Quiz 2. in class.

Ch4-Ch5:1.
regression, discrete random variable.
Special Continuous Random Variables
bring calculator: 2-page chear sheet.

Yifan Zhu

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Special Continuous Random Variables

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Overview

Normal Probabil

Normal Quantiles

The Student t

The Chi-square Distribution

The F Distribution

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▶ A random variable X is Normal (μ, σ^2) if its pdf is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

Using calculus, one can verify that:

•
$$E(X) = \mu$$

• $Var(X) = \sigma^2$

The standard normal distribution

► A standard normal random variable, usually called Z,

$$\phi(z) = \frac{1}{\sqrt{2\pi}}e^{-z^2/2}$$

- ▶ The standard normal pdf is usually denoted $\phi(z)$.
- ▶ The standard normal cdf is usually denoted $\Phi(z)$.

$$\frac{1}{2}(2) = \left((2 \leq 2)\right)$$

$$= \int_{-\infty}^{2} \phi(x) dx$$

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has the pdf:

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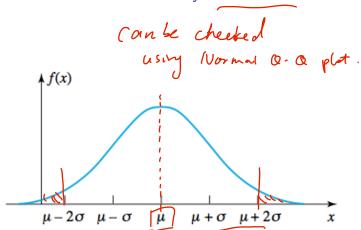
The Student *t* Distribution

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- A normal random variable is (often) a finite average of many repeated, independent, identical trials.
- Examples:
 - Mean width of the next 50 hexamine pellets.
 - ▶ Mean height of the next 30 students.
 - ► Your SAT score.
 - ► Total % yield of the next 40 runs of a chemical process.
 - ▶ The next blood pressure reading.
 - ▶ Several kinds of measurement error.
 - Corrosion resistance of carbon/carbon composites.

A look at the normal density: a bell curve



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As usual, areas denote probabilities



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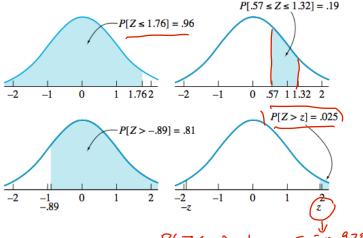
Vormal

The Student t

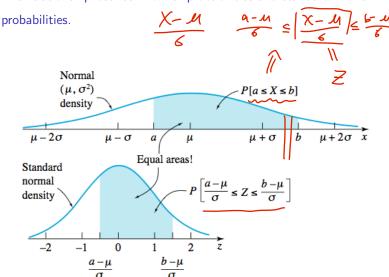
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P(ZEZ)= 1-0.025=0.975. => Z= Q(0.975)=1.91 The relationship between normal probabilities and standard normal



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$$E(x) = M$$
, $Var(X) = 6^{\frac{1}{2}}$.
 $Verify : \int_{-\infty}^{\infty} \int_{12\pi6}^{\infty} e^{-\frac{(x-M)^{2}}{26^{2}}} dx = 1$.
 $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi6}} e^{-\frac{(x-M)^{2}}{26^{2}}} dx = 1$

$$\frac{1}{2} = \frac{1}{\sqrt{2\pi6}} e^{-\frac{(x-M)}{26^2}} dx = 1$$

$$\frac{1}{2} = \frac{x-M}{\sqrt{2\pi6}} = \frac{1}{2} \times \frac{$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}6} e^{-\frac{1}{26^2}} dx = 1$$

$$= \frac{X - M}{6} = X = 62 + M.$$

 $\int_{\sqrt{2\pi}}^{\infty} \frac{1}{6} e^{-\frac{1}{2}z^2} d(6z+n)$

 $\int_{-\infty}^{\infty} \int_{-\infty}^{1/2\pi} dx = \int_{-\infty}^{\infty} \int_{-\infty}^{1/2\pi} dx = \int_{-$

Verify:
$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = 1$$

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dz = \sqrt{2\pi}$$

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx \cdot \left(\int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dy \right)$$

$$\left(\int_{-\infty}^{\infty} e^{-\frac{1}{2}x^{2}} dx\right) \cdot \left(\int_{-\infty}^{\infty} e^{-\frac{1}{2}y^{2}} dy\right)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x^{2}+y^{2})} dx dy$$

$$\frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2+y^2)} dx dy}{\int_{-\infty}^{\infty} \frac{1}{2} \int_{-\infty}^{\infty} \frac{1$$

$$\int_{0}^{2\pi} \int_{0}^{\infty} e^{-\frac{1}{2}y^{2}} r dr d\theta$$

$$= \int_{0}^{2\pi} \int_{0}^{2\pi} e^{-\frac{1}{2}r^{2}} dr d\theta$$

$$= \int_{0}^{2\pi} \left(-e^{-\frac{1}{2}r^{2}}\right) \Big|_{0}^{2\pi} d\theta$$

$$= \int_{0}^{2\pi} 1 d\theta = 2\pi.$$

 $\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^{2}} dz = \sqrt{2\pi}.$

$$\int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \frac{(x-u)^2}{6^2}} dx \right)$$

$$= \int_{-\infty}^{\infty} \left(62+u \right) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$= \int_{-\infty}^{\infty} \left(62+u \right) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$= \int_{-\infty}^{\infty} \left(62+u \right) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$= \int_{-\infty}^{\infty} \left(62+u \right) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$= \int_{-\infty}^{\infty} 2 \phi(z) dz + u \int_{-\infty}^{\infty} \phi(z) dz = \int_{-\infty}^{\infty} dz dz$$

N

$$\int_{-\infty}^{\infty} x^{2} \frac{1}{\sqrt{2\pi}6} e^{-\frac{1}{2} \frac{(x-u)^{2}}{6^{2}}} dx \quad (2 = \frac{x-u}{6}).$$

