

Scaling Up DPLL(T) String Solvers Using Context-Dependent Simplification

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Importance of String Solvers

- Automated string solvers are essential for formal methods applications
- Security applications (e.g. finding XSS attacks) require *string solvers* that:
 - Are *highly efficient*
 - Reason about strings with *unbounded length* (not just bounded ones)
 - Accept a *rich language* of string constraints

In This Paper:

- DPLL(T) string solvers for *extended string constraints*
- New technique, *context-dependent simplification*, improves *scalability* of current string solvers
- Implemented in SMT solver *CVC4*
- *Experiments* show advantages using CVC4 as backend to *symbolic execution* engine PyEx

Basic String Constraints

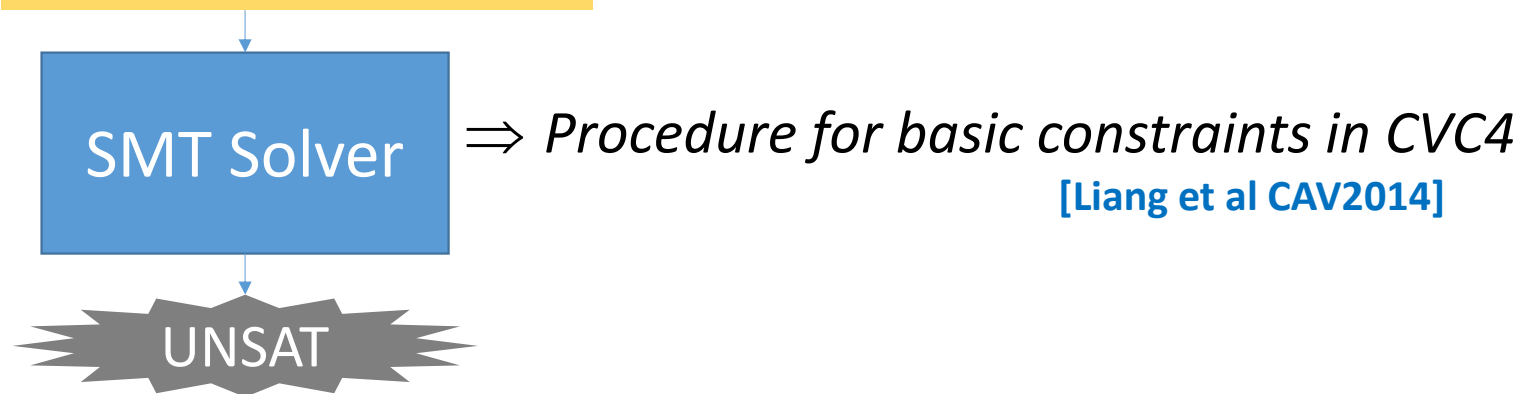
- Equalities and disequalities between:
 - *Basic string* terms
 - String constants: ϵ , "abc"
 - Concatenation: $x \cdot \text{"abc"}$
 - Length: $|x|$
 - *Linear arithmetic* terms: $x+4$, $y>2$

Example: $x \cdot \text{"a"} = y \wedge |y| > |x| + 2$

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Extended String Constraints

- Equalities and disequalities between:
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 - String constants: ϵ , "abc"
 - Concatenation: $x \cdot \text{"abc"}$
 - Length: $|x|$
 - *Linear arithmetic* terms: $x+4$, $y>2$
 - *Extended string* terms:
 - Substring: `substr("abcde", 1, 3)`
 - String contains: `contains("abcde", "cd")`
 - Find "index of": `indexof("abcde", "d", 0)`
 - String replace: `replace(x, "a", "b")`

Example: `contains(substr(x, 0, 3), "a") \wedge 0 \leq indexof(x, "ab", 0) $<$ 4`

?

\Rightarrow *Focus of this work*

DPLL(T) String Solvers

- Cooperation between:

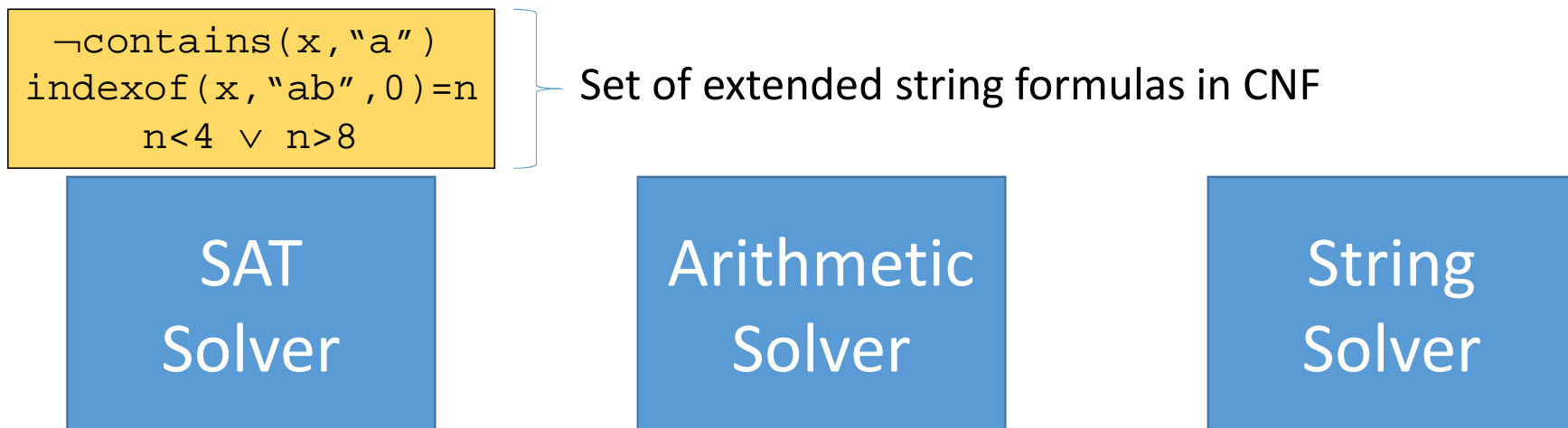


SAT
Solver

Arithmetic
Solver

String
Solver

DPLL(T) String Solvers



DPLL(T) String Solvers

```
¬contains(x, "a")  
indexof(x, "ab", 0) = n  
n < 4 ∨ n > 8
```

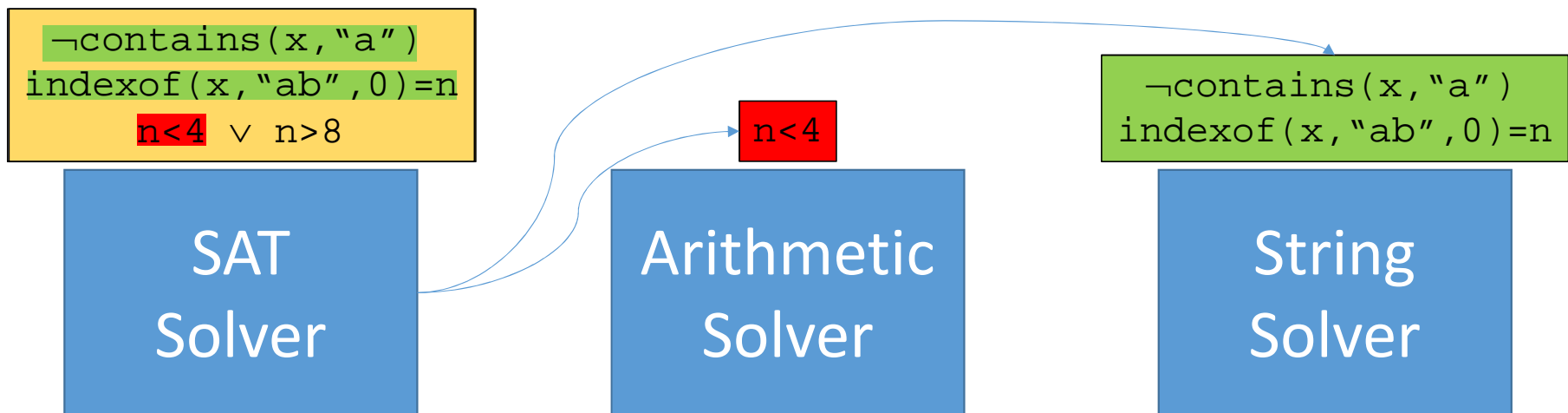
SAT
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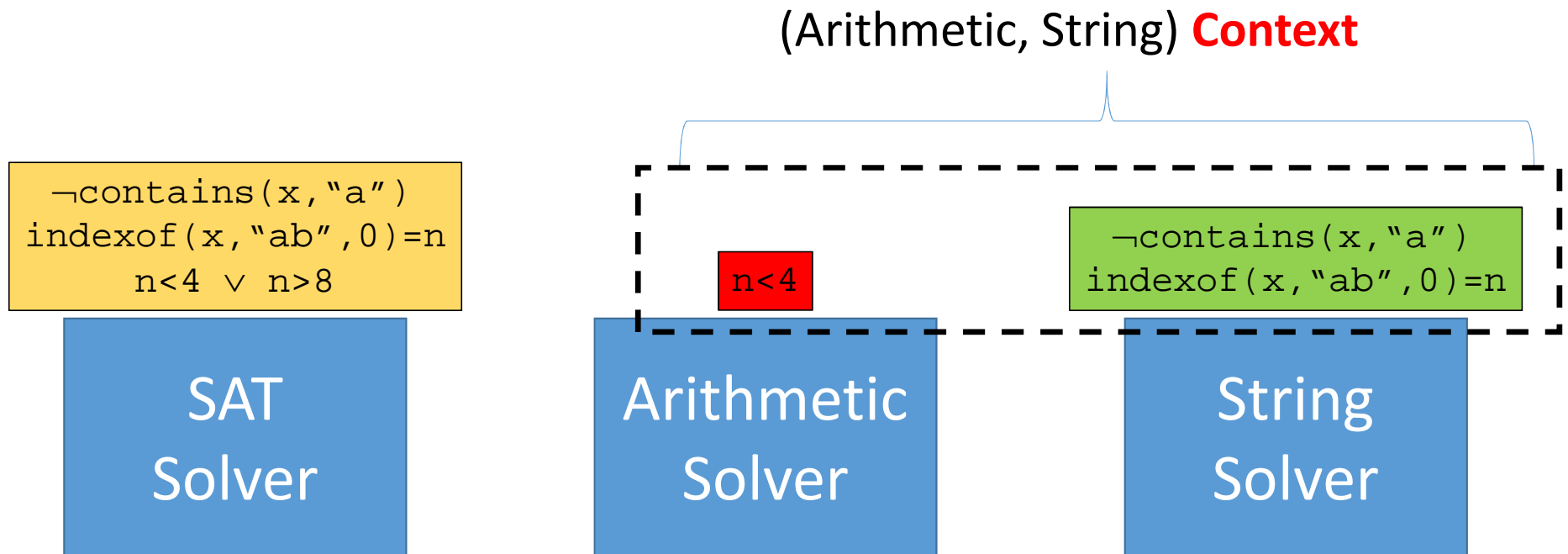
⇒ Find propositionally satisfying assignment

DPLL(T) String Solvers



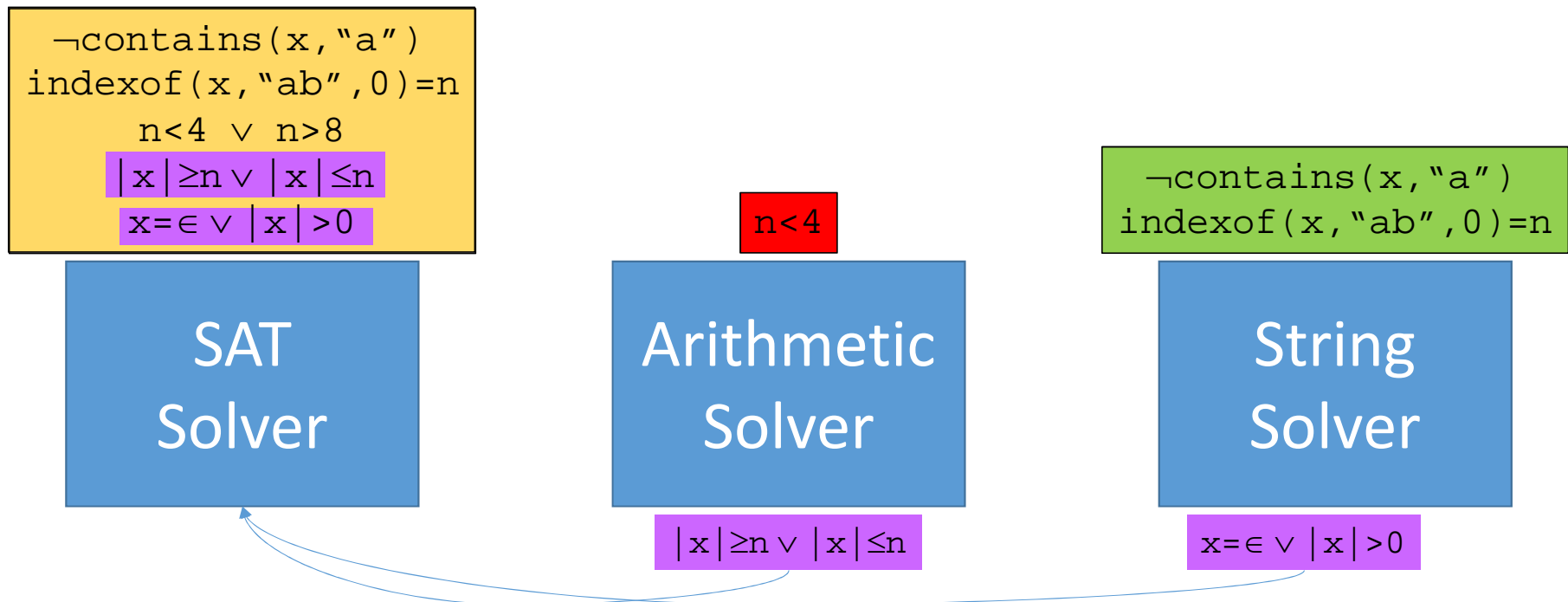
⇒ Distribute to **arithmetic** and **string** solvers

DPLL(T) String Solvers



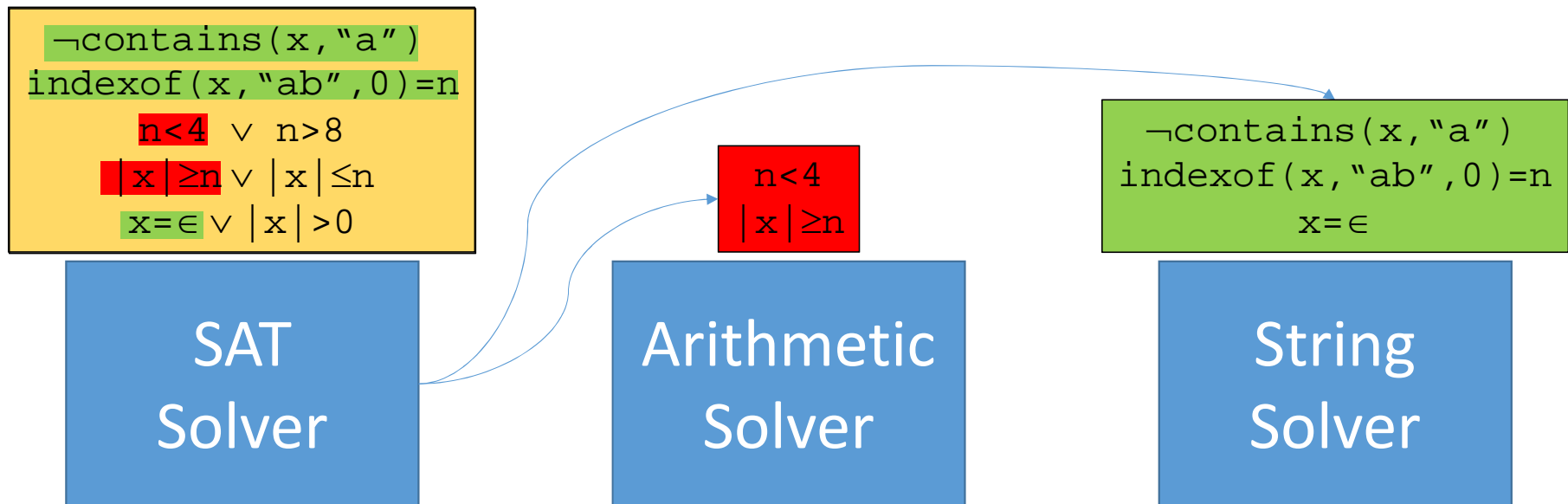
⇒ Solvers maintain a **context** (conjunction of theory literals)

