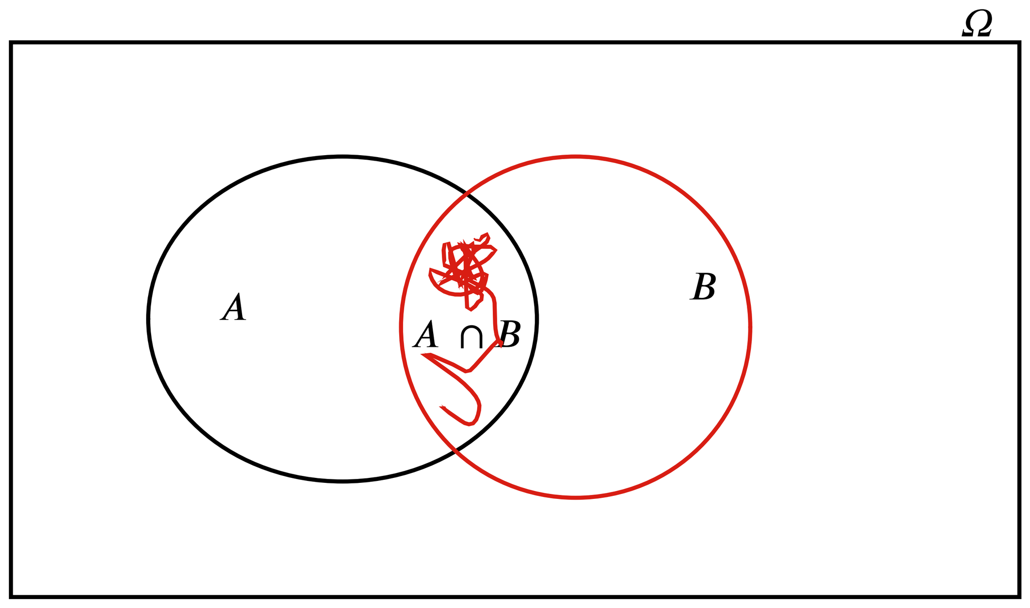


$$\textit{Bayes Rule}: \ \mathbb{P}(A|B)=\frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)} \ \textit{AND} \ \mathbb{P}(B|A)=\frac{\mathbb{P}(A\cap B)}{\mathbb{P}(A)} \ \Rightarrow \ \mathbb{P}(A|B)=\frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)}=\frac{\mathbb{P}(B|A)\cdot\mathbb{P}(A)}{\mathbb{P}(B)}=\frac{\textit{likelihood}\ \cdot \textit{prior}}{\textit{constant}}=\textit{posterior}$$

$$\textit{Alternatively}: \mathbb{P}(A\cap B)= \ \mathbb{P}(A|B)\cdot\mathbb{P}(B) \ \textit{OR} = \ \mathbb{P}(B|A)\cdot\mathbb{P}(A)$$

$$\textit{posterior} \propto \textit{likelihood} \times \textit{prior}$$

$$\textit{Bayes Factor}: \ BF_{10}=\frac{\mathbb{P}\left(\textit{Data} \mid H_1 \textit{true}\right)}{\mathbb{P}\left(\ \textit{Data}| \ H_0 \ \textit{true}\right)}$$

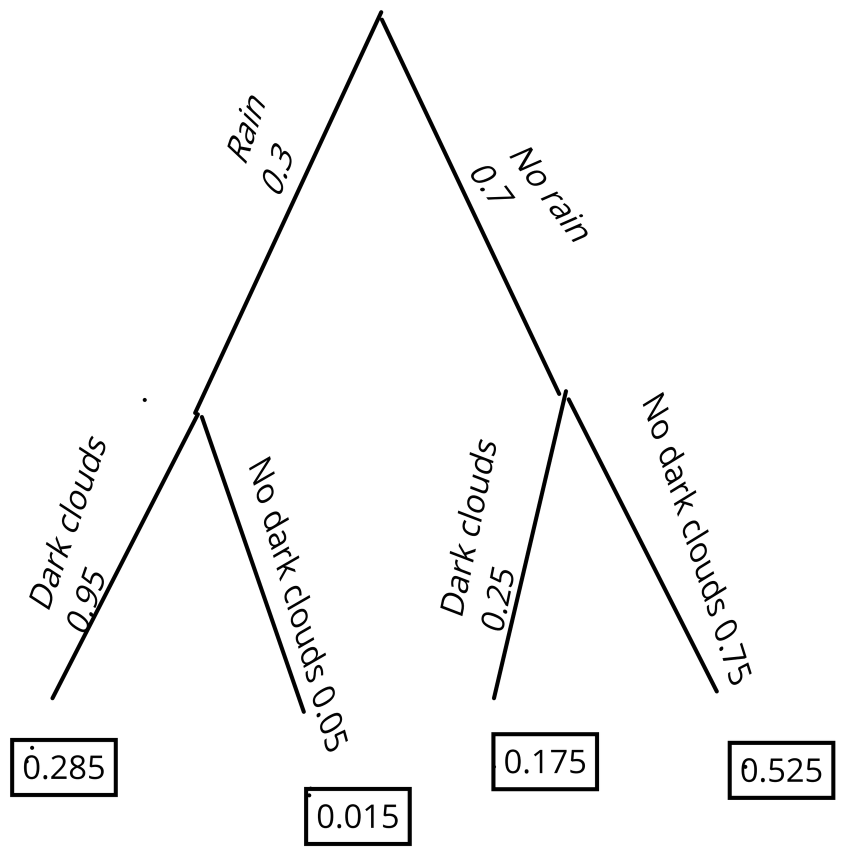
$$\Rightarrow \mathbb{P}\left(\textit{Data} \ |H_1\textit{true}\right)=\frac{\mathbb{P}\left(H_1\textit{true}\middle| \ \textit{Data}\right)\cdot\mathbb{P}(\textit{Data})}{\mathbb{P}\left(H_1 \ \textit{true}\right)}$$