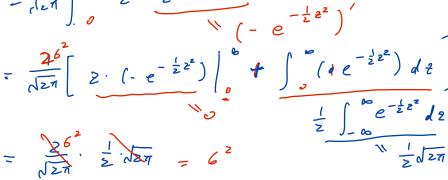
$$= 6^{2} \int_{-\infty}^{\infty} 2^{2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}2^{2}} dz.$$
even faretion

$$= \frac{26}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{1}{2}z^{2}} dz$$

$$= \frac{26}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{1}{2}z^{2}} dz$$

$$= \frac{26}{\sqrt{2\pi}} \left[2 \cdot (-e^{-\frac{1}{2}z^{2}}) \right]_{0}^{\infty} + \int_{0}^{\infty} (4e^{-\frac{1}{2}z^{2}}) dz$$

$$= \frac{26}{\sqrt{2\pi}} \left[2 \cdot (-e^{-\frac{1}{2}z^{2}}) \right]_{0}^{\infty} + \int_{0}^{\infty} (4e^{-\frac{1}{2}z^{2}}) dz$$



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Special Notation of Quantiles

Since $Z = \frac{X - \mu}{\sigma}$ is *standard* normal probability values from X can be expressed as:

$$P(a \le X \le b) = P\left(\frac{a - \mu}{\sigma} \le Z \le \frac{b - \mu}{\sigma}\right)$$
$$= \int_{(a - \mu)/\sigma}^{(b - \mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

- ► Unfortunately, the integral cannot be evaluated analytically. Instead, we use either:
 - A computer.
 - A standard normal probability table like the one in Table B.3 in Vardeman and Jobe.

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Special Notation of Quantiles

- ▶ J. Fisher, in his article *Computer Assisted Net Weight Control* (Quality Progress, June 1983), discusses the filling of food containers with strained plums with tapioca by weight. The mean of the values portrayed is about 137.2 g, the standard deviation is about 1.6 g, and data look bell-shaped.
- Let W = the next fill weight. Then, $W \sim N(\mu = 137.2, \sigma^2 = (1.6)^2)$.
- ▶ Let's find the probability that the next jar contains less food by mass than it's supposed to (declared weight = 135.05 g).

$$P(W < 135.0) = P\left(\frac{W - 137.2}{1.6} < \frac{135.05 - 137.2}{1.6}\right)$$

$$= P(Z < -1.34)$$

$$= \Phi(-1.34)$$

The approximate value of $\Phi(-1.34)$ is found to be 0.0901 in Table B.3.

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$$

(Z)	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
14	.0808	.0793	.0778	.0764	0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379