DPLL(T) String Solvers



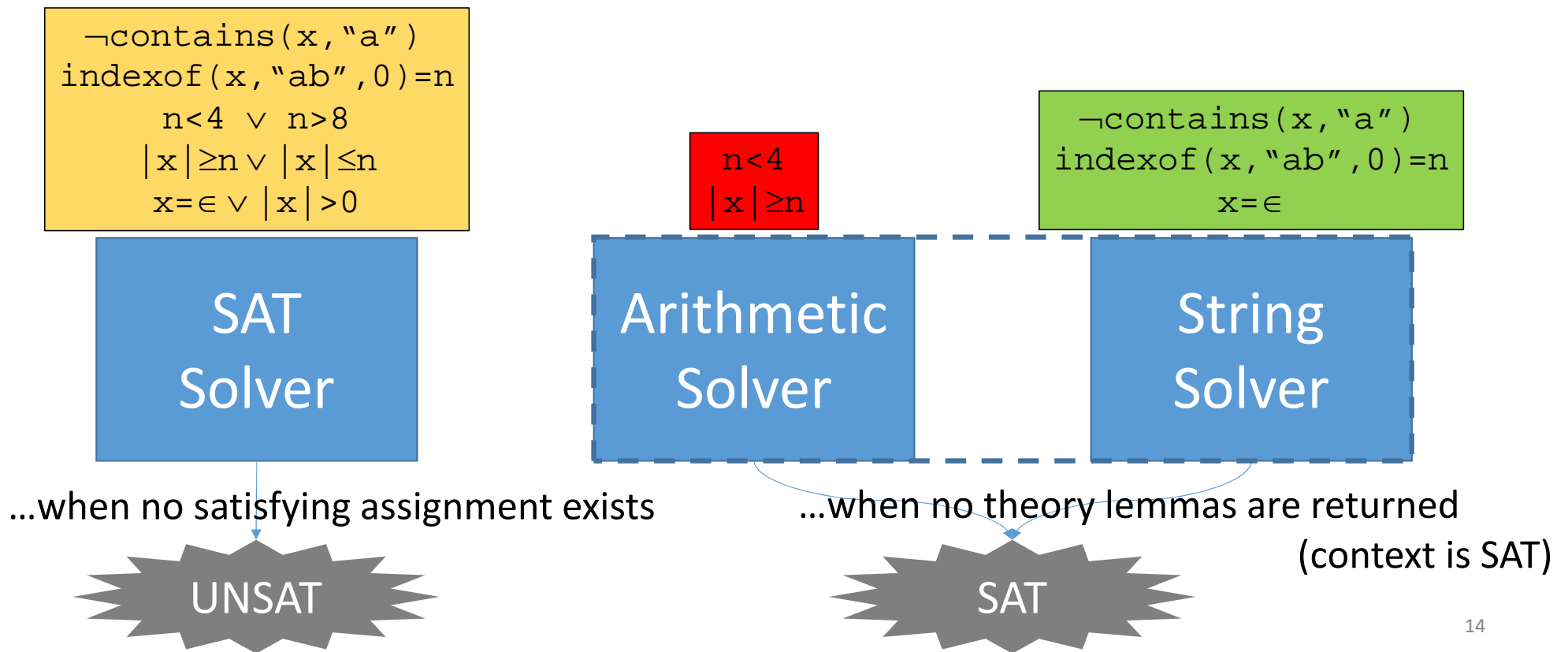
\Rightarrow String and arithmetic solvers return **theory lemmas** to SAT solver

DPLL(T) String Solvers



\Rightarrow ...and repeat

DPLL(T) String Solvers



Properties of DPLL(T) String Solvers

- For *basic* constraints, DPLL(T) string solvers:
 - Can be used for “**sat**” and “**unsat**” answers
 - Are **incomplete** and/or **non-terminating** in general
- Expected, since *decidability is unknown*
[Bjorner et al 2009, Ganesh et al 2011]
- Regardless, modern solvers are *efficient in practice*
[Zheng et al 2013, Liang et al 2014, Abdulla et al 2015, Trinh et al 2016]

How do we handle **Extended String Constraints?**

$\neg \text{contains}(x, \text{"a"})$

How do we handle Extended String Constraints?

