
HIERARCHICAL VEHICLE CLASSIFICATION WITH MULTI-TASK LEARNING

A PREPRINT

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ABSTRACT

This project investigates effective strategies for hierarchical image classification on the Stanford Cars Dataset, addressing the challenge of distinguishing between 196 classes of car models. We used a ResNet-50 backbone and evaluate three distinct output designs: a flat single-head classifier, a two-head classifier, and a three-head Multi-Task Learning (MTL) architecture that independently predicts the vehicle's Make, Type, and Model. We demonstrate that the three-head design, specifically when optimized with Curriculum Learning (CL) and Hierarchical Label Smoothing (HLS), significantly outperforms simpler, flat classification methods. The inclusion of the "Type" prediction head proved critical, acting as a necessary semantic bridge to guide the feature extractor. This structured approach enforced strong taxonomic coherence, driving prediction consistency between Make and Model to 97.66%. Our final model, stabilized by Test Time Augmentation (TTA), achieved a Top-1 accuracy of 88.57%, confirming that for complex, fine-grained tasks, structuring the learning process is paramount to achieving robust and logically coherent predictions.

Keywords First keyword · Second keyword · More

1 Introduction

Fine-grained vehicle classification, which differentiates between various types, makes, and models of cars, presents significant challenges. This difficulty arises from the large number of visually similar vehicle categories, variations in lighting and weather conditions, occlusions, and the diverse viewpoints from which vehicles may be captured. As a result, building a robust vehicle classification system requires a model capable of recognizing subtle visual cues while generalizing across highly variable real-world conditions.

In this project, we aim to explore and compare different strategies for hierarchical image classification on the Stanford Cars Dataset (Krause et al., 2013). The dataset contains 16,185 high-resolution images labeled with three hierarchical attributes: vehicle make, vehicle type, and vehicle model. These labels naturally form a multi-level taxonomy, making the dataset well-suited for studying hierarchical classification approaches.

Our baseline approach uses a ResNet-50 model pretrained on ImageNet as the backbone architecture. Initially, we treat the problem as a flat classification task by predicting the complete car label (containing make, type, and model) as a single class among 196 possible categories. This serves as our starting point for evaluating how well a standard single-head classifier performs on fine-grained classification without explicit hierarchical structure.

Building on this baseline, the primary objective of this project is to investigate how different hierarchical output designs influence classification performance. Specifically, we focus on three classification strategies:

1. **Single-head (flat) classifier:** Predicts the entire label (make, type, and model) as one combined class. This ignores the hierarchical relationships.

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2. **Two-head classifier:** One head predicts the vehicle make, while the second head predicts the combined type+model label.
3. **Three-head classifier:** Predicts make, type, and model independently using three parallel classification heads.

After we evaluated these 3 variants, we will further enhance their performance using targeted regularization and optimization strategies. These include data augmentation, dropout, weight decay, and learning-rate adjustments, all of which can help reduce overfitting.

2 Related Work

Since the Stanford Cars dataset is popular, it is possible to find plenty prior work, trying to achieve high accuracy for classification by using different types of models and strategies.

The paper A Systematic Evaluation of Recent Deep Learning Architectures for Fine-Grained Vehicle Classification by Valev et al. investigates how modern convolutional neural networks perform on the Stanford Cars dataset. The authors evaluate several well-known CNN architectures, including VGG16, Inception, MobileNet, and multiple residual networks, with an emphasis on ResNet-50, ResNet-152, and DenseNet variants. Their study focuses entirely on flat fine-grained classification, meaning that each car is classified directly into one of 196 labels. The method does not employ any form of hierarchical structure such as predicting make or type separately. ResNet-50 is tested both when trained from scratch and when fine-tuned from ImageNet weights. Training ResNet-50 from scratch yields an accuracy of 84.3%, whereas fine-tuning dramatically improves performance to 92.0%. The deeper ResNet-152 shows similar behavior and achieves only 35.3% when trained from scratch but rises to 92.6% when fine-tuned. Among all evaluated CNNs, the best performing model is DenseNet-161, which reaches 94.6% accuracy on the Stanford Cars dataset. Their experiments highlight the importance of data augmentation such as horizontal flipping and motion blur, which add up to meaningful improvements. The paper does not consider hierarchical labels or multi-head predictions that leverage the natural make-type-model structure of the dataset. (<https://arxiv.org/pdf/1806.02987>)

