STAT 580 Homework 1

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1.
```c
#include <stdio.h>
#include <math.h>

#define P0 0.01 /*lower limit of the probability (p) */
#define P1 0.5 /* upper limit of the probability (p) */
#define PLEN 10 /* number of columns */
#define N 5 /* number of experiments (n) */

int factorial(int n)
{
    if (n <= 1)
        return 1;
    else
        return (n * factorial(n - 1));
}

double binopmf(int n, int x, double p)
{
    return (factorial(n) / (factorial(n - x) * factorial(x))) * pow(p, x
        ) * pow(1 - p, n - x);
}

int main()
{
    printf("x\\p\t");
    double step = (P1 - P0) / (double) (PLEN - 1);
    for (int i = 1; i <= PLEN - 1; i++)
    {
        printf("%.4f\t", P0 + (i - 1)*step);
    }
    printf("%.4f\n", P0 + (PLEN - 1)*step);

    for (int x = 0; x <= N; x++)
    {
        printf("%d\t", x);
        for (int i = 1; i <= PLEN - 1; i++)
        {
            printf("%.4f\t", binopmf(N, x, P0 + (i - 1)*step));
        }
        printf("%.4f\n", binopmf(N, x, P0 + (PLEN - 1)*step));
    }
    return 0;
}
```

2. (a) $\int_1^{10} f(x) = 1 \Rightarrow c\int_1^{10} \frac{1}{x}\mathrm{d}x = c(\log 10 - 0) = 1 \Rightarrow \frac{1}{\log 10}$. Hence

$$F(x) = \int_1^x f(x)\mathrm{d}x = \int_1^x \frac{1}{\log 10}\frac{1}{x}\mathrm{d}x = \frac{\log x}{\log 10} = \log_{10}(x),\ 1 < x < 10$$

Hence the inverse function

$$F^{-1}(u) = 10^u,\ 0 < u < 1$$

Thus the algorithm would be

Algorithm 1 Sampling X with cdf $F(x) = \log_{10}(x)$

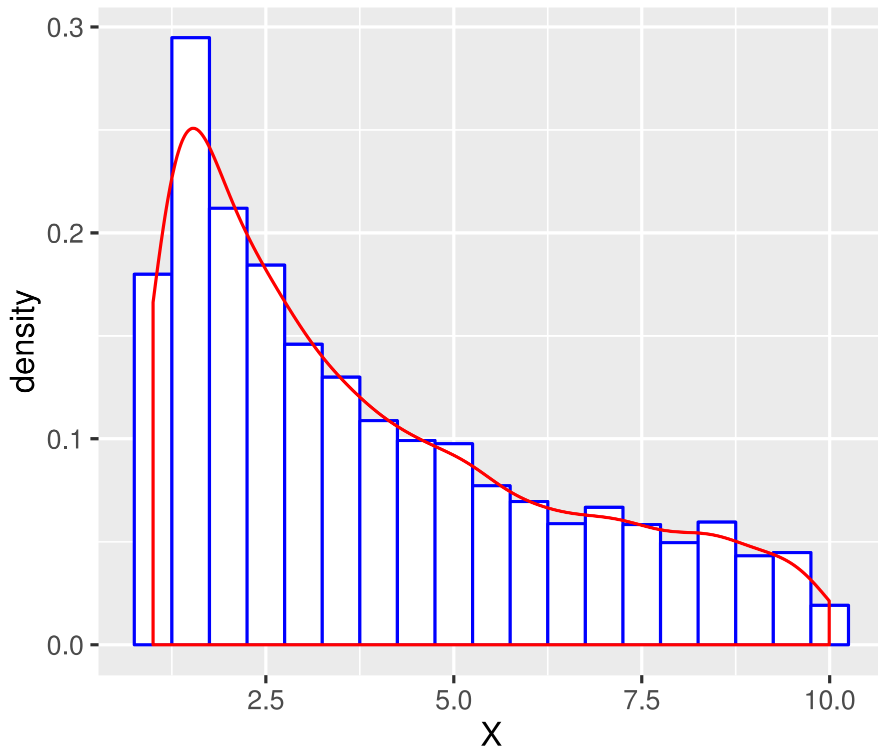
1: Generate $U \sim Unif(0, 1)$;
2: Set $X = 10^U$;

(b)

```
#include <stdio.h>
#include <time.h>
#define MATHLIB_STANDALONE
#include <Rmath.h>

int main()
{
    double u, x;

    set_seed(time(NULL), 580580); /* set seed */

    u = unif_rand();
    x = pow(10.0, u);
    printf("%f\n", x);

    return 0;
}
```

(c)

```
require(ggplot2)

U <- runif(5000, min = 0, max = 1);
X <- 10^U;

d <- data.frame(x = X);

ggplot(d, aes(x = X)) + geom_histogram(aes(y = ..density..),
    binwidth = 0.5, color = "blue", fill = "white") + geom_density(
    alpha = .2 , colour = "red",fill = "#FF6666")
```

