

More Inference for Simple Linear Regression (Ch. 9.1)

Yifan Zhu

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Outline

SLR: Inference for the Mean Response at some x

Prediction interval for a new y at some x

Simultaneous Confidence Intervals for $\mu_{y|x}$

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SLR: mean response at x

- ▶ Recall our model:

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$\varepsilon_1, \dots, \varepsilon_n \sim \text{iid } N(0, \sigma^2)$$

- ▶ Under the model, the true mean response at some observed covariate value x_i is:

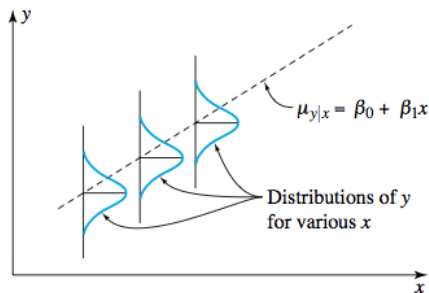
$$\mu_{y|x_i} = \beta_0 + \beta_1 x_i$$

- ▶ Now, if some new covariate value x is within the range of the x_i 's, we can estimate the true mean response at this new x :

$$\hat{\mu}_{y|x} = b_0 + b_1 x$$

SLR: mean response at x

- ▶ But how good is the estimate?



- ▶ That's why we do inference.

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SLR: mean response at x

- Under the model, $\hat{\mu}_{y|x}$ is normally distributed with:

$$E(\hat{\mu}_{y|x}) = \mu_{y|x} = \beta_0 + \beta_1 x$$
$$Var(\hat{\mu}_{y|x}) = \sigma^2 \left(\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2} \right)$$

- We can construct a $N(0, 1)$ random variable by standardizing:

$$Z = \frac{\hat{\mu}_{y|x} - \mu_{y|x}}{\sigma \sqrt{\frac{1}{n} \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}} \sim N(0, 1)$$

- Replacing σ with $s_{LF} = \sqrt{\frac{1}{n-2} \sum_i (y_i - \hat{y}_i)^2}$:

$$T = \frac{\hat{\mu}_{y|x} - \mu_{y|x}}{s_{LF} \sqrt{\frac{1}{n} \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}} \sim t_{n-2}$$

SLR: mean response at x

- ▶ To test $H_0 : \mu_{y|x} = \#$, we can use the test statistic:

$$T = \frac{\hat{\mu}_{y|x} - \#}{s_{LF} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}}$$

which has a t_{n-2} distribution if H_0 is true and the model is correct.

- ▶ A 2-sided $1 - \alpha$ confidence interval for $\mu_{y|x}$ is:

$$\left(\hat{\mu}_{y|x} - t_{n-2, 1-\alpha/2} \cdot s_{LF} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}, \right. \\ \left. \hat{\mu}_{y|x} + t_{n-2, 1-\alpha/2} \cdot s_{LF} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \right)$$

and the one-sided intervals are analogous.

Pressing pressures and specimen densities for a ceramic compound

A mixture of Al_2O_3 , polyvinyl alcohol, and water was prepared, dried overnight, crushed, and sieved to obtain 100 mesh size grains. These were pressed into cylinders at pressures from 2,000 psi to 10,000 psi, and cylinder densities were calculated.

x (pressure in psi)	y (density in g/cc)
2000.00	2.49
2000.00	2.48
2000.00	2.47
4000.00	2.56
4000.00	2.57
4000.00	2.58
6000.00	2.65
6000.00	2.66
6000.00	2.65
8000.00	2.72
8000.00	2.77
8000.00	2.81
10000.00	2.86
10000.00	2.88
10000.00	2.86

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Example: ceramics

- First, I'll make a 2-sided 95% confidence interval for the true mean density of the ceramics at 4000 psi.

