

# Tutorial on Word-Level Model Checking

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**Abstract**—In SMT bit-vectors and thus word-level reasoning is common and widely used in industry. However, it took until 2019 that the hardware model checking competition started to use word-level benchmarks. Reasoning on the word-level opens up many possibilities for simplification and more powerful reasoning. In SMT we do see advantages due to operating on the word-level, even though, ultimately, bit-blasting and thus transforming the word-level problem into SAT is still the dominant and most important technique. For word-level model checking the situation is different. As the hardware model checking competition in 2019 has shown bit-level solvers are far superior (after bit-blasting the model through an SMT solver though). On the other hand word-level model checking shines for problems with memory modeled with arrays. In this tutorial we revisit the problem of word level model checking, also from a theoretical perspective, give an overview on classical and more recent approaches for word-level model checking and then discuss challenges and future work. The tutorial covered material from the following papers.

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# World-Level Modelling

- bit-precise reasoning: bit-vector as basic modelling element
- thus in essence SMT theory QF\_BV of bit-vectors [SMTLIB]
  - sorts: bit  $\mathbb{B} = \{0, 1\}$  bit-vector  $\mathbb{B}[w] = \mathbb{B}^w$
  - constants:  $65_{10}$  decimal  $00100011_2$  binary  $\overbrace{111 \cdots 111}^{35}$  (unary)
  - variables: declared as  $b[1]$  and  $x[32]$  `bool b, x[32];`
  - comparison:  $=, \neq, <, \leq$  (signed and unsigned), ...
  - bit-wise operators:  $\sim, -, \wedge, \vee, \oplus, \dots$  shifting operators: shift, rotate ...
  - arithmetic operators:  $+, -, *, /, \dots$  string operators: slicing, append, extend, ...
- plus array theory QF\_ABV to model memory main memory, caches, etc.
  - sorts: array  $\mathbb{B}[r][2^d] = (\mathbb{B}^d \rightarrow \mathbb{B}^r) = \mathbb{B}^{r2^d} = \mathbb{B}[r \cdot 2^d]$
  - constants: ? zero, range initializers, lambdas, quantifiers, ...
  - variables: declared as  $c[64][1024]$  8KB cache  $m[8][2^{64}]$  main memory

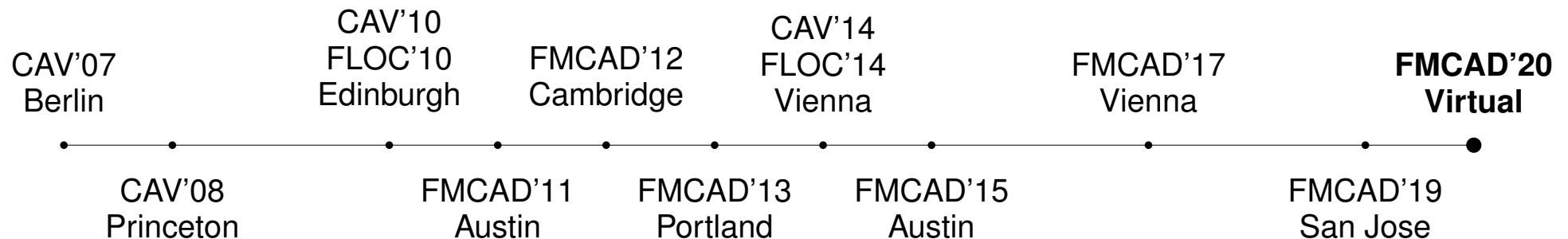
```
(declare-fun c () (Array (_ BitVec 10) (_ BitVec 64)))
(declare-fun m () (Array (_ BitVec 64) (_ BitVec 8)))
```
  - operators: read, write (update) `select, store`