The paper FineGrained Image Classification for Vehicle Makes & Models using CNNs explores the challenge of distinguishing highly similar car models within the Stanford Cars dataset. For their approach, they rely on transfer learning with a VGG-16 backbone pretrained on ImageNet. Two model variants are evaluated: a baseline that finetunes only the fully connected layers, and an improved architecture that removes one of the original fully connected layers, reduces the dimensionality of the remaining layers, and introduces dropout for regularization. The authors apply extensive data augmentation. Their method predicts all 196 classes as a single flat classification task. The experimental results show that the baseline VGG-16 achieves only 52.1% accuracy on the test set, while the modified VGG-16 reaches a much stronger accuracy of 84.0. These findings demonstrate that fine-tuned CNN backbones, combined with aggressive augmentation and streamlined classification heads, can perform competitively on fine-grained vehicle recognition tasks. (https://cs230.stanford.edu/projects_spring_2019/reports/18681590.pdf)

Our approach follows a similar foundation in that we also rely on transfer learning, starting from a ResNet-50 backbone pretrained on ImageNet. However, instead of only treating the Stanford Cars dataset as a flat 196-class problem, we additionally explore hierarchical classification strategies. Concretely, we design and compare models with one, two, and three prediction heads, where each head independently predicts one of the hierarchical levels: make, type, and model. Additionally, our approach incorporates a broader set of regularization and stabilization techniques, such as data augmentation, dropout, weight decay, and learning-rate scheduling. Finally, we also evaluate the impact of test-time augmentation (TTA).

3 Methods

3.1 Dataset and Pre-Processing??

- The dataset contains 16185 images of 196 classes of cars. The dataset is split into 8144 training images and 8041 test images. Each class has been split around a 50-50 split.
- Hierarchical Label Definiton:

3.2 Model Architecture

- **Backbone:** ResNet-50 (pretrained on ImageNet) without any modifications.
- **Multi-Task Heads:** The Data Architecture is based on the Shared-Bottom Multi-Task Learning paradigm. This approach uses a single Convolutional Neural Network (CNN) backbone to extract a universal, high-

Hierarchy Level	Description	Example	Classification Difficulty
Head 1: Make	The manufacturer or brand of the vehicle. This is the coarse-grained class.	BMW	Easy (Highly distinct features/logos)
Head 2: Type	The general body style of the vehicle, inferred from the model name.	Sedan	Medium (Defined by shape/proportions)
Head 3: Model	The specific model and year of the vehicle (the original 196 classes).	3-Series Sedan 2012	Hard (Subtle visual differences)

Table 1: Hierarchical Label Definition

dimensional feature representation from the input image, which is then fed into multiple, distinct "heads," each responsible for a single classification task (e.g., Make, Model, or Type).

- **Multi-Head Architecture:** The architecture utilizes a multi-head design, based on the Shared-Bottom Multi-Task Learning paradigm, where the feature vector, F_{shared} , extracted by the backbone, is split and fed simultaneously into three independent, fully-connected (Dense) classification heads: Make, Type, and Model. This split allows the backbone to learn general, low-level visual characteristics shared by all tasks, while each head refines the features necessary for its specific, hierarchical classification level. The entire network is trained end-to-end by optimizing a single Combined Training Objective, L_{Total} , which is the weighted sum of the Categorical Cross-Entropy losses (L_{CCE}) from each head:

$$L_{Total} = w_{Make} \cdot L_{CCE}(y_{Make}, \hat{y}_{Make}) + w_{Type} \cdot L_{CCE}(y_{Type}, \hat{y}_{Type}) + w_{Model} \cdot L_{CCE}(y_{Model}, \hat{y}_{Model})$$

