Spring 2020 **Stat 305 (Section 4)** **Quiz 4**

Name:
Solution

Total points for the exam is 50. Points for individual questions are given at the beginning of each problem. Show all your calculations clearly. Put final answers in the box at the right (except for the diagrams!).

1. [5+5+5+10=25 points]
B. Choi tested the stopping properties of various bike tires on various surfaces. For one thing, he tested both treaded and smooth tires on dry concrete. The lengths of skid marks produced in his study under these two conditions were as follows (in cm). Denote the true mean length of skid marks for treaded tires μ_1 and the true mean length of skid marks for smooth tires μ_2.

Treaded	Smooth
365, 374, 376, 391, 401, 402	341, 348, 349, 355, 375, 391
$\bar{x}_1 = 384.8333$	$\bar{x}_2 = 359.8333$
$s_1 = 15.38072$	$s_2 = 19.16681$

(a) Give a 95% upper confidence bound for the mean length of skid marks for treaded tires in this study. (No need to simplify.)

upper conf bd = $384.8333 + 2.015 \times \frac{15.38072}{\sqrt{6}} = 397.4858$

$n_1 = 6$, $\alpha = 1 - 0.95 = 0.05$.
So the quantile used is $t_{n_1-1,1-\alpha} = t_{5,0.95} = 2.015$
upper confidence bound is

$$\bar{x}_1 + t_{n_1-1,1-\alpha}\frac{s_1}{\sqrt{n_1}} = 384.8333 + 2.015 \times \frac{15.38072}{\sqrt{6}}$$
$$= 397.4858$$

(b) Give a 90% lower prediction bound for the length of skid mark in a single additional test for treaded tires in this study. (No need to simplify.)

lower pred bd = $384.8333 - 1.476 \times 15.38072\sqrt{1+\frac{1}{6}} = 360.3124$

$n_1 = 6$, $\alpha = 1 - 0.9 = 0.1$.
So the quantile used is $t_{n_1-1,1-\alpha} = t_{5,0.9} = 1.476$
lower prediction bound is

$$\bar{x}_1 - t_{n_1-1,1-\alpha}\, s_1\sqrt{1+\frac{1}{n_1}} = 384.8333 - 1.476 \times 15.38072\sqrt{1+\frac{1}{6}}$$
$$= 360.3124$$

(c) Assuming the true standard deviations for treaded tires and smooth tires are equal ($\sigma_1 = \sigma_2$), give a 95% two-sided confidence interval for the mean difference in lengths of skid marks for treaded tire and smooth tire (i.e. $\mu_1 - \mu_2$). (No need to simplify.)

conf. interval = $25 \pm 2.228(17.37719)\sqrt{\frac{1}{3}} = (2.641402, 47.3586)$

$n_1 = 6$. $n_2 = 6$. $\alpha = 1 - 0.95 = 0.05$. So the quantile used is

$t_{n_1+n_2-2,\,1-\alpha/2} = t_{10,\,0.975} = 2.228$

the pooled sample variance is

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{5(15.38072)^2 + 5(19.16681)^2}{10} = 301.9666$$

so $S_p = \sqrt{301.9666} = 17.37719$

the two-sided confidence interval is

$$\bar{x}_1 - \bar{x}_2 \pm t_{n_1+n_2-2,\,1-\alpha/2}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$= (384.8333 - 359.8333) \pm 2.228(17.37719)\sqrt{\frac{1}{6}+\frac{1}{6}}$$

$$= 25 \pm 2.228(17.37719)\sqrt{\frac{1}{3}} = (2.647088, 47.35291)$$

(d) Supposing the equal variance assumption in (c) still holds, use the hypothesis testing with p-value to assess the strength of evidence to support that there is a mean difference in lengths of skid marks for treaded tire and smooth tire. (Show all the steps)

1. $H_0: \mu_1 = \mu_2$, $H_a: \mu_1 \neq \mu_2$

2. The test statistic is

$$T = \frac{\bar{x}_1 - \bar{x}_2 - 0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

Assuming

- H_0 is true.
- Sample 1 $\overset{iid}{\sim} N(\mu_1, \sigma^2)$
- Sample 2 $\overset{iid}{\sim} N(\mu_2, \sigma^2)$
- Sample 1 and Sample 2 are independent.

$T \sim t_{n_1+n_2-2} = t_{10}$.

3. The observed test statistic is

$$t = \frac{384.8333 - 359.8333 - 0}{17.38161\sqrt{\frac{1}{6}+\frac{1}{6}}} = 2.491212$$

The p-value is

$$P(|T| > |t|) = P(|T| > 2.491212) = 2P(T > 2.491212)$$
$$= 0.0319286$$

4. The p-value is small. We reject H_0 and conclude H_a

5. There is significant evidence that the mean lengths of skid marks for treaded and smooth tires are different.

2. [9+1+5+5+5=25 points]

The data below were collected for studying the physical property of concrete made using burned clay aggregates. 10 batches of this kind of concrete are collected. The response y is compressive strength (in psi) and x is splitting tensile strength (in psi).

x	207	233	254	328	325	302	258	335	315	302
y	1420	1950	2230	3070	3060	3110	2650	3130	2960	2760

Use the JMP output on page 6 answering the following questions.
Note that $\bar{x} = 285.9$, and $\sum(x_i - \bar{x})^2 = 17896.9$.

(a) Use the hypothesis testing with F statistic and the corresponding p-value to assess the strength of evidence to support that the slope β_1 is different from 0. (Show all steps.)

1. $H_0: \beta_1 = 0$, $H_a: \beta_1 \neq 0$

2. The test statistic is

$$F = \frac{SSR/1}{SSE/(n-2)} = \frac{SSR}{SSE/8}$$

Assuming
- H_0 is true
- $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, $\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$.

