Statistics I

Exam 2 Example

1. Someone studies the effectiveness of a certain drug against headache by means of an experiment with 102 people, and she wants to know how much lower people scored on a pain scale (with scores ranging from 0-100) one day after taking the drug, compared to half of the group who had received a placebo drug. She finds that the placebo group scored 65 points (standard deviation 10), and the experimental group scores 55 (standard deviation 9) She is not so much interested in whether the effect is there (she assumes it is), but in the size of the effect.

    a. Write down the null hypothesis and the alternative hypothesis in words and in formal language

Given: $\bar{x}_a = 65; s_a = 10; N = 51$. $\bar{x}_b = 55; s_b = 9; N = 51$.

H0: $\mu 1 = \mu 2$
H1: $\mu 1 \neq \mu 2$

We want to know if both groups are the same or if they are different. As the ***σ's*** are known we can use the z-test. Based on the n1 > 30 and n2 > 30 and the ratio of the sample variance between 0,5 and 2 ($10^2/9^2$=1,23). If the ***σ's*** would not be known then we would use the t-test.

However, to estimate the effect size we would be using Cohen's *d:*

$$Cohen's\ d = \frac{\bar{x}_a - \bar{x}_b}{\sqrt{\frac{s_a^2 + s_b^2}{2}}}$$

Cohen's d = (55 - 65)/9.513149 = 1.051177.

H_0:= Cohen's $|d| = 1$;
H_1:= $|d| < 1$.
Cohen's d measures how many standard deviations the means are away from each other, if they were plotted on the same graph. It calculates the difference of means, over a pooled standard deviation. If the absolute value of d is small ($\tilde{}0.2$) or medium ($\tilde{}0.5$), then so is the effect size. If the absolute value of d is near 1, then the difference between the means is 1 standard deviation, so the effect size is large. Our null hypothesis is that that the effect size is large (or substantial), and the alternative hypothesis is that the effect size is not large (either small or medium).

    b. Calculate a 95%-CI for the difference in means

95% CI = $$\boxed{\bar{x}_1 - \bar{x}_2 \pm 1.96\sqrt{\frac{\sigma^2_1}{n_1} + \frac{\sigma^2_2}{n_2}}}$$

95%CI = 10 +/- 1.196
= 8.804 - 11,196

If we would use a pooled estimate then we would use:

$$SE(\bar{x}_1 - \bar{x}_2) = S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

If $n_1 > 30$ and $n_2 > 30$, we can use the z-table:

$$(\bar{x}_1 - \bar{x}_2) \pm z\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Use Z table for standard normal distribution

If $n_1 < 30$ or $n_2 < 30$, use the t-table:\

$$(\bar{x}_1 - \bar{x}_2) \pm t\, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Use the t-table with degrees of freedom = n_1+n_2-2

c. Conduct the significance test for the differences in means (alpha = 5%)

For the z-statistic we would get:

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Z = 5.31 I Think I made here somewhere a mistake ☺

d. After the experiment, she doubts whether to analyze the collected data by means of a confidence interval or a significance test. How would you advise her, assuming you have to pick one? Explain your answer.

2. Significance test are often criticized for not providing the answer that the researcher wants. It is often argued that researchers would like to know the probability that the null hypothesis is true (and hence, what the probability is that it is false). The p-value, however, is not providing that probability. Explain why not, by focusing on which question *can* be answered by means of the p-value. (Hint: Look at the Slides 7 19-Oct).

P-value is the probability that for a given statistical model that, when the null hypothesis is true, the statistical summary (such as the sample mean difference between two compared groups) would be the same as or of greater magnitude than the actual observed results.
It is the probability of observing a particular event given your hypothesis.
$P(H)$ tells us the probability that we observe the value X as being greater than or equal to some value x, given our hypothesis H (right-tailed). Similarly, for left-tailed. If we wish to evaluate a two-tailed event, then you would take double the minimum of either left- or right-tailed: $2\times\{P(H), P(H)\}$.