3.3 Training Strategies

- **Baseline:** First we set up a baseline for training which contained batch size = 32 with 15 epochs and a 0.0001 learning rate, where the ImageNet layers were frozen and only the fully connected layer was unfrozen. Then we unfroze the rest of the model parameters, to help the model recognize the shapes of the elements of the car models. Then we added the Make classification head that only predicts the make of the car (fx. BMW) while the first head was predicting the full label to make sure that the model is learning the hierarchy. Then later we added a third, Type classification head that only predicts the type of the car (fx. Sedan)
- **Data Augmentation, Normalization:** We added 4 different types of data augmentation and normalization
 - Random Resized: and Cropped Images were first resized to a larger dimension (256x256) and then a 224×224 area was randomly cropped. This combination forces the model to learn features that are robust to variations in position, scale, and perspective (i.e., the car is not always perfectly centered).
 - Random Horizontal Flip: Flips the image along the vertical axis with a 50
 - Random Rotation: Images were randomly rotated up to ± 15 degrees. This provides rotational invariance, which is particularly useful for handling slight tilts or oblique angles in real-world images.
 - Color Jitter: Randomly alters the brightness, contrast, and saturation of the images to make the model invariant to lighting conditions.
- Normalization: Input images were normalized using the mean and standard deviation of the ImageNet dataset, matching the pre-training conditions of the ResNet backbone.
- **Curriculum Learning:** We addressed curriculum learning with the 3 model head to make the model first learn easy (Make) and medium (Type) tasks before focusing on the hard (Model) task. To achieve it, we increased the number of epochs to 20 and in the 5 epochs, we froze the Model head, so it only trains on the Make and Type heads.
- **Hierarchical Label Smoothing (HLS):** While we were trying to squeeze out the last percentages of accuracy in training, we used HLS, to make the model penalize more the easy misses than the hard misses. Which means that for example mistaking a BMW M3 for a BMW 328i (Sibling error) shouldn't give the same loss as mistaking a BMW M3 for a Ford F-150 (Distant error). The technique we used for it is instead of a hard [0, 1, 0, 0] target, use soft targets that give partial credit to siblings. While the hard target gives 100% loss for the correct car, while the soft targets give 90% to the correct car and 10% distributed among other cars in the same Make

3.4 Inference Strategies:

To evaluate the robustness of our model, we employ Test Time Augmentation (TTA) during the inference phase. Standard inference utilizes a single center crop of the input image. In contrast, our TTA protocol averages the softmax probability distributions of $N = 2$ views: the original image (x) and a horizontally flipped version ($flip(x)$). The final prediction \hat{y} is computed as:

$$\hat{y} = \operatorname{argmax} \left(\frac{1}{2}(P(x) + P(flip(x))) \right)$$

This approach exploits the model's learned invariance to geometric transformations (due to the data augmentation in Section 5.4) to stabilize and often improve final predictions. We assumed that it is going to slightly increase our accuracy during testing.

4 Results

Organize results by "Research Question" rather than date.

4.1 Establishing the "Hierarchy Gap" (Baselines)

- **Objective:** Show that flat classifiers fail to capture relationships
- **Data:** Compare the Frozen Baseline (41.72% Acc) vs. Unfrozen (75.92% Acc).
- **Key Finding:** Even with decent accuracy, the "Gap" between Make Accuracy (85.49%) and Model Accuracy (75.92%) was nearly 10%, indicating the model was guessing Models without knowing the Brand.

4.2 The Impact of Regularization & Class Balancing

- **Objective:** Solving the overfitting problem.
- **Data:** Show the jump from 75.92% (Phase 3) to 85.65% (Phase 4) just by adding Data Augmentation.
- **Observation:** We gained nearly 10% accuracy on the hardest task without changing the architecture or training longer. This demonstrates why Data Augmentation is standard practice in Deep Learning: it converts "memorization" into "generalization."