# Sequential Modelling = State Machines / Kripke Structures / Automata

- use “logic” (e.g., bit-vector formulas) to describe sequential semantics symbolically
- Kripke structure flavor think “SMV”
  - initialization and (total) transition relation
  - non-deterministic modelling thus inputs are part of the state
  - still usually variable based: state space = possible variable assignments
  - constraints (invariants / fairness) and properties (temporal logic)
- automata or circuit flavor think “Verilog” or AIGER on the bit-level
  - initialization and transition function partial initialization important in AIGER
  - separate variables for inputs and states
  - non-determinism modelled with inputs “ $\dots = *;$ ” in SLAM, oracle / Choueka construction
  - constraints, properties and explicit outputs for simple compositional semantics
  - **clear semantics** close to actual HW / SW
- thus in summary we prefer the second “functional” view as in AIGER and BTOR
  - also gives a faster and simpler to implement model checker [JussilaBiere’07]

# AIGER

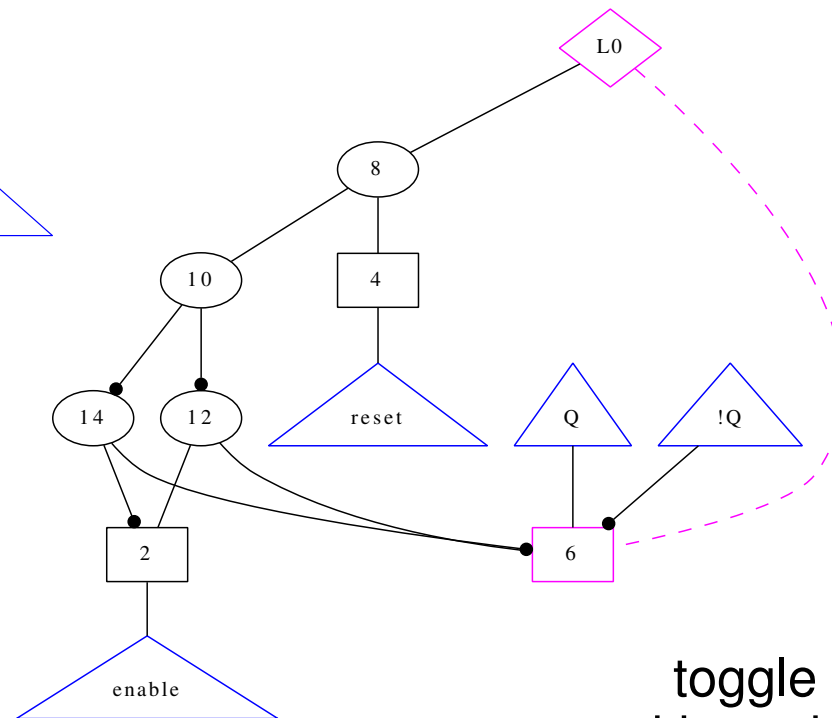
