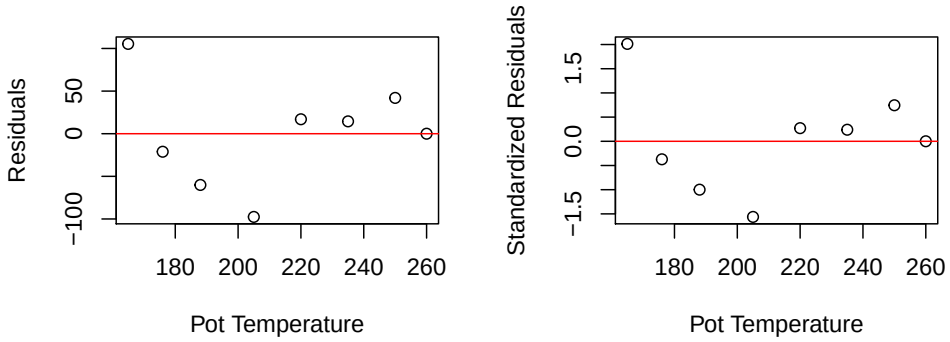
Homework 11

Due April 16, 2020 at 11:59 PM

1. P. 674: 1 (ignore (e), (g)) (10 points for (b), 8 points for others), dataset: `polyol.jmp`

   (a) $s_{LF} = 67.01$. Thus measures the baseline variation in the molecular weight that would be observed for any fixed pot temperature, assuming the SLR model is appropriate.

   (b) The residuals versus x and the standard residual versus x plots are almost the same.



   (c) For a 90% two-sided confidence interval, we use $t_{n-2,1-\alpha/2} = t_{6,0.95} = 1.943$. We calculate $\bar{x} = 212.375, \sum_i (x_i - \bar{x})^2 = 8469.875$. So we have

$$\begin{aligned} & b_1 \pm t_{n-2,1-\alpha/2} \frac{s_{LF}}{\sqrt{\sum_i (x_i - \bar{x})}} \\ =& 23.49827 \pm 1.943 \frac{67.01}{\sqrt{8469.875}} \\ =& (22.08, 24.91) \end{aligned}$$

   (d) For a 90% two-sided confidence interval, we use $t_{n-2,1-\alpha/2} = t_{6,0.95} = 1.943$. So for the mean at $x = 212$,

$$\begin{aligned} & b_0 + b_1 x \pm t_{n-2,1-\alpha/2} s_{LF} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \\ =& 1807.063 \pm 1.942 \cdot 67.01 \sqrt{\frac{1}{8} + \frac{0.140625}{8469.875}} \\ =& (1761.03, 1853.10) \end{aligned}$$

For the mean at $x = 250$,

$$\begin{aligned} & b_0 + b_1 x \pm t_{n-2,1-\alpha/2} s_{LF} \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \\ =& 2699.997 \pm 1.942 \cdot 67.01 \sqrt{\frac{1}{8} + \frac{1415.641}{8469.875}} \\ =& (2629.63, 2770.37) \end{aligned}$$

(f) For a 90% one-sided prediction interval, we use $t_{n-2,1-\alpha} = t_{6,0.9} = 1.440$. So at $x = 212$, the lower prediction bound is

$$\begin{aligned} & b_0 + b_1 x - t_{n-2,1-\alpha} s_{LF} \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \\ =& 1807.063 - 1.440 \cdot 67.01 \sqrt{1 + \frac{1}{8} + \frac{0.140625}{8469.875}} \\ =& 1704.72 \end{aligned}$$

For $x = 250$,

$$\begin{aligned} & b_0 + b_1 x - t_{n-2,1-\alpha} s_{LF} \sqrt{1 + \frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \\ =& 2699.997 - 1.440 \cdot 67.01 \sqrt{1 + \frac{1}{8} + \frac{1415.641}{8469.875}} \\ =& 2590.31 \end{aligned}$$

(h) The ANOVA table is

Source	SS	df	MS	F
Regression	4676798	1	4676798	1041.58
Error	26941	6	4490	
Total	4703739	7		

The p-value is

$$P(F_{1,6} > 1041.58)$$

which is less than 0.001, according to Table B.6E, the 0.999 quantile of $F_{1,6}$ is $35.51 < 1041.58$. This is overwhelming evidence that the mean average molecular weight is related to the pot temperature.