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Some facts about $\Phi(z)$

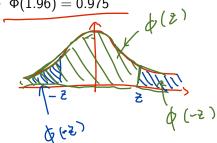
$$\Phi(z) - (-\Phi(z))$$

$$= Z\Phi(z) - 1$$

$$\Phi(z) + \Phi(-z) = 1$$

•
$$\Phi(z) - \Phi(-z) = 2\Phi(z) - 1$$

 $\Phi(1.96) = 0.975$



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z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	8000.	.0008	8000.	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
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0.1 5.398 5.438 5.478 5.517 5.557 5.596 5.636 .5675 5.714 .5753 0.2 5.793 5.822 5871 5.910 5.984 5.987 .6026 .6064 .6103 .6141 0.3 6.799 6212 6255 6293 6331 .6368 .6406 .6434 .6480 .6879 0.5 .6915 .6950 .6965 .6664 .6700 .6736 .772 .6808 .6844 .6879 0.6 .7257 .7291 .7324 .7357 .7389 .7422 .7454 .7466 .7517 .7249 0.6 .7257 .7291 .7324 .7357 .7389 .7422 .7454 .7764 .7754 0.8 .7881 .7911 .7324 .7357 .7389 .7422 .7454 .7764 .7784 0.8 .8186 .8187 .8526 .8023 .8021 .8080 .8333 .83	Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.2 5.793 5.832 5.871 5.910 5.948 5.987 6.026 6.064 6.103 6.141 0.3 6.179 6217 6255 6299 6.331 6.368 6.606 6.404 6.613 6.141 0.4 6.554 6.6591 6.628 6.664 6.700 6.736 6.772 6.808 6.644 6.879 0.5 6.915 6.950 6.985 7.019 7.054 7.088 7.712 7.7190 7.224 0.6 7.257 7.291 7.324 7.357 7.399 7.422 7.442 7.444 7.446 7.749 7.823 7.944 7.821 7.764 7.734 7.744 7.843 8.810 8.813 8.8	0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.33 6.179 6.217 6.255 66.93 6.631 6.331 6.368 6.406 6.443 6.480 6.517 0.4 6.554 6.691 6.628 6.664 6.700 6.736 6.772 6.808 .6844 .6879 0.5 6.915 6.950 6.985 7.019 7.054 7.088 .7123 7.157 7.190 7.224 7.357 7.389 7.422 7.454 .7486 .7517 .7590 7.611 7.642 7.673 .704 .7734 .7764 .7794 .7823 .7852 .7050 .8113 .8106 .8106 .8133 .8951 .8078 .8106 .8133 .918 .8106 .8133 .8264 .8289 .8315 .8540 .8365 .8869 .8888 .8907 .8979 .8770 .8790 .810 .8330 .934 .9406 .9418 .9441 .9418 .9449 .966 .9082 .9999 .9115 .9131 .9147	0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.4 6.554 .6591 .6628 .6664 .6700 .6736 .6772 .6808 .6844 .6879 0.5 .6915 .6950 .6985 .7019 .7054 .7088 .7123 .7157 .7190 .7224 0.6 .7257 .7291 .7324 .7357 .7389 .7422 .7454 .7486 .7517 .7549 0.7 .7580 .7611 .7642 .7673 .7704 .7734 .7764 .7794 .7823 .7852 0.8 .7881 .7910 .7939 .7967 .7995 .8023 .8051 .8106 .8133 0.9 .8159 .8186 .8212 .8238 .8264 .8289 .8315 .8340 .8365 .8363 1.0 .8413 .8438 .8461 .8485 .8508 .8531 .8554 .8577 .8599 .8611 1.1 .8463 .8665 .8666 .8708 .8729 .8749 <	0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.5 6.915 6.995 6.985 .7019 .7054 .7088 .7123 .7157 .7190 .7224 0.6 .7257 .7291 .7324 .7357 .7399 .7422 .7454 .7486 .7517 .7549 0.7 .7580 .7911 .7324 .7573 .7704 .7734 .7764 .7794 .7823 .7852 .7852 .8353 .8378 .8106 .8133 .8186 .8128 .8289 .8315 .8340 .8365 .8389 1.0 .8413 .8438 .8461 .8485 .8584 .8574 .8577 .8599 .8611 1.1 .8643 .8665 .8686 .8708 .8729 .8749 .8770 .8790 .8810 .8897 .9015 1.2 .8849 .8869 .8888 .8907 .8925 .8944 .8962 .890 .8997 .9015 1.2 .9032 .9040 .9066 .9082 .9915	0.3	.6179	.6217	.6255	6293	.6331	.6368	.6406	.6443	.6480	.6517
0.6 7.257 7.291 7.324 7.357 7.389 7.422 7.454 7.486 7.517 7.549 0.7 7.580 7.511 7.642 7.673 7.704 7.734 7.764 7.794 8.23 7.852 0.8 7.881 7.911 7.995 7.995 8.023 8.051 8.078 8.106 8.133 0.9 8.159 8.186 8.212 8.284 8.289 8.315 8.340 8.365 8.389 1.0 8.413 8.438 8.461 8.485 8.508 8.531 8.574 8.790 8.810 8.389 1.2 8.849 .8665 .8688 8.8907 .8925 .8944 .8962 .8900 .9917 1.3 .9032 .9049 .966 .9082 .9099 .9115 .9131 .9147 .9162 .9171 1.4 .9192 .9207 .9222 .9236 .9294 .9406 .9418 .9429 <t< td=""><th>0.4</th><td>.6554</td><td>.6591</td><td>.6628</td><td>.6664</td><td>.6700</td><td>.6736</td><td>.6772</td><td>.6808</td><td>.6844</td><td>.6879</td></t<>	0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.7 7.580 .7611 .7642 .7673 .7794 .7734 .7744 .7823 .7823 .7852 .882 .8811 .7810 .7939 .7935 .8023 .8051 .8078 .8166 .8138 .8242 .8284 .8289 .8315 .8340 .8365 .8339 1.0 .8413 .8438 .8461 .8485 .8508 .8531 .8554 .8577 .8599 .8621 1.1 .8643 .8666 .8708 .8729 .8749 .8770 .8790 .8101 .8301 1.2 .8489 .8866 .8888 .8907 .8925 .8944 .8962 .8890 .8997 .9112 1.3 .9032 .9049 .9666 .9082 .9099 .9115 .9131 .9147 .9162 .9171 1.4 .9192 .9207 .9222 .9236 .9251 .9265 .9279 .9292 .9306 .9318 .9414 .9462 .9463 <th></th> <td></td>											
0.8 .7881 .7910 .7993 .7967 .7995 .8023 .8051 .8078 .8106 .8133 0.9 .8159 .8166 .8212 .8238 .8264 .8289 .8315 .8340 .8365 .8369 1.1 .8643 .8468 .8488 .8907 .8922 .8749 .8770 .8790 .8810 .8897 .917 1.2 .8849 .8866 .8888 .8907 .8925 .8944 .8962 .8980 .8997 .9172 1.4 .9192 .9049 .9066 .9082 .9949 .9151 .9131 .9147 .9162 .917 .414 .9162 .9270 .9222 .9236 .9251 .9265 .9279 .9292 .9306 .9312 1.6 .9452 .9461 .9573 .9582 .9594 .906 .9418 .9429 .9441 1.6 .9452 .9563 .9573 .9582 .9591 .9596											
0.9 8.159 8.186 8.212 8.238 8.264 8.289 8.315 8.340 8.365 8.389 1.0 8.413 8.438 8.461 8.485 8.508 8.531 8.574 8.577 8.999 .8611 1.1 8.643 8.665 8.686 8.708 8.729 8.744 .870 .8790 .8810 .890 .9015 1.2 8.849 .8869 .8888 .8907 .8925 .8944 .8962 .8900 .9917 .917 1.4 .9192 .9207 .9222 .9236 .9591 .9515 .9213 .9147 .9162 .9171 1.5 .9332 .9434 .9448 .9495 .9551 .9525 .9533 .9541 .9418 .9429 .9418 1.6 .9452 .9453 .9547 .9484 .9495 .9551 .9525 .9533 .9541 1.8 .9641 .9564 .9573 .9581											
1.0 8.413 8.438 8.461 .8485 .8508 .8531 .8554 .8577 .8599 .8611 1.1 .8643 .8665 .8666 .8708 .8729 .8749 .8770 .8790 .8110 .830 1.2 .8849 .8869 .8888 .8907 .8925 .8944 .8962 .8980 .8997 .9172 1.4 .9192 .9207 .9222 .9236 .9251 .9265 .9279 .9222 .9302 .9312 1.5 .9332 .9345 .9357 .9370 .9382 .9344 .9406 .9418 .9429 .9411 1.6 .9452 .9463 .9474 .9484 .9495 .9505 .9515 .9525 .9525 .9526 .9525 .9533 .9616 .9525 .953 .9616 .9525 .9541 .969 .9666 .9661 .9673 .9666 .9671 .9678 .9688 .9671 .9678 .9680	8.0	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