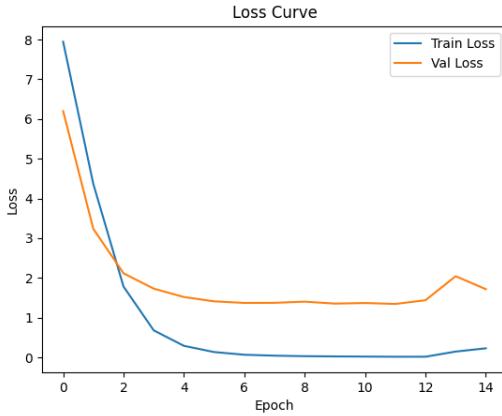


Figure 1: *
Without Data Augmentation

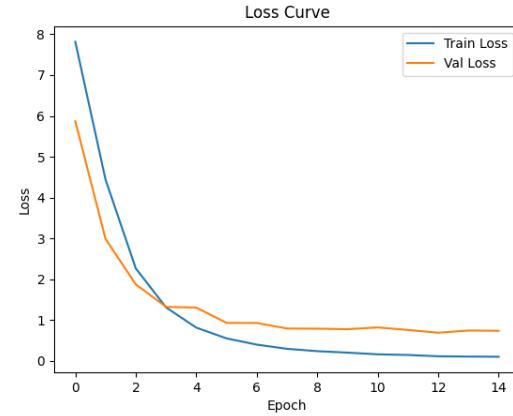


Figure 2: *
With Data Augmentation

4.3 Data Augmentation and Class Balancing on 2 Head (Model, Make)

- **Objective:** Make the model to perform better on more rare classes

- **Data:** While the 2 head model with Data augmentation the Model Head got 85.65% and the Make Head got 92,71% accuracy after class balancing the Model Head increased to 86.02% the Make Head decreased to 91.77%.
- **Key Finding:** While Class Balancing helped rare classes but slightly hurt overall consistency (The "Robin Hood" effect).

4.4 Optimization Dynamics: Interference vs. Curriculum

- **Objective:** Solving the "Task Interference" problem in Multi-Task Learning.
- **Data:** Contrast Experiment 6 (3-Head without Curriculum Learning: 85.36%) vs. Experiment 7 (3-Head with Curriculum Learning: 86.15%).
- **Key Finding:** Without Curriculum Learning, gradients conflicted. Freezing the hard head allowed the backbone to learn stable features first.

4.5 Architectural Ablation: The Necessity of "Type"

- **Objective:** Solving the overfitting problem.
- **Data:** Compare Experiment 8 (2-Head Curriculum: 85.51%) against Experiment 7 (3-Head Curriculum: 86.15% - initial) and Experiment 9/10 (Final 3-Head: 87,95%).
- **Key Finding:** While 3-Head initially struggled due to interference, once optimized (see next section), it outperformed the 2-Head approach, proving that the "Type" layer acts as a necessary semantic bridge.

4.6 SOTA Performance: Hierarchical Label Smoothing (HLS) & Learning Rate Scheduler (LR Scheduler)

- **Objective:** Pushing the limit.
- **Data:** Present the final training model (Experiment 10) achieving 87.95% Top-1 Accuracy. The Make-Model Consistency also improved from 96.60% to 97.66% and the Type-Model Consistency from 95.93% to 96.58%
- **Key Finding:** Learning Rate Scheduler and Hierarchical Label Smoothing closed pushed the Consistencies to the limit.

4.7 The Impact of Inference Strategies (Test Time Augmentation (TTA)):

- **Objective:** Trying to get the final accuracy improvement in testing phase.
- **Data:** After implementing 5 Test Time Augmentation the model reached 88,57% accuracy which is the best we could squeeze out.
- **Key Finding:** This confirms that the model's errors were partly due to view-specific noise which ensembling effectively mitigated.

Model	Top - 1 error rate
Frozen Baseline	58,28%
Unfrozen Baseline	24,08%
Regularization and Class Balancing on 1 Head	14,35%
Data Augmentation and Class Balancing on 2 Head	13,98%
3 Head without Curriculum Learning	14,64%
2 Head with Curriculum Learning	14,49%
3 Head with Curriculum Learning	13,85%
3 Head Hierarchical Label Smoothing and Learning Rate Scheduler	12,05%
3 Head Test Time Augmentation	11,43%

Table 2: Top-1 error rates through experiments

4.8 Compare our results to other benchmarks:

Paper	Model Architecture	Top - 1 Accuracy
A Systematic Evaluation of Recent Deep Learning Architectures for Fine-Grained Vehicle Classification	ResNet-50 (FineTuned on ImageNet)	92,0%
	ResNet-152 (FineTuned on ImageNet)	92,6%
	DenseNet-161 (FineTuned on ImageNet)	94,6%
Fine-Grained Image Classification for Vehicle Makes & Models using CNNs	VGG-16 backbone (pre-trained on ImageNet)	84,0%
Our Model	ResNet-50	88,57%

Table 3: Compare accuracy to related benchmarks

4.9 Qualitative evaluation



Figure 3: Five Stanford-Cars test images and the five labels considered most probable by our model (3 head, HLS, LRS). All 3 head predictions are wrong. The correct label and the five labels considered most probable are written under each image.