$$\Rightarrow BF_{10}=\frac{\mathbb{P}\left(H_1\textit{true}\middle| \ \textit{Data}\right)\cdot\mathbb{P}(\textit{Data})}{\mathbb{P}\left(H_1 \ \textit{true}\right)}\cdot\frac{\mathbb{P}\left(H_0 \ \textit{true}\right)}{\mathbb{P}\left(H_0\textit{true}\middle| \ \textit{Data}\right)\cdot\mathbb{P}(\textit{Data})}=\frac{\mathbb{P}\left(H_1\textit{true}\middle| \ \textit{Data}\right)\cdot\mathbb{P}\left(H_0 \ \textit{true}\right)}{\mathbb{P}\left(H_0\textit{true}\middle| \ \textit{Data}\right)\cdot\mathbb{P}\left(H_1 \ \textit{true}\right)}$$

$\textit{Large } BF_{10}=100 \ \Rightarrow$ *the probability of the data under the alternative hypothesis is* 100 *times more likely than the null hypothesis*

$\textit{Small } BF_{10}=0.0001 \ \Rightarrow$ *the prob of the data under the alt is* 0.0001 *times less likely than the null, or the null is* $1/0.0001=10000\left(=BF_{01}\right)$ *times more likely than the alt*

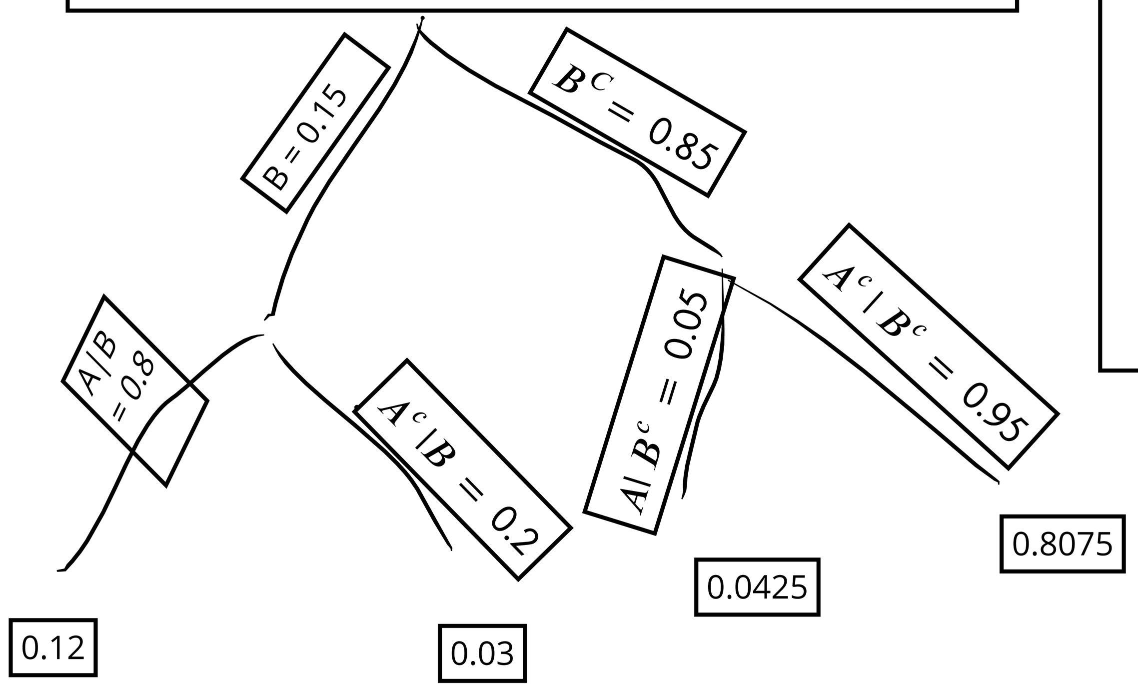
$$\mathbb{P}(\textit{Rain})=0.3$$
$$\mathbb{P}(\textit{No rain})=0.7$$
$$\mathbb{P}(\textit{Dark clouds} \mid \textit{Rain})=0.95$$
$$\mathbb{P}(\textit{Dark clouds} \mid \textit{No rain})=0.25$$
$$\mathbb{P}(\textit{Rain} \mid \ \textit{Dark clouds}) \ =\frac{\mathbb{P}(\textit{Dark clouds} \mid \ \textit{Rain})\cdot\mathbb{P}(\textit{Rain})}{\mathbb{P}(\textit{Dark clouds})}$$