1.1 8.643 .8665 .8686 .8708 .8729 .8749 .8770 .8790 .8810 .8810 .8830 1.2 .8849 .8866 .8886 .8897 .8925 .8944 .8962 .8980 .8979 .9175 1.3 .9322 .9066 .9082 .9999 .9115 .9131 .9147 .9162 .9179 1.4 .9192 .9207 .9222 .9236 .9251 .9265 .9279 .9292 .9306 .9139 1.5 .9332 .9345 .9377 .9372 .9382 .9344 .9406 .9418 .9429 .9411 1.6 .9482 .9564 .9563 .9551 .9590 .9608 .966 .9623 .953 1.8 .9641 .9649 .9656 .9664 .9671 .9788 .9794 .9750 .9756 .9761 .9767 1.9 .9713 .9778 .9783 .9788 .9793 <td< td=""><th>0.9</th><td>.8159</td><td>.8186</td><td>.8212</td><td>.8238</td><td>.8264</td><td>.8289</td><td>.8315</td><td>.8340</td><td>.8365</td><td>.8389</td></td<>	0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.2 8.849 .8869 .8888 .8907 .8925 .8944 .8962 .8980 .8997 .9015 1.3 .9032 .9049 .9066 .9082 .9099 .9115 .9131 .9147 .9162 .9177 1.4 .9192 .9207 .9222 .9236 .9251 .9265 .9279 .9292 .9306 .9314 1.6 .9352 .9345 .9357 .9370 .9382 .9354 .9406 .9418 .9429 .9441 1.6 .9542 .9463 .9474 .9484 .9495 .9505 .9515 .9525 .9535 .9543 1.7 .9554 .9564 .9673 .9582 .9591 .9599 .9608 .9661 .9625 .9533 1.8 .9641 .9649 .9666 .9672 .9732 .9738 .9744 .9750 .9756 .9673 .9676 2.0 .9773 .9778 .9783 .9744 <	1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.3 9.032 .9049 .9066 .9082 .9099 .9115 .9131 .9147 .9162 .9177 1.4 .9192 .9207 .9222 .9236 .9251 .9265 .9279 .9292 .9366 .9379 1.5 .9332 .9345 .9347 .9484 .9495 .9595 .9515 .9525 .9535 .9541 1.7 .9554 .9564 .9573 .9582 .9591 .9599 .9608 .9616 .9625 .9531 1.8 .9641 .9649 .9656 .9664 .9671 .9678 .9686 .9693 .9699 .9760 2.0 .9731 .9719 .9728 .9733 .9788 .9783 .9784 .9500 .9750 .9750 .9761 .9762 2.0 .9733 .9783 .9788 .9933 .9908 .9803 .9808 .9812 .9812 .9812 .9812 .9812 .9812 .9812 .9812	1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.4 9.192 9.207 9.222 9.236 9.251 9.265 9.279 9.292 9.306 9.319 1.5 9.332 9.345 9.357 9.370 9.382 9.394 9.406 9.418 9.429 9.441 1.6 9.452 9.463 9.474 9.484 9.495 9.505 9.515 9.525 9.535 9.543 1.7 9554 9.564 9.561 9.591 9.599 9.608 9.616 9.626 9.611 9.678 9.606 9.661 9.625 9.561 1.70 9.733 9.713 9.712 9.732 9.732 9.732 9.784 9.750 9.566 9.661 9.661 9.606 9.610 9.610 9.610 9.676 9.767 2.0 9.733 9.778 9.733 9.788 9.793 9.794 9.606 9.650 9.676 9.676 9.676 9.676 9.871 9.872 9.878 9.881 9.881 9.882 9.89	1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.6 9.332 9.345 9.357 9.370 9.382 9.394 9.406 9.418 9.429 9.411 1.6 9.452 9.463 9.474 9.484 9.495 9.505 9.515 9.525 9.535 9.545 1.7 9.584 9.648 9.673 9.582 9.591 9.598 9.608 9.661 9.625 9.633 1.8 9.641 9.649 9.664 9.671 9.578 9.686 9.693 .969 .976 9.76 9.76 .981 .986 .981 .981 .984 .981 .984 .984 .984 .984 .984 .984 <	1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1.5	0222	02.45	0257	0270	0202	0204	0.400	0410	0.430	0.441
1.7 9.554 9.564 9.573 9.582 9.591 9.599 9.608 9.616 9.623 9.633 1.8 9.641 9.649 9.664 9.671 9.678 9.688 9.693 9.909 9.706 1.9 9.713 9.712 9.726 9.732 9.738 9.744 9.750 9.766 9.671 9.738 9.744 9.750 9.766 9.671 9.738 9.748 9.750 9.760 9.672 9.733 9.748 9.748 9.752 9.733 9.748 9.748 9.803 9.808 9.812 9.817 2.817 2.817 9.812 9.844 9.867 9.864 9.864 9.867 9.878 9.881 9.881 9.881 9.881 9.881 9.981 9.911 9.913 9.911 9.913 9.912 9.911 9.913 9.914 9.914 9.914 9.914 9.914 9.916 9.911 9.913 9.916 9.911 9.913 9.916 9.914											
1.8 9.641 .9649 .9656 .9664 .9671 .9678 .9686 .9693 .9699 .9706 1.9 .9713 .9718 .9726 .9732 .9738 .9744 .9750 .9756 .9761 .9767 2.1 .9731 .9778 .9783 .9788 .9782 .9803 .9803 .9812 .9812 .9812 .9812 .9812 .9812 .9812 .9812 .9814 .9838 .9812 .9841 .9850 .9854 .9850 .9854 .9850 .9854 .9850 .9854 .9850 .9861 .9861 .9861 .9861 .9861 .9861 .9861 .9861 .9861 .9862 .9971 .9872 .9993 .9911 .9913 .9934 .9961 .9961 .9934 .9960 .9901 .9911 .9913 .9934 .9936 .9931 .9932 .9934 .9936 .9961 .9932 .9952 .9952 .9961 .9962 .9962											
1.9 9.713 9.719 9.726 9.732 9.738 9.744 9.750 9.750 9.761 9.767 2.0 9.733 9.778 9.783 9.788 9.798 9.908 9.808 9.812 9.817 2.1 9.821 9.826 9.830 9.834 9.838 9.942 9.846 9.854 9.869 2.2 9.861 9.668 9.9671 9.875 9.788 8.981 9.884 9.887 9.960 2.3 9.833 9.960 9.961 9.906 9.901 9.911 9.913 9.916 2.4 9.918 9.920 9.922 9.925 9.927 9.929 9.931 9.932 9.916 9.912 9.913 9.916 9.916 9.962 9.963 9.963 9.964 9.948 9.949 9.951 9.952 9.964 9.948 9.942 9.962 9.962 9.962 9.962 9.962 9.962 9.962 9.962 9.962 9.962 </td <th></th> <td></td>											
2.0 9.973 9.778 9.783 9.788 9.978 9.978 9.981 9.980 9.803 9.808 9.812 9.817 2.1 9821 9826 9830 9.984 9.883 9.842 9.846 9.864 9.867 9.862 2.2 3.981 9.864 9.888 9.901 9.906 5.990 9.911 1.913 .916 2.4 9.918 9.920 9.922 9.922 9.927 9.929 9.931 .9932 .9934 .9946 2.5 9.933 .9945 .9943 .9945 .9946 .9948 .9941 .9951 .9951 .9951 .9951 .9951 .9951 .9951 .9951 .9952 .9963 .996 .9962 .9962 .9963 .9964 .9948 .9949 .9949 .9962 .9962 .9963 .9964 .9949 .9971 .9972 .9973 .9974 .9971 .9971 .9972 .9973 .9974 .9974 </td <th></th> <td></td>											
2.1 9.821 .9826 .9830 .9834 .9838 .9842 .9848 .9861 .9867 .9857 2.2 .9861 .9864 .9868 .9901 .9974 .9976 .9891 .9848 .9910 .9916 .9994 .9918 .9911 .9913 .9912 .9926 .9927 .9929 .9921 .9931 .9934 .9934 .9946 .9948 .9949 .9951 .9952 .9963 .9969 .9914 .9934 .9946 .9948 .9949 .9951 .9951 .9952 .9963 .9949 .9951 .9951 .9952 .9963 .9969 .9961 .9962 .9963 .9964 .9948 .9949 .9951 .9952 .9964 .9948 .9949 .9951 .9952 .9960 .9961 .9962 .9963 .9961 .9962 .9963 .9971 .9971 .9971 .9972 .9971 .9971 .9971 .9972 .9973 .9974 .9974 .9974 </td <th>1.9</th> <td></td> <td></td> <td></td> <td>.9732</td> <td></td> <td></td> <td></td> <td>.9756</td> <td>.9761</td> <td></td>	1.9				.9732				.9756	.9761	