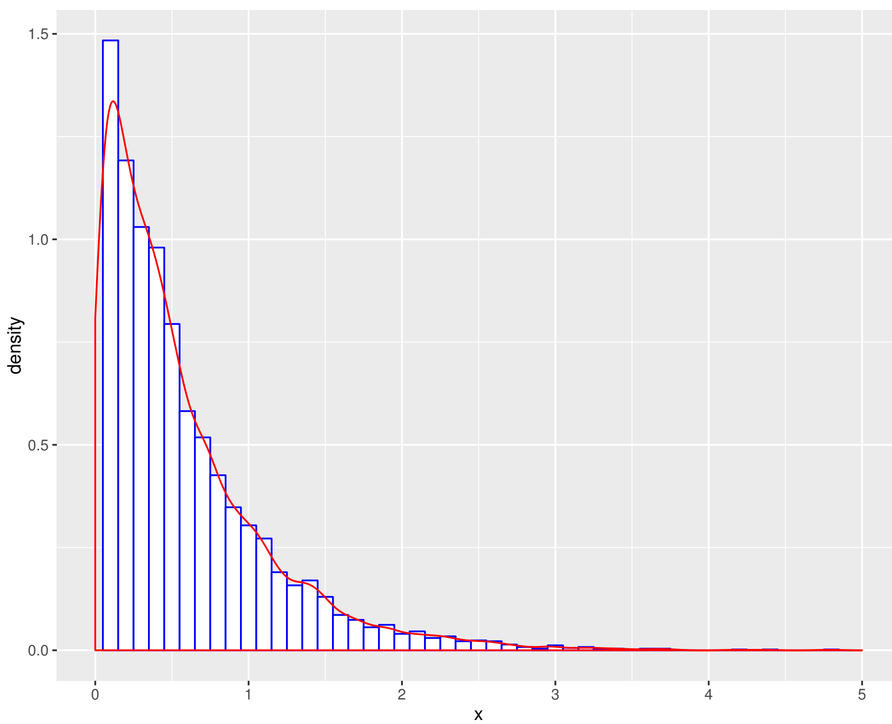


3. (a)

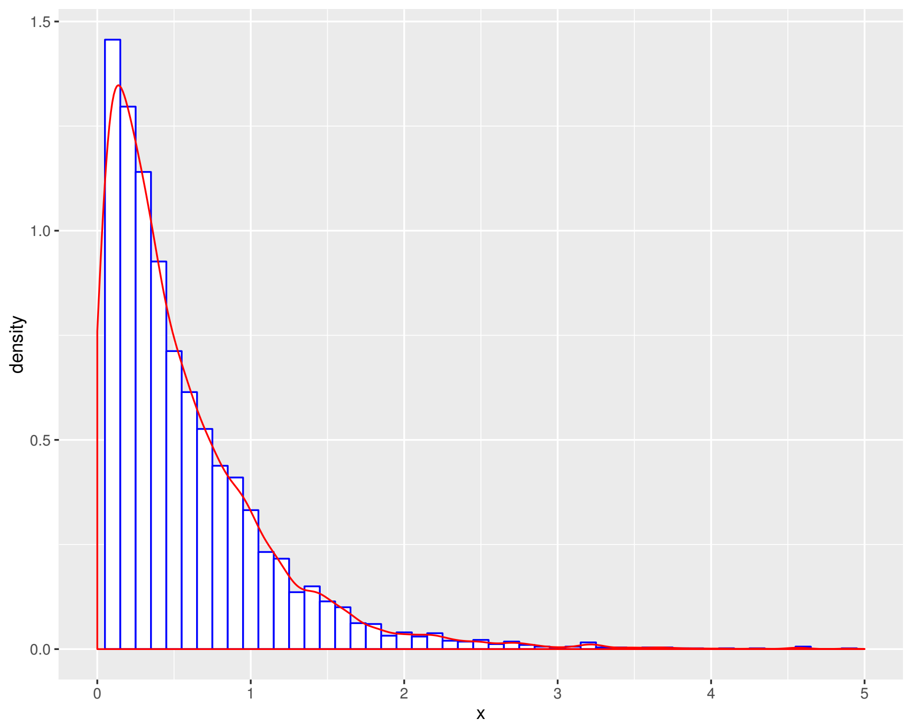
```
require(ggplot2)

sample1 <- function (n)
{
  size <- 0;
  O <- rep(0,5000);
  while (size < 5000)
  {
    U <- runif(1);
    X <- rexp(1);
    if (U <= 1/(1 + X^2))
    {
      size <- size + 1;
      O[size] <- X;
    }
  }

  return(O)
}

sample2 <- function (n)
{
  size <- 0;
  O <- rep(0,5000);
  while (size < 5000)
  {
    U <- runif(1);
    X <- abs(rcauchy(1));
```

```
    if (U <= exp(-X))
    {
      size <- size + 1;
      O[size] <- X;
    }
  }

  return(O)
}

d1 <- data.frame(x = sample1(5000));
ggplot(d1, aes(x = x)) + geom_histogram(aes(y = ..density..),
    binwidth = 0.1, color = "blue", fill = "white") + geom_density(
    alpha = .2 , colour = "red",fill = "#FF6666") + xlim(0,5)

d2 <- data.frame(x = sample2(5000));
ggplot(d2, aes(x = x)) + geom_histogram(aes(y = ..density..),
    binwidth = 0.1, color = "blue", fill = "white") + geom_density(
    alpha = .2 , colour = "red",fill = "#FF6666") + xlim(0,5)

system.time(sample1(5000))

system.time(sample2(5000))
```

The density when using $g_1(x) = \mathrm{e}^{-x}$:



The density when using $g_2(x) = \frac{\pi(1+x^2)}{2}$:



(b) The density we get using g_1 and g_2 are almost the same. And the time using g_2 is longer than the time using g_1.

The `system.time(sample1(5000))` returns the time when using g_1:

```
user   system elapsed
0.04     0.00    0.04
```

The `system.time(sample2(5000))` returns the time when using g_2:

```
user   system elapsed
0.07     0.00    0.08
```

4.

Algorithm 2 Sampling from $f(x, y) \propto x^\alpha y$ with support being $x^2 + y^2 \leq 1$ in the first quadrant

1: Generate $X \sim Beta(\alpha + 1, 1)$ and $Y \sim Beta(2, 1)$ independently;
2: **if** $X^2 + Y^2 > 1$ **then**
    Go to Step 1;
3: **else**
    Return (X, Y);
4: **end if**