STAT 510 Homework 12

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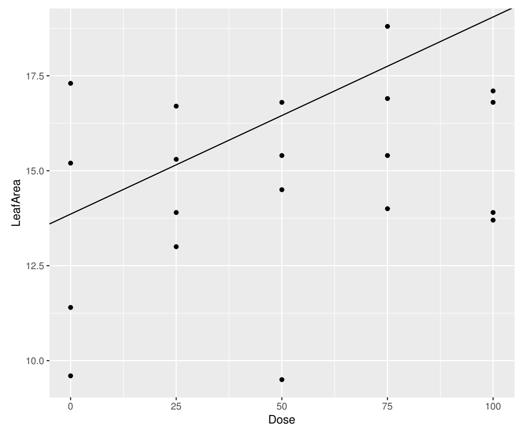
1. (a) $\hat{\sigma}_e^2 = 3.949$

(b) $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{b}} = \begin{bmatrix} 10.49 & 1.46 \times 10^{-3} \\ 1.46 \times 10^{-3} & 5.62 \times 10^{-5} \end{bmatrix}$

(c)



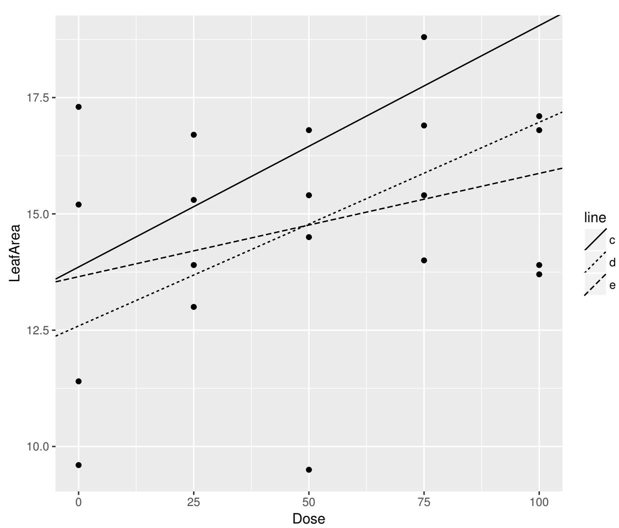
(d)

$$12.591 + 0.044x$$

(e)

$$13.650 + 0.022x$$

(f)



(g)

$$-2\log\Lambda = 30.73227$$

(h) $AIC = 1345.905$

(i) $AIC = 1342.693$

(j) $AIC = 1650.107$

(k) Model in part (i) is preferred. The AIC of that model is the smallest.

2.

$$\boldsymbol{X} = \begin{bmatrix} \mathbf{1}_{300}, \mathbf{1}_{15} \otimes \begin{bmatrix} 0 \\ 25 \\ 50 \\ 75\ 100 \end{bmatrix} \otimes \mathbf{1}_4 \end{bmatrix}$$

$$\boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\boldsymbol{Z} = \boldsymbol{I}_{15\times 15} \otimes \begin{bmatrix} \mathbf{1}_{20}, \begin{bmatrix} 0 \\ 25 \\ 50 \\ 75\ 100 \end{bmatrix} \otimes \mathbf{1}_4 \end{bmatrix}$$

$$\boldsymbol{u} = \begin{bmatrix} b_{11} \\ b_{12} \\ b_{21} \\ b_{22} \\ \vdots \\ b_{15,1} \\ b_{15,2} \end{bmatrix}$$

$$\boldsymbol{G} = \boldsymbol{I}_{15\times 15} \otimes \boldsymbol{\Sigma_b}$$

$$\boldsymbol{R} = \sigma_e^2 \boldsymbol{I}_{300\times 300}$$

3. (a)

$$\boldsymbol{X} = \begin{bmatrix} \mathbf{1}_{n_1} \otimes \boldsymbol{I}_{t\times t} & & \\ & \mathbf{1}_{n_2} \otimes \boldsymbol{I}_{t\times t} & \\ & & \mathbf{1}_{n_3} \otimes \boldsymbol{I}_{t\times t} \end{bmatrix}$$

(b)

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{I}_{n_1\times n_1} \otimes \boldsymbol{W} & & \\ & \boldsymbol{I}_{n_2\times n_2} \otimes \boldsymbol{W} & \\ & & \boldsymbol{I}_{n_3\times n_3} \otimes \boldsymbol{W} \end{bmatrix} = \boldsymbol{I}_{(n_1+n_2+n_3)\times(n_1+n_2+n_3)} \otimes \boldsymbol{W}$$