2.2 9.861 .9864 .9868 .9871 .9878 .9881 .9881 .9884 .9890 .9901 2.3 .9833 .9960 .9901 .9904 .9906 .9903 .9913 .9913 .9913 .9913 .9913 .9913 .9913 .9913 .9913 .9914 .9916 .9927 .9929 .9331 .9929 .9924 .9923 .9934 .9936 .996 .996 .996 .9962 .9949 .9951 .996 .996 .996 .9960 .9961 .9949 .9953 .9964 .994 .9949 .9963 .996 .9960 .9970 .9971 .9972 .9973 .9974 .9973 .9974 .9978 .9979 .9979 .9974 .9984 .9990 .9974 .9978 .9978 .9978 .9986 .9986 .9984 .9984 .9985 .9985 .9986 .9986 .9984 .9984 .9985 .9985 .9986 .9986 .9986		.9773	.9778		.9788		.9798	.9803	.9808	.9812	
2.3 9.893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 .916 2.4 .9918 .9920 .9225 .927 .9929 .9931 .9944 .9934 .9934 .9934 .9934 .9934 .9934 .9934 .9934 .9934 .9934 .9934 .9934 .9949 .9951 .9951 .9952 .9964 .9948 .9949 .9952 .9962 .9964 .9962 .9962 .9964 .9962 .9962 .9964 .9964 .9962 .9962 .9974 .9974 .9974 .9979 .9979 .9973 .9974 .9974 .9978 .9979 .9979 .9979 .9978 .9985 .9986 .9964 .9985 .9985 .9980 .9986 .9986 .9984 .9985 .9985 .9980 .9981 .9984 .9985 .9986 .9963 .9984 .9985 .9986 .9984 .9984 .9984 .9984 .9984 <th>2.1</th> <td>.9821</td> <td>.9826</td> <td>.9830</td> <td>.9834</td> <td>.9838</td> <td>.9842</td> <td>.9846</td> <td>.9850</td> <td>.9854</td> <td>.9857</td>	2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.4 9.918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .9943 .9946 .9948 .9949 .9951 .9952 .9952 .9960 .9961 .9962 .9953 .9954 .9953 .9964 .9969 .9970 .9971 .9972 .9973 .9974 .9971 .9972 .9973 .9974 .9978 .9979 .9979 .9979 .9971 .9972 .9981 .9984 .9984 .9985 .9985 .9986 .9981 .9981 .9982 .9994 .9985 .9986 .9986 .9986 .9986 .9986 .9986 .9986 .9986 .9986 .9989 .9989 .9989 .9989 .9989 .9989 .9999 .9999 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9994 .9994 .9994 .9994 .9994 .9994 .9994 .9994 .9996 .9996 .9996 .9996 .9996 .	2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.5 9.938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 2.7 .9955 .9966 .9967 .9977 .9978 .9979 .9979 .9973 .9973 .9973 .9978 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9982 .9983 .9983 .9984 .9984 .9985 .9985 .9986 .9986 .9984 .9984 .9985 .9985 .9986 .9986 .9984 .9984 .9985 .9985 .9986 .9986 .9986 .9986 .9989 .9989 .9999 .9990 .9990 .9990 .9990 .9990 .9990 .9992 .9992 .9992 .9992 .9992 .9994 .9994 .9994 .9994 .9994	2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.6 9.953 .9955 .9956 .9957 .9959 .9961 .9961 .9962 .9963 .9964 2.7 .9955 .9967 .9968 .9969 .9970 .9971 .9973 .9973 .9973 .9973 .9973 .9973 .9974 .9981 .9991 .9997 .9990 .9990 .9990 .9990 .9980 .9986 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9981 .9991 .9992 .9992 .9992 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9994 </td <th>2.4</th> <td>.9918</td> <td>.9920</td> <td>.9922</td> <td>.9925</td> <td>.9927</td> <td>.9929</td> <td>.9931</td> <td>.9932</td> <td>.9934</td> <td>.9936</td>	2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.7 9.965 .9966 .9967 .9978 .9969 .9970 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9977 .9977 .9978 .9979 .9980 .9981 .9980 .9981 .9980 .9981 .9980 .9986 .9986 .9986 .9986 .9986 .9986 .9986 .9980 .9981 .9991 .9991 .9991 .9992 .9992 .9992 .9993 .9993 .9993 .9934 .994 .994 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 <th>2.5</th> <td>.9938</td> <td>.9940</td> <td>.9941</td> <td>.9943</td> <td>.9945</td> <td>.9946</td> <td>.9948</td> <td>.9949</td> <td>.9951</td> <td>.9952</td>	2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.8 9.974 .9975 .9976 .9977 .9978 .9979 .9979 .9980 .9981 .9981 2.9 .9981 .9982 .9983 .9983 .9984 .9984 .9985 .9985 .9986 .9966 3.0 .9987 .9987 .9988 .9988 .9989 .9999 .9990 .9990 3.1 .9993 .9991 .9994 .9994 .9994 .9994 .9995 .9995 .9995 .9995 .9995 .9995 .9995 .9997 .9997 .9997 .9997 .9997 .9998 .9998 .9998 .9998 .9998 .9998 .9998 .9998 .9998 .9999 .9999 .9999 .9999 .9999 .9999 .9999 .9999 .9999 .9999 .9998 .9999 .9998 .9999 .9998 .9999 .9998 .9999 .9999 .9999 .9999 .9999 .9999 .9999 .9999 .9999 .9999	2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.9 9.981 .982 .983 .983 .9984 .9984 .9985 .9985 .9986 .9986 3.0 .9987 .987 .9988 .9988 .9989 .9989 .9990 .9990 .9990 .9990 .9992 .9992 .9992 .9992 .9993 .9993 .9994 .9949 .9949 .9949 .9959 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .997 .997 .997 .997 .998 .998 .998 .998 .998 .9996 .9995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .995 .996 <	2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
3.0 9.987 9.987 .9987 .9988 .9988 .9989 .9989 .9990 .9990 .9991 3.1 .9990 .9991 .9991 .9992 .9992 .9992 .9993 .9933 .9933 3.2 .9993 .9993 .9994 .9994 .9994 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9997 .9997 .9997 .9997 .9997 .9998 .9998 .9998 .9998 .9998 .9998 .9998 .9998 .9998 .9998 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9993 .9994 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996	2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
3.1 9990 9991 9991 9992 9992 9993 9993 9993 3.2 9993 9994 9994 9994 9994 9994 9995 9995 9996	2.9	.9981	.9982	.9983	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.2 9.993 .9993 .9994 .9994 .9994 .9994 .9994 .9995 .9995 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9997 .9997 .9997 .9997 .9997 .9997 .9997 .9998 .	3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.3 9.995 .9995 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9997 .9997 .9997 .9997 .9997 .9997 .9997 .9997 .9997 .9998 .	3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.4 9.997 9.997 9.997 9.997 9.997 9.997 9.997 9.997 9.999 9.999	3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
	3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
Vifan 7hu Jawa State University	3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
			Yifa <u>n</u>	Zhu					owa St	ate Uni	iversit <u>y</u>

