# STAT 510 Homework 9

Yifan Zhu

April 2, 2017

**1.** (a)

| fert | geno 1 | 2 | 3 |
|---|---|---|---|
| 0 | 125 | 140 | 115 |
| 50 | 141.25 | 156.25 | 141.25 |
| 100 | 150 | 165 | 160 |
| 150 | 151.25 | 166.25 | 171.25 |

(b) Not true.

(c) Not true.

(d) Not true.

(e) Geno Type 1:

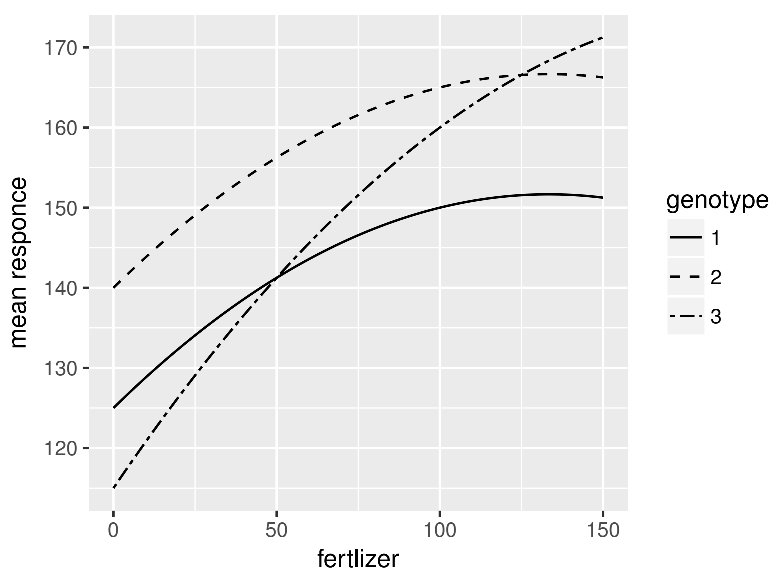
$$E(y) = 125 + 0.4f - 0.0015f^2$$

Geno Type 2:

$$E(y) = 140 + 0.4f - 0.0015f^2$$

Geno Type 3:

$$E(y) = 115 + 0.6f - 0.0015f^2$$




(f) $\bar{y}_{11\cdot} - \bar{y}_{12\cdot} = -13.75$, $SE = \sqrt{\frac{\hat{\sigma}_e^2}{2}} = \frac{6.30128}{\sqrt{2}}$.

$CI = (\bar{y}_{11\cdot} - \bar{y}_{12\cdot} - SE \cdot t_{27,0.975}, \bar{y}_{11\cdot} - \bar{y}_{12\cdot} + SE \cdot t_{27,0.975}) = (-22.8923, -4.607704)$.

(g) $\mu_{11} - \mu_{12} = -15 \in CI = (-22.8923, -4.607704)$.

(h) $\bar{y}_{11} - \bar{y}_{21} = -22.5$, $SE = \frac{\sqrt{\hat{\sigma}_e^2 + \hat{\sigma}_w^2}}{\sqrt{2}} = \sqrt{\frac{39.70613+67.2981}{2}} = 7.314514$, $df = \frac{\left(\frac{1}{4} MS_{Block\times Geno} + \frac{3}{4} MS_{Error}\right)^2}{\frac{1}{16}\frac{MS^2_{Block\times Geno}}{6} + \frac{9}{16}\frac{MS^2_{Error}}{27}} = 8.88$.

$CI = (\bar{y}_{11\cdot} - \bar{y}_{21\cdot} - SE \cdot t_{27,0.975}, \bar{y}_{11\cdot} - \bar{y}_{12\cdot} + SE \cdot t_{8.88,0.975}) = (-39.08073, -5.919272)$.

(i) $\mu_{11} - \mu_{21} = -16.25 \in CI = (-39.08073, -5.919272)$.

(j) $SE = \frac{\hat{\sigma}_b^2}{4} + \frac{\hat{\sigma}_w^2}{12} + \frac{\hat{\sigma}_e^2}{48} = \frac{MS_{Block}}{48}$, $df = 4 - 1 = 3$.

**2.** (a)

$$\frac{MS_{\text{WL}}}{MS_{\text{GH:WL}}} = \frac{160.9}{19.4} = 8.29$$

(b)

$$\frac{MS_{\text{GENO}}}{MS_{\text{Error}}} = \frac{MS_{\text{GENO}}}{\frac{SS_{\text{GH:GENO}} + SS_{\text{GENO:WL:GENO}}}{3+6}} = \frac{2.5}{\frac{11.7+14.5}{3+6}} = \frac{2.5}{2.92} = 0.856$$

(c)

$$\frac{MS_{\text{WL:GENO}}}{MS_{\text{Error}}} = \frac{37.5}{2.92} = 12.84$$

**3.** (a) $avg_{ik} = a_{ik} = \bar{\mu}_{i\cdot} + p_k + \bar{e}_{i\cdot k}$, then $\bar{a}_{1\cdot} = \bar{\mu}_{1\cdot} + \frac{1}{6}\sum_{k=1}^{6} p_k + \frac{1}{6}\sum_{k=1}^{6} \bar{e}_{1\cdot k}$, $\bar{a}_{2\cdot} = \bar{\mu}_{2\cdot} + \frac{1}{12}\sum_{k=7}^{18} p_k + \frac{1}{12}\sum_{k=7}^{18} \bar{e}_{2\cdot k}$. Hence we have

$$Var\left(\bar{a}_{1\cdot}\right) = \frac{1}{6}\sigma_p^2 + \frac{1}{6}\frac{1}{2}\sigma_e^2 = \frac{\sigma_p^2}{6} + \frac{\sigma_e^2}{12},\ Var(\bar{a}_{2\cdot}) = \frac{1}{12}\sigma_p^2 + \frac{1}{12}\frac{1}{2}\sigma_e^2 = \frac{1}{2}Var(\bar{a}_{1\cdot})$$