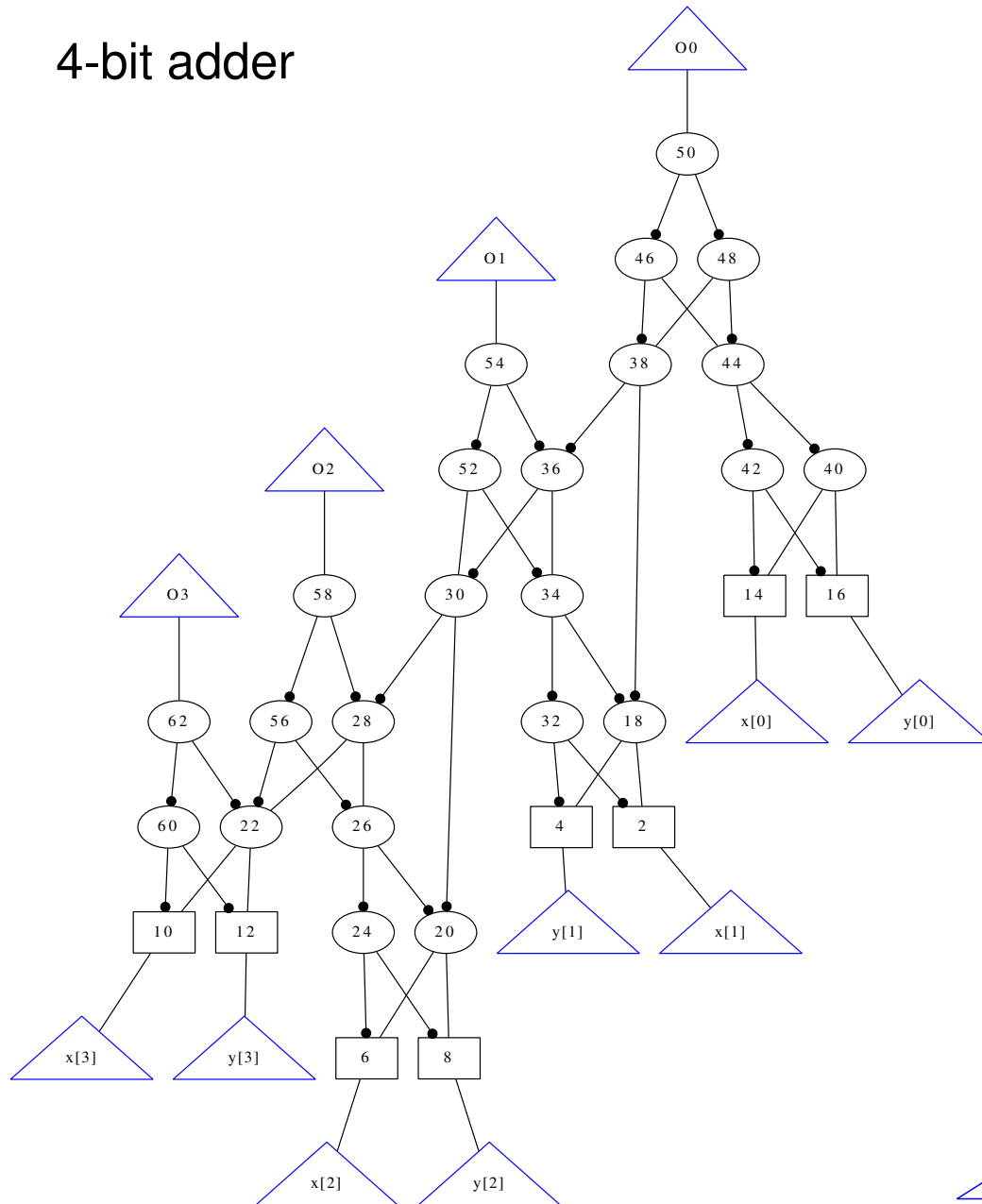
- bit-level (propositional) functional model checking format
- bootstrapped first hardware model checking competition (HWMCC'07)
- witness / trace format, tool set for simulation / witness checking , splitting, unrolling ...
- simple and clean semantics, common denominator of model checkers [Biere'07]
- constraints, more general properties and synthesis support [BiereHeljankoWieringa'11]
- now supported by many HW tools as (binary) exchange format (such as ABC)
- AIG means And-Inverter Graph (formulas with AND and NOT only)
- used since 2007 in the hardware model checking competition (HWMCC)  
[Cabodi et.al. : HWMCC'14] [BiereVanDijkHeljanko'17]
- collected and selected benchmark sets used in many papers



# AIGER

## 4-bit adder

<http://fmv.jku.at/aiger>



toggle flip-flop  
with enable & reset

- BTOR 1.0 [BrummayerBiereLonsing'09]
  - word-level generalization of the initial AIGER format from 2007 (ASCII version)
  - supports bit-vectors and arrays (again quantifier-free formulas only)
  - sequential functional extensions as in AIGER
  