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Your turn: using the standard normal table, calculate the following.

- 1. $P(X \le 3), X \sim N(2, 64)$
- 2. $P(X > 7), X \sim N(6, 9)$
- 3. $P(|X-1| > 0.5), X \sim N(2,4)$
- 4. P(X is within 2 standard deviations of its mean.) $X \sim N(\mu, \sigma^2)$

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Answers: normal probabilities

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1. $P(X < 3), X \sim N(2, 64)$

$$P(X \le 3) = P\left(Z \le \frac{3-2}{\sqrt{64}} = 0.125\right)$$

$$= \Phi(0.125)$$

$$= 0.5597 \text{ from the standard normal table}$$

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2. $P(X > 7), X \sim N(6, 9)$

$$P(X > 7) = P\left(Z > \frac{7 - 6}{\sqrt{9}} = 0.33\right)$$
= 1 - P(Z \le 0.33)
= 1 - \Phi(0.33)
= 1 - 0.6293 from the standard normal table
= 0.3707

Answers: normal probabilities

3. $P(|X-1| > 0.5), X \sim N(2,4)$ P(|X-1| > 0.5) = P(X-1 > 0.5 or X-1 < -0.5)= P(X - 1 > 0.5) + P(X - 1 < -0.5)= P(X > 1.5) + P(X < 0.5) $=P\left(\frac{X-2}{2}\right)+P\left(\frac{X-2}{2}\right)$ = P(Z > -0.25) + P(Z < -0.75) $=1-P(Z \le -0.25)+P(Z \le -0.75)$ $=1-\Phi(-0.25)+\Phi(-0.75)$ = 1 - 0.4013 + 0.2266 from the standard normal table = 0.8253

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4. $P(X \text{ is within 2 standard deviations of its mean.}) X \sim N(\mu, \sigma^2)$

$$P(|X - \mu| < 2\sigma) = P(-2\sigma < X - \mu < 2\sigma)$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma)$$

$$= P\left(\frac{(\mu - 2\sigma) - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{(\mu + 2\sigma) - \mu}{\sigma}\right)^{\frac{2}{5}}$$

$$= P(-2 < Z < 2)$$

$$= P(Z < 2) - P(Z < -2)$$

$$= \Phi(2) - \Phi(-2)$$

$$= 0.9773 - 0.0228$$

$$= 0.9545$$

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Normal quantiles

 I can find standard normal quantiles by using the standard normal tabl:e in reverse.

