STAT 510 Homework 9

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1. (a)

fert	geno 1	2	3
0	125	140	115
50	141.25	156.25	141.25
100	150	165	160
150	151.25	166.25	171.25

(b) Not true.

(c) Not true.

(d) Not true.

(e) Geno Type 1:

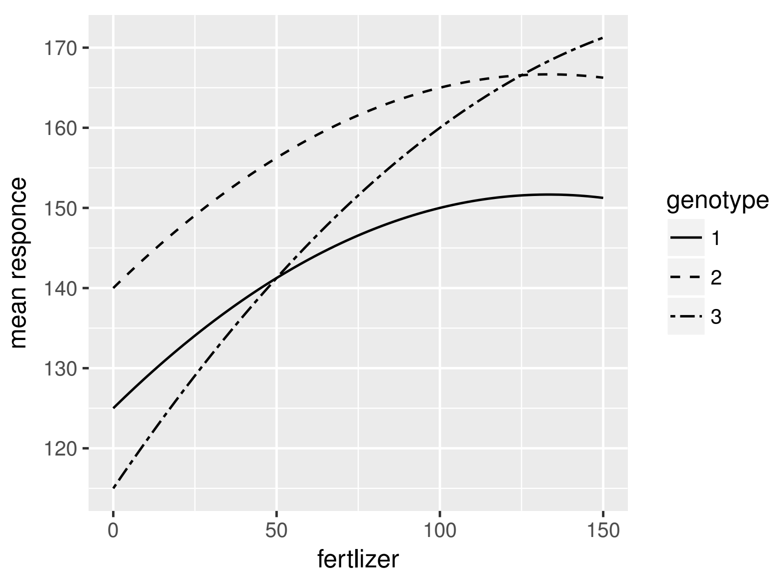
$$E(y) = 125 + 0.4f - 0.0015f^2$$

Geno Type 2:

$$E(y) = 140 + 0.4f - 0.0015f^2$$

Geno Type 3:

$$E(y) = 115 + 0.6f - 0.0015f^2$$



(f) $\bar{y}_{11\cdot} - \bar{y}_{12\cdot} = -13.75$, $SE = \sqrt{\frac{\hat{\sigma}_e^2}{2}} = \frac{6.30128}{\sqrt{2}}$.
$CI = (\bar{y}_{11\cdot} - \bar{y}_{12\cdot} - SE \cdot t_{27,0.975}, \bar{y}_{11\cdot} - \bar{y}_{12\cdot} + SE \cdot t_{27,0.975}) = (-22.8923, -4.607704)$.

(g) $\mu_{11} - \mu_{12} = -15 \in CI = (-22.8923, -4.607704)$.

(h) $\bar{y}_{11}-\bar{y}_{21}=-22.5$, $SE=\frac{\sqrt{\hat{\sigma}_e^2+\hat{\sigma}_w^2}}{\sqrt{2}}=\sqrt{\frac{39.70613+67.2981}{2}}=7.314514$, $df=\frac{\left(\frac{1}{4}MS_{Block\times Geno}+\frac{3}{4}MS_{Error}\right)^2}{\frac{1}{16}\frac{MS^2_{Block\times Geno}}{6}+\frac{9}{16}\frac{MS^2_{Error}}{27}}=$ 11.15.

$CI=(\bar{y}_{11\cdot}-\bar{y}_{21\cdot}-SE\cdot t_{11.15,0.975},\bar{y}_{11\cdot}-\bar{y}_{12\cdot}+SE\cdot t_{11.15,0.975})=(-38.57272,-6.427279)$.

(i) $\mu_{11}-\mu_{21}=-16.25\in CI=(-38.57272,-6.427279)$.

(j) $SE=\frac{\hat{\sigma}_b^2}{4}+\frac{\hat{\sigma}_w^2}{12}+\frac{\hat{\sigma}_e^2}{48}=\frac{MS_{Block}}{48}$, $df=4-1=3$.

2. (a)

$$\frac{MS_{\text{WL}}}{MS_{\text{GH:WL}}}=\frac{160.9}{19.4}=8.29$$

(b)

$$\frac{MS_{\text{GENO}}}{MS_{\text{Error}}}=\frac{MS_{\text{GENO}}}{\frac{SS_{\text{GH:GENO}}+SS_{\text{GENO:WL:GENO}}}{3+6}}=\frac{2.5}{\frac{11.7+14.5}{3+6}}=\frac{2.5}{2.92}=0.856$$

(c)

$$\frac{MS_{\text{WL:GENO}}}{MS_{\text{Error}}}=\frac{37.5}{2.92}=12.84$$

3. (a) $avg_{ik}=a_{ik}=\bar{\mu}_{i\cdot}+p_k+\bar{e}_{i\cdot k}$, then $\bar{a}_{1\cdot}=\bar{\mu}_{1\cdot}+\frac{1}{6}\sum_{k=1}^{6}p_k+\frac{1}{6}\sum_{k=1}^{6}\bar{e}_{1\cdot k}$, $\bar{a}_{2\cdot}=\bar{\mu}_{2\cdot}+\frac{1}{12}\sum_{k=7}^{18}p_k+\frac{1}{12}\sum_{k=7}^{18}\bar{e}_{2\cdot k}$. Hence we have

$$Var\left(\bar{a}_{1\cdot}\right)=\frac{1}{6}\sigma_p^2+\frac{1}{6}\frac{1}{2}\sigma_e^2=\frac{\sigma_p^2}{6}+\frac{\sigma_e^2}{12},\ Var(\bar{a}_{2\cdot})=\frac{1}{12}\sigma_p^2+\frac{1}{12}\frac{1}{2}\sigma_e^2=\frac{1}{2}Var(\bar{a}_{1\cdot})$$