3. Alex wants to study whether UCG students are smarter than other RuG students. From other students, we know that the mean IQ is 120 (population standard deviation is 15). What is the power of Alex's study if he uses 100 people to study this question? Assume that, in reality, UCG students do have a slightly higher mean IQ of 123. Use a 5% test of significance.

$$H_0 := \mu = 123;$$

$$H_1 := \mu < 123.$$

$$\bar{x} = 120; \sigma = 15; \alpha = 0.05.$$

$$\Rightarrow z = \frac{120-123}{15} = -\frac{3}{15} = -0.2.$$

$$p = P(Z < -0.2) = 1 - P(Z > 0.2) = 1 - 0.5793 = 0.4207.$$

Assume that H_0 is true. We find the value of z_b that corresponds to $\alpha = 0.05$, i.e. z_b=1.645. We need to find the value of b that corresponds to α, i.e.

$$P(X < b) = \alpha$$

$$\Rightarrow b = \mu + z\times\frac{\sigma}{\sqrt{n}} = 123 + (-0.2)\times\frac{15}{\sqrt{100}} = 122.7$$

Assume that H_0 is false, let $\mu = 120$. Calculate:

$$z = \frac{b-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{122.7-120}{\frac{15}{10}} = \frac{2.7}{1.5} = 1.8$$

$$Power = P(Z < 1.8) = 0.9641.$$

This tells us that we have a 96.41% chance of correctly rejecting the null hypothesis, which is false.

We could also assume that we test the hypothesis, as:

$$H_0 \coloneqq \mu = 123;$$
$$H_1 \coloneqq \mu > 123.$$

This would measure the other side of the distribution, however we are missing the standard deviation of the UCG students.

4. A certain variable has a distribution that is slightly skewed to the right. Someone wants to take a sample of 200 people from this population. How does the sample distribution look like? And how does the sample distribution look like for samples of n = 1? Explain both answers (briefly).

For samples of size 200, you would observe a standard distribution centered around the mean of the skewed distribution, with a smaller standard deviation. For samples of size 1, you would have a single point.

5. An experienced statistics teacher assumes that male students have higher grades for his course than female students. He uses a grade system ranging from 0-10, with 10 being the best grade possible. He knows that the population of male students scores on average a 7.1. He finds in a random sample of 20 of his female students that they score a 7.4 on average. He assumes that the scores of females follow a normal distribution, with a population standard deviation of 1, and he uses an alpha of .10. *Hint: it can be very helpful to make a picture for this question*