- Naively, by **reduction** to basic constraints + bounded \forall

```
¬contains(x, "a")
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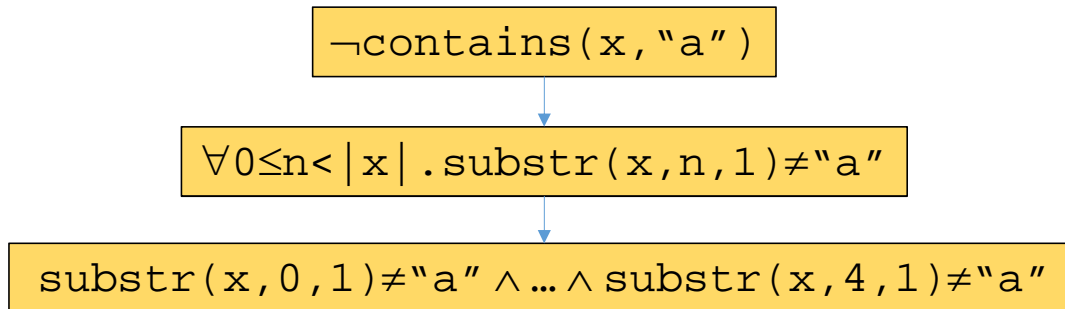


$\forall 0 \leq n < |x| . \text{substr}(x, n, 1) \neq "a"$

Expand contains

How do we handle Extended String Constraints?

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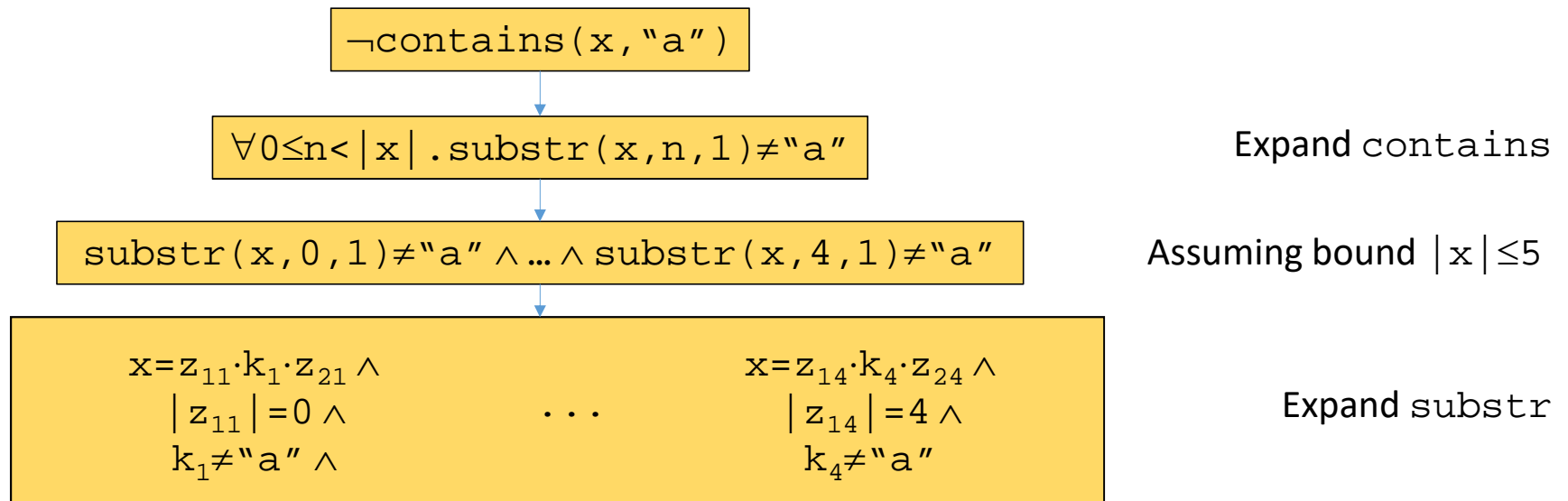


Expand contains

Assuming bound $|x| \leq 5$

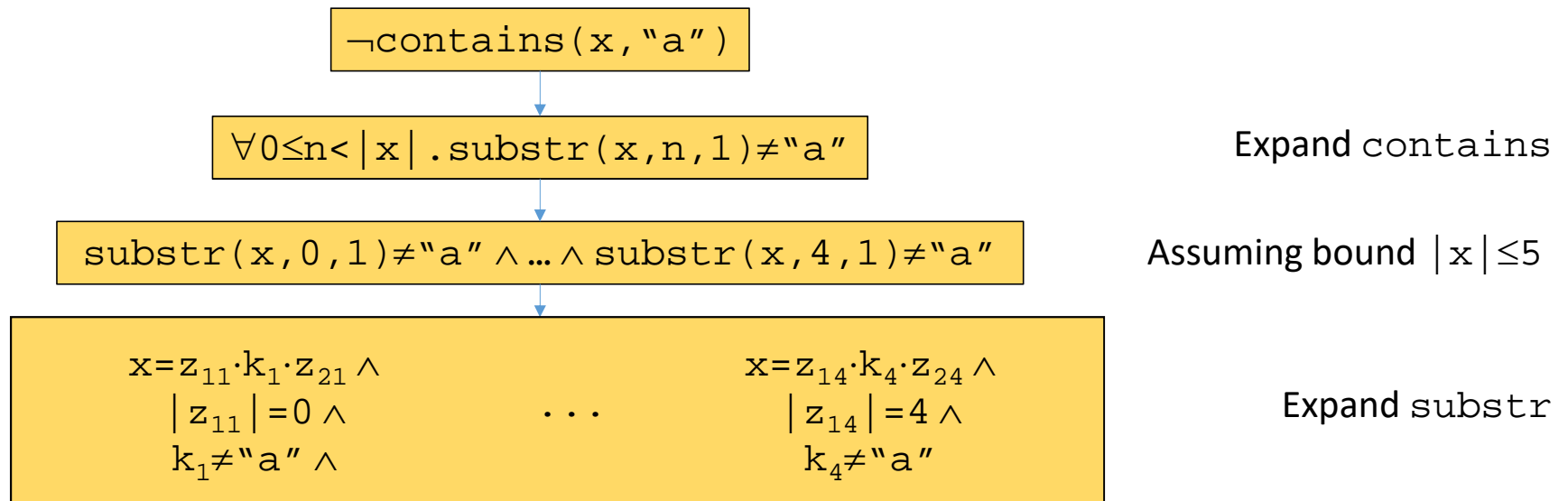
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How do we handle Extended String Constraints?

- Naively, by **reduction** to basic constraints + bounded \forall



- Approach used by many current solvers
[Bjorner et al 2009, Zheng et al 2013, Li et al 2013, Trinh et al 2014]

(Eager) Expansion of Extended Constraints

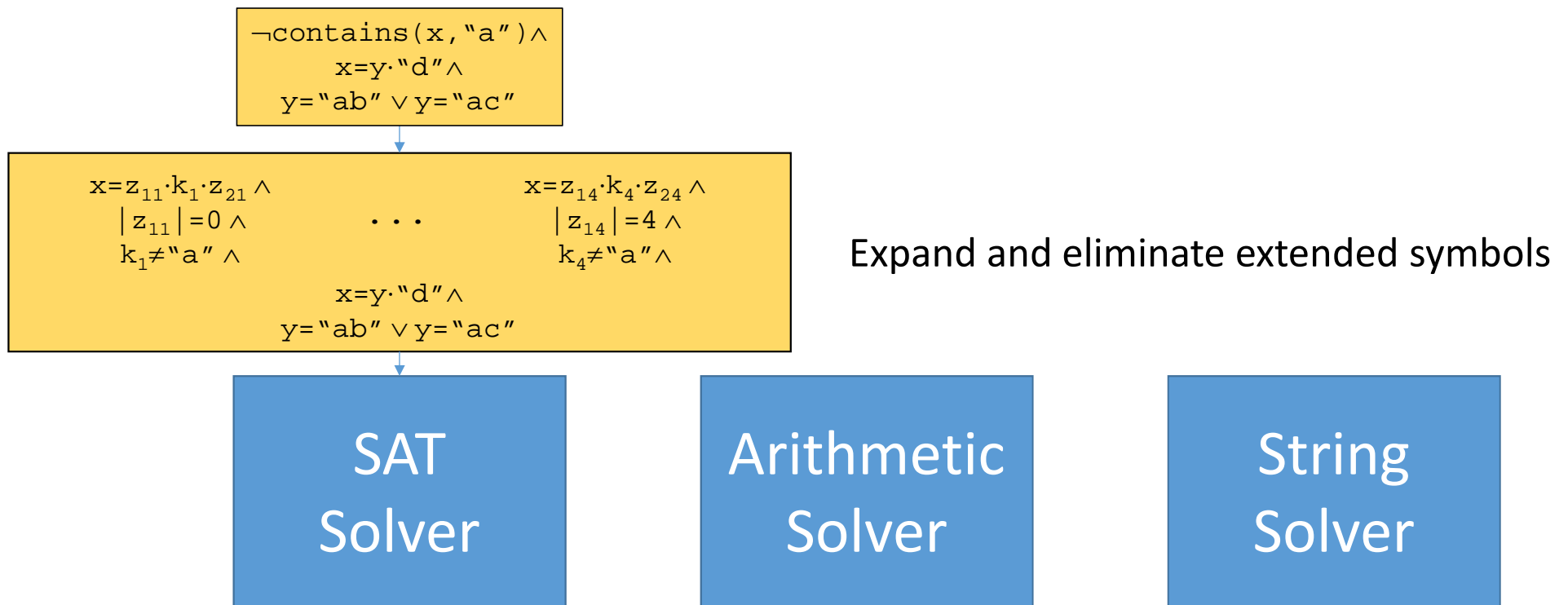
```
¬contains(x, "a") ∧  
  x=y·"d" ∧  
  y="ab" ∨ y="ac"
```

SAT
Solver

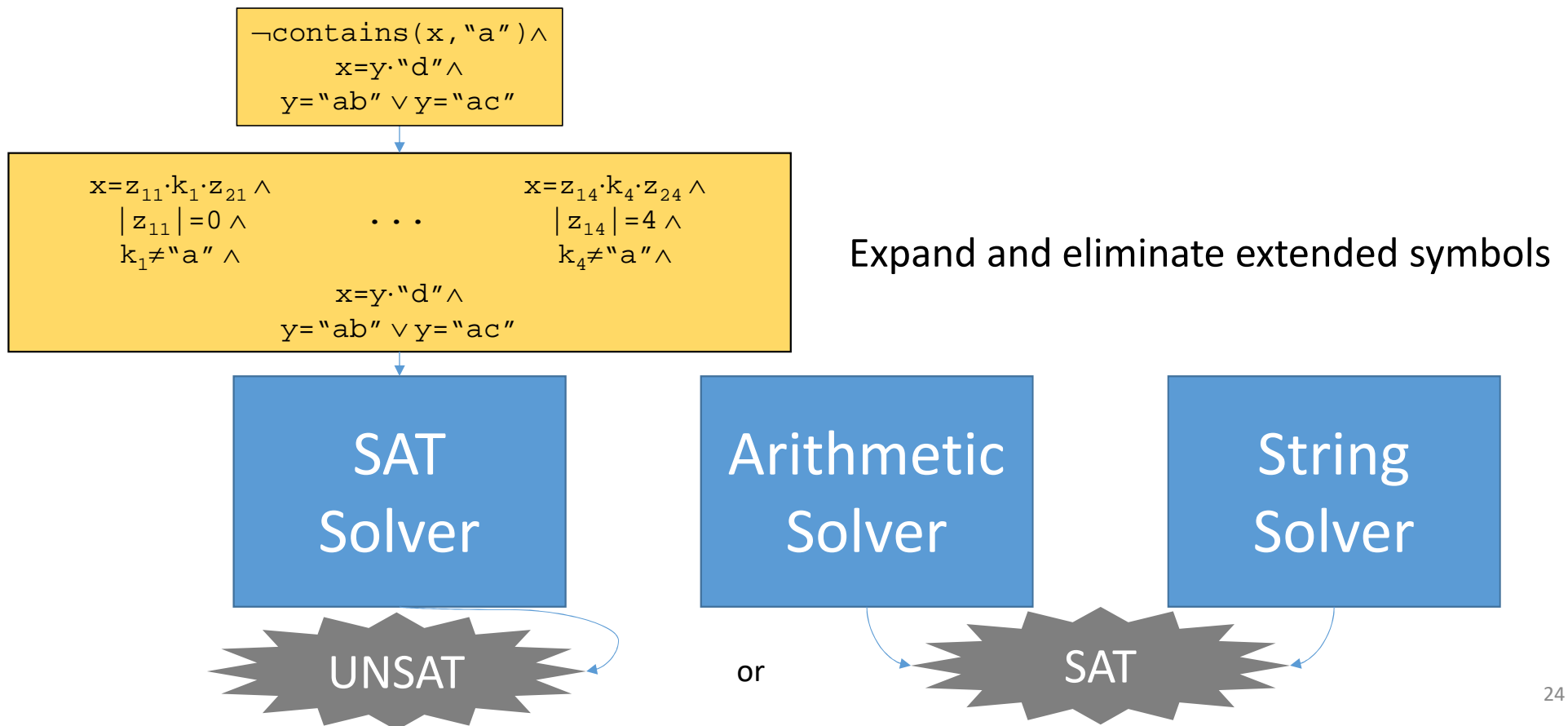
Arithmetic
Solver

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Solver

(Eager) Expansion of Extended Constraints



(Eager) Expansion of Extended Constraints



(Lazy) Expansion of Extended Constraints

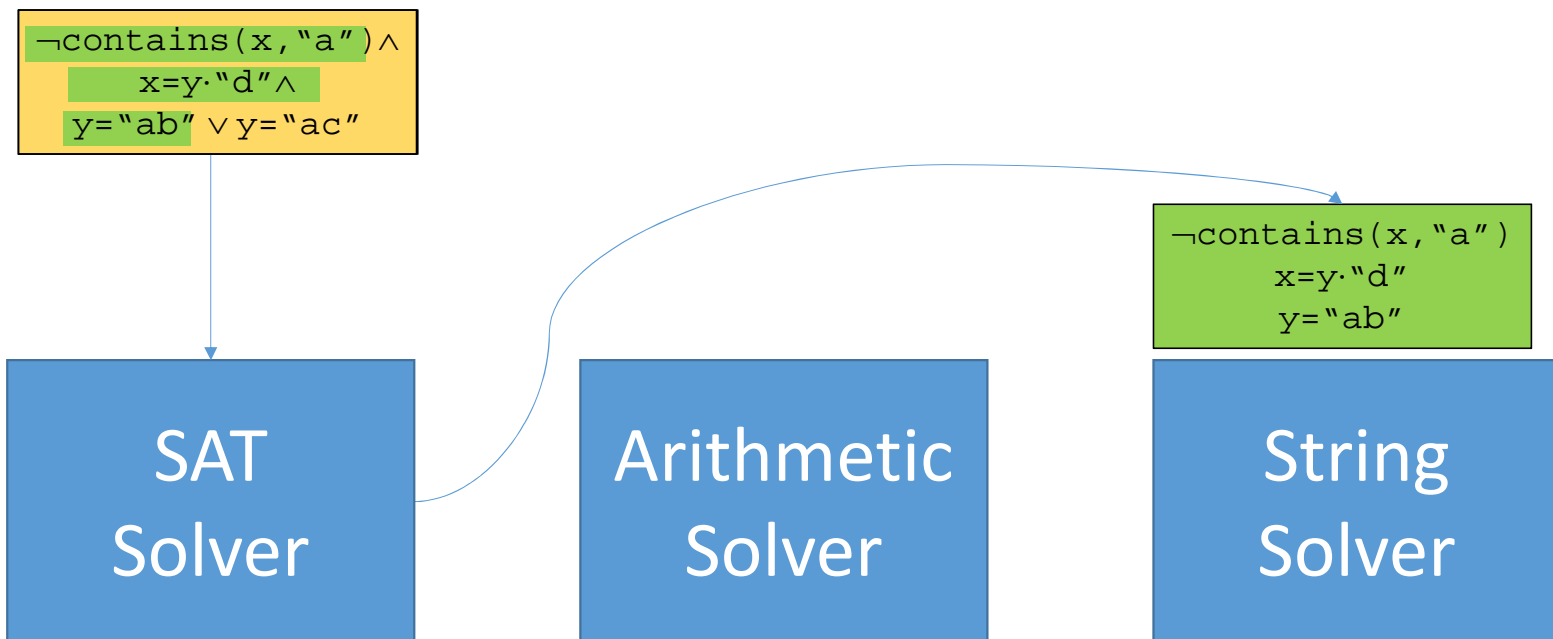
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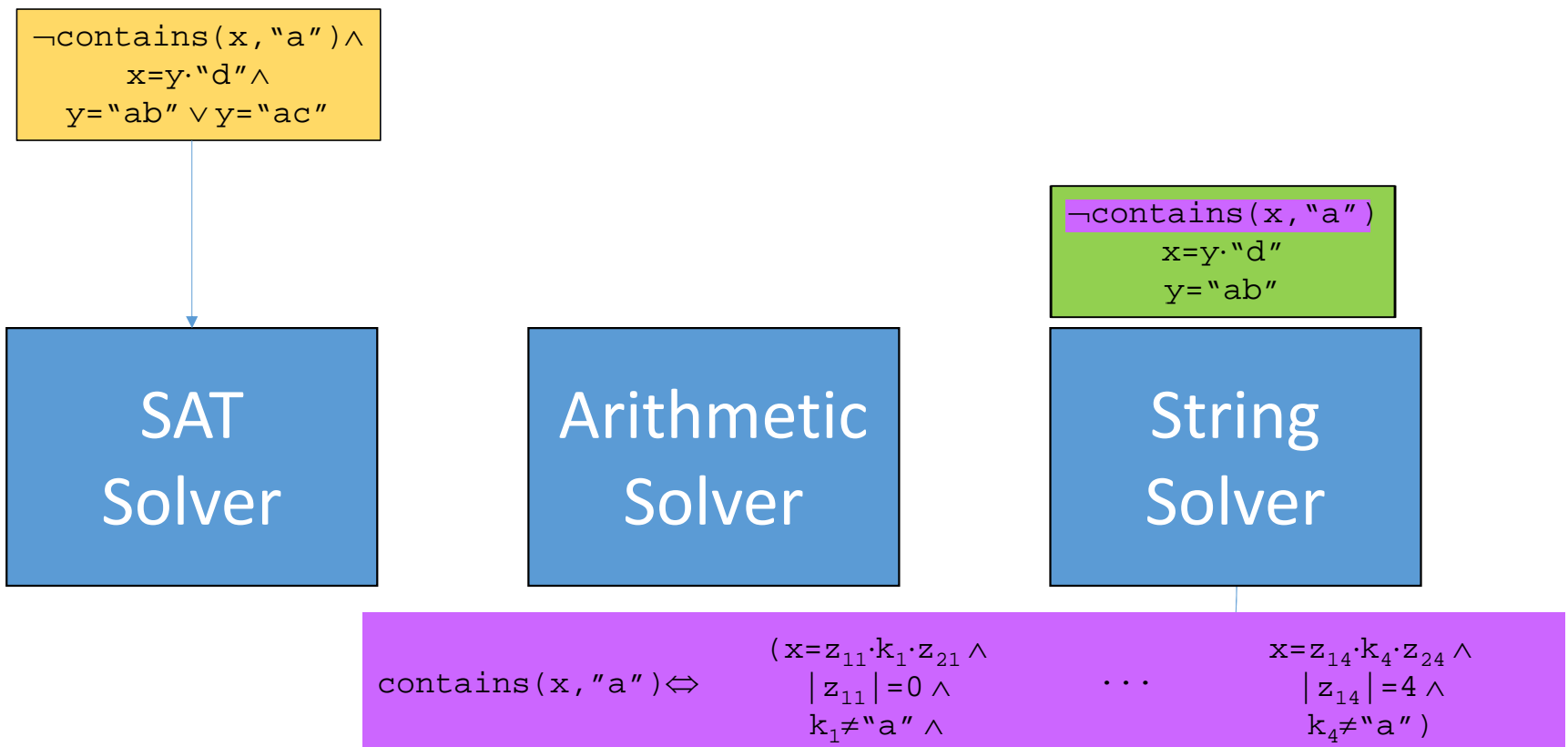
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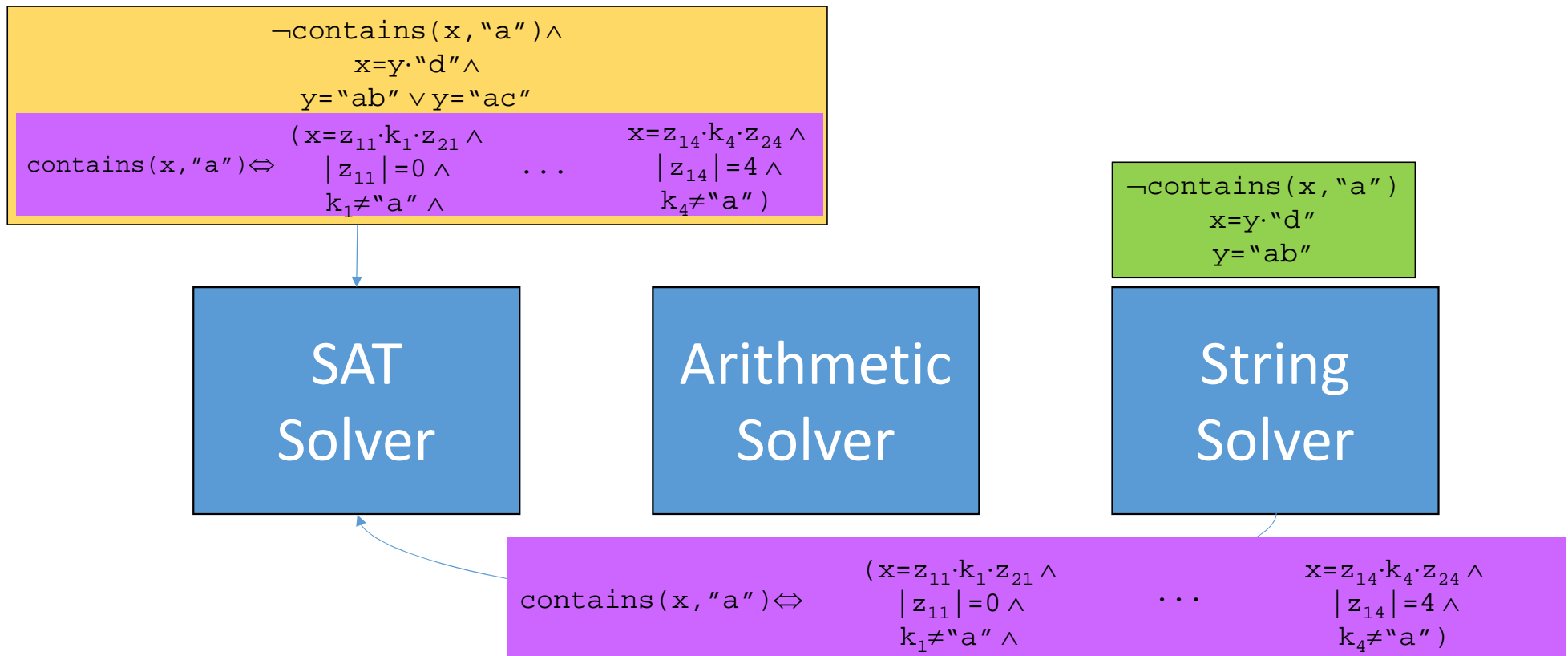
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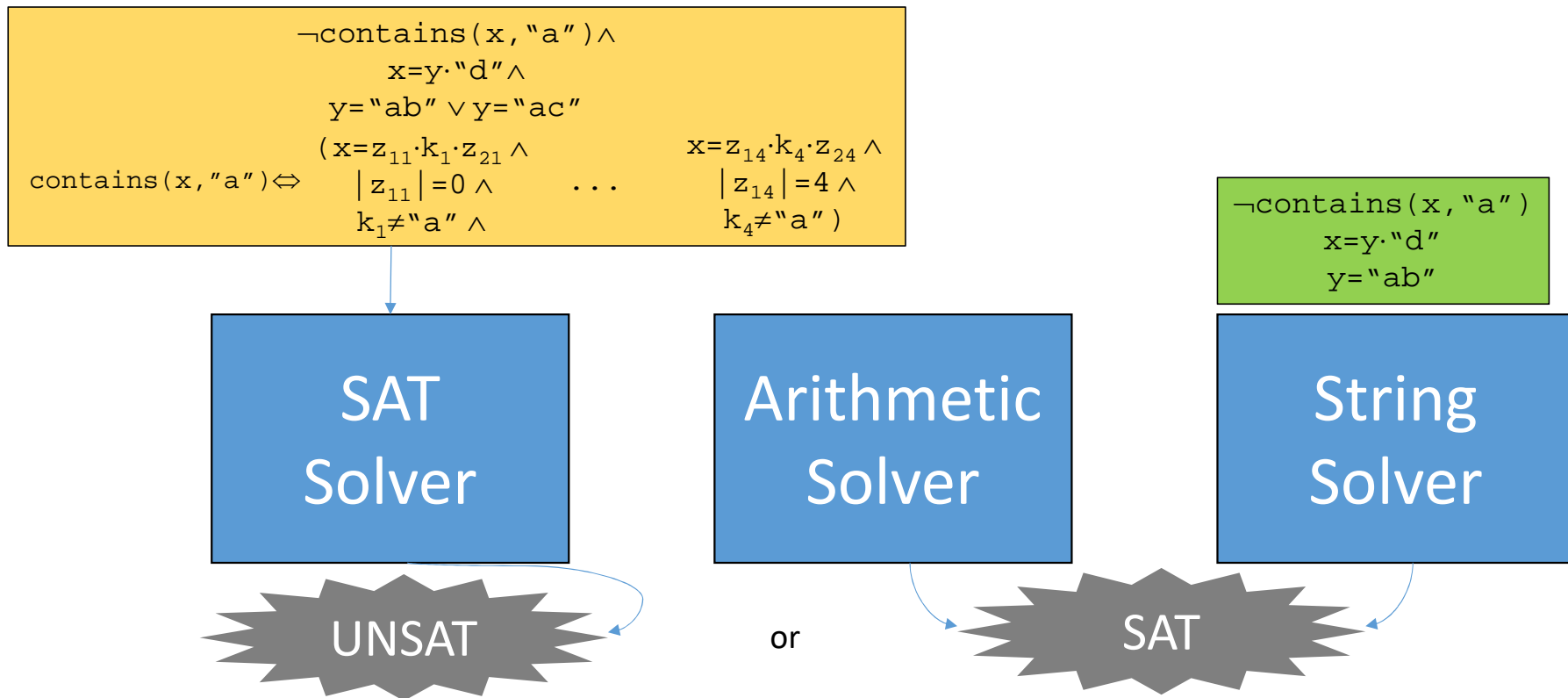
(Lazy) Expansion of Extended Constraints



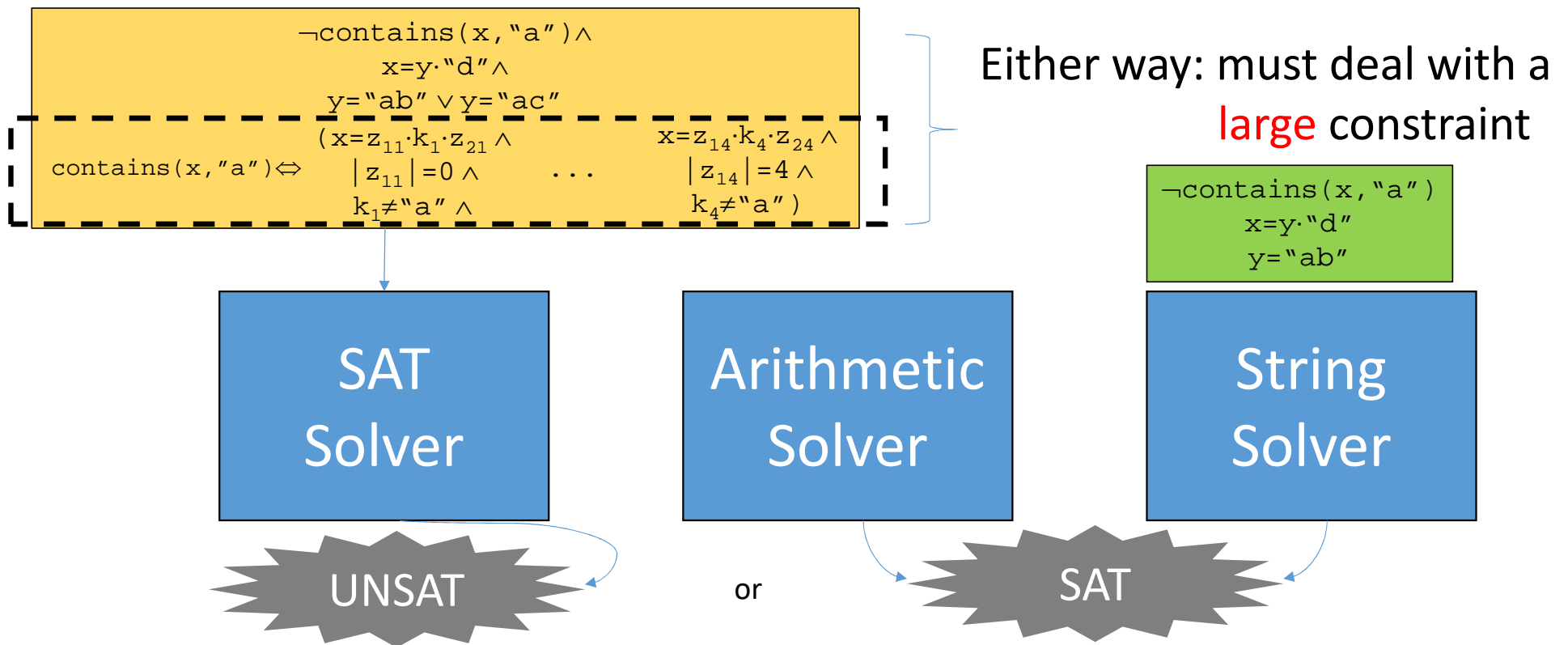
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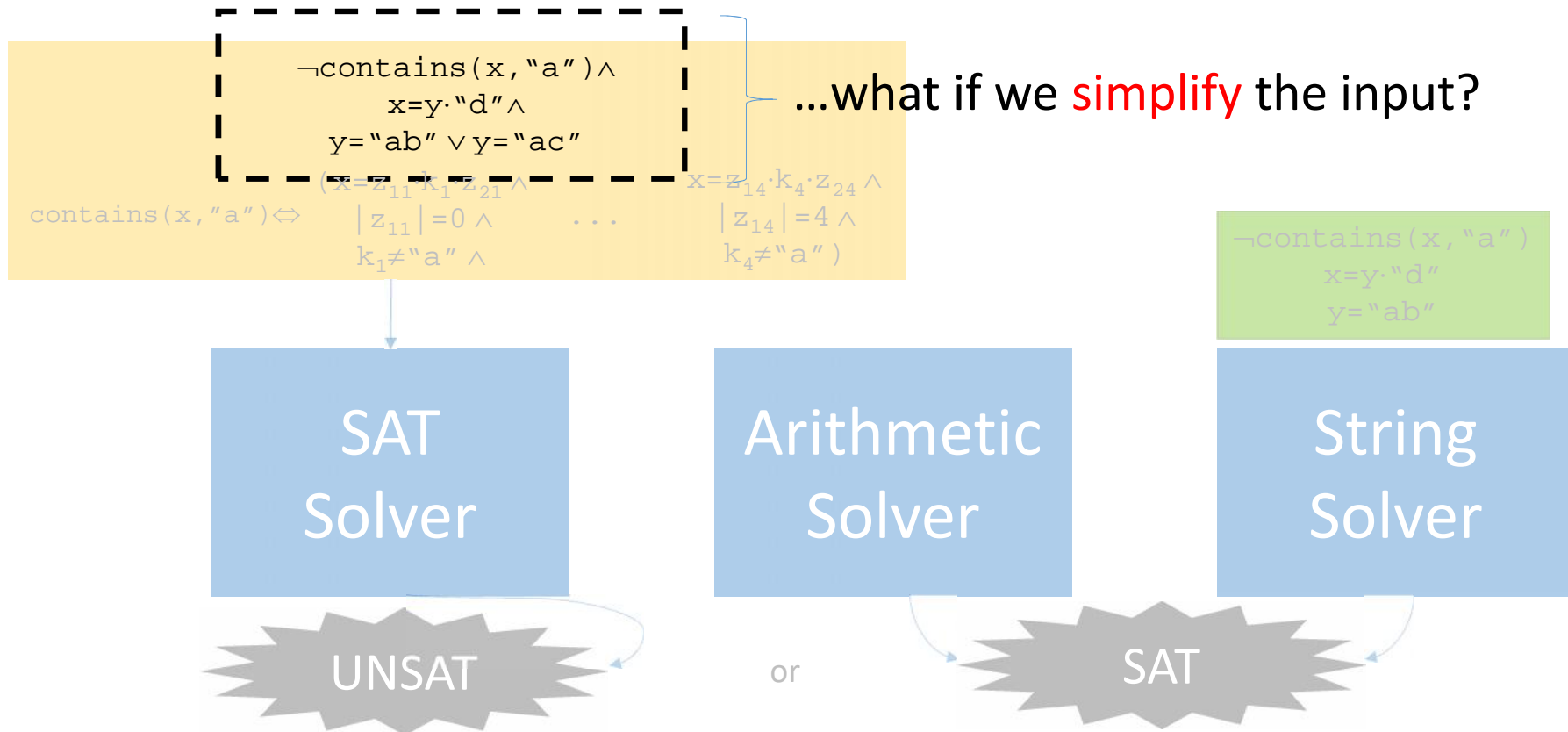
(Lazy) Expansion of Extended Constraints



(Lazy) Expansion of Extended Constraints



(Lazy) Expansion of Extended Constraints



SMT Solvers + Simplification

- All SMT solvers implement *simplification* techniques
(also called *normalization* or *rewrite rules*)

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$\neg \text{contains}(x, \text{"a"}) \wedge$