$$\hat{\mu}_{y|x} = 2.375 + 0.0000487(4000) = 2.5697 \text{ g/cc}$$

With $t_{n-2, 1-\alpha/2} = t_{13, 0.975} = 2.160$, the margin of error in the confidence interval is:

$$\begin{aligned} t_{n-2, 1-\alpha/2} \cdot s_{LF} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \\ = 2.160(0.0199) \sqrt{\frac{1}{15} + \frac{(4000 - 6000)^2}{1.2 \times 10^8}} = 0.0136 \text{ g/cc} \end{aligned}$$

Hence, the 95% CI is:

$$(2.5697 - 0.0136, 2.5697 + 0.0136) = (2.5561, 2.5833)$$

- We're 95% confident that the true mean density of the ceramics at 4000 psi is between 2.5561 g/cc and 2.5833 g/cc.

Your turn: ceramics

- ▶ Calculate and interpret a 2-sided 95% confidence interval for the true mean density at 5000 psi, given:
 - ▶ $\hat{\mu}_{y|x} = 2.375 + 0.0000487x$
 - ▶ The margin of error is $t_{n-2, 1-\alpha/2} s_{LF} \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{\sum_i (x_i - \bar{x})^2}}$
 - ▶ $\sum_i (x_i - \bar{x})^2 = 1.2 \times 10^8$
 - ▶ $n = 15, \bar{x} = 6000.$
 - ▶ $s_{LF} = 0.0199$
 - ▶ $t_{13, 0.975} = 2.16$
- ▶ Test $H_0 : \beta_0 = 0$ vs. $H_a : \beta_0 \neq 0$ at significance level $\alpha = 0.05$ using the method of p-values.

Answers: ceramics

- ▶ Make a 2-sided 95% confidence interval for the true mean density of the ceramics at 5000 psi:

$$\hat{\mu}_{y|x} = 2.375 + 0.0000487(5000) = 2.6183 \text{ g/cc}$$

With $t_{n-2, 1-\alpha/2} = t_{13, 0.975} = 2.160$, the margin of error in the confidence interval is:

$$\begin{aligned} t_{n-2, 1-\alpha/2} \cdot s_{LF} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \\ = 2.160(0.0199) \sqrt{\frac{1}{15} + \frac{(5000 - 6000)^2}{1.2 \times 10^8}} = 0.0118 \text{ g/cc} \end{aligned}$$

Hence, the 95% CI is:

$$(2.6183 - 0.0118, 2.6183 + 0.0118) = (2.6065, 2.6301)$$

- ▶ We're 95% confident that the true mean density of the ceramics at 5000 psi is between 2.6065 g/cc and 2.6301 g/cc.

Answers: ceramics

Now for the hypothesis test:

1. $H_0 : \beta_0 = 0, H_a : \beta_0 \neq 0$
2. $\alpha = 0.05$
3. β_0 is just $\mu_{y|x=0}$. The test statistic is:

$$T = \frac{b_0 - 0}{s_{LF} \sqrt{\frac{1}{n} + \frac{(0 - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}} = \frac{b_0}{s_{LF} \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\sum_i (x_i - \bar{x})^2}}}$$

- ▶ $T \sim t_{n-2}$ assuming:
 - ▶ H_0 is true.
 - ▶ The model $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ is correct, with $\varepsilon_1, \dots, \varepsilon_n \sim \text{iid } N(0, 1)$.

4. The observed test statistic:

$$b_0 = 2.375$$

$$t = \frac{2.375}{0.0199 \sqrt{\frac{1}{15} + \frac{6000^2}{1.2 \times 10^8}}} = 197.09$$

$$\text{p-value} = P(|t_{13}| > 197.09) \ll 0.0001$$

5. With a p-value $\ll 0.0001 < \alpha$, we reject H_0 and conclude H_a .
6. There is overwhelming evidence that the intercept of the “true” line is different from 0.