Note that $\bar{a}_{1\cdot}-\bar{a}_{2\cdot}=\bar{\mu}_{1\cdot}-\bar{\mu}_{2\cdot}+\frac{1}{6}\sum_{k=1}^{6}(p_k+\bar{e}_{1\cdot k})+\frac{1}{12}\sum_{k=7}^{18}(p_k+\bar{e}_{2\cdot k})$, and $Var(\bar{a}_{1\cdot}-\bar{a}_{2\cdot})=\frac{1}{6}\sigma_p^2+\frac{1}{12}\sigma_p^2+\frac{1}{12}\sigma_e^2+\frac{1}{24}\sigma_e^2=\frac{1}{4}\sigma_p^2+\frac{1}{8}\sigma_e^2$. Thus test statistic is

$$T=\frac{84.892-80.454}{\sqrt{2.169^2+1.534^2}}=1.6705$$

(b) $diff_{ik}=d_{ik}=\mu_{i1}-\mu_{i2}+e_{i1k}-e_{i2k}$, then $Var(\bar{d}_{1\cdot})=Var\left(\mu_{11}-\mu_{12}+\frac{1}{6}\sum_{k=1}^{6}(e_{11k}-e_{12k})\right)=\frac{1}{6}2\sigma_e^2=\frac{1}{3}\sigma_e^2$, $Var(\bar{d}_{2\cdot})=Var\left(\mu_{21}-\mu_{22}+\frac{1}{12}\sum_{k=7}^{18}(e_{21k}-e_{22k})\right)=\frac{1}{12}2\sigma_e^2=\frac{1}{6}\sigma_e^2=\frac{1}{2}Var(\bar{d}_{1\cdot})$.
Note that $\frac{1}{2}(\bar{d}_{1\cdot}+\bar{d}_{2\cdot})=\bar{\mu}_{\cdot 1}-\bar{\mu}_{\cdot 2}+\frac{1}{12}\sum_{k=1}^{6}(e_{11k}-e_{12k})+\frac{1}{24}\sum_{k=7}^{18}(e_{21k}-e_{22k})$ and $Var(\frac{1}{2}(\bar{d}_{1\cdot}+\bar{d}_{2\cdot}))=\frac{1}{8}\sigma_e^2$. Hence the test statistic

$$T=\frac{\frac{1}{2}(8.25+1.492)}{\sqrt{\frac{1}{4}(2.439^2+1.724^2)}}=3.262$$

(c) $\bar{y}_{11\cdot}-\bar{y}_{12\cdot}-\bar{y}_{21\cdot}+\bar{y}_{22\cdot}=\mu_{11}-\mu_{12}-\mu_{21}+\mu_{22}+\frac{1}{6}\sum_{k=1}^{6}p_k+\frac{1}{6}\sum_{k=1}^{6}e_{11k}-\frac{1}{6}\sum_{k=1}^{6}p_k-\frac{1}{6}\sum_{k=1}^{6}e_{12k}-\frac{1}{12}\sum_{k=7}^{18}p_k-\frac{1}{12}e_{21k}+\frac{1}{12}\sum_{k=7}^{1}8p_k+\frac{1}{12}\sum_{k=7}^{18}e_{22k}=\mu_{11}-\mu_{12}-\mu_{21}+\mu_{22}+\bar{e}_{11\cdot}-\bar{e}_{12\cdot}-\bar{e}_{21\cdot}+\bar{e}_{22\cdot}$. And $Var(\bar{y}_{11\cdot}-\bar{y}_{12\cdot}-\bar{y}_{21\cdot}+\bar{y}_{22\cdot})=\frac{1}{6}\sigma_e^2+\frac{1}{6}\sigma_e^2+\frac{1}{12}\sigma_e^2+\frac{1}{12}\sigma_e^2=\frac{1}{2}\sigma_e^2$.
We also know that $(\bar{y}_{11\cdot}-\bar{y}_{12\cdot}-\bar{y}_{21\cdot}+\bar{y}_{22\cdot})^2/(1/6+1/6+1/12+1/12)=MS_{\text{geno:infection}}\Rightarrow \bar{y}_{11\cdot}-\bar{y}_{12\cdot}-\bar{y}_{21\cdot}+\bar{y}_{22\cdot}=\sqrt{91.35/2}=6.758$. Then test statistic

$$T=\frac{6.758}{\sqrt{2.439^2+1.724^2}}=2.263$$

(d) $\hat{\sigma}_e^2=(2.439^2+1.724^2)\times 2=17.84$

(e) $\hat{\sigma}_p^2=(2.169^2+1.534^2)\times 4-17.84/2=19.31$.