Spring 2020 **Stat 305 (Section 4)** **Quiz 2**

Name:

Solution

Total points for the exam is 50. Points for individual questions are given at the beginning of each problem. Show all your calculations clearly. Put final answers in the box at the right (except for the diagrams!).

1. [6+6+6+2+4+4+2=30 points]

The data used in creating the attached JMP output (Page 5) appear in the text *Quality Control and Industrial Statistics* by Duncan (and were from a paper of L. E. Simon). The data were collected in a study of the effectiveness of armor plate. Armor-piercing bullets were fired at an angle of 40° against armor plate of thickness x_1 (in 10^{-3} in.) and Brinell hardnessnumber x_2, and the resulting so-called ballistic limit, y (in ft/sec), was measured.

(a) What were the two least square equations fit to the data ?

Equation 1: $\hat{y} =$ $8.2672166 x_1 - 886.0007$

Equation 2: $\hat{y} =$ $7.6122317 x_1 - 2.5938892 x_2 - 1674.083$

(b) What fraction of the raw variability in the *ballistic limit* is accounted for by the two equations?

For equation 1: 0.115383

For equation 2: 0.601328

(c) What is the sample correlation between y and $\hat{y}$ by the two equations? (Give a number.)

equation 1 : $\sqrt{R^2} = \sqrt{0.115383} = 0.3397$

equation 2 : $\sqrt{R^2} = \sqrt{0.601328} =$

For equation 1: 0.3397

For equation 2: 0.7755

(d) What is the sample correlation between *ballistic limit* (y) and *thinkness of the armor plate* (x_1)? (Give a number, be careful about the sign.)

$r = \sqrt{R^2} = 0.3397$

$r =$ 0.3397

(e) Using the **first equation**, what *ballistic limit* would you predict when *thinkness of the armor plate* is 251×10^{-3} in. ? Would you be willing to predict ~~*strength of wood beams*~~ ballistic limit when ~~*moisture content*~~ thickness of armor plate is 100×10^{-3} in. ? Why or why not?

$\hat{y} = 8.2672166 \times 251 - 886.0007$

predicted $y =$ 1189.071

Yes/No ? Why: No. That would be extrapolation.

(f) Using the **second equation**, find the values of the residuals for the first 2 data points (the first two data points are the two observations in the first row of the data table).

$y - \hat{y} = 927 - (7.6122317 \times 233 + 2.5938892 \times 317 - 1674.083)$

$= -147.0745$

residual for first data point = -147.0745

$y - \hat{y} = 1393 - (7.6122317 \times 253 + 2.5938892 \times 407 - 1674.083)$

$= 85.47548$

residual for second data point = 85.47548

(g) Using the **second equation**, about what **change** in average *ballistic limit* seems to accompany a 2-unit increase in both x_1 and x_2? (For x_1, one unit is 10^{-3} in.)

change in $y =$ 20.41224

$\Delta\hat{y} = 7.6122317 \times 2 + 2.5938892 \times 2$

$= 20.41224$

2. [4+4+6=14 points]

Consider a discrete random variable X with the probability mass function as specified below. The constant c is to be determined.

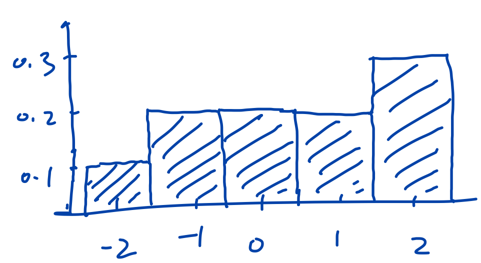
x	-2	-1	0	1	2
$f(x)$	0.1	0.2	c	0.2	0.3

(a) Determine c and make a probability histogram (barplot) for X.

$\sum_x f(x) = 1$

$c =$ 0.2

$\Rightarrow 0.1 + 0.2 + c + 0.2 + 0.3 = 1 \Rightarrow c = 0.2$



(b) Find $P(|X| < 2)$ and $P(|X - 1| > 1)$.

$P(|X|<2) = P(-2<X<2)$

$= P(X=-1) + P(X=0) + P(X=1)$

$= 0.2 + 0.2 + 0.2 = 0.6$

$P(|X| < 2) =$ 0.6

$P(|X-1| > 1) = P(X-1>1) + P(X-1<-1)$

$= P(X>2) + P(X<0)$

$= P(X=-2) + P(X=-1)$

$= 0.1 + 0.2 = 0.3$

$P(|X - 1| > 1) =$ 0.3

(c) Find the mean and standard deviation of X.

$E(X) = \sum_x x f(x) = 0.1\times(-2) + 0.2\times(-1) + 0.2\times 0 + 0.2\times 1 + 0.3\times 2$

$= 0.4$

$E(X^2) = 4\times 0.1 + 1\times 0.2 + 0\times 0.2 + 1\times 0.2 + 4\times 0.3$

$= 2$

$\mu =$ 0.4

$Var(X) = 2 - 0.4^2 = 2 - 0.16 = 1.84$

$SD(X) = \sqrt{1.84} = 1.356466$

$\sigma =$ 1.356466

3. [6 points]

Suppose that 15% of all daily oxygen purities delivered by an air-products supplier are below 99.5% purity and that it is plausible to think of daily purities as independent random variables. Evaluate the probability that in the next five-day workweek, 1 or less delivered purities will fall below 99.5%.

X: number of delivered purities that fall below 99.5% in the next 5 days.

probability = 0.83521

$X \sim \text{Binomial}(n=5, p=0.15)$

$P(X \le 1) = \binom{5}{0} \cdot 0.15^0 0.85^5 + \binom{5}{1} 0.15^1 0.85^4$

$= 0.83521$

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Data Table

x_1	x_2	y	x_1	x_2	y
253	317	927	253	407	1393
258	321	978	252	426	1401
259	341	1028	246	432	1436
247	350	906	250	469	1327
256	352	1159	242	257	950
246	363	1055	243	302	998
257	365	1335	239	331	1144
262	375	1392	242	355	1080
255	373	1362	244	385	1276
258	391	1374	234	426	1062

JMP Output

Fit 1

Response y

Whole Model — **x1**

Summary of Fit

RSquare	0.115383
RSquare Adj	0.066237
Root Mean Square Error	180.1802
Mean of Response	1179.15
Observations (or Sum Wgts)	20

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-886.0007	1348.395	-0.66	0.5194
x1	8.2672166	5.395487	1.53	0.1428

Fit 2

Response y

Whole Model — **x1** **x2**

Effect Summary

Summary of Fit

RSquare	0.601328
RSquare Adj	0.554425
Root Mean Square Error	124.4655
Mean of Response	1179.15
Observations (or Sum Wgts)	20

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-1674.083	947.4012	-1.77	0.0952
x1	7.6122317	3.729889	2.04	0.0571
x2	2.5938892	0.569824	4.55	0.0003*