- BTOR 2.0 [NiemetzPreinerWolfBiere'18]
  - resumed word-level motivated by open flows (Yosys) and open cores (RISC-V)
  - incorporated new AIGER 1.9 features from 2011
  - witness format
  - new tools:
    - witness checker / simulator
    - bounded model checker
    - new bit-blaster on top of Boolector's bit-blaster [Preiner'2019]
  - still lacking: fuzzer, delta debugger, bit-blasting of arrays
  - initialization of arrays still tricky
  - used in HWMCC'19 and HWMCC'20

## BTOR Model Example

```
1 sort bitvec 1
2 sort bitvec 3
3 zero 2      }
4 state 2 cnt  }  $cnt = 0$ 
5 init 2 4 3   }
6 input 2 in   }
7 add 2 4 6     }  $cnt' = cnt + in$ 
8 next 2 4 7    }
9 ones 2       }
10 eq 1 4 9     }  $bad : (cnt == 7)$ 
11 bad 10       }
12 constd 2 3   }
13 ulte 1 6 12  }  $in \leq 3$ 
14 constraint 13 }
```

## Witness Example

```
sat
b0
#0
@0
0 011 in@0
@1
0 010 in@1
@2
0 010 in@2
@3
0 000 in@3
.
```



<b>&lt;num&gt;</b>	::=	positive unsigned integer (greater than zero)
<b>&lt;uint&gt;</b>	::=	unsigned integer (including zero)
<b>&lt;string&gt;</b>	::=	sequence of whitespace and printable characters without '\n'
<b>&lt;symbol&gt;</b>	::=	sequence of printable characters without '\n'
<b>&lt;comment&gt;</b>	::=	';' <b>&lt;string&gt;</b>
<b>&lt;nid&gt;</b>	::=	<b>&lt;num&gt;</b>
<b>&lt;sid&gt;</b>	::=	<b>&lt;num&gt;</b>
<b>&lt;const&gt;</b>	::=	'const' <b>&lt;sid&gt;</b> [0-1] <sup>+</sup>
<b>&lt;constd&gt;</b>	::=	'constd' <b>&lt;sid&gt;</b> ['-'] <b>&lt;uint&gt;</b>
<b>&lt;consth&gt;</b>	::=	'consth' <b>&lt;sid&gt;</b> [0-9a-fA-F] <sup>+</sup>
<b>&lt;input&gt;</b>	::=	( 'input'   'one'   'ones'   'zero' ) <b>&lt;sid&gt;</b>   <b>&lt;const&gt;</b>   <b>&lt;constd&gt;</b>   <b>&lt;consth&gt;</b>
<b>&lt;state&gt;</b>	::=	'state' <b>&lt;sid&gt;</b>
<b>&lt;bitvec&gt;</b>	::=	'bitvec' <b>&lt;num&gt;</b>
<b>&lt;array&gt;</b>	::=	'array' <b>&lt;sid&gt;</b> <b>&lt;sid&gt;</b>
<b>&lt;node&gt;</b>	::=	<b>&lt;sid&gt;</b> 'sort' ( <b>&lt;array&gt;</b>   <b>&lt;bitvec&gt;</b> )   <b>&lt;nid&gt;</b> ( <b>&lt;input&gt;</b>   <b>&lt;state&gt;</b> )   <b>&lt;nid&gt;</b> <b>&lt;opidx&gt;</b> <b>&lt;sid&gt;</b> <b>&lt;nid&gt;</b> <b>&lt;uint&gt;</b> [ <b>&lt;uint&gt;</b> ]   <b>&lt;nid&gt;</b> <b>&lt;op&gt;</b> <b>&lt;sid&gt;</b> <b>&lt;nid&gt;</b> [ <b>&lt;nid&gt;</b> [ <b>&lt;nid&gt;</b> ]]   <b>&lt;nid&gt;</b> ( 'init'   'next' ) <b>&lt;sid&gt;</b> <b>&lt;nid&gt;</b> <b>&lt;nid&gt;</b>   <b>&lt;nid&gt;</b> ( 'bad'   'constraint'   'fair'   'output' ) <b>&lt;nid&gt;</b>   <b>&lt;nid&gt;</b> 'justice' <b>&lt;num&gt;</b> ( <b>&lt;nid&gt;</b> ) <sup>+</sup>
<b>&lt;line&gt;</b>	::=	<b>&lt;comment&gt;</b>   <b>&lt;node&gt;</b> [ <b>&lt;symbol&gt;</b> ] [ <b>&lt;comment&gt;</b> ]
<b>&lt;btor&gt;</b>	::=	( <b>&lt;line&gt;</b> '\n' ) <sup>+</sup>