ightharpoonup Example: for the jar weights $W\sim (137.2,1.6^2)$, I will find Q(0.1)

$$\frac{0.1 = P(X \le Q(0.1))}{P(X \le Q(0.1))} \\
= P\left(Z \le \frac{Q(0.1) - 137.2}{1.6}\right) \\
= \Phi\left(\frac{Q(0.1) - 137.2}{1.6}\right) \\
\frac{\Phi^{-1}(0.1)}{1.6} = \frac{Q(0.1) - 137.2}{1.6} \\
Q(0.1) = 137.2 + 1.6 \cdot \Phi^{-1}(0.1)$$

$$\frac{Q(0.1) = 137.2 + 1.6 \cdot \Phi^{-1}(0.1)}{Q(0.1)} = \frac{Q(0.1) - 137.2}{Q(0.1)} = \frac{Q(0.1)$$

 $\Phi^{-1}(0.1) = -1.28$ from the standard normal table. Hence:

$$Q(0.1) = 137.2 + 1.6(-1.28)$$

= 135.152

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Table B.3

Standard Normal Cumulative Probabilities

 $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$

					J-0	∞ √ 2π	(2)				
	z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
	-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
	-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
	-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
	-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
	-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
	-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
	-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
	-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
	-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
	-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
	-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
	-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
	-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
	-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
	-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
	-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
	-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
	-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
	-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
	-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
	-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
	-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
ı	-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
١	-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
	-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379

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Your turn: calculate the following:

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- 1. Q(0.95) of $X \sim N(9,3)$
- 2. c such that $P(|X-2|>c)=0.01, X\sim N(2,4)$
- 3. c such that $P(|X \mu| < \sigma c) = 0.95$, $X \sim N(\mu, \sigma^2)$

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1. Q(0.95) for $X \sim N(9,3)$

$$0.95 = P(X \le Q(0.95))$$

$$= P\left(\frac{X - 9}{\sqrt{3}} < \frac{Q(0.95) - 9}{\sqrt{3}}\right)$$

$$= P\left(Z < \frac{Q(0.95) - 9}{\sqrt{3}}\right)$$

$$0.95 = \Phi\left(\frac{Q(0.95) - 9}{\sqrt{3}}\right)$$

$$\Phi^{-1}(0.95) = \frac{Q(0.95) - 9}{\sqrt{3}}$$

$$Q(0.95) = \sqrt{3}\left(\Phi^{-1}(0.95) + 9\right)$$

$$= \sqrt{3} \cdot (1.645) + 9 \quad \text{(from the std. normal table)}$$

$$= 11.85$$

Answers

$$\left|\frac{z}{z}\right| = \left|\frac{x-z}{z}\right| > \frac{c}{z}$$

2. c such that P(|X-2|>c)=0.01, $X\sim N(2/2,4)$

 $c = -2\Phi^{-1}(0.005)$ = $-2 \cdot (-2.575)$

= 5.15

$$0.01 = P(|X - 2| > c)$$

$$= P(X - 2 > c \text{ or } X - 2 < -c)$$

$$= P(X - 2 > c) + P(X - 2 < -c)$$

$$= P\left(\frac{X - 2}{2} > \frac{c}{2}\right) + P\left(\frac{X - 2}{2} < -\frac{c}{2}\right)$$

$$= \left(-\frac{7}{2}\left(\frac{c}{2}\right)\right) = P\left(Z < -\frac{c}{2}\right) + P\left(Z < -\frac{c}{2}\right)$$

$$= P\left(Z < -\frac{c}{2}\right) + P\left(Z < -\frac{c}{2}\right) + P\left(Z < -\frac{c}{2}\right)$$

$$= 2P\left(Z < -\frac{c}{2}\right)$$

$$0.01 = 2\Phi(-c/2)$$

$$0.005 = \Phi(-c/2)$$

$$\Phi^{-1}(0.005) = -c/2$$

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(using the standard normal table)

$$0.95 = P(|X - \mu| < \sigma c)$$

$$= P(-\sigma c < X - \mu < \sigma c)$$

$$= P\left(-c < \frac{X - \mu}{\sigma} < c\right)$$

$$= P(-c < Z < c)$$

$$= P(Z < c) - P(Z < -c)$$

$$= (1 - P(Z > c)) - P(Z < -c)$$

$$= (1 - P(Z < -c)) - P(Z < -c)$$
(since $\phi(z)$ is symmetric about 0)
$$= 1 - 2P(Z < -c)$$

$$0.95 = 1 - 2\Phi(-c)$$

$$0.05 = 2\Phi(-c)$$

$$c = -\Phi^{-1}(0.025)$$

$$= -(-1.96) \text{ from the standard normal table}$$

$$= (1.96) \qquad = (3, 6, 775)$$

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The Student t distribution

A random variable T has a t_ν distribution — that is, a t distribution with ν degrees of freedom — if its pdf is:

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{(\nu\pi)^{\frac{1}{2}}} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \quad -\infty < t < \infty$$

Gamma function:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt, \, x > 0$$

- We use the t table (Table B.4 in Vardeman and Jobe) to calculate quantiles and probabilities.
- ► Like the standard normal distribution, the *t* distribution is mound-shaped and symmetric about 0.
- The *t* distribution has fatter tails than the normal, but approaches the shape of the normal as $\nu \to \infty$

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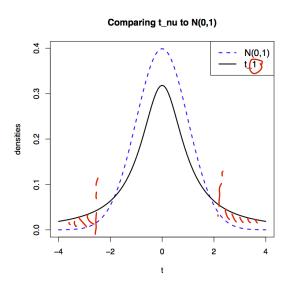
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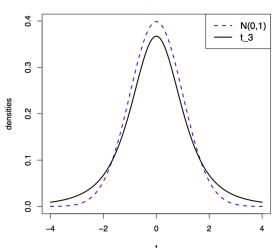
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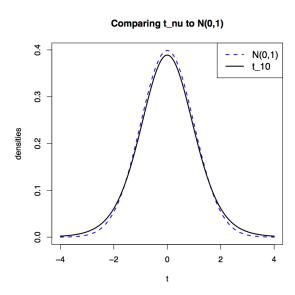
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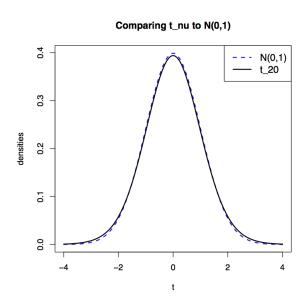
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Find probabilities and quantiles of t_{ν} with the table.