   a. Formulate the null and alternative hypothesis

$$H_0 \coloneqq \mu_M - \mu_F > 0;$$
$$H_1 \coloneqq \mu_M - \mu_F \leq 0.$$

   b. Calculate the p-value for the effect

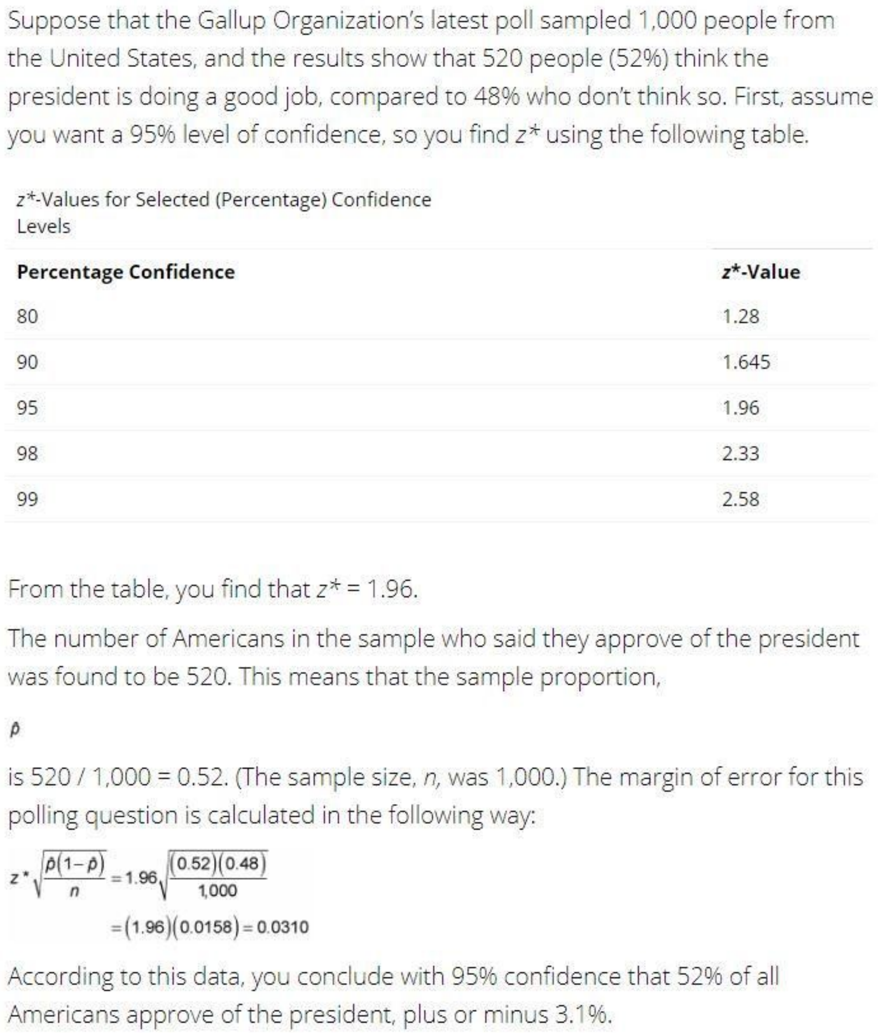
$$z = \frac{7.4 - 7.1}{1} = 0.3.$$
$$p = P\,(Z \leq z) = 0.6179.$$

   c. What do you conclude for this study?

p>0.1, hence we do not reject the null hypothesis. It is not statistically significant. Accept H_0; population mean for males is higher than population mean for females.

6. A student conducts an experiment for his bachelor thesis, and calculates a 95% confidence interval for the most important effect in his thesis. He is not satisfied with the width of the interval: according to him it should have been smaller. Explain what he could have done differently to get a narrower 95% confidence interval.

The student wishes to reduce his margin of error. In order to do so, he may increase the sample size. In general, the sample size n is a denominator in equations related to margin of error, e.g.:



Suppose that the Gallup Organization's latest poll sampled 1,000 people from the United States, and the results show that 520 people (52%) think the president is doing a good job, compared to 48% who don't think so. First, assume you want a 95% level of confidence, so you find z^* using the following table.

z^*-Values for Selected (Percentage) Confidence Levels

Percentage Confidence	**z^*-Value**
80	1.28
90	1.645
95	1.96
98	2.33
99	2.58

From the table, you find that $z^* = 1.96$.

The number of Americans in the sample who said they approve of the president was found to be 520. This means that the sample proportion,

$\hat{p}$

is 520 / 1,000 = 0.52. (The sample size, *n*, was 1,000.) The margin of error for this polling question is calculated in the following way:

$$z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}=1.96\sqrt{\frac{(0.52)(0.48)}{1{,}000}}$$

$$=(1.96)(0.0158)=0.0310$$

According to this data, you conclude with 95% confidence that 52% of all Americans approve of the president, plus or minus 3.1%.

