Theorem 2. *BC_m covers CS_m using the minimum number of intervals, h, each of duration TI.*

Proof. Suppose the minimum number of intervals required to cover CS_m is h^*. Let the spans to be covered be $cs = CS_m \neq \emptyset$, and let $rc = \max\{\omega | \omega \in cs\}$ be the highest time in cs.

Clearly, $h^* \leq h$ and $rc = b_r = q_1$. For any optimal cover solution $S_0^* = \{[s_1 - TI, s_1] \cup ... \cup [s_{h^*} - TI, s_{h^*}]\}$, with $s_1 > s_2 > ... > s_{h^*}$, we have $cs \subseteq S_0^*$.

step 1: From $cs \subseteq S_0^*$ and $cs \neq \emptyset$, it follows that $S_0^* \neq \emptyset$, and consequently, $h^* \geq 1$. For the first interval of S_0^*, we have $rc \in [s_1 - TI, s_1]$. Thus, by setting s_1 to $rc = q_1$, S_0^* is modified to $S_1^* = \{[q_1 - TI, q_1] \cup [s_2 - TI, s_2] \cup [s_3 - TI, s_3] \cup \cdots \cup [s_{h^*} - TI, s_{h^*}]\}$, which is still an optimal cover solution. After that, update $cs = cs \backslash \{[q_1 - TI, q_1]\}$ and $rc = \max\{\omega | \omega \in cs\} = q_2$.

step 2: From $cs \subseteq S_1^* \backslash \{[q_1 - TI, q_1]\}$ and $cs \neq \emptyset$, it follows that $S_1^* \backslash \{[q_1 - TI, q_1]\} \neq \emptyset$, and consequently, $h^* \geq 2$. For the second interval of S_1^*, we have $rc \in [s_2 - TI, s_2]$. Thus, by setting s_2 to $rc = q_2$, S_1^* is modified to $S_2^* = \{[q_1 - TI, q_1] \cup [q_2 - TI, q_2] \cup [s_3 - TI, s_3], [s_4 - TI, s_4] \cup \cdots \cup [s_{h^*} - TI, s_{h^*}]\}$, which is still an optimal cover solution. After that, update $cs = cs \backslash \{[q_2 - TI, q_2]\}$ and $rc = \max\ \{\omega | \omega \in cs\} = q_3$.

...

step i: From $cs \subseteq S_{i-1}^* \backslash \{[q_1 - TI, q_1] \cup ... \cup [q_{i-1} - TI, q_{i-1}]\}$ and $cs \neq \emptyset$, it follows that $S_{i-1}^* \backslash \{[q_1 - TI, q_1] \cup ... \cup [q_{i-1} - TI, q_{i-1}]\} \neq \emptyset$, and consequently, $h^* \geq i$. For the ith interval of S_{i-1}^*, we have $rc \in [s_i - TI, s_i]$. Thus, by setting s_i to $rc = q_i$, S_{i-1}^* is modified to $S_i^* = \{[q_1 - TI, q_1] \cup ... \cup [q_i - TI, q_i], [s_{i+1} - TI, s_{i+1}] \cup \cdots \cup [s_{h^*} - TI, s_{h^*}]\}$, which is still an optimal cover solution. After that, update $cs = cs \backslash \{[q_i - TI, q_i]\}$ and $rc = \max\ \{\omega | \omega \in cs\} = q_{i+1}$.

...

step h: From $cs \subseteq S_{h-1}^* \backslash \{[q_1 - TI, q_1] \cup ... \cup [q_{h-1} - TI, q_{h-1}]\}$ and $cs \neq \emptyset$, it follows that $S_{h-1}^* \backslash \{[q_1 - TI, q_1] \cup ... \cup [q_{h-1} - TI, q_{h-1}]\} \neq \emptyset$, and consequently, $h^* \geq h$. To take into account that $h^* \leq h$, we can conclude that $h = h^*$. Hence, it can be concluded that BC_m covers CS_m with the minimum number of intervals.

$\square$