⟨binary-string⟩	::=	[0-1] <sup>+</sup>
⟨bv-assignment⟩	::=	⟨binary-string⟩
⟨array-assignment⟩	::=	'[' ⟨binary-string⟩ ']' ⟨binary-string⟩
⟨assignment⟩	::=	⟨uint⟩ ( ⟨bv-assignment⟩   ⟨array-assignment⟩ ) [⟨symbol⟩]
⟨model⟩	::=	( ⟨comment⟩'\n'   ⟨assignment⟩'\n' ) <sup>+</sup>
⟨ <b>state part</b> ⟩	::=	'#' ⟨uint⟩ '\n' ⟨model⟩
⟨ <b>input part</b> ⟩	::=	'@' ⟨uint⟩ '\n' ⟨model⟩
⟨ <b>frame</b> ⟩	::=	[ ⟨ <b>state part</b> ⟩ ] ⟨ <b>input part</b> ⟩
⟨ <b>prop</b> ⟩	::=	('b'   'j')⟨uint⟩
⟨ <b>header</b> ⟩	::=	'sat\n' ( ⟨ <b>prop</b> ⟩ ) <sup>+</sup> '\n'
⟨ <b>witness</b> ⟩	::=	( ⟨comment⟩'\n' ) <sup>+</sup>   ⟨ <b>header</b> ⟩ ( ⟨ <b>frame</b> ⟩ ) <sup>+</sup> '.'

# Another Example Modelling a C program

```
#include <assert.h>
#include <stdio.h>
#include <stdlib.h>
#include <stdbool.h>
static bool read_bool () {
    int ch = getc (stdin);
    if (ch == '0') return false;
    if (ch == '1') return true;
    exit (0);
}
int main () {
    bool turn;           // input
    unsigned a = 0, b = 0; // states
    for (;;) {
        turn = read_bool ();
        assert (!(a == 3 && b == 3));
        if (turn) a = a + 1;
        else      b = b + 1;
    }
}
```

```
1 sort bitvec 1      sat
2 sort bitvec 32     b0
3 input 1 turn        #0
4 state 2 a           @0
5 state 2 b           0 1 turn@0
6 zero 2              @1
7 init 2 4 6          0 0 turn@1
8 init 2 5 6          @2
9 one 2               0 0 turn@2
10 add 2 4 9           @3
11 add 2 5 9           0 0 turn@3
12 ite 2 3 4 10        @4
13 ite 2 -3 5 11       0 1 turn@4
14 next 2 4 12         @5
15 next 2 5 13         0 1 turn@5
16 constd 2 3          @6
17 eq 1 4 16           0 0 turn@6
18 eq 1 5 16
19 and 1 17 18
20 bad 19
```

# Application Specific Sequential Word-Level Formats

- Hardware description languages (HDL): (System)-Verilog, System-C, VHDL, ...
  - “what you check is what you get”
  - usually have (very) complex semantics and undefined behaviour
  - Yosys, Reveal, Enhanced ABC, commercial model checkers
- Software languages: C, Java, JVM, GraalVM, LLVM, assembler, ...
  - “what you check is what you get”
  - usually have complex semantics and undefined behaviour
  - “Competition on Software Verification” SV-Comp
- application specific languages problematic
  - hard to reuse solver / checker technology
  - QF\_BV is pretty successful in both HW and SW applications
  - encode “undefinedness” precisely is better
  - same should apply to model checking
  - but: “v2c – A Verilog to C translator ”