Figure 1 Sourced from http://www.dummies.com/education/math/statistics/how-sample-size-affects-the-margin-of-error/

7. A teacher asks her students to collect data in order to estimate a parameter known only to her, and subsequently they calculate a 90% confidence interval for their data. Out of the 50 students, 44 had an interval that included the parameter. Is that what could reasonably be expected, or did something go wrong? Explain your answer

Since a 90% confidence interval about a point, say x, is a region where x is contained 90% of the time, it is unreasonable that (50-44)/50=12% of the class did not contain said x in their intervals.

8. Jonathan and Mathilda are discussing which of them is more inclined to mood swings. They both score their moods on a scale from -10 to +10 on 5 randomly chosen days in a year, and find the following results: Mathilda scores on average +3, with standard deviation 4, and Jonathan scores +1, with standard deviation 2.
    a. What is the F-value for Jonathan's mood compared to Mathilda's?

Remembering to always have the larger s.d. as the numerator, we have:

$$F = \frac{4^2}{2^2} = \frac{2^4}{2^2} = 2^2 = 4$$

b. What is the p-value for the F-value you found in 8a?

$$P\ (n, m = 4) \approx 0.100,\ \textit{from table E below}.$$

9.

a. A person who gambles quite a bit plays an online game in which he wins when he throws a six on a (virtual) die, and loses otherwise. He plays the game 100 times in a row. What are the mean and standard deviation for the sampling distribution? Assume for this question and the next that it is a fair die.

As the die is fair, the probability of rolling a 6 is equal to the probability of rolling any other number. As there are 6 sides to a die, we have that the probability of rolling a 6 (probability of a win) is equal to 1/6, which is approximately 0.1667. The probability of a loss (rolling numbers 1 through to 5) is 5/6, which is approximately 0.8333. Here we have that n=100. So we have,

$$\mu = n \times P\ (win) = 100 \times \frac{1}{6} \approx 16.67.$$

$$\sigma = \sqrt{n \times P\ (win) \times P\ (loss)} = \sqrt{100 \times \frac{1}{6} \times \frac{5}{6}} \approx 3.726$$

b. What is the probability that he will win exactly twice in the first three throws?

As the order of wins does not matter, only that he wins 2 games out of the first three, we want to calculate:

$$P\ (win \cap win \cap loss) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \approx 0.0231$$

c. And what is the probability that the number of eyes is maximally 3 times *odd*?

If the number of eyes on the die is odd, then the number on the die is itself odd. This question is asking us to calculate the probability that, in the first 3 throws, that we roll an odd number on 3 or less occasions. The probability of rolling an odd number is 3/6 = 0.5. Each event is I.I.D., so they are asking for the probability that on each throw, you get an odd number, which is 0.5.

Table A

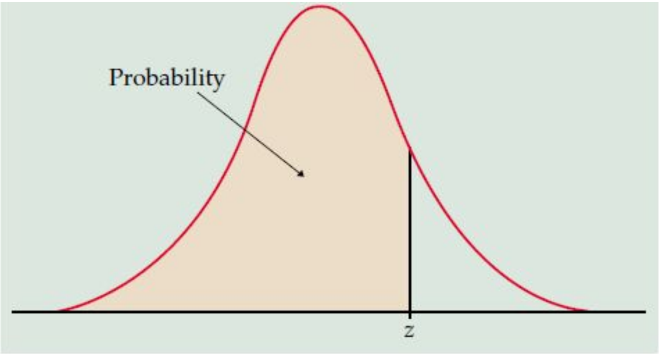


Table entry for *z* is the area under the standard normal curve to the left of *z*.