(c)

$$\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} \boldsymbol{I}_{n_1\times n_1}\otimes \boldsymbol{W}^{-1} & & \\ & \boldsymbol{I}_{n_2\times n_2}\otimes \boldsymbol{W}^{-1} & \\ & & \boldsymbol{I}_{n_3\times n_3}\otimes \boldsymbol{W}^{-1} \end{bmatrix}$$

$$\boldsymbol{X}^T = \begin{bmatrix} \boldsymbol{1}_{n_1}^T\otimes \boldsymbol{I}_{t\times t} & & \\ & \boldsymbol{1}_{n_2}^T\otimes \boldsymbol{I}_{t\times t} & \\ & & \boldsymbol{I}_{n_3}\otimes \boldsymbol{I}_{t\times t} \end{bmatrix}$$

$$\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} \boldsymbol{1}_{n_1}^T\otimes \boldsymbol{W}^{-1} & & \\ & \boldsymbol{1}_{n_2}^T\otimes \boldsymbol{W}^{-1} & \\ & & \boldsymbol{1}_{n_3}^T\otimes \boldsymbol{W}^{-1} \end{bmatrix}$$

$$\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X} = \begin{bmatrix} n_1\boldsymbol{W}^{-1} & & \\ & n_2\boldsymbol{W}^{-1} & \\ & & n_3\boldsymbol{W}^{-1} \end{bmatrix}$$

$$(\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X})^{-1} = \begin{bmatrix} \frac{1}{n_1}\boldsymbol{W} & & \\ & \frac{1}{n_2}\boldsymbol{W} & \\ & & \frac{1}{n_3}\boldsymbol{W} \end{bmatrix}$$

(d)

$$\begin{aligned} &(\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1} \\ &= \begin{bmatrix} \frac{1}{n_1}\boldsymbol{W} & & \\ & \frac{1}{n_2}\boldsymbol{W} & \\ & & \frac{1}{n_3}\boldsymbol{W} \end{bmatrix} \begin{bmatrix} \boldsymbol{1}_{n_1}^T\otimes \boldsymbol{W}^{-1} & & \\ & \boldsymbol{1}_{n_2}^T\otimes \boldsymbol{W}^{-1} & \\ & & \boldsymbol{1}_{n_3}^T\otimes \boldsymbol{W}^{-1} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{n_1}\boldsymbol{1}_{n_1}^T\otimes \boldsymbol{I}_{t\times t} & & \\ & \frac{1}{n_2}\boldsymbol{1}_{n_2}^T\otimes \boldsymbol{I}_{t\times t} & \\ & & \frac{1}{n_3}\boldsymbol{1}_{n_3}^T\otimes \boldsymbol{I}_{t\times t} \end{bmatrix} \end{aligned}$$

(e)

$$\begin{aligned} &(\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{y} \\ &= \begin{bmatrix} \frac{1}{n_1}\boldsymbol{1}_{n_1}^T\otimes \boldsymbol{I}_{t\times t} & & \\ & \frac{1}{n_2}\boldsymbol{1}_{n_2}^T\otimes \boldsymbol{I}_{t\times t} & \\ & & \frac{1}{n_3}\boldsymbol{1}_{n_3}^T\otimes \boldsymbol{I}_{t\times t} \end{bmatrix}\boldsymbol{y} \\ &= \begin{bmatrix} \frac{1}{n_1}\sum_{j=1}^{n_1}\boldsymbol{y}_{1j} \\ \frac{1}{n_2}\sum_{j=1}^{n_2}\boldsymbol{y}_{2j} \\ \frac{1}{n_3}\sum_{j=1}^{n_3}\boldsymbol{y}_{3j} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{y}}_{1\cdot} \\ \bar{\boldsymbol{y}}_{2\cdot} \\ \bar{\boldsymbol{y}}_{3\cdot} \end{bmatrix} \end{aligned}$$

(f)

$$\begin{aligned} \boldsymbol{\mu}_1 &= \left[\bar{y}_{1\cdot 1}, \bar{y}_{1\cdot 2}, \ldots, \bar{y}_{1\cdot t}\right]^T \\ \boldsymbol{\mu}_2 &= \left[\bar{y}_{2\cdot 1}, \bar{y}_{2\cdot 2}, \ldots, \bar{y}_{2\cdot t}\right]^T \\ \boldsymbol{\mu}_3 &= \left[\bar{y}_{3\cdot 1}, \bar{y}_{3\cdot 2}, \ldots, \bar{y}_{3\cdot t}\right]^T \end{aligned}$$