

Project Report

Group 41

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Theoretical Run time Analysis

Algorithm 1 Pseduocode

Enumeration(A[0...n])

```
for i = 0 to n-1
    for j = i to n-1
        sum = 0;
        aLength = 0;
        //clear the subA array
        j++

        for k = i to j
            subA[aLength] = a[k];
            sum += a[k];
            aLength++;
        k++

        if sum > maxSum
            maxSum = sum;
            maxLength = aLength;
            //copy subA array into maxSubA array
        i++

return (A[0...maxLength], sum)
```

Complexity

First for loop takes n steps, while the second does n steps as well, as well as the third

$T(n) = n^3$ or $T(n) = O(n^3)$

Algorithm 2 Pseduocode

BetterEnumeration (A[0...n])

```
SubA[]
maxSubA[]
maxSum = 0
For i = 0 to n-1
    Sum = 0
    aLength = 0
    Clear array SubA[0...n]
    For j = i to n-1
        subA[aLength] = a[j]
        sum+=a[j]
        aLength++
        if sum > maxSum
            maxSum = sum
            maxLength = aLength
            Copy SubA into maxSubA
        j++
    i++
return (a[0...maxLength],sum)
```

Complexity

We only have two for loops that run n times,

$T(n) = n^2$ or $T(n) = O(n^2)$

Algorithm 3 Pseudocode

DivideAndConquer (A[1...n])

If $n=1$

```

        return (A[1...n],sum(A))
Lhs_subArray = DivideAndConquer(A[1...n/2])
Rhs_subArray = DivideAndConquer(A[n/2+1...n])

checkRhsArray = -inf
SumRhsArray = 0
For j = n/2 +1 to n
    sumRhsArray = sumRhsArray + A[j]
    if(sumRhsArray > checkRhsArray)
        checkRhsArray = sumRhsArray
        rhs_checkLength = j+1 - (n/2)
        rhs_checkEndIndex = j
        sumToRhsEnd = sumRhsArray
    j = j+1
if Sum(Rhs_subArray) < checkRhsArray
    Rhs_subArray = Rhs_subArray[n/2 + 1...j]
    Sum(Rhs_subArray) = checkRhsArray

checkLhsArray = -inf
SumLhsArray = 0
For i = n/2 to 1
    sumLhsArray = sumLhsArray + A[i]
    if(sumLhsArray > checkLhsArray)
        checkLhsArray = sumLhsArray
        rhs_checkLength = n/2 - i
        rhs_checkBegIndex = j
        sumToLhsEnd = sumLhsArray
    i = i+1
if Sum(Lhs_subArray) < checkLhsArray
    Lhs_subArray = Lhs_subArray[i...n/2]

```

```
Sum(Lhs_subArray) = checkLhsArray
```

```
MaxsubArr =  
max(max(Lhs_subArray[i...n/2], Rhs_subArray[n/2...j]), A[checkBegIndex...check  
EndIndex])  
Return (MaxsubArr[], sum(MaxSubArr))
```

Complexity

The recursive calls require $2T(n/2)$ complexity, while the rest requires $c_1 * n$ since we are checking to see if there is a greater sub array in the left hand side then the right hand side, so we do n calculations.

$$T(n) = 2T(n/2) + c_1n$$

Apply master method:

$$a=2, b=2 \Rightarrow n^{\log_2(2)}=n, f(n) = n$$

$$f(n) = \theta(n) \text{ therefore } T(n) = \theta(n * \lg(n))$$

Algorithm 4 Pseudocode

linear_time(a[1...n])

```
    int b[] = a[]; //b is a clone of a  
    int max=a[0];  
    for i from 0 to n  
        if(b[i-1]>0)  
            b[i]=b[i-1]+a[i];  
    maxsum=max value in array b  
return maxsum
```

Complexity

The i loop track the input array elements from 1 to n . $\sum_{i=1}^n O(1) = n * O(1) = O(N)$, the theoretical run-time is $O(N)$.

