STAT 500 Homework 6

Yifan Zhu

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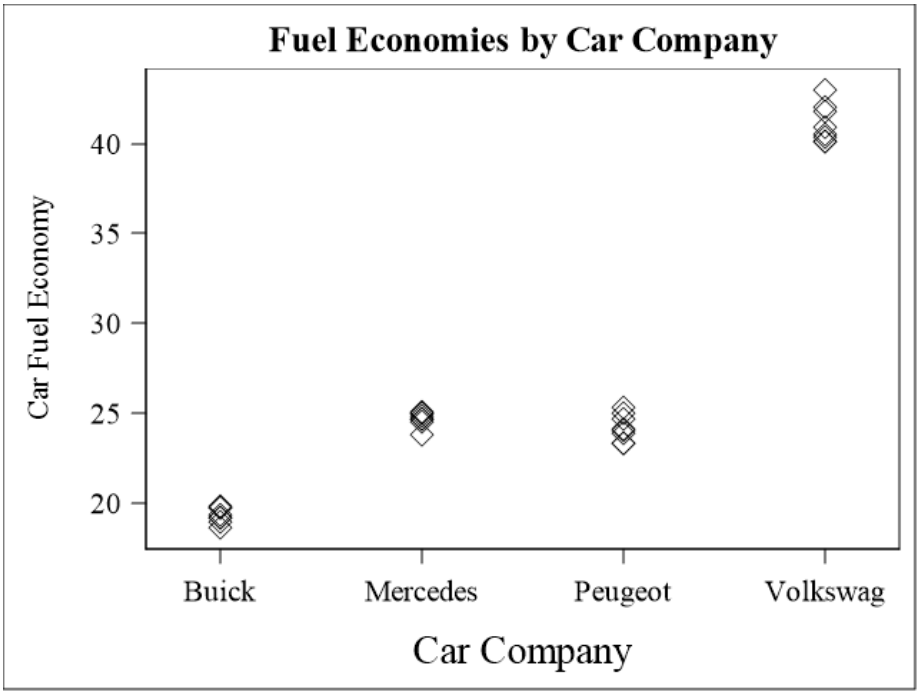
1. (a) The side-by-side dotplot of the fuel economies by car company is shown below. From the plot we can see that the fuel economies of cars from Mercedes and Peugeot are almost the same. While the fuel economy of cars from Volkswagen appears to be higher than these two and fuel economy of cars from Buick appears to be lower than these two.



(b) Denote the mean fuel economies of cars from Buick μ_1, Mercedes μ_2, Peugeot μ_3 and Volkswagen μ_4.

$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a : at least one μ_i from $\mu_1, \mu_2, \mu_3, \mu_4$ is different than others.

Test statistic : $F = 1434.06$

p-value: $< .0001$

we will reject the null hypothesis and conclude there is sufficient evidence that at least one of the mean fuel economies of cars from the four companies is different than the others.

Source	DF	Sum of Squares	Mean Square	F Value	Pr >F
Model	3	2166.460000	722.153333	1434.06	<.0001
Error	28	14.100000	0.503571		
Corrected Total	31	2180.560000			

(c) The result of Tukey's HSD method is shown below. From the result we can see that the mean fuel economies of cars from Mercedes and Peugeot, there is no significant difference in the mean fuel economy between these two brands. The mean fuel economy of Volkswagen is significantly different from the others with a higher mean. The mean fuel economy of Buick is also significantly different from the other three with a lower mean.

Alpha	0.05
Error Degrees of Freedom	28
Error Mean Square	0.503571
Critical Value of Studentized Range	3.86124
Minimum Significant Difference	0.9688

Tukey Grouping	Mean	N	company
A	41.1000	8	Volkswag
B	24.7000	8	Mercedes
B	24.2000	8	Peugeot
C	19.3000	8	Buick

(d) In the order of μ_1, μ_2, μ_3, μ_4, the contrasts should be like this

i. Volkswagen versus the other three companies: $\boldsymbol{c} = (-1/3, -1/3, -1/3, 1)$

ii. Buick versus Mercedes and Peugeot: $\boldsymbol{c} = (1, -1/2, -1/2, 0)$

iii. Mercedes versus Peugeot: $\boldsymbol{c} = (0, 1, -1, 0)$

$(-1/3) \times 1 + (-1/3) \times (-1/2) + (-1/3) \times (-1/2) + 1 \times 0 = 0$

$(-1/3) \times 1 + (-1/3) \times (-1/2) + (-1/3) \times (-1/2) + 1 \times 0 = 0$

$1 \times 0 + (-1/2) \times 1 + (-1/2) \times (-1) + 0 \times 0 = 0$

Thus these three contracts are orthogonal.

(e) This result means the two contrasts $\mu_4 - (\mu_1 + \mu_2 + \mu_3)/3$ and $\mu_1 - (\mu_2 + \mu_3)/2$ are significantly different from 0, while the contrast $\mu_2 - \mu_3$ is not significantly different from 0. Thus, the mean fuel economy of cars from Volkswagen is significant different from the mean fuel economy of the other three companies; the mean fuel economy of cars from Buick is significantly different than the mean fuel economy of Mercedes and Peugeot; but the mean fuel economy of Mercedes and Peugeot are not significantly different.

Contrast	Estimate	Standard Error	Contrast SS	F Value	Pr $> F$
Volkwagen-(Buick + Mercedes + Peugeot)/3	18.3666667	0.28970428	2024.006667	4019.30	$< .0001$
Buick-(Mercedes+Peugeot)/2	-5.1500000	0.30727779	141.453333	280.90	$< .0001$
Mercedes-Peugeot	0.5000000	0.35481383	1.000000	1.99	0.1698

(f) i. Independence: the description of the experiment did not indicate how the experiment units are selected. If the cars from the 4 companies are randomly chosen from the population, we can assume they are independent.

ii. Homogeneous variance: The ratio of the largest to smallest company standard deviation is between 2 and 3, and the Brown and Forsythe's Test fails to reject the null hypothesis of equal variances. So we will conclude the assumption of homogeneous variances is valid.

	Buick	Mercedes	Peugeot	Volkswagen
Std. Dev.	0.4208834	0.4140393	0.7425824	1.0555973

Source	DF	Sum of Squares	Mean Square	F Value	Pr $> F$
company	3	1.5400	0.5133	2.37	0.0917
Error	28	6.0600	0.2164		

iii. Normal Distribution: The normal quantile plot does not indicate a large amount of deviation from a straight line. So we will conclude the assumption of normality is valid.