Across these five fully misclassified samples in Figure 3, a consistent pattern emerges: the model often shows high internal confidence despite being wrong, especially for the Make and Type heads. One example is the Aston Martin convertible, where the model predicts Ferrari as the make with 98.86% confidence, while Aston Martin does not even appear in the top-5 list. The same happens for the Type head, where Coupe is selected with 92.64% confidence, although the car is a convertible.

In contrast, the Rolls-Royce example shows a different behavior: the model predicts Maybach as the top make, but with only 27.99% confidence, and the true class Rolls-Royce still appears in the top-5 with nearly 10% probability. Among these misclassified cases, this is the closest the model gets to identifying the correct make.

Across the examples in Figure 4, where only the Make was predicted correctly, a consistent pattern emerges: when the model correctly identifies the Make, the predictions for Type and Model tend to be noticeably better than in cases where the Make is already mistaken.

The Bentley example illustrates this well. Once the model correctly recognizes the Make as Bentley with near certainty (99.99%), the Type prediction is already better than in the fully misclassified examples and Sedan appears with 14.45%. More importantly, the Model head assigns substantial probability to the correct Model family: the correct class Bentley Continental Flying Spur Sedan 2007 receives 33.59%, and other close variants of the Continental series dominate the rest of the top-5.

The Audi TTS example, on the other hand, reveals that correct Make classification does not always guarantee strong downstream predictions. While Audi is detected with high confidence (99.29%) and the Type prediction Coupe



Figure 4: Five Stanford-Cars test images and the five labels considered most probable by our model (3 head, HLS, LRS). Only the make head's prediction is correct. The correct label and the top five labels considered most probable are written under each image.

achieves a solid 36.56%, the Model head struggles because the true Model Audi TTS Coupe 2012 appears in the top-5 but with only 4.17% confidence.

