Homework 8

Due March 12, 2020 at 11:59 PM

1. P. 323 : 7 (5 points)

   The probability of one part failing to meet inspection is equal to the long-run fraction of parts failing to meet inspection, if the part measurements are considered to be identical random variables. You need to make μ small enough so that

   $$P(X > 3.150) \leq 0.003$$

   This is equivalent to $P(X \leq 3.150) > 0.97$, which is equivalent to $P\left(Z \leq \frac{3.150-\mu}{0.002}\right) > 0.97$, where Z is a standard normal random variable. Looking up 0.97 in the body of Table B.3

   $$\frac{3.150 - \mu}{0.002} > 1.88,$$

   or $\mu < 3.14624$.

2. P. 332: 42 (2×5 points)

   (a) The probability of an individual depth being within specification is the same as the long-run fraction of depths within specifications, if successive shelf depths can be considered identical random variables.

   $$\begin{aligned} P(0.0275 \leq X \leq 0.0278) &= P(X \leq 0.0278) - P(X < 0.0275) \\ &= P(Z \leq 2) - P(Z < -1) \\ &= 0.9773 - 0.1587 = 0.8186 \end{aligned}$$

   (Z is a standard normal random variable.)

   (b) Assuming that $\mu = 0.02765$, we want

   $$P(0.0275 \leq X \leq 0.0278) = 0.98$$

   By symmetry, this is equivalent to $P(X \leq 0.0278) = 0.99$, which is equivalent to

   $$P\left(Z \leq \frac{0.0278 - 0.02765}{\sigma}\right) = 0.99$$

   Looking uo 0.99 in the body of Table B.3, this means that

   $$\frac{0.00015}{\sigma} \approx 2.33$$

   or that $\sigma \approx 0.00006438$.

3. P. 322: 3 (5×5 points)

   From P. 263:1, $E(X) = \mu = \frac{13}{27}$, $Var(X) = \sigma = 0.28808$.

(a)

$$E(\bar{(X)}) = \mu = \frac{13}{27},\ SD(\bar{X}) = \sqrt{Var(\bar{X})} = \sqrt{\frac{\sigma^2}{n}} = \frac{0.28808}{\sqrt{25}} = 0.05762.$$

(b) Because n is large and the individual X's are independe, the central limit theorem says that the distribution of $\bar{X}$ is approximatelt normal. From part (a), the distribution of $\bar{X}$ is approximately normal with mean $\frac{13}{27}$ and standard deviation 0.05762.

(c)

$$\begin{aligned} P(\bar{X} > 0.5) &= 1 - P(\bar{X} < 0.5) \\ &= 1 - P\left(\frac{\bar{X} - \frac{13}{27}}{0.05762} \leq \frac{0.5 - \frac{13}{27}}{0.05762}\right) \\ &= 1 - P(Z \leq 0.321) \quad \text{where } Z \text{ is standard normal} \\ &= 1 - 0.6255 = 0.3745. \end{aligned}$$

(d)

$$\begin{aligned} P(0.4615 \leq \bar{X} \leq 0.5015) &= P(\bar{X} \leq 0.5015) - P(\bar{X} < 0.4615) \\ &= P(Z \leq 0.35) - P(Z < -0.35) \\ &= 0.6368 - 0.3632 = 0.2736. \end{aligned}$$

(e) $\bar{X}$ is approximately normal with mean $\frac{13}{27}$ and standard deviation $\frac{0.28808}{\sqrt{100}} = 0.02881$.

$$\begin{aligned} P(\bar{X} > 0.5) &= 1 - P(\bar{X} \leq 0.5) \\ &= 1 - P\left(\frac{\bar{X} - \frac{13}{27}}{0.02881} \leq \frac{0.5 - \frac{13}{27}}{0.02881}\right) \\ &= 1 - P(Z \leq 0.64) \\ &= 1 - 0.7389 = 0.2611. \end{aligned}$$

$$\begin{aligned} P(0.4615 \leq \bar{X} \leq 0.5015) &= P(\bar{X} \leq 0.5015) - P(\bar{X} < 0.4615) \\ &= P(Z \leq 0.69) - P(Z < -0.69) \\ &= 0.7549 - 0.2451 = 0.5098 \end{aligned}$$

4. P. 324: 10 (5 points)

Let $X_1, X_2, \ldots, X_{370}$ be the 370 individual sheet thickness. The thickness of the text is then $U = \sum_i X_i$. The mean of the sum is the sum of the mean (by linearity of expectation):

$$E(U) = E(\sum_i X_i) = \sum_i E(X_i) = 370 \times 0.1 = 37$$

Assuming that the individual thickness are independent, you can say the variance of the sum is the sum of the variance (the coefficient are squares of ones, and therefore ones):

$$Var(U) = Var(\sum_i X_i) = \sum_i Var(X_i) = 370 \times (0.003)^2 = 0.00333,$$

so the standard deviation of U is $\sqrt{0.00333} = 0.577$.

5. P. 326: 20 (5 points)

Since the sample size is large, the central limit theorem says that the distribution of $\bar{X}$ is approximately normal. The mean of $\bar{X}$ is μ and the standard deviation of $\bar{X}$ is $\frac{0.1}{\sqrt{25}}$.

$$\begin{aligned} P(\mu - 0.03 \leq barX \leq \mu + 0.03) &= P\left(\frac{-0.03}{\frac{0.1}{\sqrt{25}}} \leq \frac{\bar{X} - \mu}{\frac{0.1}{\sqrt{25}}} \leq \frac{0.03}{\frac{0.1}{\sqrt{25}}}\right) \\ &= P(-1.5 \leq Z \leq 1.5) \\ &= P(Z \leq 1.5) - P(Z < -1.5) \\ &= 0.9332 - 0.0668 = 0.8664. \end{aligned}$$