Ceramics: back to the JMP output

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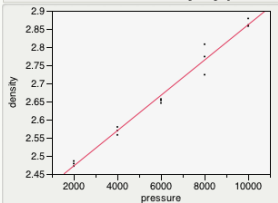
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▼ Bivariate Fit of density By pressure



Linear Fit

▼ Linear Fit

density = 2.375 + 4.8667e-5*pressure

▼ Summary of Fit

RSquare	0.982193
RSquare Adj	0.980824
Root Mean Square Error	0.019909
Mean of Response	2.667
Observations (or Sum Wgts)	15

▼ Lack Of Fit

▼ Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	0.28421333	0.284213	717.0604
Error	13	0.00515267	0.000396	Prob > F
C. Total	14	0.28936600		<.0001*

▼ Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	2.375	0.012055	197.01	<.0001*
pressure	4.8667e-5	1.817e-6	26.78	<.0001*

Ceramics: back to the JMP output

▼ Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	2.375	0.012055	197.01	<.0001*
pressure	4.8667e-5	1.817e-6	26.78	<.0001*

- ▶ The observed test statistic t is under “t Ratio” for the intercept.
- ▶ “Prob> |t|” for the intercept is the p-value for the significance test you just did.
- ▶ “Estimate” for the intercept is b_0 .
- ▶ “Std Error” for the intercept is:

$$\widehat{SD}(b_0) = s_{LF} \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\sum_i (x_i - \bar{x})^2}}$$

Be careful with Inference on β_0

- ▶ In this case and many others, $\beta_0 = \mu_{y|x=0}$ is beyond the range of our data.
- ▶ Estimating beyond the range of our covariate values is called **extrapolation**, which is dangerous for linear regression.
- ▶ Only extrapolate when:
 - ▶ You know your process or system well, and can describe it with the right equations.
 - ▶ You estimate the parameters of the resulting model using *nonlinear* regression:
 - ▶ Example: special case of the Michaelis-Menten model for enzyme kinetics with reaction speed y and substrate concentration x :

$$Y_i = \frac{\theta_1 x_i}{\theta_2 + x_i} + \varepsilon_i$$

- ▶ See Nonlinear Regression Analysis and Its Applications by Bates and Watts for more information on nonlinear regression.

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- ▶ The prediction interval in SLR is trying to capture the next response at a given value of predictor variable.
- ▶ A 2-sided $1 - \alpha$ prediction interval for a new response y at some x is:

$$\left(\hat{\mu}_{y|x} - t_{n-2, 1-\alpha/2} \cdot s_{LF} \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}, \right. \\ \left. \hat{\mu}_{y|x} + t_{n-2, 1-\alpha/2} \cdot s_{LF} \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \right)$$

and the one-sided intervals are analogous.

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Example: ceramics

- ▶ We will make a 2-sided 95% prediction interval for the next density of the ceramics at 4000 psi.

$$\hat{\mu}_{y|x} = 2.375 + 0.0000487(4000) = 2.5697 \text{ g/cc}$$

With $t_{n-2, 1-\alpha/2} = t_{13, 0.975} = 2.160$, the margin of error in the confidence interval is:

$$\begin{aligned} t_{n-2, 1-\alpha/2} \cdot s_{LF} \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}} \\ = 2.160(0.0199) \sqrt{1 + \frac{1}{15} + \frac{(4000 - 6000)^2}{1.2 \times 10^8}} = 0.0451 \text{ g/cc} \end{aligned}$$

Hence, the 95% CI is:

$$(2.5697 - 0.0451, 2.5697 + 0.0451) = (2.5246, 2.6148)$$

- ▶ We're 95% confident that the next collected density of the ceramics at 4000 psi is between 2.5246 g/cc and 2.6148 g/cc.

