# Answer Key Example Exam

1. D
   - 5% means $P(Z \leq z)=0.05$
   - Check Table A for the corresponding z-value
     - 0.05 not in Table A, but $P(Z \leq -1.64)=0.0505$ and $P(Z \leq -1.65)=0.0495$
     - Therefore $P(Z \leq \frac{-1.64+ -1.65}{2})=0.05$, thus $P(Z \leq -1.645)=0.05$
   - Now we know that $z = -1.645$ and we already knew that $\mu=62$ and $\sigma=11$
   - We need these numbers to <u>unstandardize</u>:
     - $X=\mu+Z\sigma = 62 + (-1.645*11) = 43.905$
   - Thus, answer D is correct

2. A
3. B
4. A
   - Expected value=n*p= 200 * 0.8 = 160
5. D
6. A
7. A
8. A
   - P(next three babies are of the same sex) = P(boy/boy/boy or girl/girl/girl)=
   - P(bbb) + P(ggg) = (P(boy)* P(boy)* P(boy)) + (P(girl)* P(girl)* P(girl))=
   - (0.5*0.5*0.5) + (0.5*0.5*0.5) = 0.125 + 0.125 = 0.250
9. D
   - Three children aged 3, 4, 5; mean=$\frac{3+4+5}{3}$= 4; variance=$\frac{(3-4)^2+(4-4)^2+(5-4)^2}{3-1} = \frac{2}{2} = 1$
   - Four children aged 3, 4, 4, 5; mean=$\frac{3+4+4+5}{4}$= 4;
     variance=$\frac{(3-4)^2+(4-4)^2+(4-4)^2+(5-4)^2}{4-1} = \frac{2}{3}$
   - So the mean stays the same and the variance decreases >> answer D
10. B
    - $P(W > -5.5) = 1 - P(W< -5.5)$
    - Step 1: standardize
      - $Z = \frac{-5.5-5}{14.4} = -0.73$
    - Step 2: check table A
      - $P(Z< -0.73) = 0.2327$
    - Therefore: $P(W< -5.5) = 1 - 0.2327 = 0.7673$
11. C
12. B
13. A
14. B
15. D
16. A

17. B

- According to the central limit theorem when $n$ is large: $\bar{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$
- In this case: $\bar{x} \sim N(\mu=400, \frac{\sigma}{\sqrt{n}} = \frac{80}{\sqrt{100}} = 8)$
- P(X>425) = 1 – P(X<425)
- Step 1: standardize
    - $Z = \frac{425-400}{8} = 3.125$
- Step 2: check table A
    - P(Z<3.125) = 0.9991
- Therefore: P(X>425) = 1 – 0.9991 = 0.0009

18. A

- Based on the information you can draw the following table

| | | | | | |
|---|---|---|---|---|---|
| | | Airline | | | |
| | | A | B | C | |
| weapon detected? | Yes | .9*.5=0.45 | .5*.3=0.15 | .4*.2=0.08 | 0.68 |
| | No | | | | |
| | | 50% = 0.5 | 30% = 0.3 | 20% = 0.2 | |

- The question can be rephrased as:
- P(traveling with airline B | weapon detected)
- Since we know (it is given) that a weapon is detected, we are only interested in that part of our sample. In the table below the cells of interest are marked yellow:

| | | | | | |
|---|---|---|---|---|---|
| | | Airline | | | |
| | | A | B | C | |
| weapon detected? | Yes | .9*.5=0.45 | .5*.3=0.15 | .4*.2=0.08 | 0.68 |
| | No | | | | |
| | | 50% = 0.5 | 30% = 0.3 | 20% = 0.2 | |

-
- In general: $P(B|A) = \frac{P(\text{A and B})}{P(\text{A})}$
- In this case event B is traveling with airline B, event A is that a weapon is detected
- P(A and B) = 0.15   P(A) = 0.68
- Therefore, P(traveling with airline B | weapon detected) = $\frac{0.15}{0.68}$ = 0.22

19. C

- P(A and B) is impossible with disjoint events
- With disjoint events P(A or B) = P(A) + P(B) = 0.2 + 0.8 = 1

20. B

- Np greater than about 10 is a requirement before you can apply the normal approximation to a binomial setting

21. C

22. A

- Event B: P( at least one of the next two babies is a boy)
- The complement of B means the probability of the other events than event B, in this case no boys. This can be written as: P(girl/girl)=P(girl)*P(girl)=.5*.5=.25
- You can also use the complement rule: $P(B^C) = 1 - P(B)$
- $P(B^C)$ = 1 – (P(boy/girl) + P(girl/boy) + P(boy/boy))
- = 1 – ((.5*.5)+(.5*.5)+(.5*.5))= 1 – 0.75 = 0.25

23. B

24. C

25. D
    - Using the Expected value and variance rules:
        - the mean is $E(\sum_{i=1}^{36} X_i) = \sum_{i=1}^{36} E(X_i) = \sum_{i=1}^{36} 2.5 = 36 * 2.5 = 90$
        - the variance is $V(\sum_{i=1}^{36} X_i) = \sum_{i=1}^{36} V(X_i) = \sum_{i=1}^{36} 0.2^2 = 36 * 0.04 = 1.44$
        - therefore, the standard deviation is $\sqrt{1.44} = 1.2$
26. D
    - According to the central limit theorem when $n$ is large: $\mu_{\bar{X}} = \mu = 400$
27. B
28. D
    - $X$ is number of divorces; $X \sim B(n,p) = B(10,0.55)$
    - p = 0.55 isn't displayed in Table C, therefore rephrase as number of failures
    - $Y$ is number of non-divorces; $Y \sim B(n,p) = B(10,0.45)$
    - $P(X<2) = P(Y>8)$
    - $P(Y>8) = P(Y = 9) + P(Y = 10) = 0.0042 + 0.0003 = 0.0045$
29. C
30. D
31. D
32. D
33. A
    - $\mu_W = \mu_X - \mu_Y = 75 – 70 = 5$
    - ${\sigma_W}^2 = {\sigma_X}^2 + {\sigma_Y}^2 = 8^2 + 12^2 = 208$
    - $\sigma_W = \sqrt{208} = 14.4$
34. D
35. B
36. B
    - P(at least one of the next three babies is a boy) means 1 boy, 2 boys or 3 boys. Which is the same as 1 – 0 boys
    - 1 – P(no boys) = 1 – P(three girls) = 1 – (P(girl)* P(girl)* P(girl))= 1 – (0.5*0.5*0.5)= 0.875
37. A
    - 19 and 20 are the two numbers in the middle, (19+20)/2=19.5
38. C
39. D
    - 1st quartile: 16 and 17 are the two middle numbers in the first half: (16+17)/2=16.5
    - 3rd quartile: 24 and 26 are the two middle numbers in the second half: (24+26)/2=25
    - IQR: 3rd quartile – 1st quartile: 25-16.5=8.5