▶ Say
$$T \sim t_5$$
. $P(T \le 1.476) = 0.9$

Table B.4
t Distribution Quantiles

ν		Q(.9)	Q(.95)	Q(.975)	Q(.99)	Q(.995)	Q(.999)	Q(.9995)
1		3.078	6.314	12.706	31.821	63.657	318.317	636.607
2	1	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3		1.638	2.353	3.182	4.541	5.841	10.215	12.924
4		1.533	2.132	2.776	3.747	4.604	7.173	8.610
(5)		1.476	2.015	2.571	3.365	4.032	5.893	6.869

You can find quantiles labeled in the top row.

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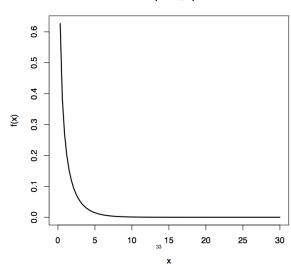
A random variable $S \sim \chi^2_{\nu}$ (is chi-square with ν degrees of freedom) if its pdf is:

 $f(x) = \begin{cases} 0 & : x \le 0\\ \frac{1}{\Gamma(\nu/2)2^{\nu/2}} \cdot x^{\nu/2 - 1} \cdot e^{-x/2} : 0 < x < \infty \end{cases}$

- ► Use Table B.5 in Vardeman and Jobe to find chi-square probabilities and quantiles.
- ightharpoonup A chi-square random variable is the sum of squares of ν independent standard normal random variables.
- ▶ A chi-sugare distribution is not symmetric.

A look at the chi-square density





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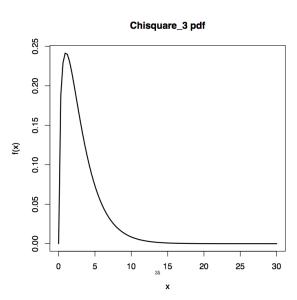
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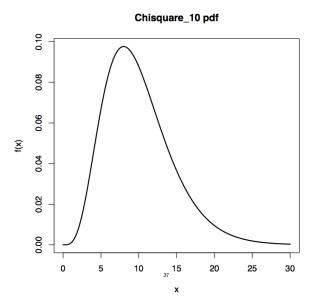
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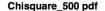
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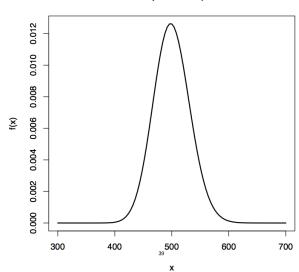
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Use Table B.5 to find chi-square probabilities and quantiles.

• Q(0.9) of χ_6^2 is 10.645.

Table B.5
Chi-Square Distribution Quantiles

ν	Q(.005)	Q(.01)	Q(.025)	Q(.05)	Q(.1)	Q(.9)	Q(.95)	Q(.975)	Q(.99)	Q(.995)
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12,592	14.449	16.812	18,548

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 \triangleright X has an F_{ν_1,ν_2} distribution if it has pdf:

: x < 0 $f(x) = \begin{cases} \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \cdot \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{x^{\nu_1/2 - 1}}{[1 + (\nu_1/\nu_2)x]^{(\nu_1 + \nu_2)/2}} : 0 < x < \infty \end{cases}$

- An F_{ν_1,ν_2} random variable is a $\chi^2_{\nu_1}$ RV divided by an independent $\chi^2_{\nu_2}$ RV. That's why ν_1 is the **numerator** degrees of freedom and ν_2 is the denominator degrees of freedom.
- Use Tables B.6A-B.6E to find probabilities and quantiles.



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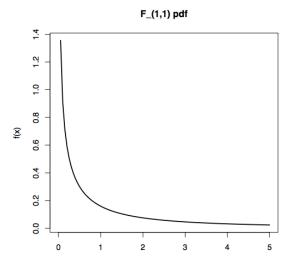
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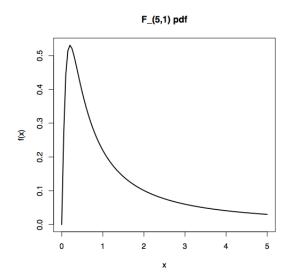
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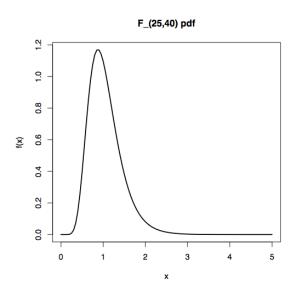
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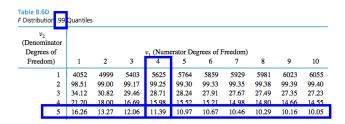
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Find probabilities and quantiles of the F distribution with Tables B.6A-B.6E

▶ The 0.99 quantile of the $F_{4.5}$ distribution is 11.39.



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- 1. Q(p) for N(0,1) is often denoted z_p .
- 2. Q(p) for t_{ν} is often denoted $t_{\nu,p}$.
- 3. Q(p) for χ^2_{ν} is often denoted $\chi^2_{\nu,p}$.
- 4. Q(p) for F_{ν_1,ν_2} is often denoted $F_{\nu_1,\nu_2,p}$.