The test statistic $F \sim F_{1,8}$.

3. From the JMP output, the observed test statistic is
observed $F = 75.5912$
and the p-value is
$P(F_{1,8} > \text{observed } F) < 0.0001$.

4. The p-value is very small. We reject H_0 and conclude H_a.

5. There is significant evidence that the slope is different from 0.
The compressive strength would change if the splitting tensile strength of this kind of concrete changes.

(b) Is the splitting tensile strength (x) a useful predictor for the compressive strength (y)? (Give a simple yes or no answer.)

answer: Yes.

(c) Give a 95% two-sided confidence interval for the expected change in compressive strength for a 1 unit increase in splitting tensile strength. (No need to simplify.)

conf. interval = $12.457688 \pm 2.306 \frac{196.6861}{\sqrt{17896.9}} = (9.153529, 15.76185)$

$n = 10$, $\alpha = 1 - 0.95 = 0.05$. So the quantile used is

$t_{n-2, 1-\alpha/2} = t_{8, 0.975} = 2.306$.

The two-sided confidence interval is

$b_1 \pm t_{n-2, 1-\alpha/2} \frac{S_{LF}}{\sqrt{\Sigma(x_i - \bar{x})^2}}$

$= 12.457688 \pm 2.306 \times \frac{(91.6861)}{\sqrt{17896.9}}$

$= 12.457688 \pm 2.306 \times 1.432853$

$= (9.153529, 15.76185)$

(c) Give a 95% lower confidence bound for the mean compressive strength when splitting tensile strength is 300 psi. (No need to simplify.)

lower conf bd = $2809.653 - 1.860\,(196.6861)\sqrt{\frac{1}{10} + \frac{(300 - 285.9)^2}{17896.9}} = 2687.709$

$\hat{\mu}_{y|x} = b_0 + b_1 x = -927.6531 + 12.457688 \times 300 = 2809.653$

$t_{n-2, 1-\alpha} = t_{8, 0.95} = 1.860$.

The lower confidence bound is

$\hat{\mu}_{y|x} - t_{n-2, 1-\alpha} S_{LF} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$

$= 2809.653 - 1.860\,(196.6861)\sqrt{\frac{1}{10} + \frac{(300 - 285.9)^2}{17896.9}}$

$= 2687.709$

(d) Give a 95% two-sided prediction interval for the next compressive stength when the splitting tensile strength is 300 psi. (No need to simplify.) $= (2331.56, 3287.744)$

pred. interval = $2809.652 \pm 2.306\,(196.6861)\sqrt{1 + \frac{1}{10} + \frac{(300 - 285.9)^2}{17896.9}}$

the quantile used is $t_{n-2, 1-\alpha/2} = t_{8, 0.975} = 2.306$

The two-sided prediction interval is

$\hat{\mu}_{y|x} \pm t_{n-2, 1-\alpha/2} S_{LF} \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$

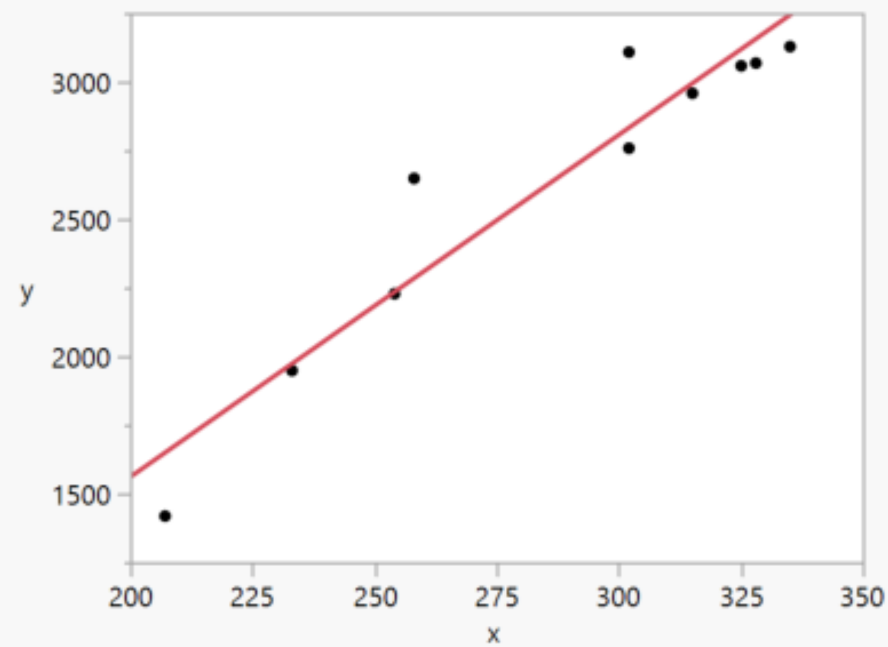
$= 2809.652 \pm 2.306\,(196.6861)\sqrt{1 + \frac{1}{10} + \frac{(300 - 285.9)^2}{17896.9}}$

$= (2331.56, 3287.744)$

This page is intended to be blank.

JMP Output

Bivariate Fit of y By x



Linear Fit

Linear Fit

y = -927.6531 + 12.457688*x

Summary of Fit

RSquare	0.904296
RSquare Adj	0.892333
Root Mean Square Error	191.6861
Mean of Response	2634
Observations (or Sum Wgts)	10

Lack Of Fit

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	2777491.4	2777491	75.5912
Error	8	293948.6	36744	**Prob > F**
C. Total	9	3071440.0		<.0001*

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-927.6531	414.1133	-2.24	0.0554
x	12.457688	1.432853	8.69	<.0001*