Course summary

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Statistics 3
PSBE2-12

1 Exercises

1.1 Power

I Nationally, performance on a mathematics test for fourth graders is reported to be normally distributed with a mean of 40.0 and a standard deviation of 9.0.

1. You would like to know whether the average performance of children in your school district differs from the national average. You believe that a difference as small as 3.0 points is important to detect. How many randomly selected students do you need to include in your sample to have power of 80% to detect a difference of 3.0 points using a two-tailed test with alpha = .05?
   (a) What is the null and the (two-sided) alternative hypothesis for a difference of 3.0 points? (Draw a sketch.)
   (b) What are the z-scores which correspond to $\alpha = 0.05$ under your null and alternative hypotheses? Denote these z-scores as $\pm z_\alpha$.
   (c) Find the z-scores which correspond to 80% power. Denote these z-scores as $\pm z_\beta$.
   (d) You know that
   $$\pm Z_\alpha = \frac{X - \mu_0}{\sigma/\sqrt{n}} \implies X = \mu_0 \pm \frac{Z_\alpha \sigma}{\sqrt{n}},$$
   and you have the z-scores from part (b) ($\pm z_\alpha$), with $\sigma = 9$ (above) and μ_0 is the mean under your null hypothesis. What is your cut-off value X? (Note: X will be a function of $\sqrt{n}$.)
   You will have a lower and upper cut-off value X, why is this?
   Denote the upper X_U and lower X_L.
   (e) Taking the upper cut-off value X_U, you can use one of your z_β's, along with σ (given above) and the mean under the alternative hypothesis μ_A, to calculate the sample size required to have 80% power:
   $$Z_\beta = \frac{X_U - \mu_A}{\sigma/\sqrt{n}} = \frac{\left(\mu_0 + \frac{Z_\alpha \sigma}{\sqrt{n}}\right) - \mu_A}{\sigma/\sqrt{n}} = Z_\alpha + \frac{\mu_0 - \mu_A}{\sigma/\sqrt{n}} \implies n = \left(\frac{\sigma\left(Z_\beta - Z_\alpha\right)}{\mu_0 - \mu_A}\right)^2.$$
   Cohen's d is a measure of effect size, here we have that $d = \dfrac{\mu_0 - \mu_A}{\sigma}$.
   (f) Can you also do this for the lower bound X_L?
2. Your friend Bumble collected data from a sample of 25 children from a large school where the population mean is 45.0. If he uses a one-tailed test with alpha = .01, how likely is he to attain statistical significance? That is, what is his statistical power?
   (a) Given that both the national population and the large school Bumble sampled from have a standard deviation of 9.0, sketch the distributions side by side.
   (b) On your sketch from part (a), shade the area representing α and find the z-score which corresponds to this. Denote this z-score as z_α.
   (c) Define your null and alternative hypotheses.
   (d) You know that
   $$Z_\alpha = \frac{X - \mu_0}{\sigma/\sqrt{n}} \implies X = \mu_0 + \frac{Z_\alpha \sigma}{\sqrt{n}},$$
   and you have the z score from part (b) (z_α), with $\sigma = 9$ (above), $n = 25$ (given) and μ_0 is the mean under the null hypothesis. What is your cut-off value X?

(e) Calculate the power of your test:

$$\textbf{Power} = \mathbb{P}\left(Z \geq z_\beta\right),$$

where

$$Z_\beta = \frac{X - \mu_A}{\sigma / \sqrt{n}} = \frac{\left(\mu_0 + \frac{Z_\alpha \sigma}{\sqrt{n}}\right) - \mu_A}{\sigma / \sqrt{n}} = Z_\alpha + \frac{\mu_0 - \mu_A}{\sigma / \sqrt{n}}.$$

3. For each of the questions above, identity the **BEAN**; **B**eta β, **E**ffect size (Cohen's d), **A**lpha α and **N** (sample size n). How are these four related?