 $x = y \cdot \text{"d"} \wedge$
 $y = \text{"ab"} \vee y = \text{"ac"}$

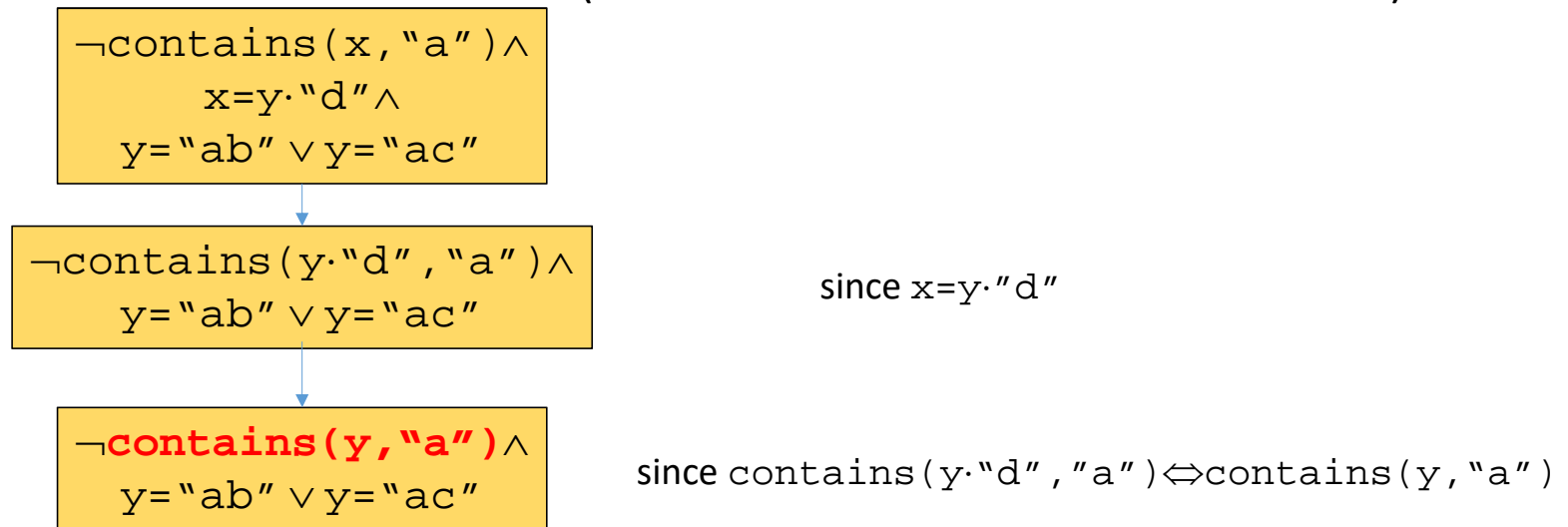


$\neg \text{contains}(\text{y} \cdot \text{"d"}, \text{"a"}) \wedge$
 $y = \text{"ab"} \vee y = \text{"ac"}$

since $x = y \cdot \text{"d"}$

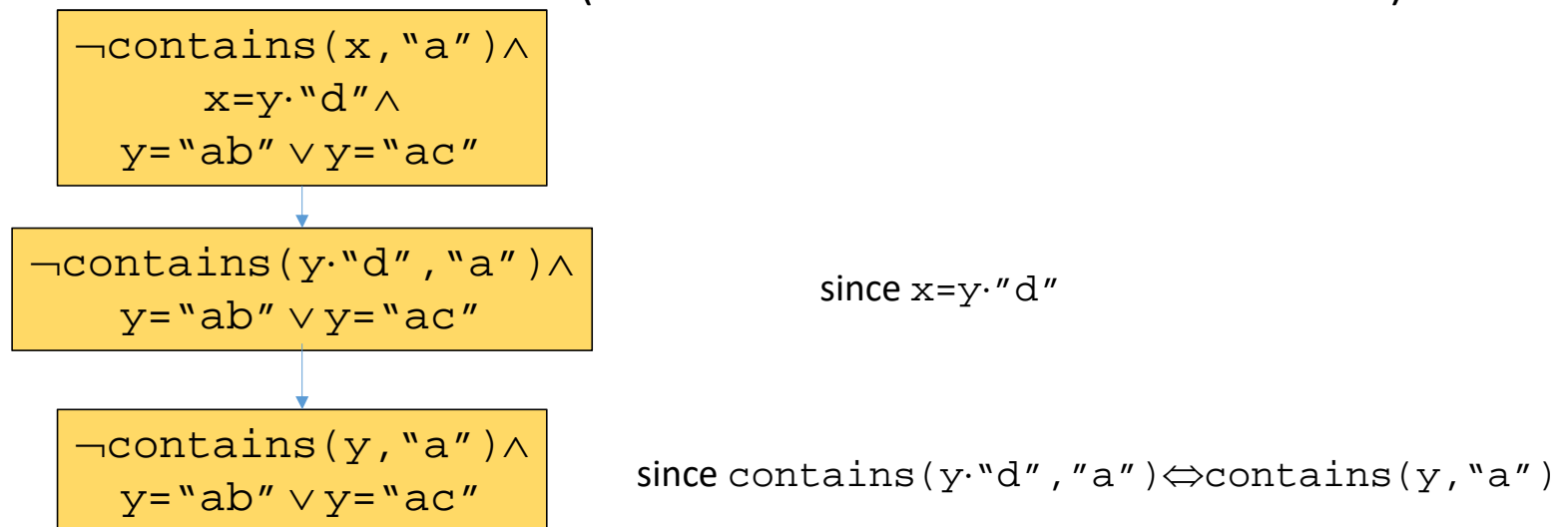
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(also called *normalization* or *rewrite rules*)



- Leads to smaller inputs, simpler procedures

(Lazy) Expansion + Simplification

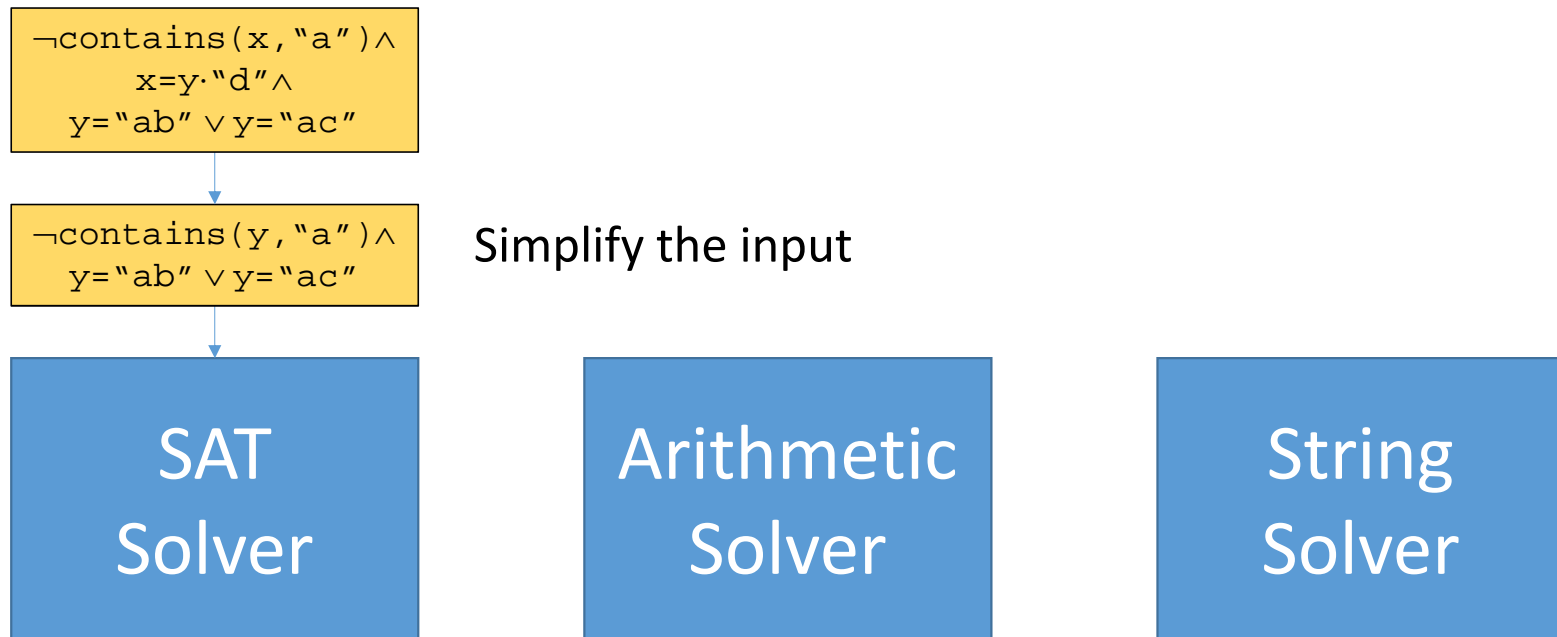
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SAT
Solver

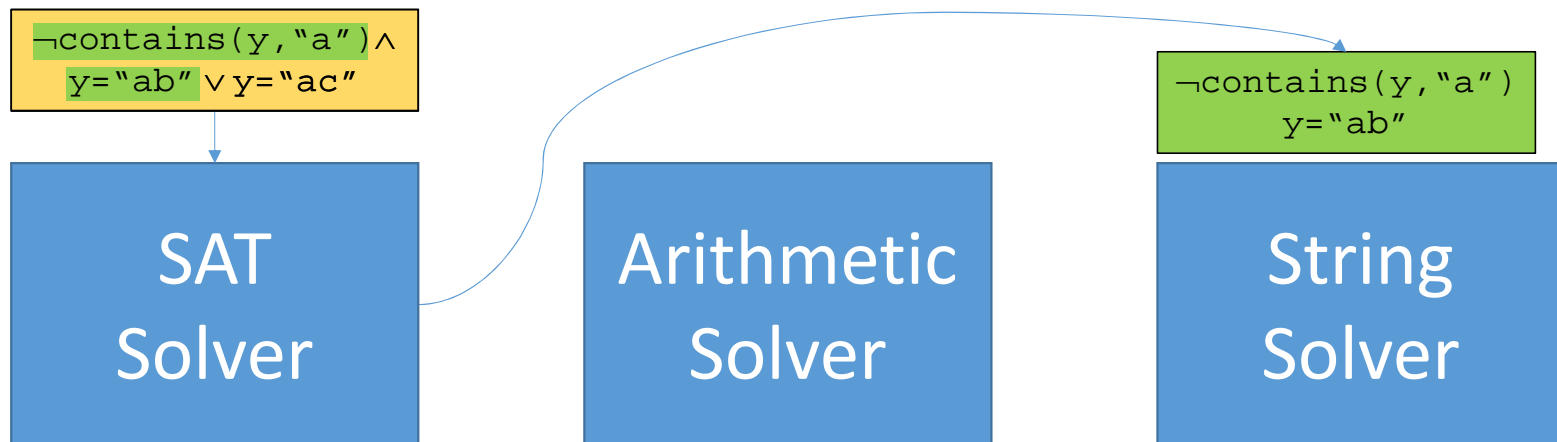
Arithmetic
Solver

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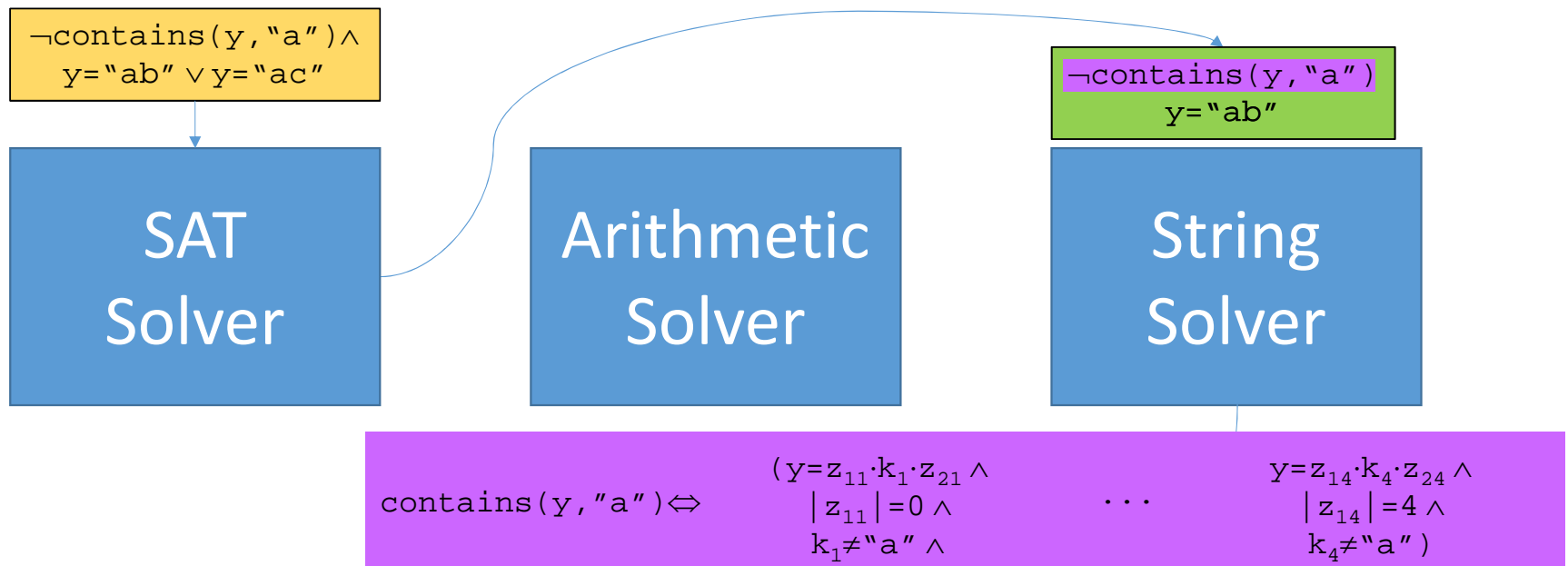
(Lazy) Expansion + Simplification



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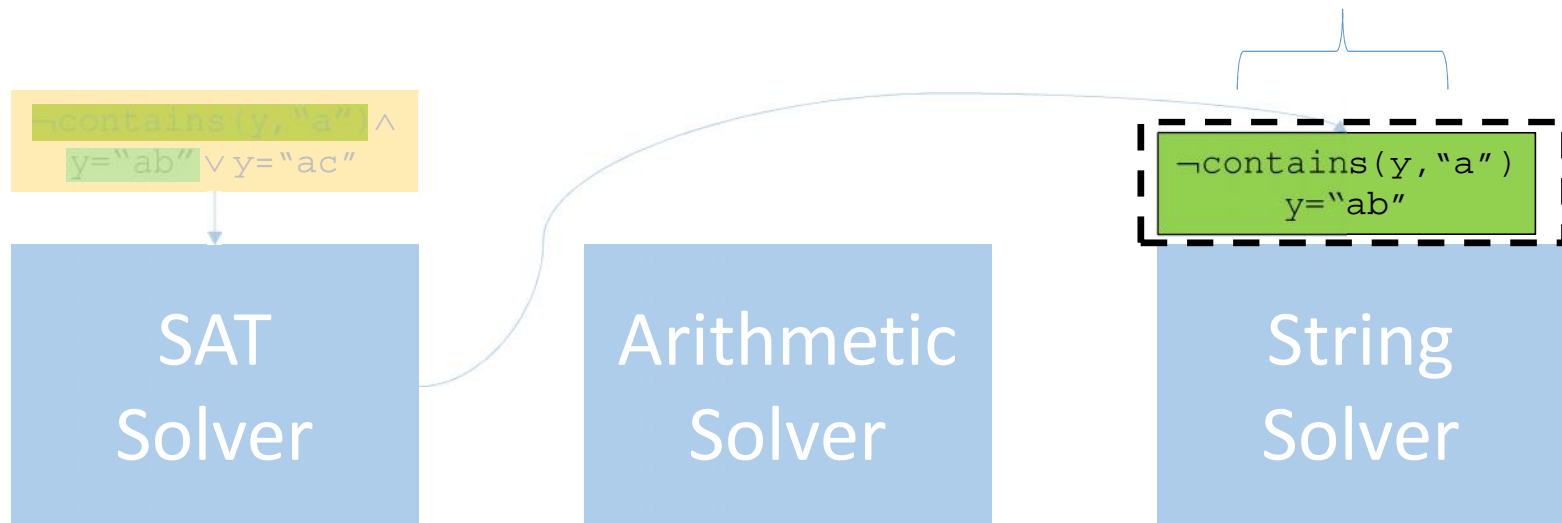


Still have a large constraint

$$\text{contains}(y, "a") \Leftrightarrow \begin{array}{l} (y = z_{11} \cdot k_1 \cdot z_{21} \wedge |z_{11}| = 0 \wedge \dots \wedge y = z_{14} \cdot k_4 \cdot z_{24} \wedge |z_{14}| = 4 \wedge k_1 \neq "a" \wedge \dots \wedge k_4 \neq "a") \end{array}$$

(Lazy) Expansion + Simplification

What if we simplify based on the **context**?



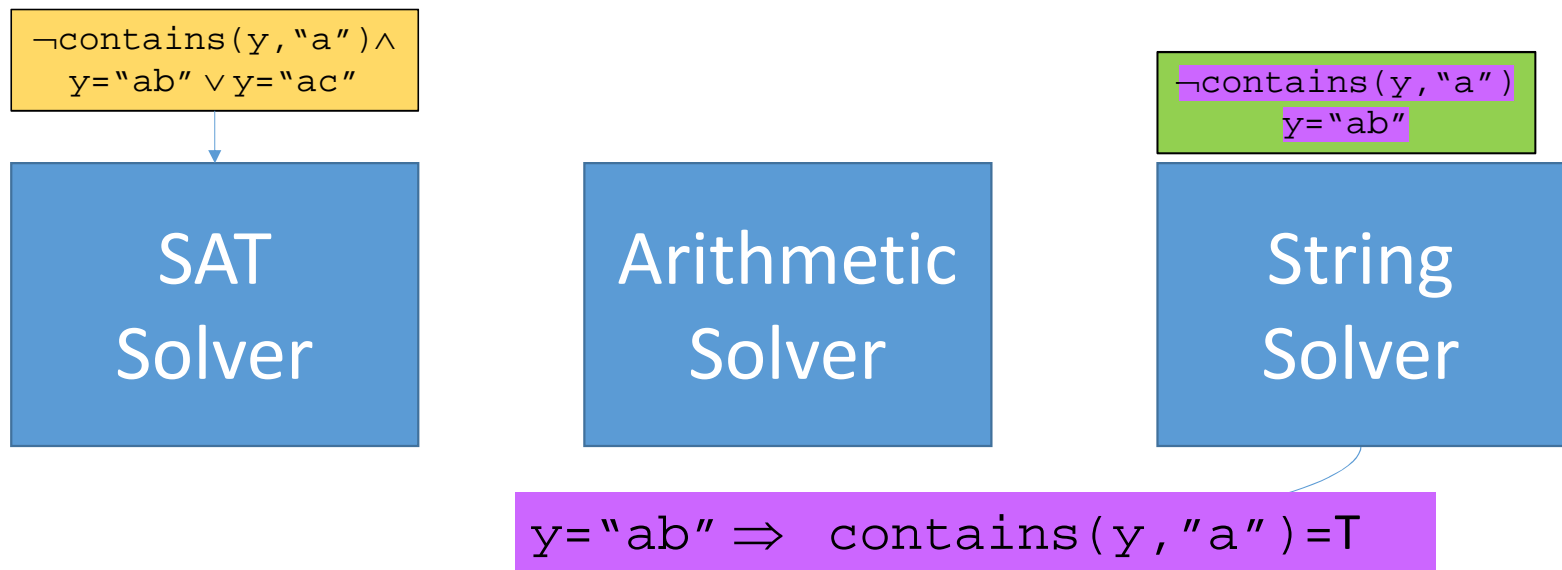
$$\text{contains}(y, "a") \Leftrightarrow (y = z_{11} \cdot k_1 \cdot z_{21} \wedge |z_{11}| = 0 \wedge k_1 \neq "a" \wedge \dots \wedge y = z_{14} \cdot k_4 \cdot z_{24} \wedge |z_{14}| = 4 \wedge k_4 \neq "a")$$

(Lazy) Expansion + **Context-Dependent** Simplification

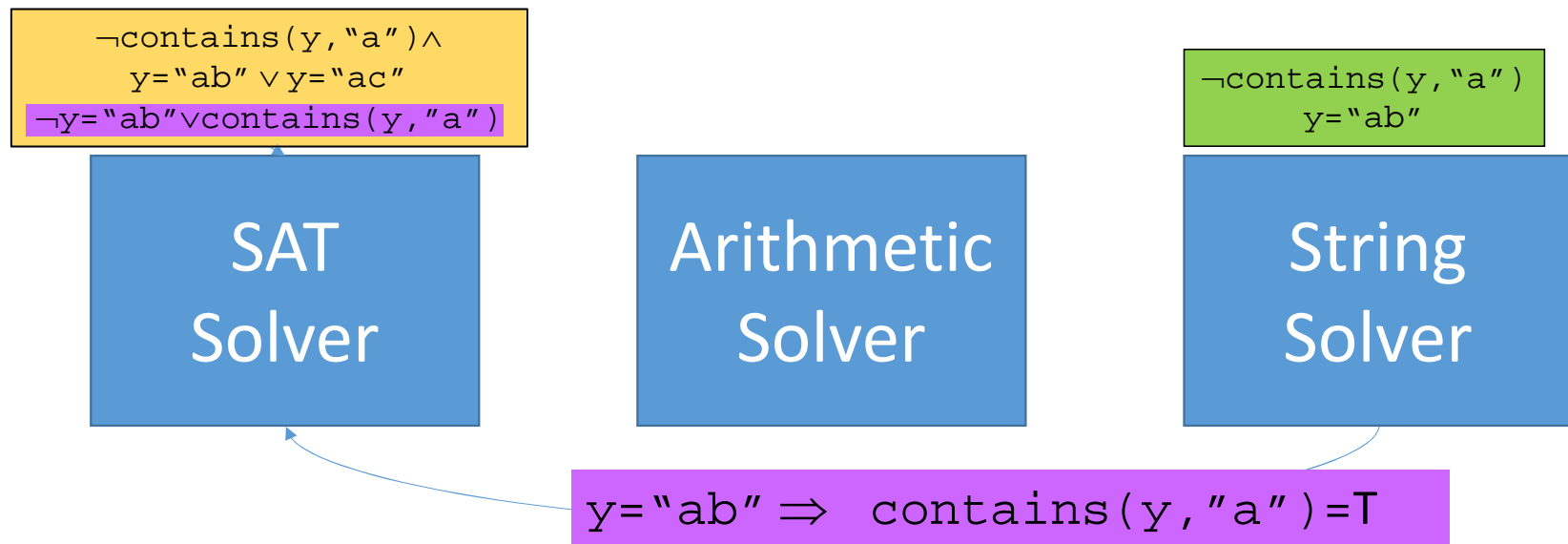


Since $\text{contains}(y, "a")$ is true when $y = "ab"$...

(Lazy) Expansion + Context-Dependent Simplification



(Lazy) Expansion + Context-Dependent Simplification



(Lazy) Expansion + **Context-Dependent** Simplification

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¬contains(y, "a") ∧  
y = "ab" ∨ y = "ac"  
¬y = "ab" ∨ contains(y, "a")
```

