statement} \\
& \mid & \mathit{statement}\ \ \mathit{requirements} \\
& \mid & \texttt{with}\ \mathit{id}\ \texttt{from}\ \mathit{expr}\ \ \mathit{requirements}
\end{array}
$$

$$
\text{😺}[\![\mathit{requirements}]\!] \quad \in \quad \mathit{Candidate} \to \mathcal{R} \to \wp(\mathcal{R})
$$

$$
\text{😺}[\![\mathit{requirements}]\!]\ X\ \mathsf{error} \quad \triangleq \quad \mathsf{error}
$$

$$
\text{😺}[\![\mathit{requirements}]\!]\ X\ \langle j,\ f,\ \rho,\ \omega\rangle \quad \triangleq \quad \mathsf{if}\ \ j = \texttt{forbidden}\ \ \mathsf{then}\ \langle j,\ f,\ \rho,\ \omega\rangle
$$

$$
\begin{array}{l}
\mathsf{else\ match}\ \ \mathit{requirements}\ \ \mathsf{with} \\
\mid \mathit{statement}\ \ \rightarrowtriangle\ \ \{\text{😺}[\![\mathit{statement}]\!]\ X\ \langle \texttt{allowed},\ f,\ \rho,\ \omega\rangle\} \\
\mid \mathit{statement}\ \ \mathit{requirements}'\ \ \rightarrowtriangle \\
\quad \mathsf{let}\ r = \text{😺}[\![\mathit{statement}]\!]\ X\ \langle \texttt{allowed},\ f,\ \rho,\ \omega\rangle\ \ \mathsf{in} \\
\qquad \mathsf{if}\ \ (r = \mathsf{error})\ \ \mathsf{then}\ \ \mathsf{error} \\
\qquad \mathsf{else}\ \ \mathsf{let}\ \langle j,\ f',\ \rho',\ \omega'\rangle = r\ \ \mathsf{in} \\
\qquad\quad \mathsf{if}\ \ (j = \texttt{allowed})\ \ \mathsf{then} \\
\qquad\qquad \text{😺}[\![\mathit{requirements}']\!]\ X\ \langle \texttt{allowed},\ f \cup f',\ \rho',\ \omega'\rangle \\
\qquad\quad \mathsf{else}\ \ \{r\} \\
\mid \texttt{with}\ \mathit{id}\ \texttt{from}\ \mathit{expr}\ \ \mathit{requirements}'\ \ \rightarrowtriangle \\
\quad \mathsf{let\ type}{:}\ S\ =\ \text{😺}[\![\mathit{expr}]\!]\ X\ \rho\ \ \mathsf{in} \\
\qquad \mathsf{if}\ \ (\mathsf{type} \neq \mathsf{set})\ \ \mathsf{then}\ \ \mathsf{error} \\
\qquad \mathsf{else}\ \ \bigcup_{e \in S} \text{😺}[\![\mathit{requirements}']\!]\ X\ \langle \texttt{allowed},\ f,\ \rho[\mathit{id} := e],\ \omega \cup \{\mathit{id}\}\rangle
\end{array}
$$

Figure 25: Semantics of requirements

5 cat library functions

For reference, we give the code of three library functions that operate over relations and sets (`fold`, `map` and `cross`).

5.1 Definition of `fold`

Given a function f, a set $S = \{e_1, e_2, \ldots, e_n\}$ and an element y, the call `fold f (S, y)` returns the value $f(e_{i_1}, f(e_{i_2}, \ldots f(e_{i_n}, y)))$, where $i_1, i_2, \ldots, i_n$ is a permutation of $1, 2, \ldots, n$:

```
let fold f =
  let rec fold_rec (es,y) = match es with
  || {} -> y
  || e ++ es -> fold_rec (es, f(e,y))
  end in
  fold_rec
```

5.2 Definition of `map`

Given a function f and a set $S = \{e_1, \ldots, e_n\}$, the call `map f S` returns the set $\{f(e_1), \ldots, f(e_n)\}$. This function can be implemented directly or more concisely by calling the `fold` function:

```
let map f = fun es -> fold (fun (e,y) -> f e ++ y) (es,{})
```

5.3 Definition of `cross`

The function `cross` takes a set of sets $S = \{S_1, S_2, \ldots, S_n\}$ as argument and returns all possible unions built by picking elements from each of the S_i:

$$\{\, e_1 \cup e_2 \cup \cdots \cup e_n \mid e_1 \in S_1, e_2 \in S_2, \ldots, e_n \in S_n \,\}$$

Note that if S is empty, then `cross` should return one relation exactly: the empty relation $\emptyset$, *i.e.*, the neutral element of the union operator. This choice for `cross (∅) = ∅` is natural when we define `cross` inductively:

$$\texttt{cross}(S_1 \texttt{ ++ } S) = \bigcup_{e_1 \in S_1, t \in \texttt{cross}(S)} \{e_1 \cup t\}$$

In this specification, we simply build `cross (`S_1 `++` S`)` by building the set of all unions of one relation e_1 picked in S_1 and of one relation t picked in `cross`(S). From this inductive specification for `cross`, one writes the following concise code:

```
let rec cross S = match S with
  || {} -> { 0 }
  || S1 ++ S ->
      let yss = cross S in
      fold
        (fun (e1,r) -> map (fun t -> e1 | t) yss | r)
        (S1,{})
   end
```

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