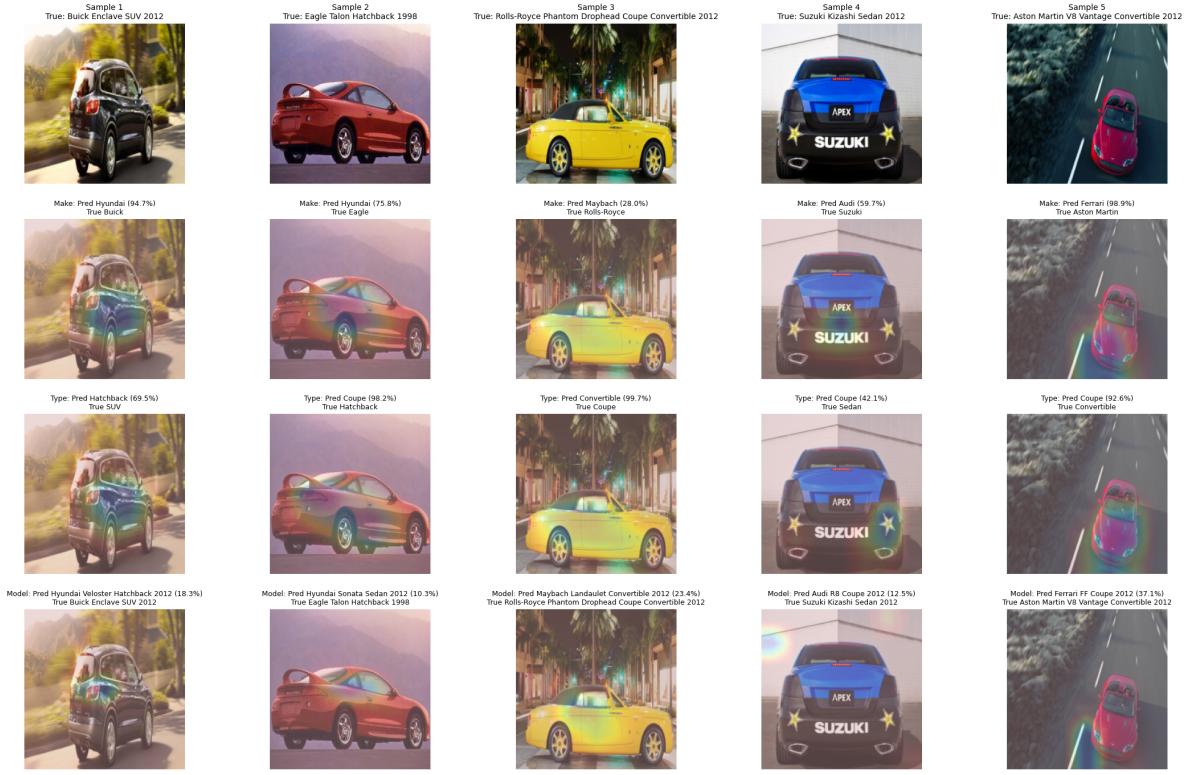


Figure 5: Five Stanford-Cars test images and the heatmaps showing on which image part our model (3 head, HLS, LRS) focused on when producing labels for each head. All 3 head predictions are wrong. The correct label and the heatmaps for each head are displayed under each image.

In the heatmaps of the fully misclassified samples shown in Figure 5, a consistent pattern becomes visible: whenever the image is captured from the side or slightly from behind (Samples 1-3), all three heads tend to focus on the central region of the vehicle. The Suzuki example shows this clearly. The image is taken entirely from the rear, and the Make head concentrates on the middle of the car. However, the Type head fixates on the large Rockstar Energy sticker on the right side of the trunk, which is completely unrelated to the vehicles actual structural features. The Model

head performs even worse, directing its attention away from the car entirely and focusing mainly on the left-hand background.

The Aston Martin sample provides a different perspective, as the photo is taken from above and toward the front. The Make and Type heads focus primarily on the bonnet, which is a reasonable region for high-level recognition. However, the Model head barely attends to the vehicle, only touching the right headlight and parts of the road next to the car.