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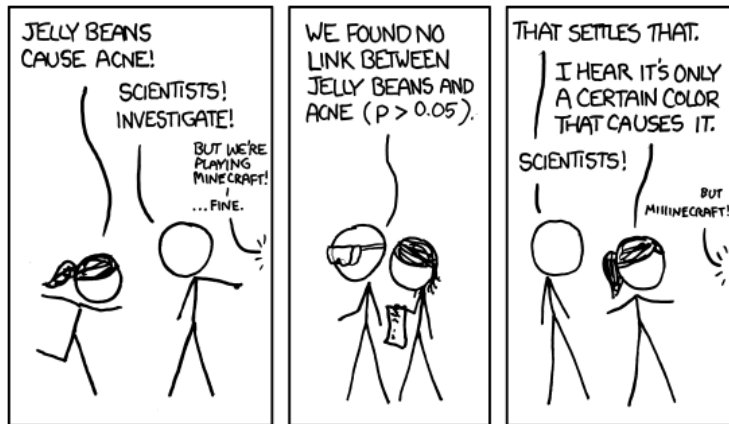
Simultaneous
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- ▶ Situations will arise when you'll want to do inference on $\mu_{y|x=2000}, \mu_{y|x=4000}, \mu_{y|x=6000}, \dots$, all at once.
- ▶ When you compute several confidence intervals at once or do multiple tests at once, you need to account for the simultaneity.
- ▶ On average, for every 20 tests you do independently at $\alpha = 0.05$, we expect 1 of those tests to conclude H_a by chance alone.
 - ▶ Remember: $\alpha = P(\text{reject } H_0 \text{ assuming } H_0 \text{ is true})$.

Example: <http://xkcd.com/882/>

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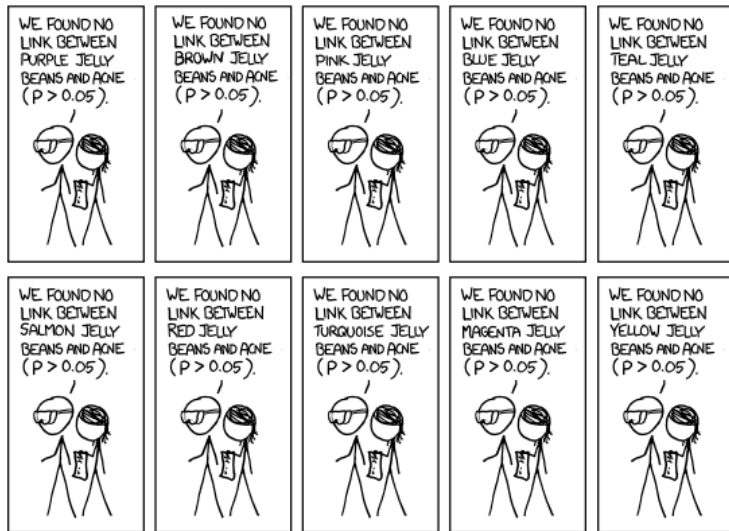


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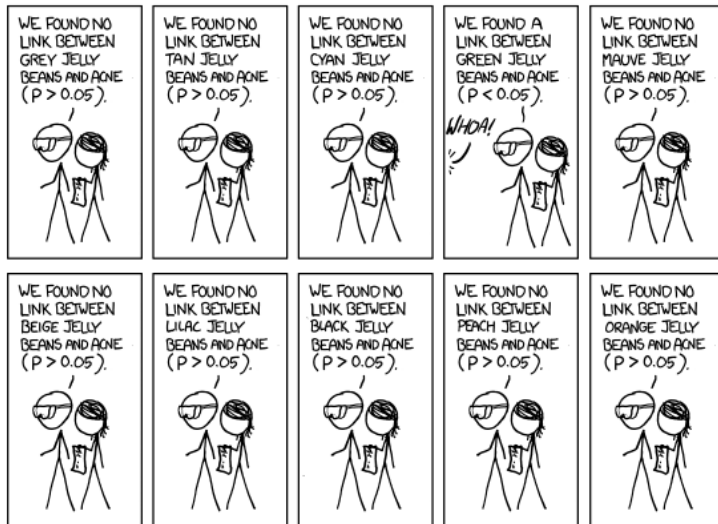
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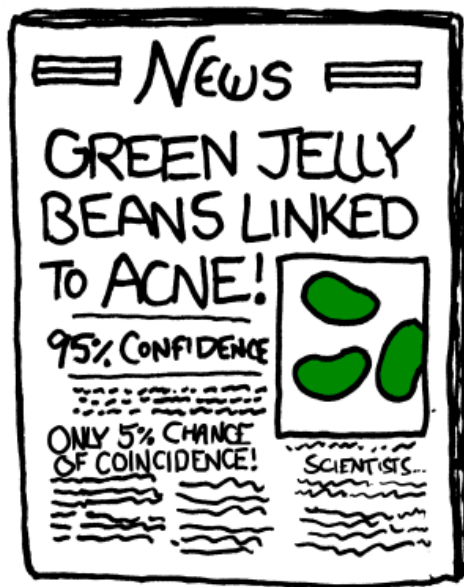
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Simultaneous confidence interval

