

$$t = \frac{X - \mathbb{E}(X)_0}{SE[\mathbb{E}(X)_0]} \Rightarrow t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{r}{\left(\frac{\sqrt{1-r^2}}{\sqrt{n-2}}\right)} \Rightarrow r_Z \sim N\left(0, \sqrt{\frac{1-r^2}{n-2}}\right) \text{ when } H_0 : \rho = 0$$

$$\text{However, when } H_0 : \rho = \rho_0 \text{ and } \rho_0 \neq 0 \Rightarrow r_Z \sim N\left(\rho_{0Z}, \sqrt{\frac{1}{n-3}}\right)$$

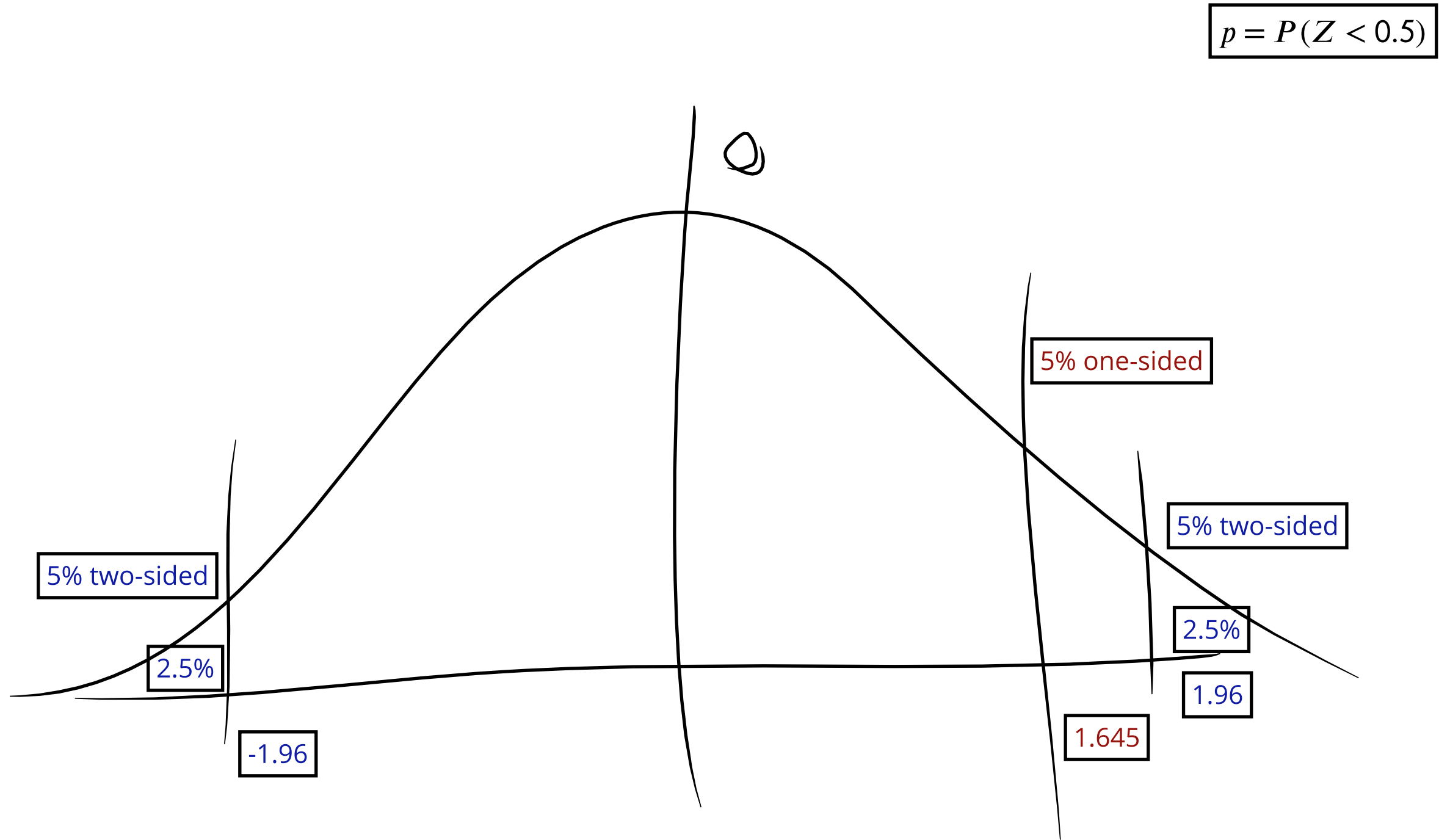
$$y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \dots + \beta_p X_{pi} + \varepsilon_i; \quad i = 1, \dots, n$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{bmatrix} \beta_0 + \beta_1 X_{11} + \beta_2 X_{21} + \dots + \beta_p X_{p1} + \varepsilon_1 \\ \beta_0 + \beta_1 X_{12} + \beta_2 X_{22} + \dots + \beta_p X_{p2} + \varepsilon_2 \\ \vdots \\ \beta_0 + \beta_1 X_{1n} + \beta_2 X_{2n} + \dots + \beta_p X_{pn} + \varepsilon_n \end{bmatrix} = X\vec{\beta} + \vec{\varepsilon} = \begin{bmatrix} 1 & X_{11} & X_{21} & \dots & X_{p1} \\ 1 & X_{12} & X_{22} & \dots & X_{p2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{1n} & X_{2n} & \dots & X_{pn} \end{bmatrix}$$