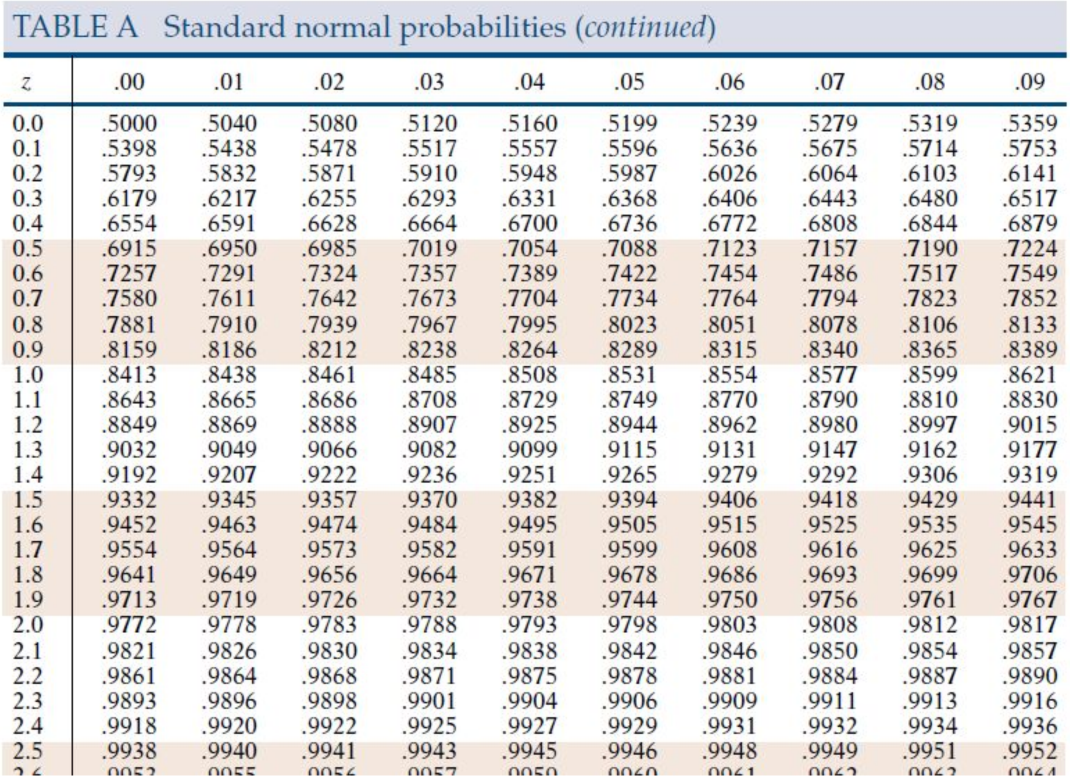


TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

Table C

TABLE C Binomial probabilities (*continued*)

		p								
n	*k*	.10	.15	.20	.25	.30	.35	.40	.45	.50
9	0	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461
	5	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7			.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8				.0001	.0004	.0013	.0035	.0083	.0176
	9						.0001	.0003	.0008	.0020
10	0	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7		.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172
	8			.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9					.0001	.0005	.0016	.0042	.0098
	10							.0001	.0003	.0010
12	0	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002
	1	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029
	2	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161
	3	.0852	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537
	4	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208
	5	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934
	6	.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256
	7		.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934
	8		.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208
	9			.0001	.0004	.0015	.0048	.0125	.0277	.0537
	10					.0002	.0008	.0025	.0068	.0161
	11						.0001	.0003	.0010	.0029
	12								.0001	.0002

Table E

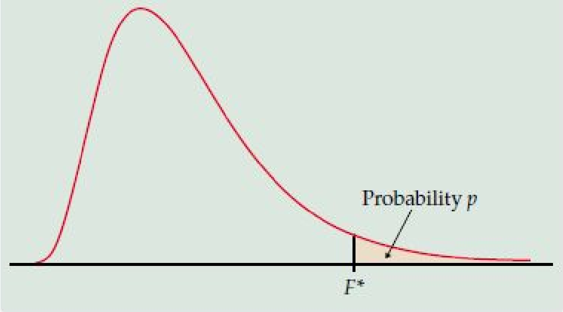


Table entry for p is the critical value F^* with probability p lying to its right.

TABLE E *F* critical values

		Degrees of freedom in the numerator								
Degrees of freedom in the denominator	p	1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69