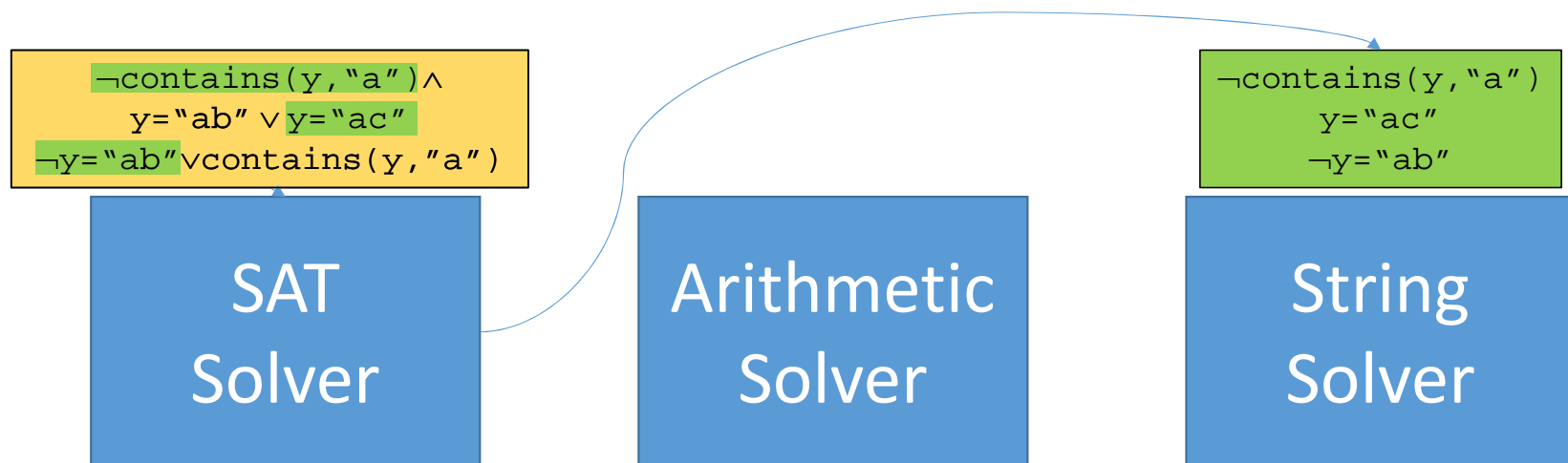
SAT
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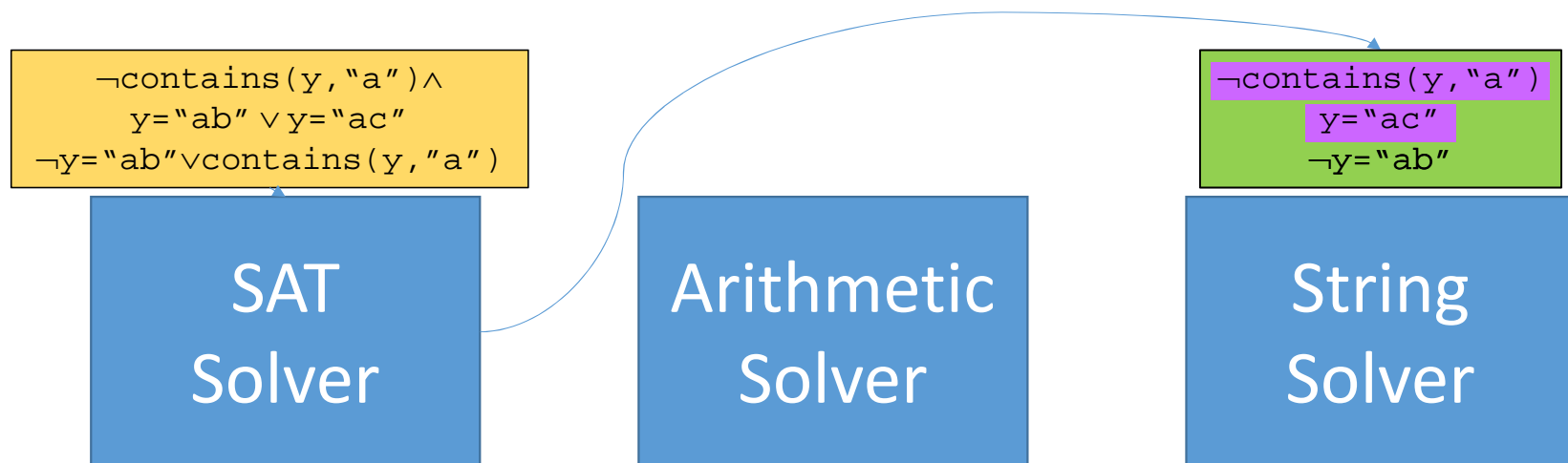
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String
Solver