[MukherjeeTautschnigKroening'16] [MukherjeeSchrammelKroeningMelham'16]

# Other Generic Word-Level Model Checking Formats

- UCLID [BryantLahiriSeshia]
  - early SMT solving (UF, lambdas, memory) targetting processor verification
  - bounded model checking in essence (manual inductive verification)
- SAL from SRI [DeMouraOwreShankar'03] Yices [Duherte'14]
  - focus was originally on infinite systems
  - sofar not-much interest in bit-precise reasoning
- constrained horn clauses  $\mu Z$  [HoderBjornerDeMoura'11]
  - basically extends an SMT solver (Z3) with (second order) least fix-points
  - active community: workshops, competition, ...
  - sofar not-much interest in bit-precise reasoning
- VMT nuXmv [CAV'14] Verilog2SMV [DATE'16] from FBK IRST in Trento
  - SMTLIB with annotations to mark initialization and transition predicates
  - built around (nu)SMV using MathSAT as word-level engine
  - actively supports bit-vectors
- related "Model Checking Competition" (MCC) has Petri nets models (in PNML)
- "classical" protocol modelling languages: Promela (SPIN), Murphi, ...

# Bit-Blasting Explodes

- show *commutativity of bit-vector addition* for bit-width 1 million:

```
(set-logic QF_BV)
(declare-fun x () (_ BitVec 1000000))
(declare-fun y () (_ BitVec 1000000))
(assert (distinct (bvadd x y) (bvadd y x)))
```

- size of SMT2 file: 138 bytes
- bit-blasting with our SMT solver Boolector
  - rewriting turned off
  - except structural hashing
  - produces AIGER circuits of file size 103 MB
- Tseitin transformation leads to CNF in DIMACS format of size 1 GB

# Complexity Classification Results for Bit-Vector Logics

our results from [KovásznaiFröhlichBiere-SMT'12] paper extended version in our TOCS'16 article

		quantifiers			
		<u>no</u>		<u>yes</u>	
		uninterpreted functions		uninterpreted functions	
		<u>no</u>	<u>yes</u>	<u>no</u>	<u>yes</u>
encoding	<u>unary</u>	NP QF_BV1 obvious	NP QF_UFBV1 Ackermann	PSPACE BV1 [TACAS'10]	NEXPTIME UFBV1 [FMCAD'10]
	<u>binary</u>	<b>NEXPTIME QF_BV2 [SMT'12]</b>	NEXPTIME QF_UFBV2 [SMT'12]	AEXP(poly) BV2 [JonášStrejček-IPL'18]	2NEXPTIME UFBV2 [SMT'12]

QF = “quantifier free”      UF = “uninterpreted functions”      BV = “bit-vector logic”  
BV1 = “unary encoded bit-vectors”      BV2 = “binary encoded bit-vectors”

# Complexity Classification Results for Arrays and Word-Level Model Checking

- AIGER problems are PSPACE complete
  - since “symbolicl reachability” is PSPACE complete [Savitch’70]
- now assume (for instance) sequential BTOR 2.0 as input
  - without arrays but sequential problems (model checking)
    - unary encoding (or bit-width as fixed parameter): PSPACE complete
    - binary encoding: EXSPACE complete [KovasznaiVeithFröhlichBiere’MFCS14]
  - with arrays and sequential problems (model checking)
    - unary encoding: ? EXPSPACE complete?
    - binary encoding: ? 2EXPSPACE complete?
- benefits of complexity characterizations
  - gives hints what solvers (SAT,SMT, AIGER) can be used as oracles
  - and how many times they have to be called
  - sometimes gives restricted classes PSPACE sub-class of QF\_BV2



