7.1

a)

→ Converting coordinate to Index:

$$I = x_2 \cdot L_1 + x_1$$

→ Converting Index to Coordinate (x_1, x_2) :

$x_2 = \lfloor I / L_1 \rfloor$, where $\lfloor \cdot \rfloor$ rounds the integer downside.

eg: $\lfloor 3/4 \rfloor = 0$; $\lfloor 11/4 \rfloor = 2$

then,

$$x_1 = I - x_2 \cdot L_1$$

7.2

a)

→ Converting coordinate to index:

$$I = \sum_{i=1}^{d} \left(x_i \cdot \prod_{j=0}^{i-1} L_j \right)$$, where $L_0 = 1$

$(L_1, L_2 .., L_j)$ is given.

→ Converting index to coordinate:

$$x_i = \left\lfloor \frac{I - \sum_{j=i+1}^{d} \left(x_j \cdot \prod_{k=0}^{j-1} L_k \right)}{\prod_{j=0}^{i-1} L_j} \right\rfloor$$, where $L_0 = 1$;

$\lfloor \cdot \rfloor$ rounds the integer downside.

Draft Paper:

$$I \cdot x_1 = I - x_2 \cdot L_1$$

$$L_0 \cdot x_1 = I - x_2 \cdot L_1$$

$$\cdots \cdot L_{i-1}\, x_i = I - x_{i+1} L_{j-1} - \cdots \cdots$$

$$x_i = \frac{I - \sum_{j=i+1}^{d} x_j \cdot \prod_{K=0}^{j} L_K}{\prod_{j=0}^{i-1} L_j}$$

eg.

$$x_2 = \frac{23 - \sum_{j=3}^{3} \left(x_j \cdot \prod_{K=0}^{2} L_K\right)}{\prod_{j=0}^{1} L_j \cdot L_1}$$

$$x_2 = \frac{(23 - [x_3 \cdot L_2 L_1 L_0])/(1 \cdot 4)}{= 23 - (1 \cdot 3 \cdot 4 \cdot 1)/4 = [2.25] = (2) \checkmark}$$

$$I = \chi_3 L_2 L_1 L_0 + \chi_2 L_1 L_0 + \chi_1 L_0$$

$$\chi_0 = \frac{I - \chi_2 L_1 L_0 - \chi_3 L_2 L_1 L_0}{L_0}$$

$$\chi_2 = \frac{I - \chi_3 L_2 L_1 L_0 - \chi_1 L_0}{\chi_2 L_1 L_0}$$

$$\chi_3 = \frac{I - \chi_2 L_1 L_0 - \chi_1 L_0}{L_2 L_1 L_0}$$