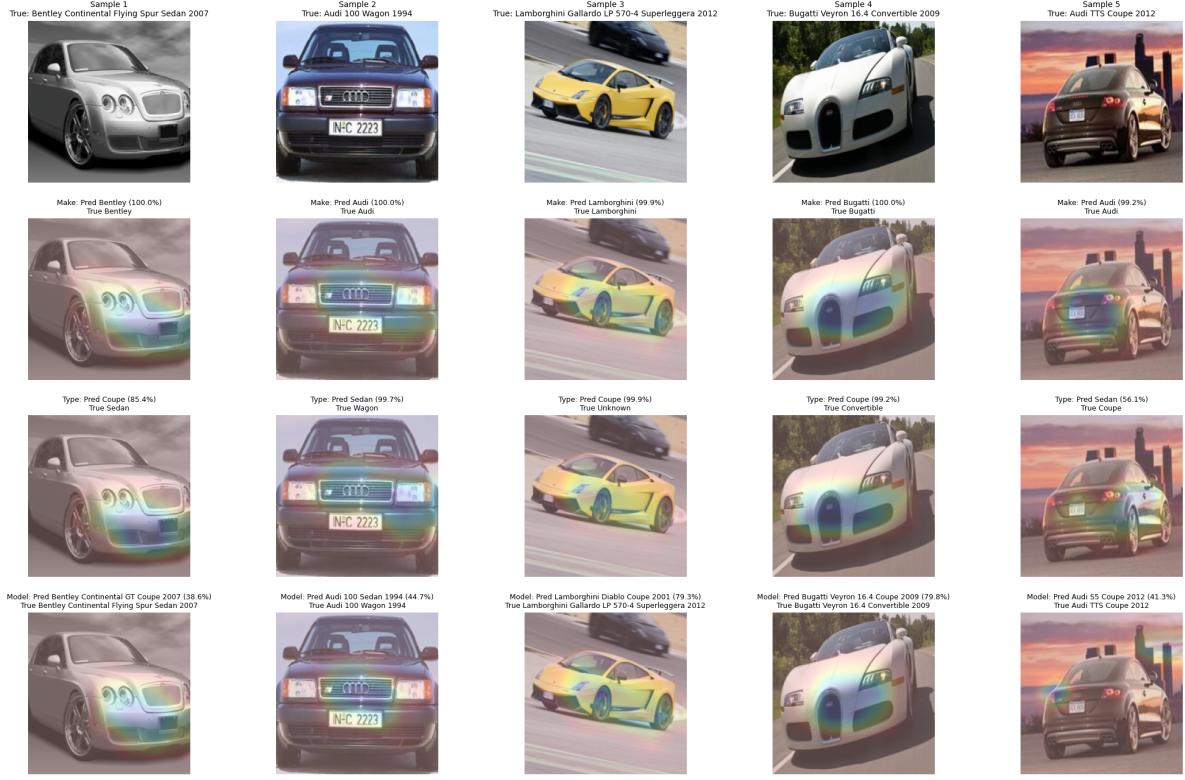


Figure 6: Five Stanford-Cars test images and the heatmaps showing on which image part our model (3 head, HLS, LRS) focused on when producing labels for each head. Only the make head's prediction is correct. The correct label and the heatmaps for each head are displayed under each image.

In Figure 6, the heatmaps for the samples where only the Make head predicted the correct class are shown. For the first four samples, each photographed from the front or slightly from the side, all three heads concentrate their attention on the central area of the cars front section.

The only notable deviation appears in the Audi TTS example, where the image is captured from the rear. The Make and Type heads still fixate on the central region of the rear end, which is consistent with their behavior in similar viewpoints. However, the Model head diverges completely and focuses primarily to the left rear light and even to the skyscrapers in the background.

5 Discussion

In the following section, we interpret the results of our project and critically examine the factors that shaped the models performance. We outline which components of our approach contributed meaningfully to improvements and where limitations were faced.

5.1 The "Frankenstein Car" Problem

- Discuss Hierarchical Consistency.
- In early experiments, consistency was low (90%). The model would predict "Toyota" (Make) and "Honda Civic" (Model).
- By Experiment 9 (HLS), consistency reached 96.6%, and finally 97.66% with the scheduler. This proves the model learned the taxonomy, not just pixel patterns.

A question in hierarchical classification is whether a model learns the underlying taxonomy or whether it simply memorizes pixel-level patterns without understanding the semantic relationships between labels. Early versions of our model frequently produced predictions where the make, type, and model belonged to different, incompatible vehicles. For example, the make head might output Toyota, while the model head simultaneously predicted Honda Civic. Such inconsistencies mean that each head is treated as an isolated classification task.

In the initial experiments, hierarchical consistency remained relatively low, hovering around 90%, meaning that in roughly one out of ten images, the predicted model did not belong to the predicted make. By introducing Hierarchical Label Smoothing (HLS) the consistency improved substantially. By Experiment 9, the model reached 96.6% consistency which demonstrates a marked improvement in aligning predictions across heads. The final experiment, which integrated both HLS and a cosine learning rate scheduler, further increased consistency to 97.66%.

This progression shows that the model was no longer merely fitting to visual textures or superficial cues but had begun to internalize the hierarchical taxonomy of make, type, and model. The reduction of incompatible predictions illustrates that the network learned meaningful relationships between car brands and their associated models which resulted in coherent outputs.

5.2 The Role of "Type" as Scaffolding

- Analyze why 3_head_curriculum eventually beat 2_head_curriculum.
- The "Type" head (Sedan, SUV, Coupe) provides Intermediate Scaffolding. It is easier to learn than "Model" but provides more structural information than "Make." It bridges the semantic gap.

A key insight from our experiments is the importance of the Type head (e.g., Sedan, SUV, Coupe) as an intermediate level of supervision. When comparing the two-head curriculum (Make + Model) to the three-head setup (Make + Type + Model), the latter consistently achieved higher accuracy and stability. This is because Type acts as a bridge because it is easier to learn than the fine-grained model classification but more informative than the broad make category.

By giving the model an additional intermediate task, the network receives structured guidance that reduces the complexity of learning the full 196-class model label. The Type head therefore provides scaffolding that helps the model organize visual information hierarchically. This effect explains why the three-head curriculum outperformed the two-head version in both accuracy and consistency.

5.3 The "Free Lunch" of Training Schedules

- Discuss the final experiment (LR Scheduler).
- ou gained 1.5% accuracy ($87.9\% \rightarrow 89.07\%$) just by changing the learning rate schedule. This indicates the architecture was sound, but the optimizer needed "fine-grained" control to settle into the sharp minima of the loss landscape.

Another finding was how strongly the learning-rate schedule influenced performance. After the architecture and curriculum strategy were already working well, switching to a learning-rate scheduler brought an additional improvement of roughly 1.5 percentage points (from 87.9% to 89.07% model accuracy) without modifying the model or adding new data.