Testing

For testing we tested our algorithms against the given test text file, we also used the online tool provided and checked irregular arrays and any arrays that we thought may give us trouble, such as all negatives in the beginning and at the end with a large positive number in the middle, a single positive number, and several larger arrays

Analysis

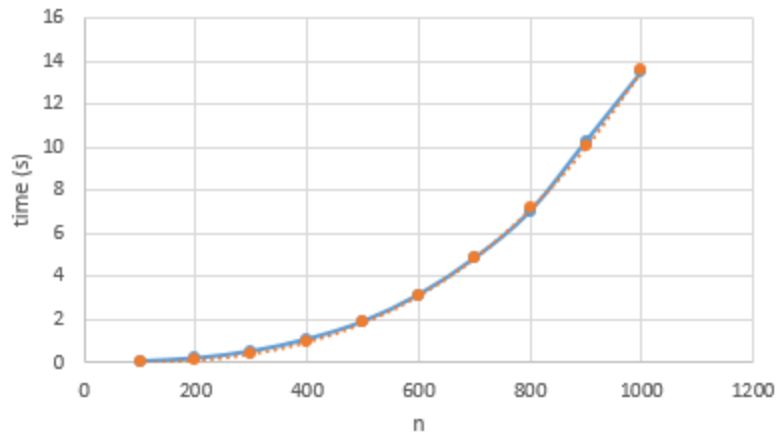
To get more accurate run time across all algorithms, we ran the following tests on a single computer:

| enumeration | | | better enumeration | | |
|--|---------|------------|-------------------------------|---------|------------|
| n | time | regression | n | time | regression |
| 100 | 0.025 | 0.01823 | 5000 | 1.39279 | 1.0215313 |
| 200 | 0.1627 | 0.13346 | 6000 | 1.8322 | 1.54692121 |
| 300 | 0.48942 | 0.42765 | 7000 | 2.3817 | 2.19704568 |
| 400 | 1.04523 | 0.97705 | 8000 | 3.08695 | 2.97734223 |
| 500 | 1.8723 | 1.85456 | 9000 | 3.74957 | 3.89270864 |
| 600 | 3.1159 | 3.13076 | 10000 | 4.78972 | 4.94761557 |
| 700 | 4.81477 | 4.8744 | 11000 | 5.89917 | 6.14618599 |
| 800 | 7.00556 | 7.15277 | 12000 | 7.30852 | 7.4922535 |
| 900 | 10.2581 | 10.0319 | 13000 | 8.97589 | 8.98940645 |
| 1000 | 13.5017 | 13.5769 | 14000 | 10.9915 | 10.6410224 |
| $(3.287*10^{-8})*(x^{2.872})$ | | | $(3.894*10^{-9})*(x^{2.276})$ | | |
| should be x^3 | | | should be x^2 | | |
| | | | | | |
| divide and conquer | | | linear-time | | |
| n | time | regression | n | time | regression |
| 20000 | 0.03994 | 0.04224 | 20000 | 0.00283 | 0.0030798 |
| 25000 | 0.05429 | 0.05277 | 25000 | 0.00377 | 0.0038068 |
| 30000 | 0.06421 | 0.06349 | 30000 | 0.00464 | 0.0045338 |
| 35000 | 0.07497 | 0.07437 | 35000 | 0.0054 | 0.0052608 |
| 40000 | 0.08687 | 0.08539 | 40000 | 0.00615 | 0.0059878 |
| 45000 | 0.09646 | 0.09652 | 45000 | 0.00683 | 0.0067148 |
| 50000 | 0.10901 | 0.10777 | 50000 | 0.00746 | 0.0074418 |
| 55000 | 0.12026 | 0.11911 | 55000 | 0.00809 | 0.0081688 |
| 60000 | 0.12915 | 0.13053 | 60000 | 0.00873 | 0.0088958 |
| $(4.4*10^{-7})*x*(\text{LOG}(x))+0.004391$ | | | $(1.454*10^{-7})*x+0.0001718$ | | |
| Rsquared = 0.99 excellent | | | Rsquared = 0.99 excellent | | |

