Formulas

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Statistics 1A
PSBE1-08

For each of the following questions, draw a sketch of the distribution and mark off the mean and the values corresponding to the 68-95-99.7 rule, or any other useful information. For example, if $X \sim \mathcal{N}(50, 5^2)$ (mean 50, sd 5) then

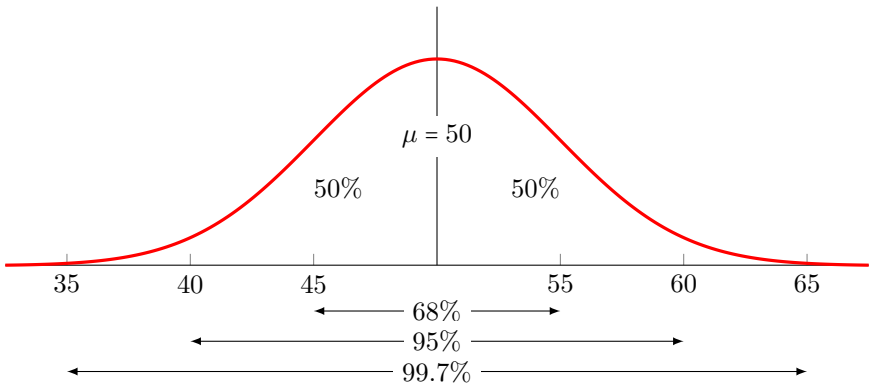

Figure 1: A normally distributed random variable X with mean 50 and variance 25. 50% of the data lies either side of 50, and we can approximate the middle proportion using the 68-95-99.7 rule due to **symmetry about the mean**.

This way, if I am asked what is the probability of having a value greater than 45, I can do some simple addition to find an approximate answer:

$$\mathbb{P}(X > 45) = 50\% + \frac{68\%}{2} = 50\% + 34\% = 84\%.$$

The exact value is 84.1345%, but our estimate is close enough so that we can be sure of our answer when looking at the z-table for the probability.

1. Let X be the IQ of a randomly selected American. Assume $X \sim \mathcal{N}(100, 16^2)$.
   (a) What is the probability that a randomly selected American has an IQ below 90?
   (b) What is the probability that a randomly selected American has an IQ above 140?
   (c) What is the probability that a randomly selected American has an IQ between 92 and 114?
2. Suppose X, the grade on a midterm exam, is normally distributed with mean 70 and standard deviation 10.
   (a) The instructor wants to give 15% of the class an A. What cutoff should the instructor use to determine who gets an A?
   (b) The instructor now wants to give 10% of the class an A$^-$. What cutoff should the instructor use to determine who gets an A$^-$?
   (c) The instructor has determined that a grade of 35 or lower is an F. What is the probability of a failing grade?
3. It was found that the mean length of 100 parts produced by a factory was 20.05 mm with a standard deviation of 0.02 mm. Find the probability that a part selected at random would have length

(a) between 20.03 mm and 20.08 mm;

(b) between 20.06 mm 20.07 mm;

(c) less than 20.01 mm;

(d) greater than 20.09 mm.

4. A company pays its employees an average wage of \$3.25 an hour with a standard deviation of 60 cents. If the wages are approximately normally distributed deteremine

   (a) the percentage of the workers getting wages between \$2.75 and \$3.69 an hour;

   (b) the minimum wage of the highest 5%.

5. The average life of a certain type of motor is 10 years, with a standard deviation of 2 years. If the manufacturer is willing to replace on 3% of the motors because of failures, how long a guarantee should she offer? Assume that the lives of the motors follow a normal distribution.