- ▶ If we have k simultaneous tests, each with type I error α , then the type I error for the simultaneous tests is

$$P(\text{at least one rejection in these } k \text{ tests}) > \alpha$$

If these tests are independent, the actual type I error is

$$\begin{aligned} & P(\text{at least one rejection in these } k \text{ tests}) \\ &= 1 - P(\text{no rejection in these } k \text{ tests}) \\ &= 1 - \prod_{i=1}^k P(\text{fail to reject the } i\text{-th test}) = 1 - (1 - \alpha)^k \end{aligned}$$

- ▶ For k confidence intervals for $\mu_1, \mu_2, \dots, \mu_k$, denote the corresponding random intervals I_1, I_2, \dots, I_k . If the confidence level is $1 - \alpha$, then

$$P(\mu_i \in I_i) = 1 - \alpha, \quad i = 1, 2, \dots, k$$

And the simultaneous confidence level would be

$$P(\mu_1 \in I_1 \text{ and } \mu_2 \in I_2 \text{ and } \dots \mu_k \in I_k \text{ at the same time}) < 1 - \alpha$$

- ▶ To get the $1 - \alpha$ simultaneous confidence level, the simultaneous confidence intervals should be **wider** than individual confidence interval.

Simultaneous confidence intervals for $\mu_{y|x}$

- ▶ Let I_x be the random intervals for the simultaneous $1 - \alpha$ confidence intervals for $\mu_{y|x}$. Then we want

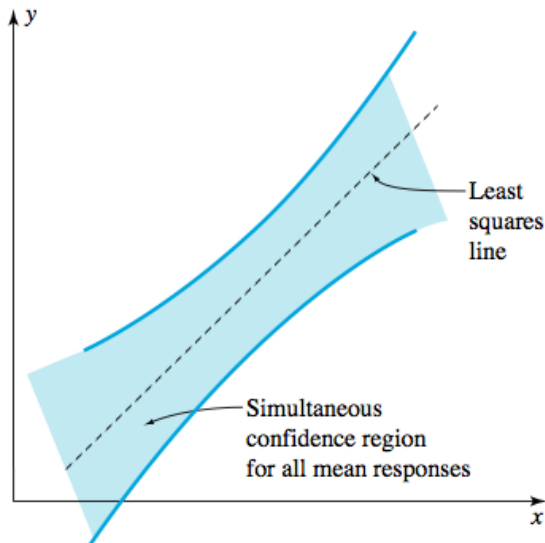
$$P(\mu_{y|x} \in I_x \text{ at the same time for all } x) = 1 - \alpha$$

- ▶ The simultaneous confidence intervals for $\mu_{y|x}$ are given by:

$$b_0 + b_1x \pm \sqrt{2F_{2,n-2,1-\alpha}} \cdot s_{LF} \cdot \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}$$

- ▶ This formula accounts for the fact that we're computing k confidence intervals at the same time.