(Lazy) Expansion + Context-Dependent Simplification

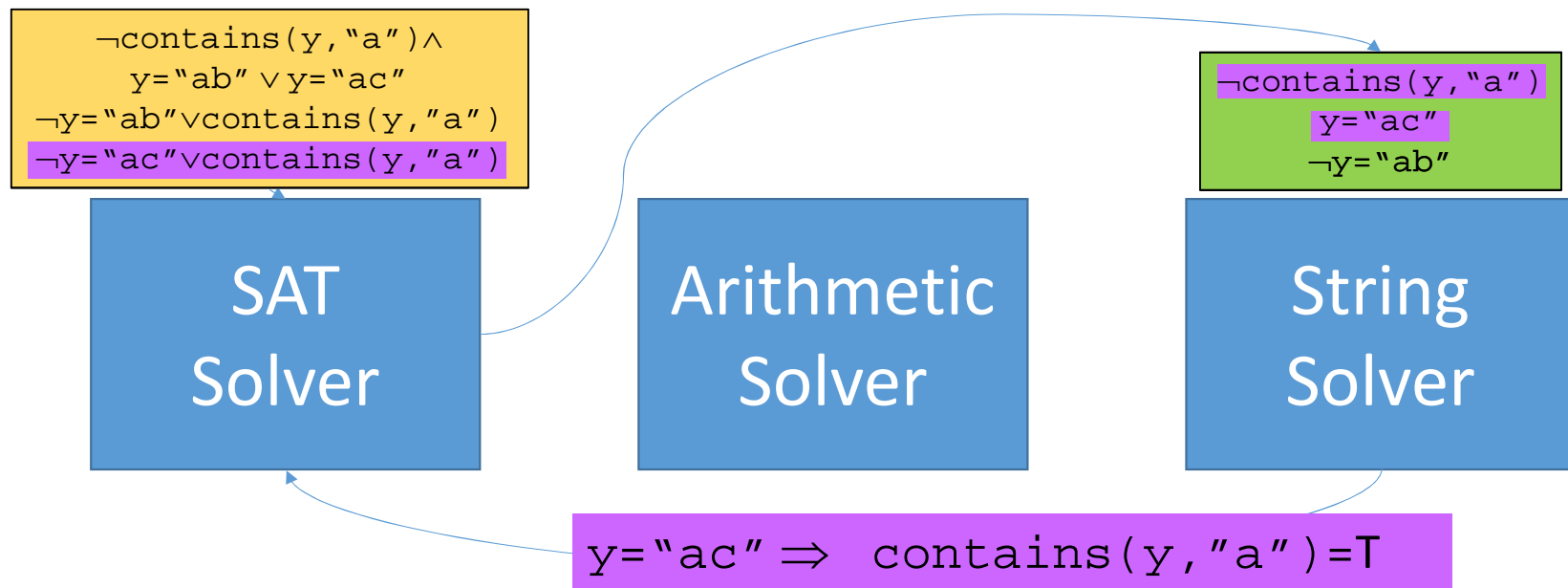


(Lazy) Expansion + **Context-Dependent** Simplification

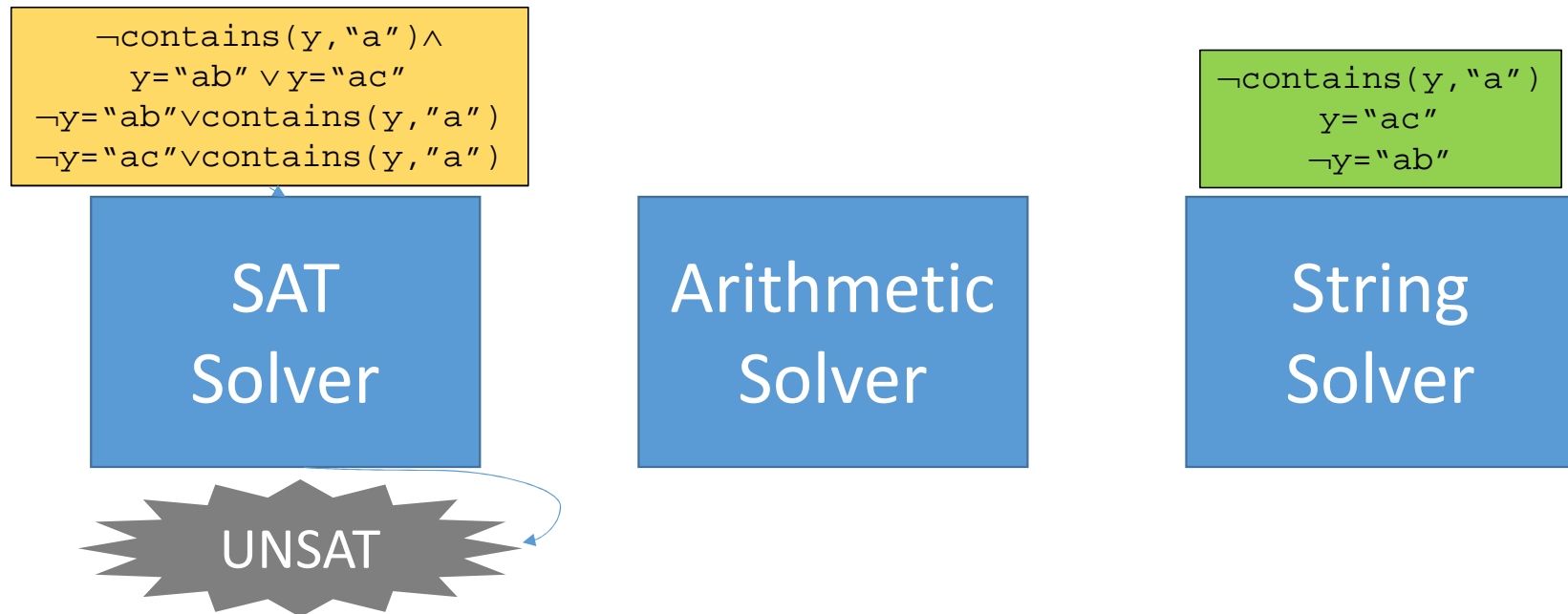


`contains(y, "a")` is also true when `y = "ac"` ...

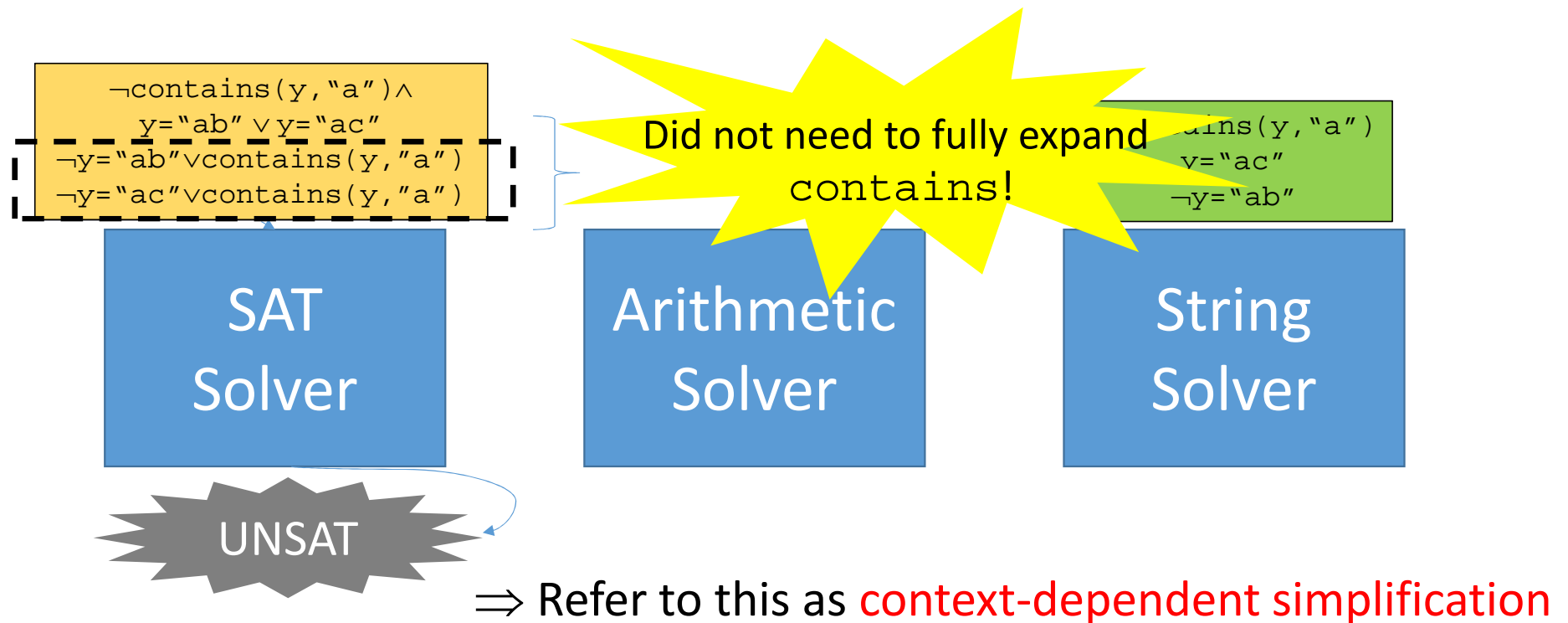
(Lazy) Expansion + Context-Dependent Simplification



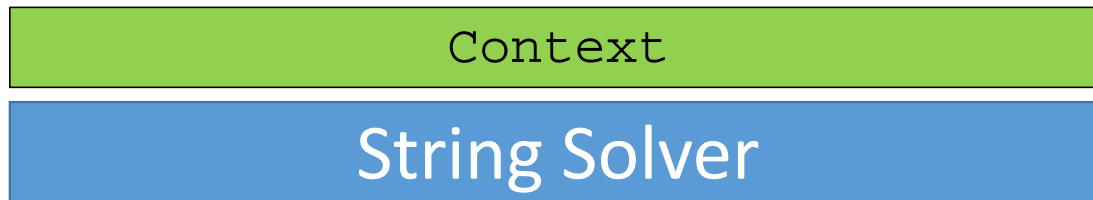
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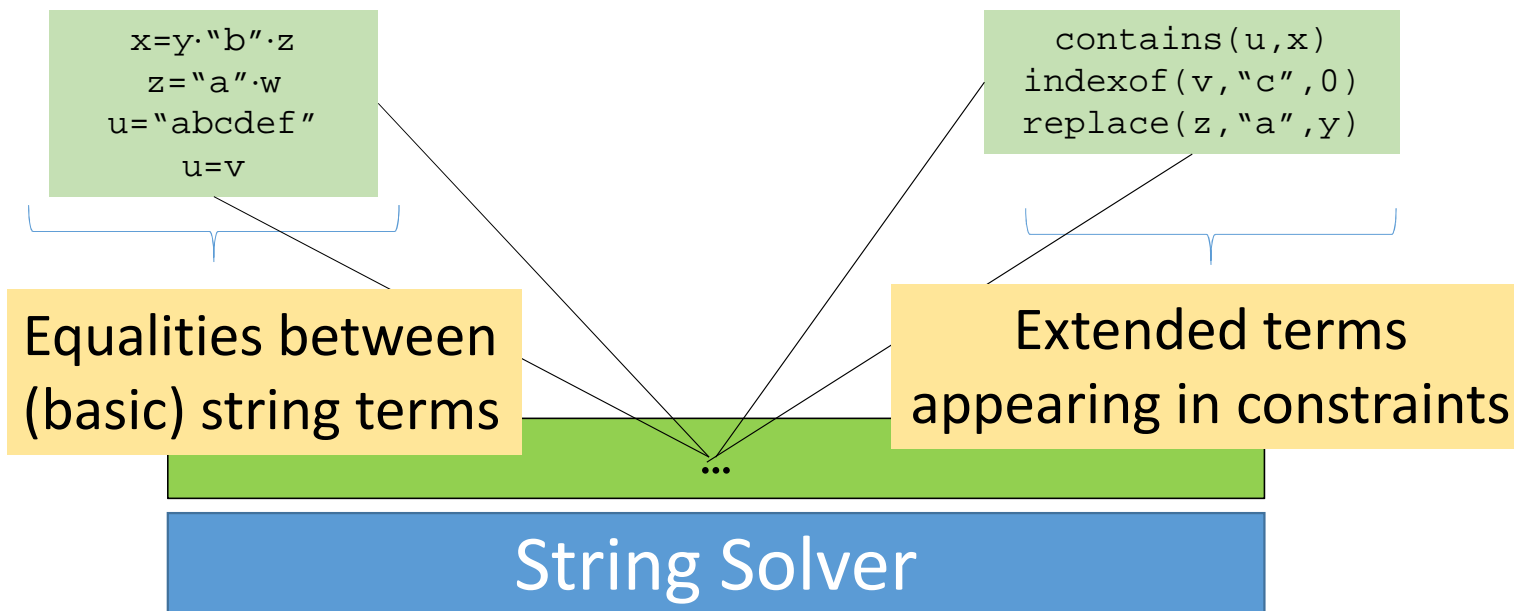
(Lazy) Expansion + Context-Dependent Simplification



Context-Dependent Simplification



Context-Dependent Simplification



Context-Dependent Simplification

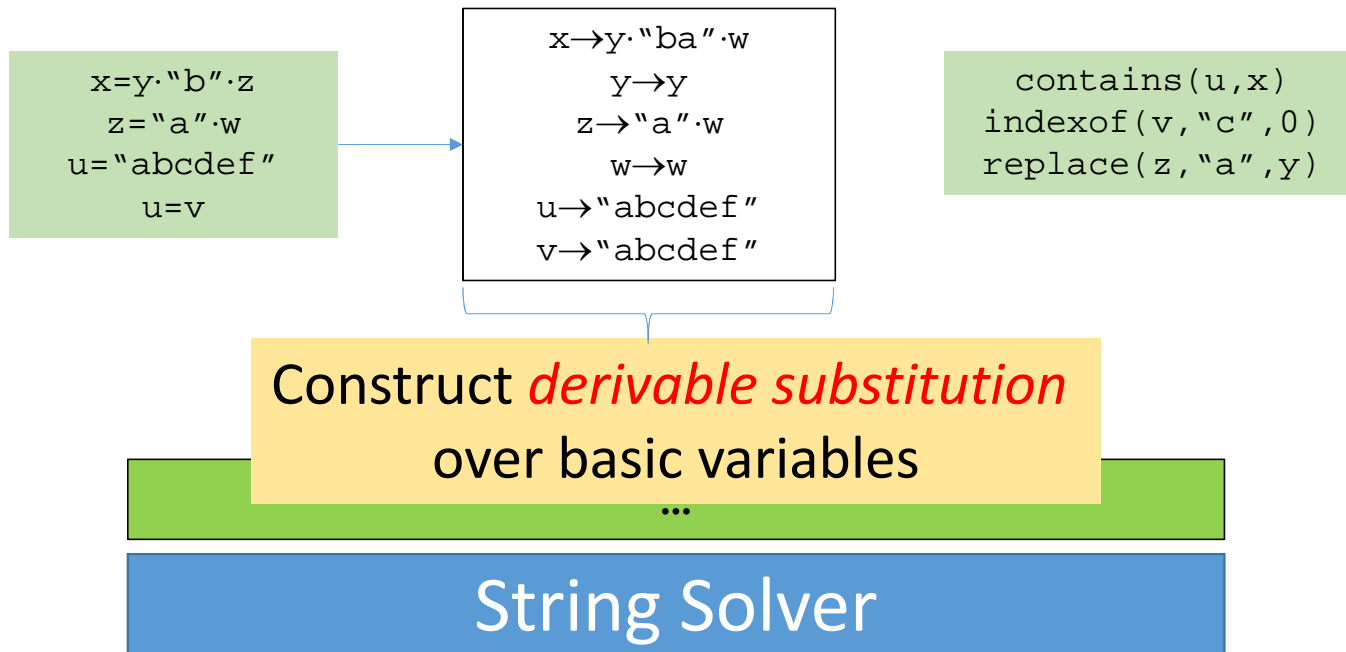
```
x=y·"b"·z  
z="a"·w  
u="abcdef"  
u=v
```

```
contains(u,x)  
indexOf(v,"c",0)  
replace(z,"a",y)
```