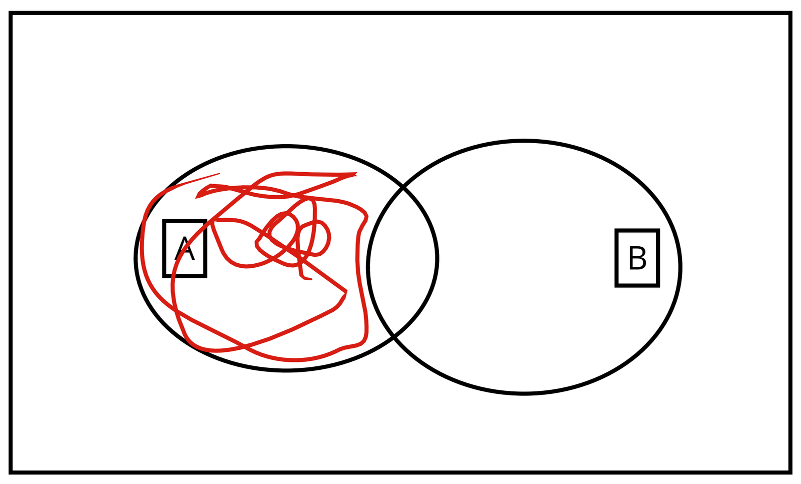


$$\boxed{\begin{array}{c}\textit{Marginal}: \ \mathbb{P}(\textit{Dark clouds})=\displaystyle\sum_{A\in\Omega}\mathbb{P}(\textit{Dark clouds} \mid A) \ \cdot\mathbb{P}(A)=\sum_{A\in\Omega}\mathbb{P}(\textit{Dark clouds} \cap A)\\ \textit{Conditional}: \ \mathbb{P}(\textit{Dark clouds} \mid A)\\ \textit{Joint}: \ \mathbb{P}(\textit{Dark clouds} \cap A)\end{array}}$$

	Rain	No Rain	
Dark clouds	$=0.95 \cdot 0.3$	$=0.25 \cdot 0.7$	Σ
No dark clouds	$=0.05 \cdot 0.3$	$=0.75 \cdot 0.7$	Σ
	0.3	0.7	

A : *computer flags IRS form*
B : *IRS form contains mistakes or errors*
$A \mid B$: *computer correctly flags the mistake*
$A^C \mid B$: *computer incorrectly doesn't flag a mistake*
$A \mid B^C$: *computer incorrectly flags the mistake*
$A^C \mid B^C$: *computer correctly does not flag a mistake*
$B \mid A$: *mistake found in file flagged by the computer*
$B^C \mid A$: *no mistake but flagged by the computer*
$B \mid A^C$: *mistake not flagged by the computer*
$B^C \mid A^C =$ *no mistake but not flagged by the computer*





$$\boxed{\mathbb{P}(B|A)=\frac{\mathbb{P}(A|B) \ \cdot\mathbb{P}(B)}{\mathbb{P}(A)}=\frac{0.12}{0.12 \ + \ 0.0425}=0.739}$$

$$\mathbb{P}(A \ \cap B)=\mathbb{P}(A \mid B) \ \cdot\mathbb{P}(B)=\mathbb{P}(B \mid A) \ \cdot\mathbb{P}(A)$$

$$\boxed{\begin{array}{c}\textit{posterior} \ =\dfrac{\textit{likelihood} \ \cdot \textit{prior}}{\textit{constant}}=\dfrac{0.5 \ \cdot 0.75}{0.3975}\\ \textit{prior}: \ \mathbb{P}(A)=0.75\\ \textit{likelihood} \ : \ \mathbb{P}(B|A) \ = \ 0.5\\ \textit{constant} \ : \ \mathbb{P}(B)=0.3975\\ \textit{posterior} \ : \ \mathbb{P}(A|B)\end{array}}$$

	B	B^c	Marginal
A	$\mathbb{P}(A \ \cap B)$ $=0.5\cdot 0.75$ $=0.375$	$\mathbb{P}\left(A \ \cap B^C\right)$ $=0.75-0.375$	$\mathbb{P}(A)=0.75$
A^c	$\mathbb{P}(A^C\cap B)$ $=0.09\cdot 0.25$ $=0.0225$	$\mathbb{P}\left(A^C \ \cap B^C\right)$ $=0.25 \ - \ 0.0225$	$\mathbb{P}(A^C)=0.25$
Marginal	$\mathbb{P}(B)=0.3975$	$\mathbb{P}(B^C)$	