Note that $\bar{a}_{1\cdot} - \bar{a}_{2\cdot} = \bar{\mu}_{1\cdot} - \bar{\mu}_{2\cdot} + \frac{1}{6}\sum_{k=1}^{6}(p_k + \bar{e}_{1\cdot k}) + \frac{1}{12}\sum_{k=7}^{18}(p_k + \bar{e}_{2\cdot k})$, and $Var(\bar{a}_{1\cdot} - \bar{a}_{2\cdot}) = \frac{1}{6}\sigma_p^2 + \frac{1}{12}\sigma_p^2 + \frac{1}{12}\sigma_e^2 + \frac{1}{24}\sigma_e^2 = \frac{1}{4}\sigma_p^2 + \frac{1}{8}\sigma_e^2$. Thus test statistic is

$$T = \frac{84.892 - 80.454}{\sqrt{2.169^2 + 1.534^2}} = 1.6705$$

(b) $diff_{ik} = d_{ik} = \mu_{i1} - \mu_{i2} + e_{i1k} - e_{i2k}$, then $Var(\bar{d}_{1\cdot}) = Var\left(\mu_{11} - \mu_{12} + \frac{1}{6}\sum_{k=1}^{6}(e_{11k} - e_{12k})\right) = \frac{1}{6}2\sigma_e^2 = \frac{1}{3}\sigma_e^2$, $Var(\bar{d}_{2\cdot}) = Var\left(\mu_{21} - \mu_{22} + \frac{1}{12}\sum_{k=7}^{18}(e_{21k} - e_{22k})\right) = \frac{1}{12}2\sigma_e^2 = \frac{1}{6}\sigma_e^2 = \frac{1}{2}Var(\bar{d}_{1\cdot})$. Note that $\frac{1}{2}(\bar{d}_{1\cdot} + \bar{d}_{2\cdot}) = \bar{\mu}_{\cdot 1} - \bar{\mu}_{\cdot 2} + \frac{1}{12}\sum_{k=1}^{6}(e_{11k} - e_{12k}) + \frac{1}{24}\sum_{k=7}^{18}(e_{21k} - e_{22k})$ and $Var(\frac{1}{2}(\bar{d}_{1\cdot} + \bar{d}_{2\cdot})) = \frac{1}{8}\sigma_e^2$. Hence the test statistic

$$T = \frac{\frac{1}{2}(8.25 + 1.492)}{\sqrt{\frac{1}{4}(2.439^2 + 1.724^2)}} = 3.262$$

(c) $\bar{y}_{11\cdot} - \bar{y}_{12\cdot} - \bar{y}_{21\cdot} + \bar{y}_{22\cdot} = \mu_{11} - \mu_{12} - \mu_{21} + \mu_{22} + \frac{1}{6}\sum_{k=1}^{6} p_k + \frac{1}{6}\sum_{k=1}^{6} e_{11k} - \frac{1}{6}\sum_{k=1}^{6} p_k - \frac{1}{6}\sum_{k=1}^{6} e_{12k} - \frac{1}{12}\sum_{k=7}^{18} p_k - \frac{1}{12}e_{21k} + \frac{1}{12}\sum_{k=7}^{1} 8p_k + \frac{1}{12}\sum_{k=7}^{18} e_{22k} = \mu_{11} - \mu_{12} - \mu_{21} + \mu_{22} + \bar{e}_{11\cdot} - \bar{e}_{12\cdot} - \bar{e}_{21\cdot} + \bar{e}_{22\cdot}$. And $Var(\bar{y}_{11\cdot} - \bar{y}_{12\cdot} - \bar{y}_{21\cdot} + \bar{y}_{22\cdot}) = \frac{1}{6}\sigma_e^2 + \frac{1}{6}\sigma_e^2 + \frac{1}{12}\sigma_e^2 + \frac{1}{12}\sigma_e^2 = \frac{1}{2}\sigma_e^2$. We also know that $(\bar{y}_{11\cdot} - \bar{y}_{12\cdot} - \bar{y}_{21\cdot} + \bar{y}_{22\cdot})^2/(1/6 + 1/6 + 1/12 + 1/12) = MS_{\text{geno:infection}} \Rightarrow \bar{y}_{11\cdot} - \bar{y}_{12\cdot} - \bar{y}_{21\cdot} + \bar{y}_{22\cdot} = \sqrt{91.35/2} = 6.758$. Then test statistic

$$T = \frac{6.758}{\sqrt{2.439^2 + 1.724^2}} = 2.263$$

(d) $\hat{\sigma}_e^2 = (2.439^2 + 1.724^2) \times 2 = 17.84$

(e) $\hat{\sigma}_p^2 = (2.169^2 + 1.534^2) \times 4 - 17.84/2 = 19.31$.