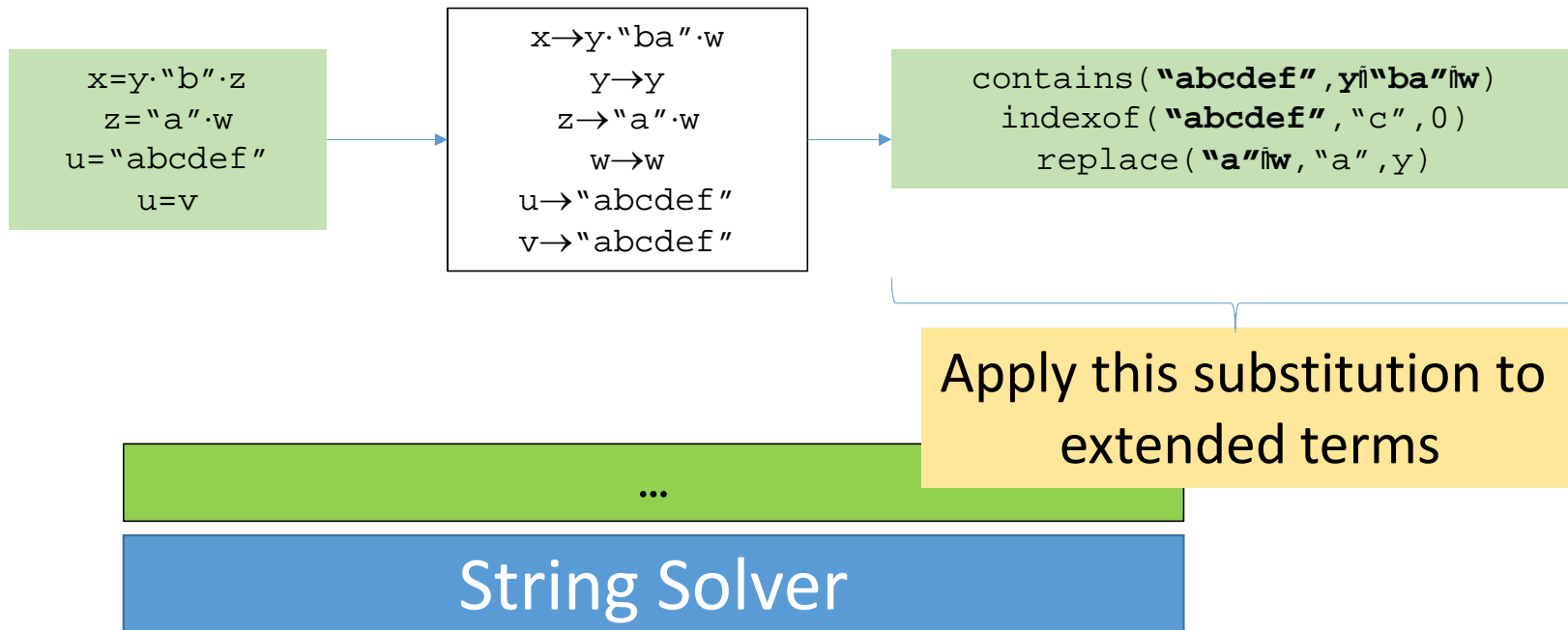
...