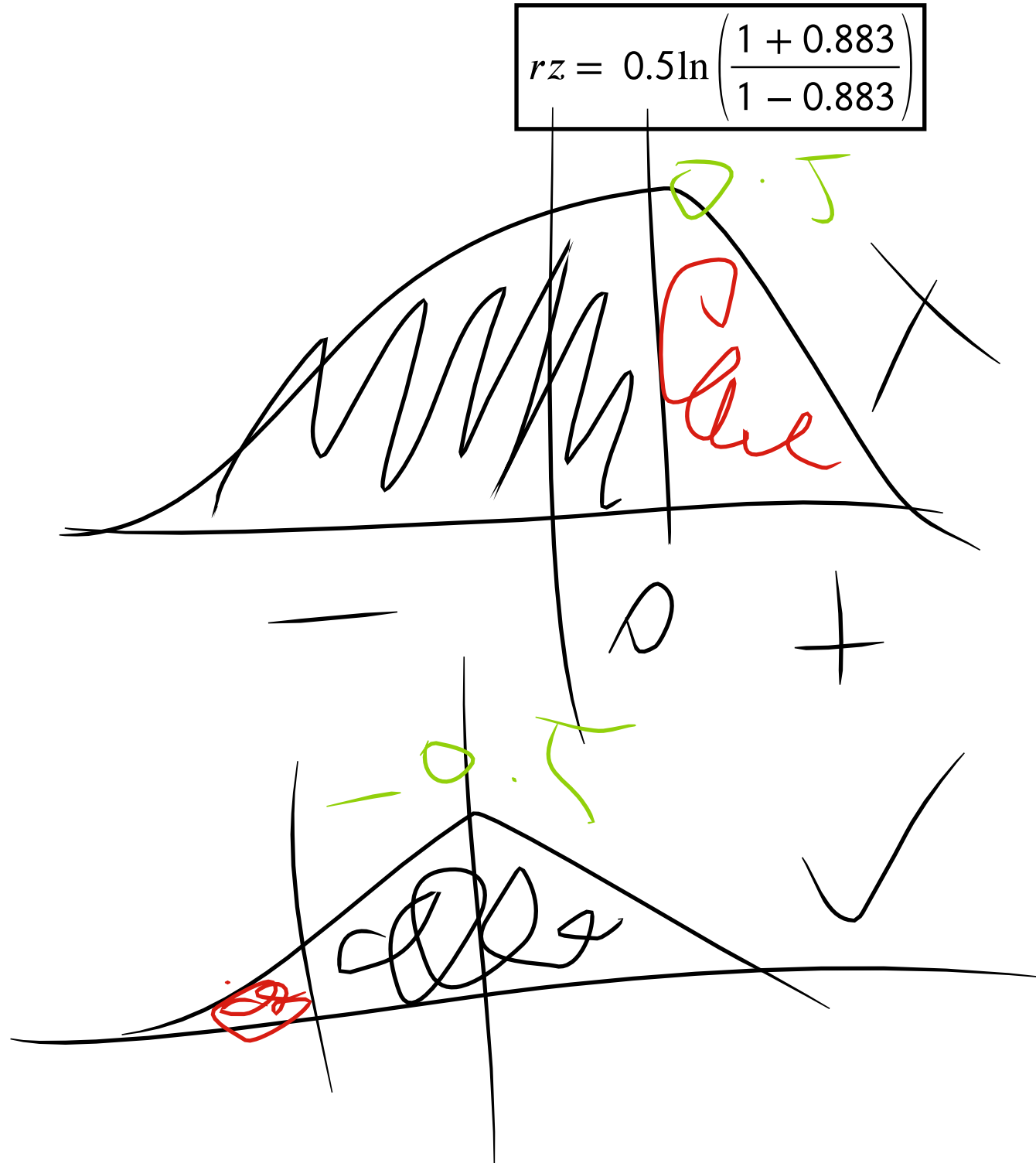
we have n equations and $(p + 1)$ unknowns. In general if $n = p + 1$, then there is no problem

If $n > p + 1$, then it is called an "overdetermined system" – this is not realistic

If $n < p + 1$, then it is an "underdetermined system" – then cannot determine the β coefficients



$$p = P(Z < 0.5)$$



For SLR $y = \alpha + \beta x$
album sales $= \beta_0 + \beta_1 \text{adverts}$
 $H_0 : \rho = 0$ equivalent $\beta_1 = 0$
 $H_A : \rho \neq 0$ equivalent $\beta_1 \neq 0$

$$9.6 = t_\rho = t_{\beta_1} = \frac{b_1}{SE_{b_1}}$$

p – value for β_1 is the same p – value for ρ

For MLR $y = \alpha + \sum \beta x$

$H_0 : \rho = 0$ equivalent $\beta_1 = \beta_2 = \dots = \beta_p$ (and statement)
 $H_A : \rho \neq 0$ equivalent (or statement) one of the β 's $\neq 0$

$H_0 : \rho = 0$ and $r = 0.578$

$$t = \frac{0.578\sqrt{398}}{\sqrt{1-0.335}} = \frac{r-0}{SE_r} \Rightarrow SE_r = \sqrt{\frac{1-0.335}{398}} = 0.041$$

$$r_z = 0.5 \ln\left(\frac{1+0.578}{1-0.578}\right) = 0.659 \sim N\left(0, \sqrt{\frac{1-r^2}{n-2}}\right) = N(0, 0.041)$$

CI for $\rho_z : r_z \pm z^* \cdot SE_{r_z} = 0.659 \pm 1.96 \cdot 0.041 = (LB = 0.578, UB = 0.739)$

$$CI \text{ for } \rho : \left(\frac{e^{2 \cdot LB} - 1}{e^{2 \cdot LB} + 1}, \frac{e^{2 \cdot UB} - 1}{e^{2 \cdot UB} + 1} \right) = (0.522, 0.629)$$