# Why do we want to do word-level model checking?

- use word-level “structure” for rewriting / simplification
  - allows (shallow) arithmetic reasoning as in the complexity example
  - word-level local search [NiemetzPreinerBiere’16/17] [NiemetzPreiner’20]
  - make full use of functional representation
    - global substitution pass instead of congruence closure
    - CNF preprocessing lacks some benefits of circuit representations
    - bit-level circuit intermediate formats (thus bit-level rewriting)
    - BDD / SAT / SMT / cut sweeping to eliminate equivalent expressions
- data and memory abstraction
  - bit-blasting of arithmetic expensive  $*_{32}$  has 8000 AIG nodes,  $*_{64}$  has 32 000
  - protocols only “move data around”: bit-precise reasoning redundant
  - properties often argue about some “reads” and “writes” only
  - bit-blasting memory is often impossible  $m_{32}[8][2^{32}]$   $m_{64}[8][2^{64}]$
- sequential and non-sequential rewriting and abstraction techniques

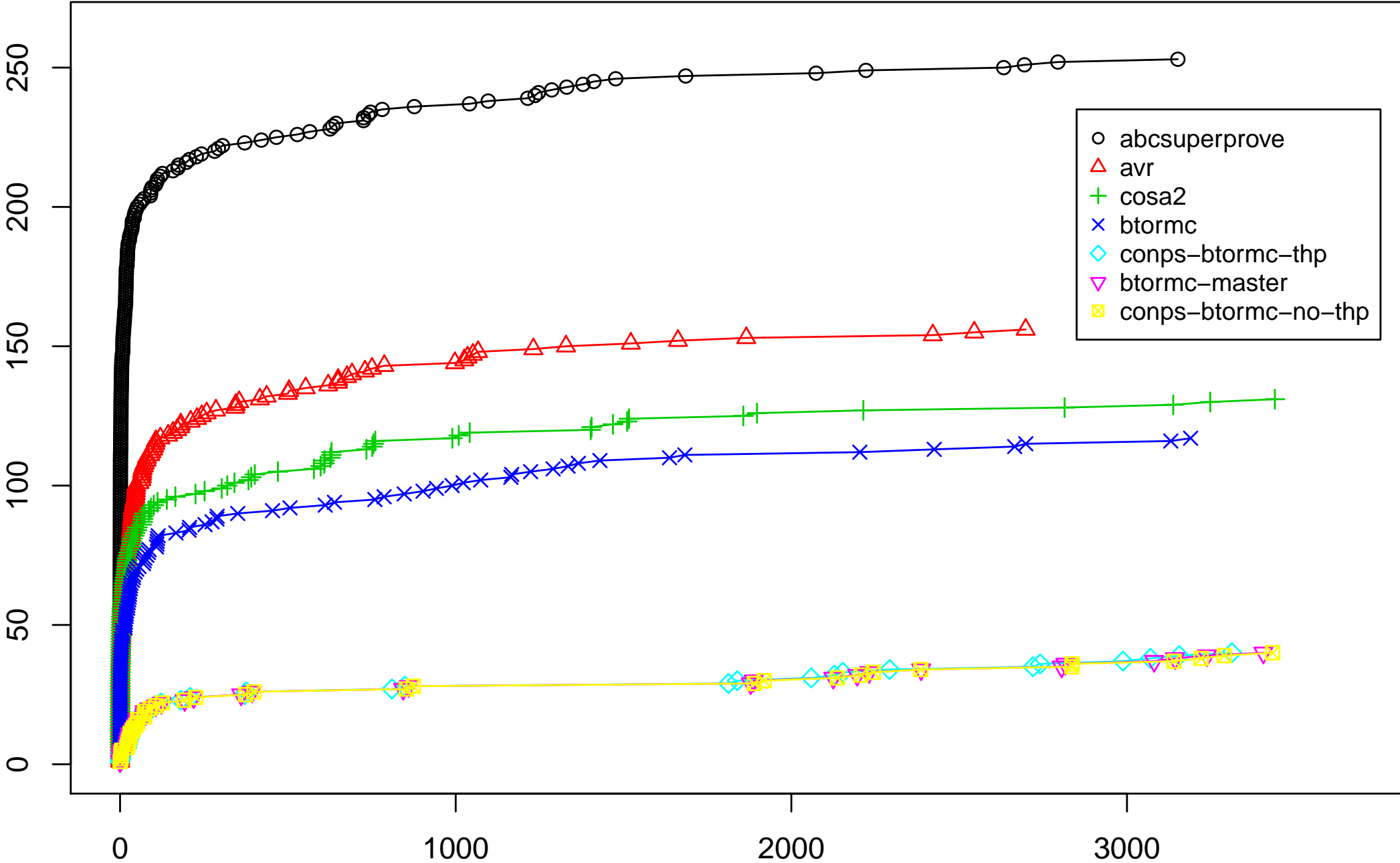
# Eager Data Abstraction

- 1-bit abstractions
  - verify sorting using only “compare & swap” on 0/1 input      zero-one principle [Knuth’73]
  - data independence of protocols [Wolper’86]
- small domain encoding      part of Ackermann’s reduction
  - if you only compare  $n$  variables then interpret them on the domain  $0, \dots, n - 1$
  - reduce those variables to bit-width  $\lceil \log n \rceil$
  - eager translation to SAT possible      [PnueliRodehShtrichmanSiegel’99]
  - plain bit-vectors [Johannsen’01/02], model checking [HojatiBrayton’95] [Bjesse’08]
  - need to “slice” bit-vectors in HW to have compatible widths      next state functions too
  - can use different domain size for each “cluster” of compared variables
- abstract uninterpreted functions (UF) through Ackermann eagerly      transformation
  - extends to memories / arrays      (exponentially) eliminate read & write as in UCLID
  - works for plain bit-vectors (thus BMC) but then lazy SMT (QF\_AUFBV) is better  