String Solver

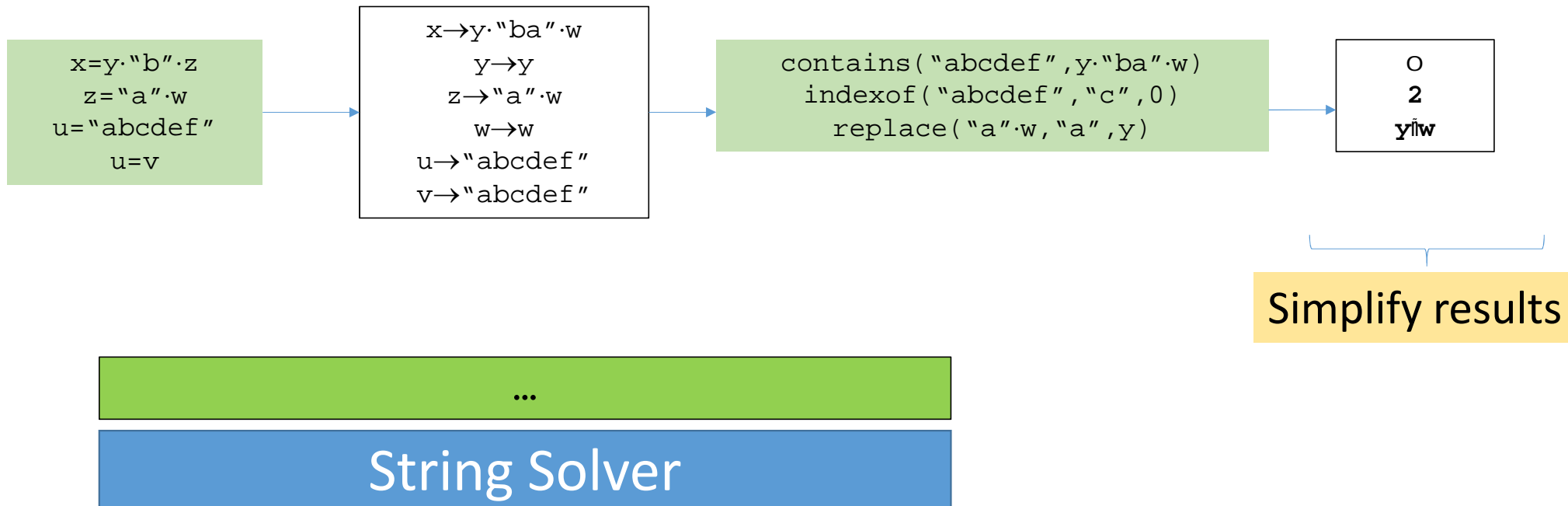
Context-Dependent Simplification



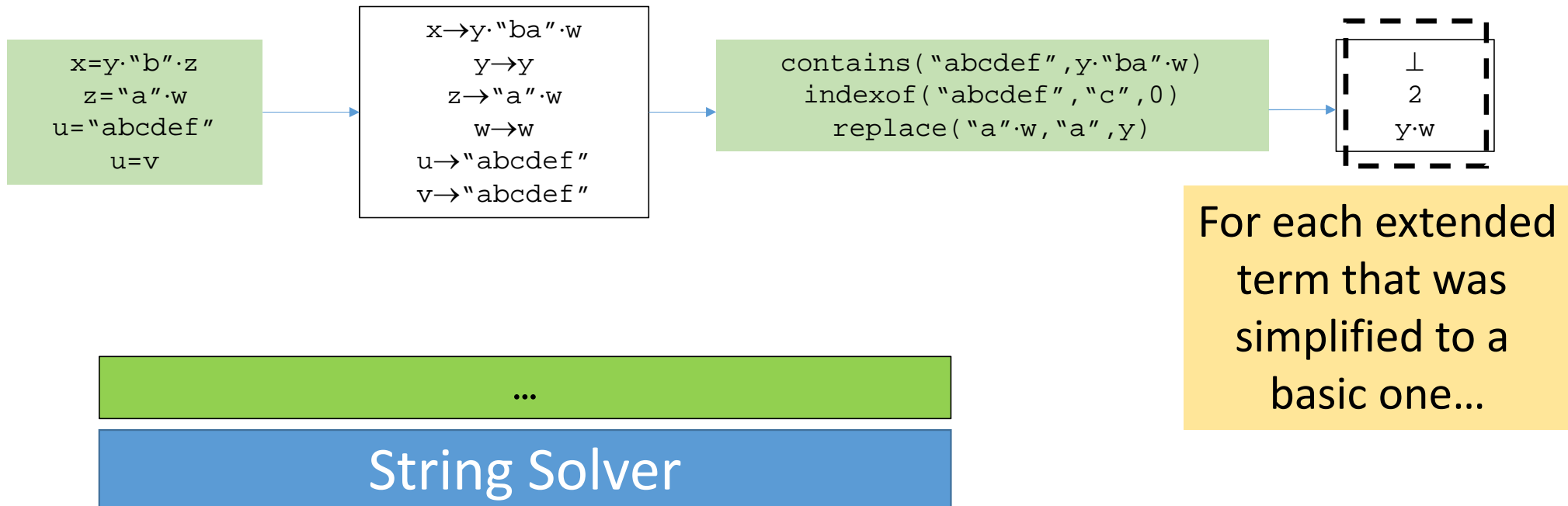
Context-Dependent Simplification



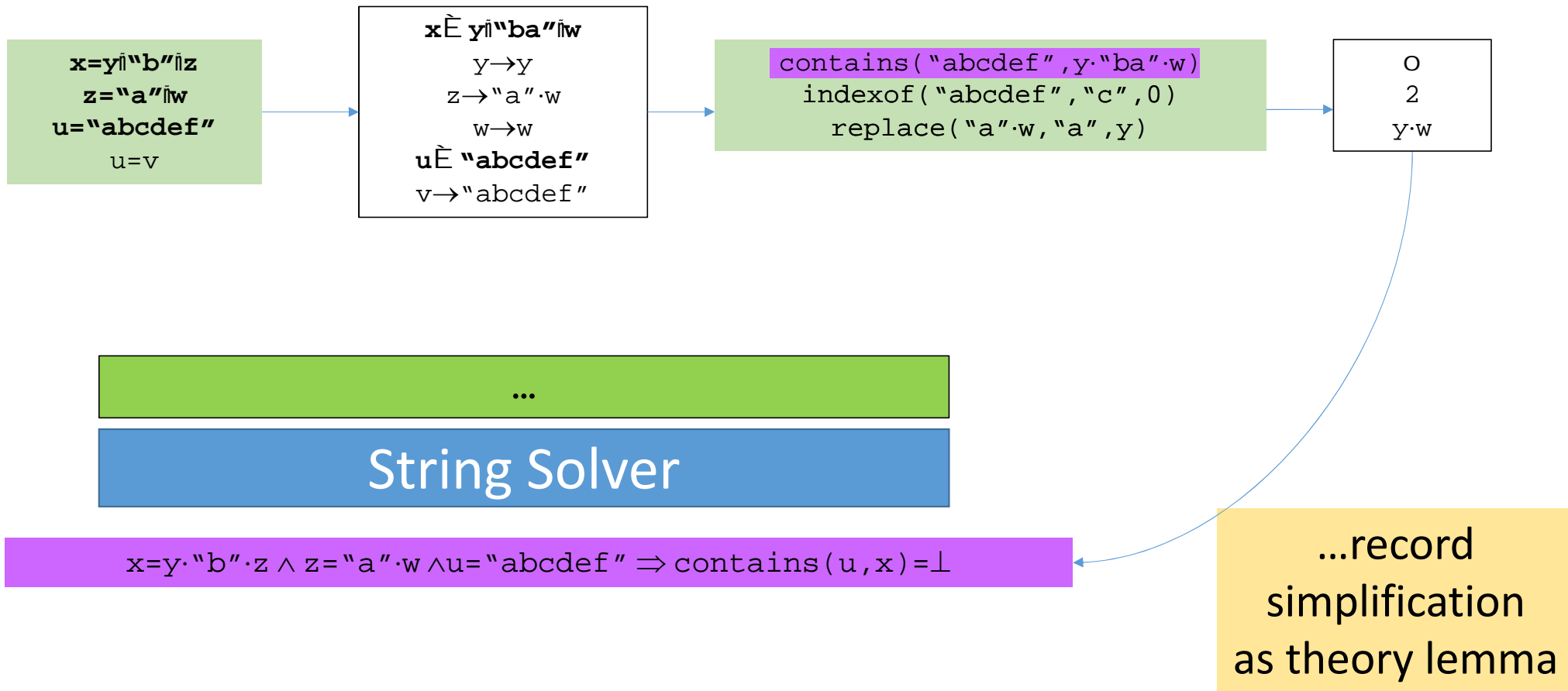
Context-Dependent Simplification



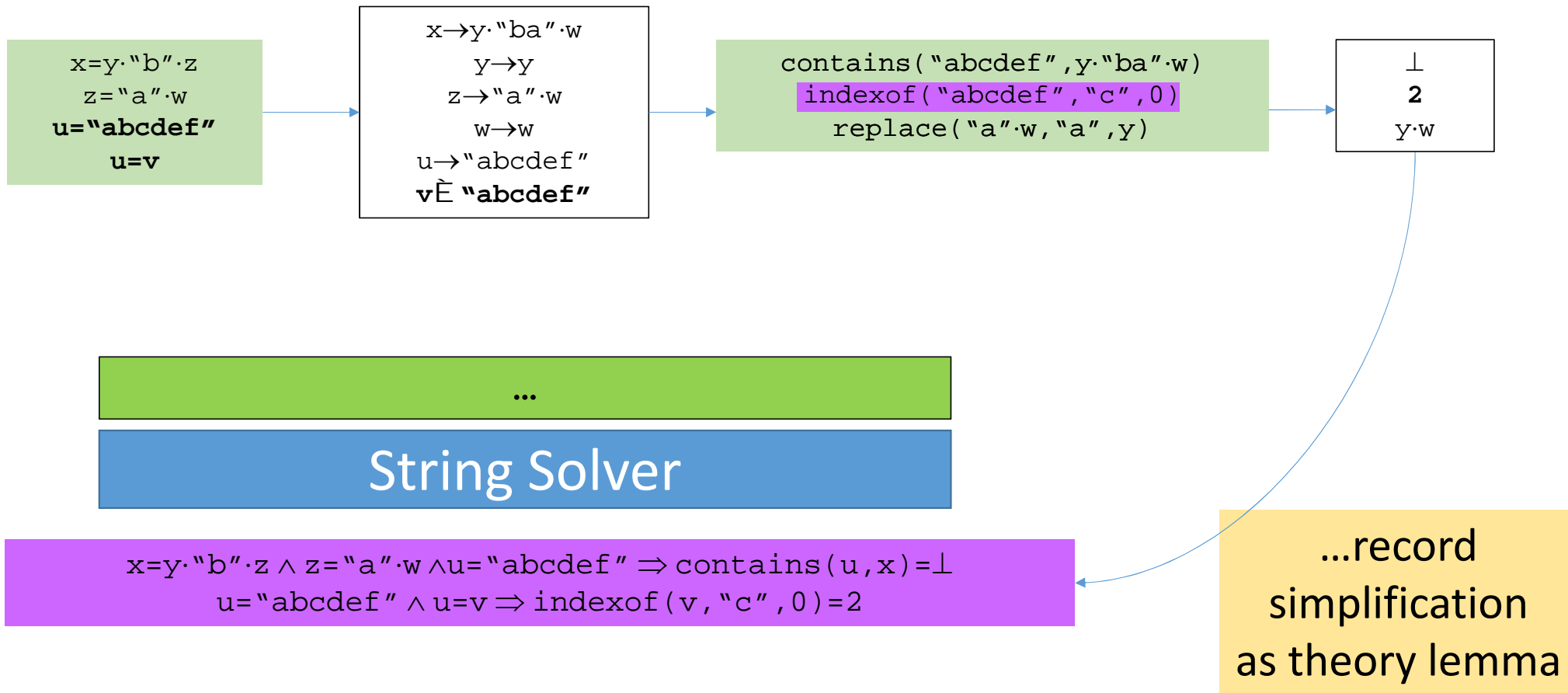
Context-Dependent Simplification



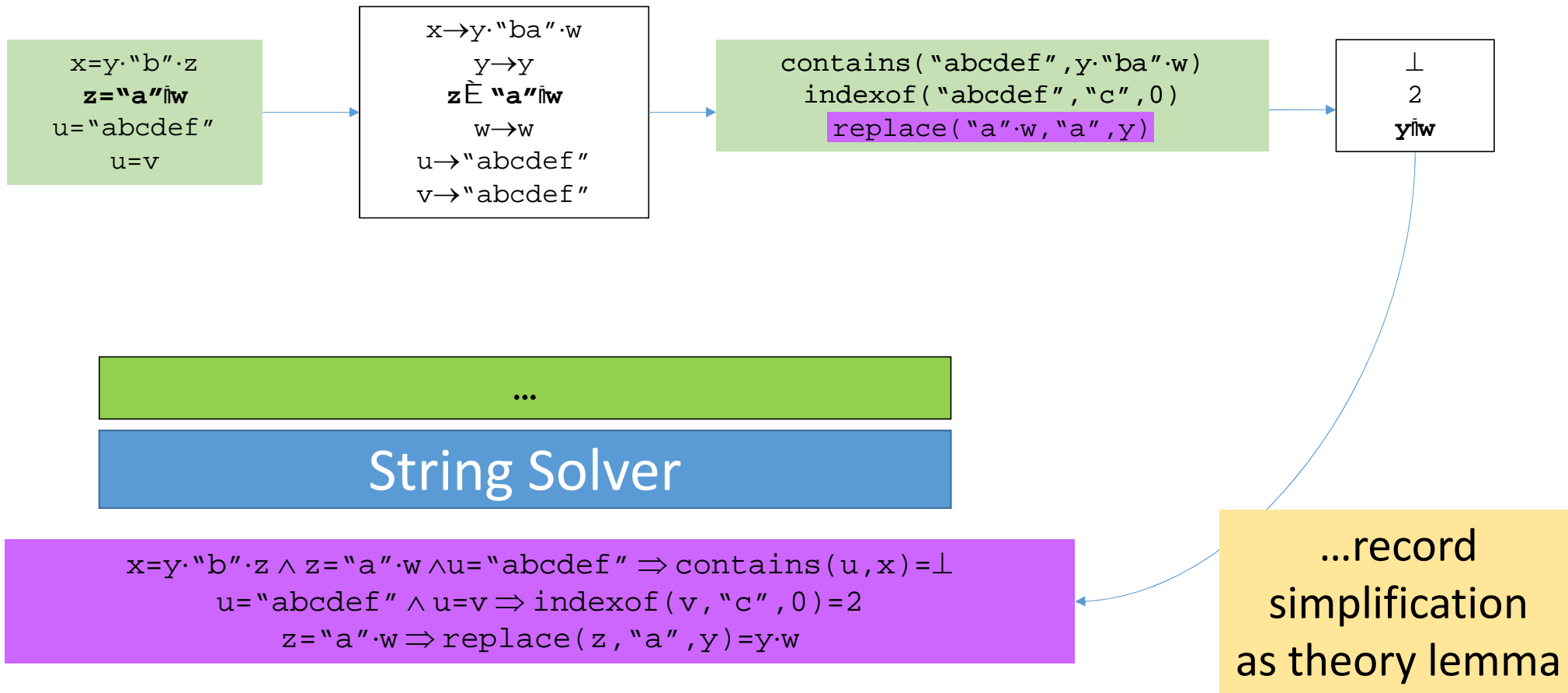
Context-Dependent Simplification



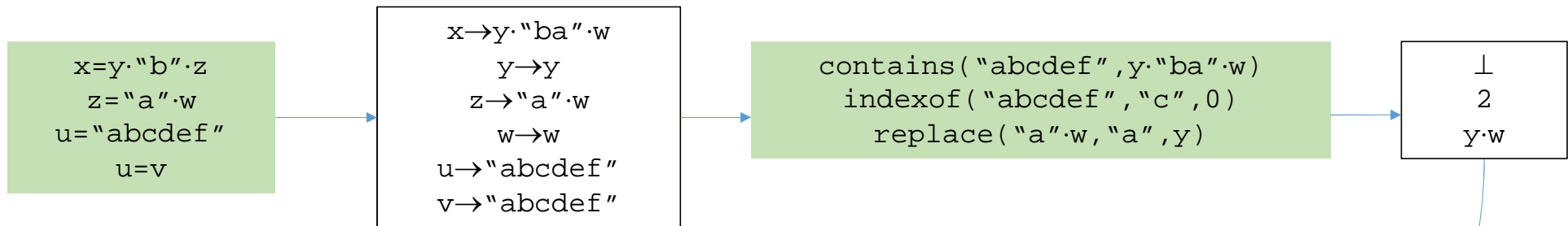
Context-Dependent Simplification



Context-Dependent Simplification



Context-Dependent Simplification



$x=y\cdot"b"\cdot z \wedge z="a"\cdot w \wedge u="abcdef" \Rightarrow \text{contains}(u, x) = \perp$
 $u="abcdef" \wedge u=v \Rightarrow \text{indexof}(v, "c", 0) = 2$
 $z="a"\cdot w \Rightarrow \text{replace}(z, "a", y) = y\cdot w$

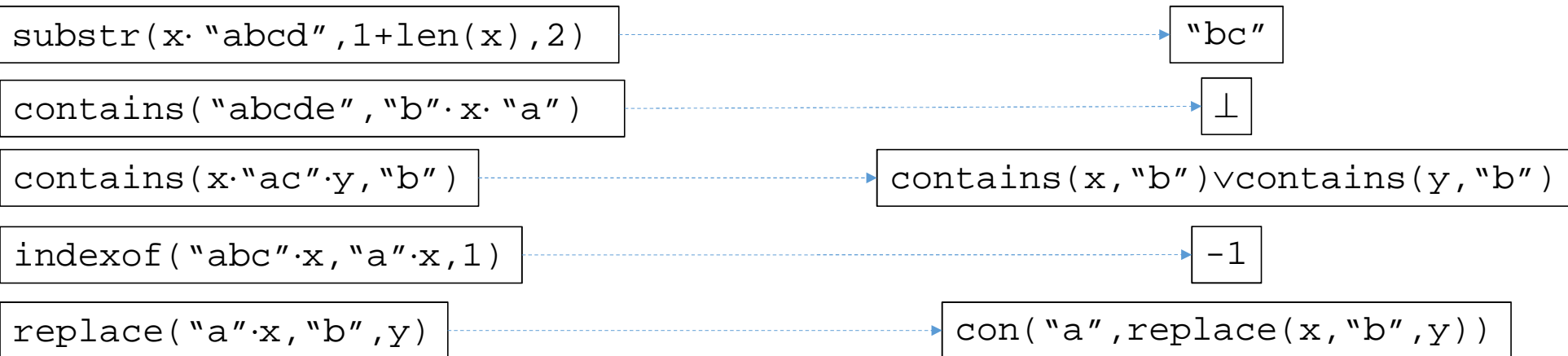
In our dataset,
inference of this
form is possible in
98% of contexts

Simplification Rules for Strings

- Unlike arithmetic:

$$x+x+7*y=y-4 \quad \longrightarrow \quad 2*x+6*y+4=0$$

...**simplification** rules for **strings** are **highly non-trivial**:



- Implemented in 3000+ lines of C++ code

Theoretical Contribution

- Approach described as a rule-based *calculus*, e.g.:

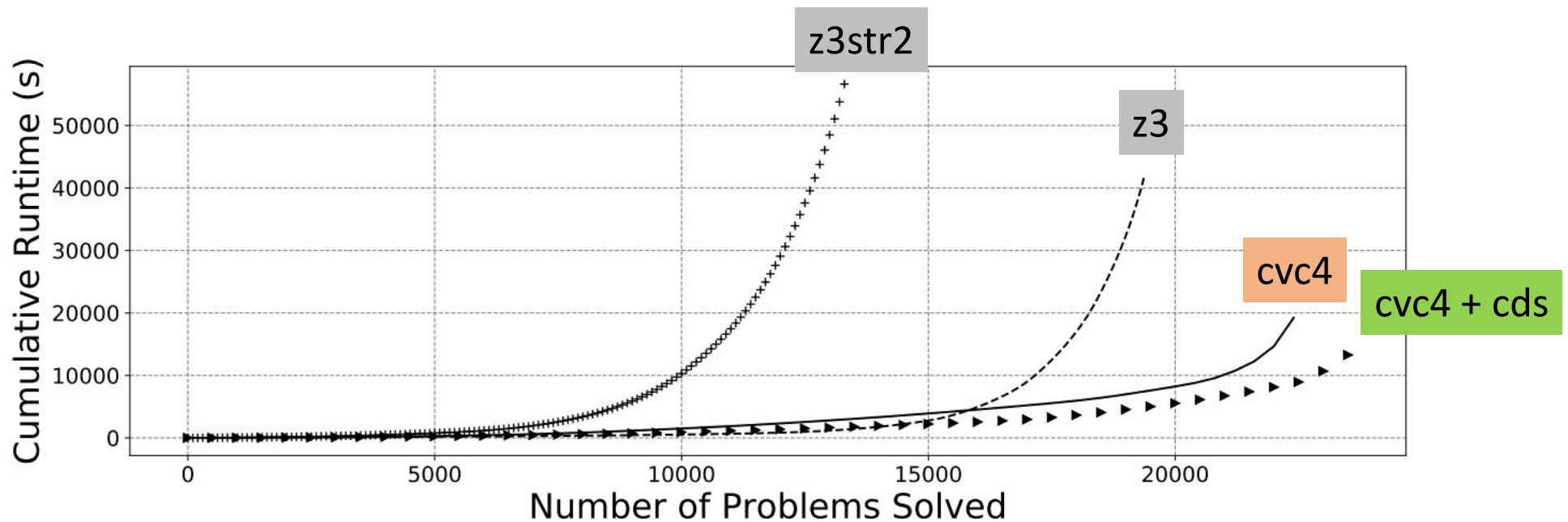
$$\text{Ext-Simplify} \frac{\begin{array}{c} \dots \\ x \approx t \in X \quad E \models \mathbf{y} \approx \mathbf{s} \quad (t\{\mathbf{y} \mapsto \mathbf{s}\})\downarrow \text{ is a } \Sigma_{\text{AS}}\text{-term} \end{array}}{\begin{array}{c} G := G, [x \approx (t\{\mathbf{y} \mapsto \mathbf{s}\})\downarrow] \quad X := X \setminus \{x \approx t\} \\ \dots \end{array}}$$

- Calculus is:
 - **Refutation-sound**
 - **Model-sound**
 - **Not terminating** in general (decidability is still unknown)

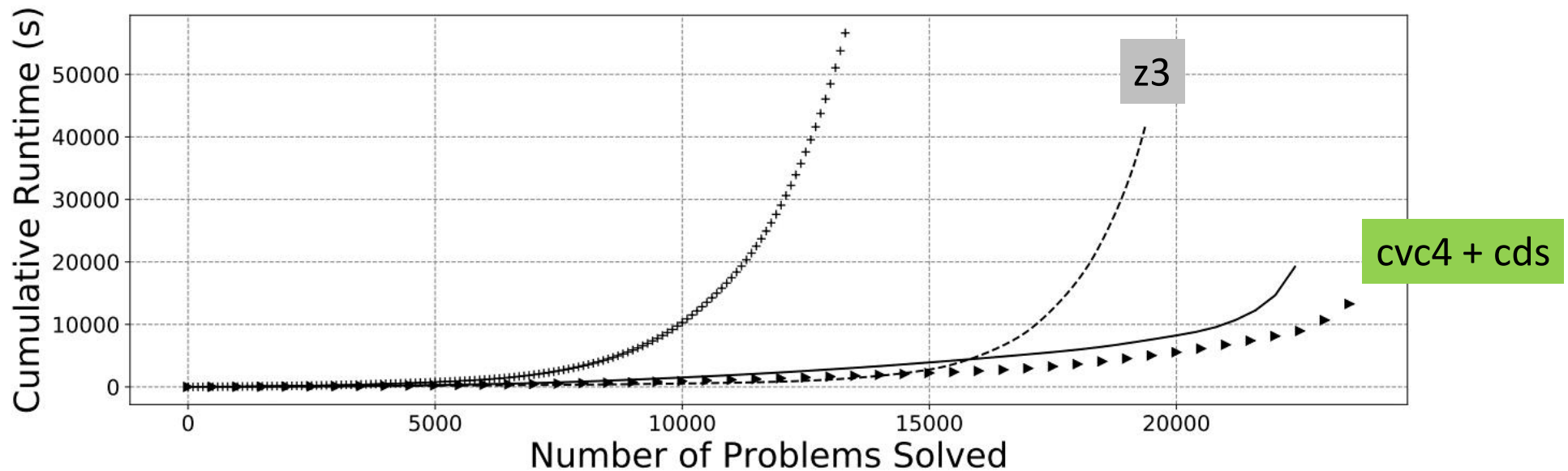
Experimental Results : PyEx Symbolic Execution

- Logged queries from **PyEx symbolic execution engine** (successor of PyExZ3)
 - Using z3str2, z3 and cvc4 as path constraint solver
- Total of **25,421 benchmarks** over 3 runs
- Compared z3str2, z3, cvc4 w, w/o context-dependent simplification (cds)

Results : PyEx Symbolic Execution Benchmarks (25,421)

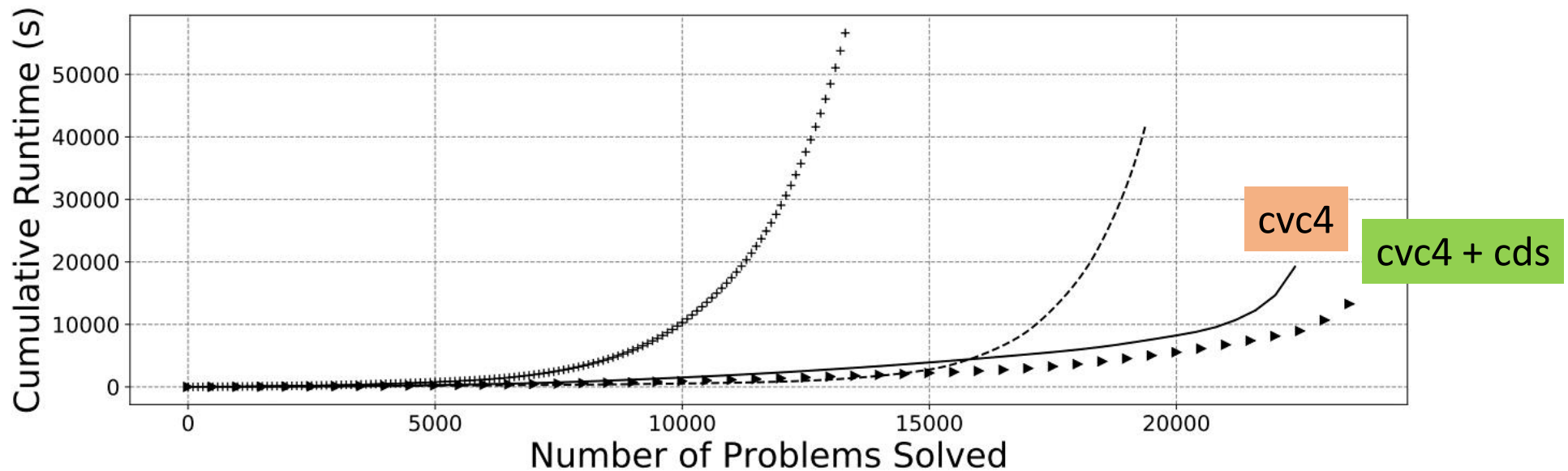


Results : PyEx Symbolic Execution Benchmarks (25,421)



- **cvc4+context-dependent simplification** solves 23,802 benchmarks in 5h8m
 - Nearest competitor **z3** solves 19,368 benchmarks in 11h33m

Results : PyEx Symbolic Execution Benchmarks (25,421)



- By using context-dependent simplification:
 - **cvc4+cds** solves 536 benchmarks (+582 -46) w.r.t default **cvc4**
 - **cvc4+cds** expands 4.2x fewer extended terms per benchmark

Impact on PyEx Symbolic Execution

- Considered regression tests for 4 Python packages:
 - `httplib2`, `pip`, `pymongo`, `requests`
- Tested PyEx using different SMT backends:

Config	Time	Branch Coverage	Line Coverage
PyEx+z3str2	13h49m	3,500	8.34%
PyEx+z3	11h57m	3,895	8.41%
PyEx+cvc4	4h55m	3,612	8.48%

⇒ PyEx+cvc4 achieves comparable program coverage, much faster, wrt other solvers

Summary

- New technique **context-dependent simplification** implemented in CVC4's string solver
- Improves **scalability** on extended string constraints
- PyEx + CVC4 achieves **comparable program coverage** using **41% of the runtime** as PyEx + nearest competitor

Future Work

- More **aggressive simplification rules** for strings
 - More powerful rules → better performance
- **Quality of models** for PyEx symbolic execution
 - Which models lead to higher code coverage?
- Apply context-dependent simplification to **other theories**:
 - Non-linear arithmetic
 - Lazy bit-blasting approaches to bit-vectors
 - ⇒ See [\[Reynolds et al FroCoS 2017\]](#) for details
- Simplification directly benefits **Syntax-Guided Synthesis** for strings in CVC4

- String solver in CVC4

- Open source
- Available at : <http://cvc4.cs.stanford.edu/web/>



- 25,421 new benchmarks from PyEx (*.smt2)

- Available at : <http://cvc4.cs.stanford.edu/papers/CAV2017-strings/>

- ...Thanks for listening!