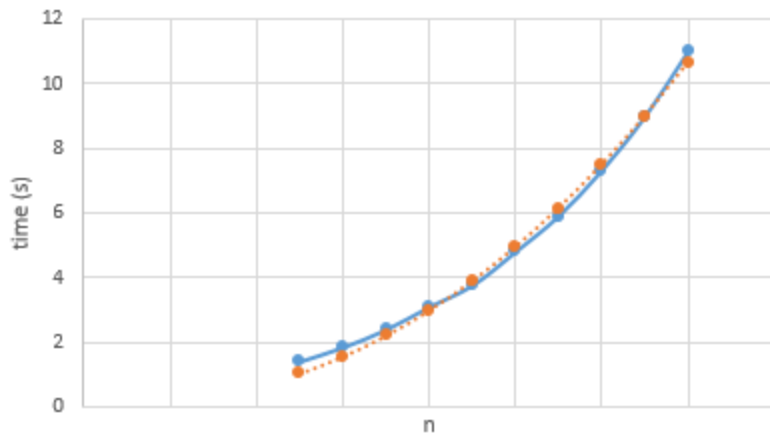
| Times(s) | Enumeration | Better_enumeration | Divide and conquer | Linear_time |
|----------|-------------|--------------------|--------------------|-------------|
| 10 | $8.99*10^2$ | $1.36*10^4$ | $1.59*10^6$ | $6.88*10^7$ |
| 30 | $1.32*10^3$ | $2.21*10^4$ | $4.45*10^6$ | $2.06*10^8$ |
| 60 | $1.68*10^3$ | $2.99*10^4$ | $8.54*10^6$ | $4.12*10^8$ |

The functions at the bottom are the functions that were the best fit, as you can see we got very close on every algorithm. Also, you can see that we had to use widely different sized algorithms, we had to do this since some of the algorithms would require much less work than the others. The following are the graphs of the above data set with the orange as the best fit:

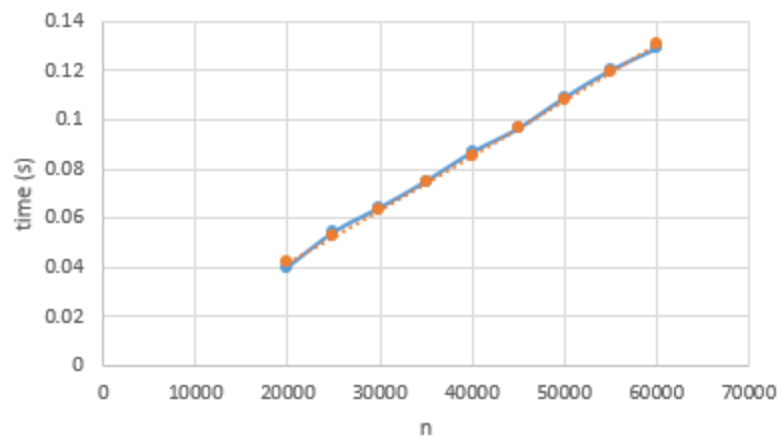
Enumeration

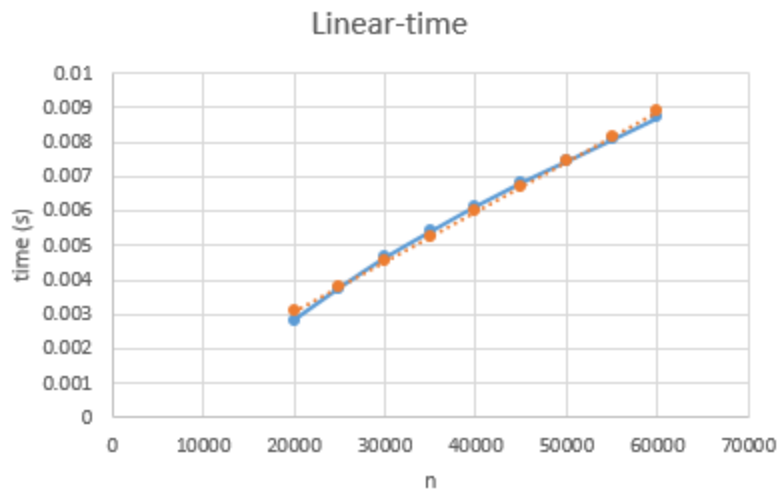


Better Enumeration



Divide and Conquer





Since we had widely different array sizes, we did a log-log plot of all four on the same graph

