Statistics 601, Spring 2018

Assignment 2

Likelihood ratio tests are nearly always presented in terms of *nested parameter spaces* for reduced and full models (e.g., Casella and Berge, page 375). This is techinically correct if one first fixes the joint distribution of all random variables involved in the problem (recall our definition of a model from Stat 520). But that can be a slippery concept in some applications.

Consider a simple situation in which we have two binomial random variables Y_1 and Y_2, say, and our objective is to test whether the binomial parameters for these are the same or not (we will assume binomial sample sizes are known and fixed). The usual presentation for this that I get in Stat 520 or Stat601 assignments or exams lists the full and reduced models as follows:

- Full Model: Two binomial distributions,

$$Y_1 \sim \text{Binom}(p_1, n_1) \quad Y_2 \sim \text{Binom}(p_2, n_2),$$

  where n_1 and n_2 are given.

- Reduced Model: One binomial distribution,

$$(Y_1 + Y_2) \sim \text{Binom}(p, n_1 + n_2).$$

Dislcuss whether this is or is not correct. Do these models have nested parameter spaces? If we form log likelihoods from these full and reduced models, will the results of estimation and testing be correct?
Hint: Do this once dropping constants from the log likelihoods and then again without dropping constants from the log likelihoods.

What general issue is involved here?

Hint: Consider what you would tell students if you were teaching this topic in a class. That is, what would you tell students about how to avoid gettnig confused by the prescription that likelihood ratio tests only require nested parameter spaces for the full and reduced models.