This highlights that the model was already capable of achieving higher performance, but the optimizer was not navigating the loss landscape efficiently. The scheduler allowed the network to make larger exploratory steps early on and gradually settle into more stable minima toward the end of training. In that sense, learning-rate scheduling was a simple change that is responsible for additional accuracy while keeping training cost and model complexity unchanged.

5.4 Robustness via Inference Ensembling

Explain how TTA serves as a proxy for measuring model robustness, argue that this proves our model is robust and "production-ready"

Test-time augmentation (TTA) provided an additional perspective on the robustness of the trained model. By averaging predictions over several augmented versions of each test image, such as horizontal flips or cropped resizes, the model effectively performs a lightweight ensembling step during inference. This reduces the influence of single-view biases, such as lighting, framing, or minor pose variations.

In our experiments, the TTA results demonstrated that the model maintained highly stable predictions under these perturbations which confirms that it had learned generalizable visual features rather than overfitting to specific image conditions. Although the absolute performance gain from TTA was modest, the consistency across augmented views indicates that the model is resilient and can be considered production-ready for real-world scenarios.

6 Conclusion

The pursuit of fine-grained vehicle classification in this project demonstrated that while a powerful backbone like ResNet-50 is essential for feature extraction, overcoming the challenge of subtle visual differences requires more than just raw model capacity; it demands a structured learning methodology. Our initial single-head baseline, while functional, failed to capture the intrinsic make-type-model hierarchy.

The key to success lay in adopting the Shared-Bottom Multi-Task Learning architecture, specifically the three-head design. The "Type" head proved invaluable, acting as a semantic bridge that provided crucial intermediate scaffolding for the fine-grained "Model" prediction, resolving initial task interference when combined with Curriculum Learning. Furthermore, the introduction of Hierarchical Label Smoothing (HLS) and a fine-tuned Learning Rate Scheduler were critical for squeezing out the final performance gains and enforcing logical coherence, driving prediction consistency to 97.66% and virtually eliminating the "Frankenstein Car" problem.

Achieving a final Top-1 accuracy of 88.57% (with TTA) validates that our strategy was sound. This project confirms that for complex, hierarchical tasks, structuring the learning process and enforcing taxonomy coherence is paramount, yielding a final model that is not only accurate but also highly resilient and logically robust against real-world variations.

7 Headings: first level

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7.1 Headings: second level

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^N \sum_{j=1}^N \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \quad (1)$$

7.1.1 Headings: third level

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Paragraph Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

8 Examples of citations, figures, tables, references

8.1 Citations

Citations use `natbib`. The documentation may be found at

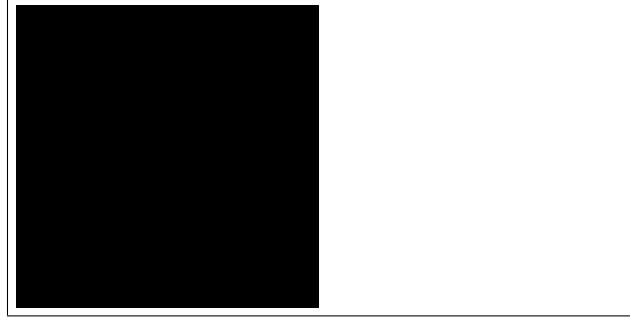


Figure 7: Sample figure caption.

Table 4: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	~ 100
Axon	Output terminal	~ 10
Soma	Cell body	up to 10^6

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Here is an example usage of the two main commands (`citet` and `citep`): Some people thought a thing (??) but other people thought something else (?). Many people have speculated that if we knew exactly why ? thought this...

8.2 Figures

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdierat lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetur odio sem sed wisi. See Figure ???. Here is how you add footnotes.² Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetur eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdierat justo nec dolor.

8.3 Tables

See awesome Table ??.

The documentation for `booktabs` ('Publication quality tables in LaTeX') is available from:

<https://www.ctan.org/pkg/booktabs>

8.4 Lists

- Lorem ipsum dolor sit amet
- consectetur adipiscing elit.
- Aliquam dignissim blandit est, in dictum tortor gravida eget. In ac rutrum magna.

²Sample of the first footnote.