[BurchDill’96] [VelevBryantJain’97] [ManoliosSrinivasanVroon’06] [GanaiGuptaAshar’04/05]
  - model checking requires to change properties [Bjesse’08/09] [German’11]

# Lazy Data Abstraction

- akin to “lazy SMT” or CEGAR / Localization
- for instance replace expensive operations (multiplication) with UF
  - abstraction refinement loop using SMT [AndrausLiffitonSakkalah’06/08]
  - conservative: if abstracted model passes property then original passes it too
  - spurious counter example: refine  $\text{“}mult(x,y)\text{”}$  to  $\text{“}(x = 0 ? 0 : mult(x,y))\text{”}$
  - refinement can make use of cores or MUS
- combine with IC3 / PDR [LeeSakallah’14] [GoelSakallah’19/20]
  - predicate abstraction existing predicates, new predicates?
  - syntax guided abstraction equality between existing expressions, new expressions?
- how to interpolation into the mix is still unclear  
bit-vectors [Griggio’16] [BackemanRümmerZeljic’18] [OkudonoKing’20] arrays ?
- also still needs to be combined with successful bit-level techniques
  - sweeping / temporal decomposition / retiming
  - local search / simulation

# HWMCC'19 Results on Bit-Vectors (BV) without arrays



# Challenges

- benchmarks: Yosys, open cores, RISC-V already helped a lot, **but need more!**
- apply HW word-level model checkers to SW (from SV-COMP) or vice versa
- symbolic execution of both SW and HW
  - modelling (slices of) programs linearly in a word-level model
  - “Selfie” by Christoph Kirsch has a BTOR2 model of RISC-U
- smart contracts
  - bit-precise semantics lends itself to word-level models
  - as discussed in invited SMT’20 talk by Mooly Sagiv
- certificates:
  - UNSAT proofs in SAT very useful “biggest math proof ever” by Marijn Heule
  - certificates for (passing properties) in AIGER with Zhengqi Yu and Keijo Heljanko
  - certificates for UNSAT proofs in QF\_BV [CVC4 team]
  - combine to provide word-level certificates
- make word-level model checkers faster than bit-level checkers ⇒ HWMCC’20?