STAT 500 Homework 7

Yifan Zhu

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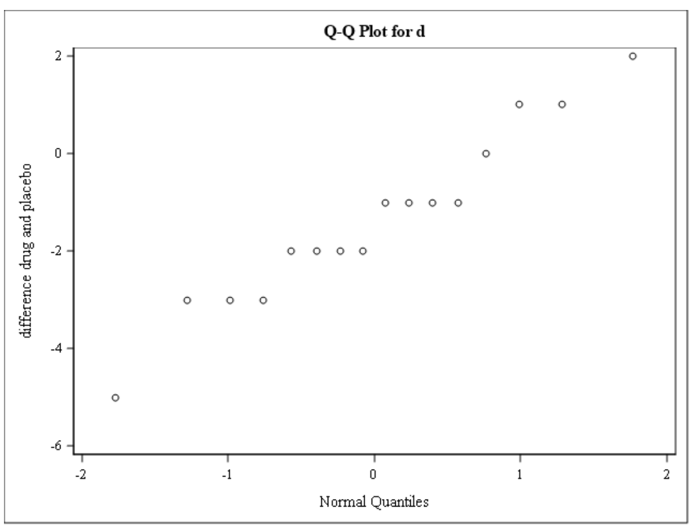
1. (a) Each student is a block.

(b) The treatments in this experiment are listening to classical music and no music.

(c) Each student received two treatments.

(d) Denote μ_1 the mean score students goy with no music, μ_2 the mean score with music.
$H_0 = \mu_1 = \mu_2$
$H_a = \mu_1 < \mu_2$
The t-test statistic is -3.08263. The SAS output reports the two-sided p-value. Since we need the one-sided p-value, we will divide this p-value from the SAS output by 2 and we have $0.0076/2 = 0.0038$. With a large p-value $0.0038 < \alpha = 0.05$, we will reject the null hypothesis and conclude there is significant evidence to claim the mean score is greater when studying while listening to classical music.

(e) The assumptions are that the differences are independent and normally distributed. The differences are independent from the description of the experiment design, because these students are randomly sampled and are independent. The normal probability plot of the differences is given below. From the plot we do not see a lot of variation from a straight line, so the normal assumption should be met.



(f) The two treatments (music and no music) should be assigned randomly in 2 periods of the experiment. In this way we can balance out some potential bias.

2. (a) i. it is a randomized complete block experiment.

ii. experimental units: all plots of a certain kind of crop.
treatments: six plant densities. (7, 8, 9, 10, 11, and 12 plant/m^2)
blocks: five fields with different soil fertility.

iii.

Source	DF	Sums of Squares
treatment	5	
block	4	
error	20	
total	29	

(b) i. it is a completely randomized experiment.

ii. experimental units: 15 different boards.
treatments: three cutting speeds (50, 70, and 90 rpm).

iii.

Source	DF	Sums of Squares
treatment	2	
error	12	
total	14	

(c) i. it is a randomized complete block experiment.

ii. experimental units: all workers in 8 plants.
treatments: four distinct music programs and no music.
blocks: 8 plants.

iii.

Source	DF	Sums of Squares
treatment	4	
block	7	
error	28	
total	39	

3. (a)

Source	DF	Sum of Squares	Mean Square	F Value	$Pr > F$
Subject	14	371.5066667	26.5361905	18.45	$< .0001$
Feet Placement	2	91.6000000	45.8000000	31.84	$< .0001$
Error	28	40.2733333	1.4383333		
Corrected Total	44	503.3800000			

(b) Significant differences in mean torque for the three feet placements is determined through testing for the treatment effect (feet placements). The F-test from the ANOVA Table gives the F statistic value as 31.84 with a p-value of < 0.0001. This means there is a statistically significant difference in the mean torque levels for the three feet placements.

(c) Below is the output for the Tukey HSD method for pairwise comparisons of the treatment means. In this case, the mean torque when feet placement is neutral is significantly different from the mean torque for other two feet placements with the highest mean torque level. There is no significant differencce in the mean torque for feet placements of back and staggered.

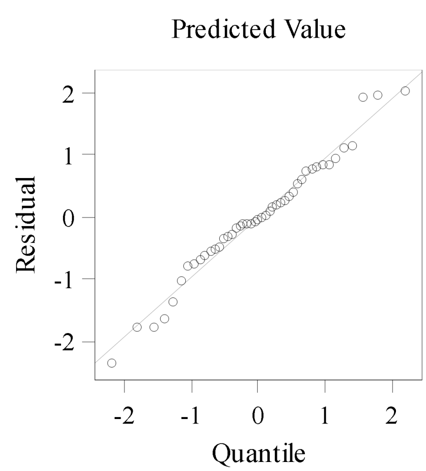
Tukey Grouping	Mean	N	Feet Placement
A	24.1000	15	N
B	21.7000	15	B
B	20.7000	15	S

(d) Two contracts used are $-\mu_B + \mu_S$ and $-\mu_B + 2\mu_N - \mu_S$. We can see $-1 + 1 = 0$ and $-1 + 2 - 1 = 0$, so they are contrast. Also, $(-1) \cdot (-1) + 0 \cdot 2 + 1 \cdot (-1) = 0$, thus they are orthogonal contrasts.

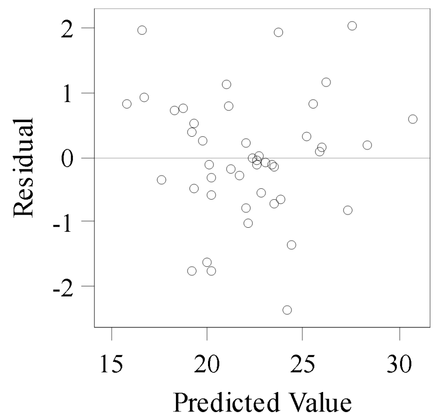
Contrast	DF	Contrast SS	Mean Square	F Value	$Pr > F$
$-\mu_B + \mu_S$	1	7.50000000	7.50000000	5.21	0.0302
$-\mu_B + 2\mu_N - \mu_S$	1	84.10000000	84.10000000	58.47	$< .0001$

The F-test statistic for the $-\mu_B + \mu_S$ contrast is 5.21 with a pvalue 0.032 and for the $-\mu_B + 2\mu_N - \mu_S$ contrast is 58.47 with a p-value < 0.0001. This means that $-\mu_B + 2\mu_N - \mu_S$ is significantly different from 0, which supports the result in (c). The p-value for $-\mu_B + \mu_S$ is also low, but it is much bigger then that of $-\mu_B + 2\mu_N - \mu_S$. We can still conclude that $-\mu_B + \mu_S$ is significantly different than 0, which does not support (c).

(e) The normal probability plot suggests the residuals are from a normal distribution, because the values arein a straight line.



(f) The residuals in this plot should be randomly scattered, but there appears to be a pattern. The residuals for low and high predicted values are positive, and the residuals for many of the middle predicted values are negative or positive. This indicates that the model is not capturing some particular feature of the data.



(g)

$$\hat{\sigma}_{CRD} = \frac{(n-1)MS_{blocks} + n(J-1)MS_{error}}{nJ-1} = \frac{14 \times 26.54 + 15 \times 2 \times 1.44}{15 \times 3 - 1} = 9.43$$

$$\hat{\sigma}_{RCBD} = MS_{error} = 1.44$$

$$\text{Estimated Efficiency} = \frac{(28+3)9.43}{(28+1)1.44} = 7.00$$

This means to have the same efficiency, $n_{CRD} = 7.00 n_{RCBD}$. Thus blocking is a efficient method in this case.