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Example: ceramics

► Given:

- $n = 15$
- $\bar{x} = 6000$
- $\sum_i (x_i - \bar{x})^2 = 1.2 \times 10^8$
- $\hat{y} = 2.375 + 4.87 \times 10^{-5}x$, $s_{LF} = 0.0199$.
- The simultaneous confidence interval formula is:

$$b_0 + b_1x \pm \sqrt{2F_{2,k,1-\alpha/2} \cdot s_{LF}} \cdot \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum_i (x_i - \bar{x})^2}}$$

- I will calculate simultaneous 95% confidence intervals for the mean responses $\mu_{y|x}$ at $x = 2000, 4000, 6000, 8000$, and 10000 .

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- Using $F_{2,n-2,1-\alpha} = F_{2,13,0.95} = 3.81$, the intervals are of the form:

$$\begin{aligned} & 2.375 + 4.87 \times 10^{-5}x \pm \sqrt{2 \cdot 3.81 \cdot 0.0199} \cdot \sqrt{\frac{1}{15} + \frac{(x - 6000)^2}{1.2 \times 10^8}} \\ & = 2.375 + 4.87 \times 10^{-5}x \pm 0.0549 \sqrt{0.066 + 8.33 \times 10^{-9}(x - 6000)^2} \end{aligned}$$

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x , pressure	CI, compact form	CI
2000	2.4723 ± 0.0246	(2.4477, 2.4969)
4000	2.5697 ± 0.0174	(2.5523, 2.5871)
6000	2.6670 ± 0.0142	(2.6528, 2.6812)
8000	2.7643 ± 0.0174	(2.7469, 2.7817)
10000	2.8617 ± 0.0246	(2.8371, 2.8863)

Ceramics; plotting simultaneous confidence regions

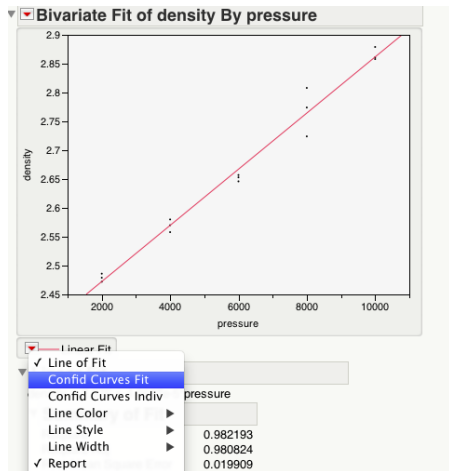
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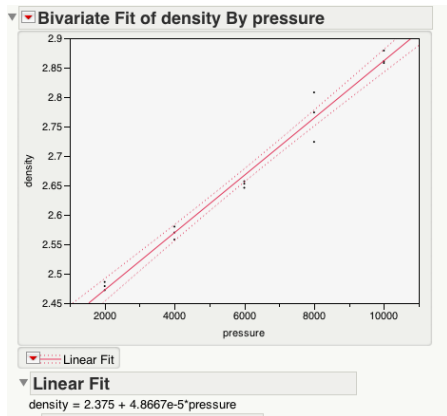
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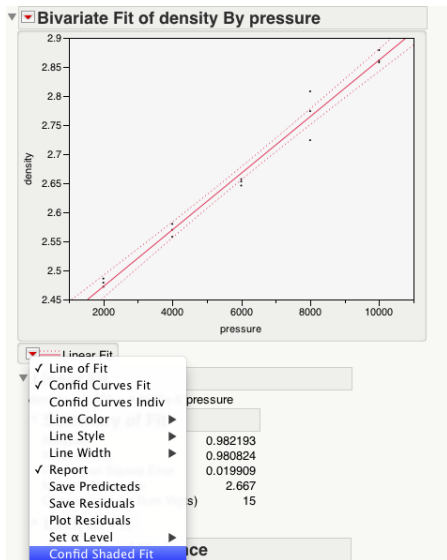
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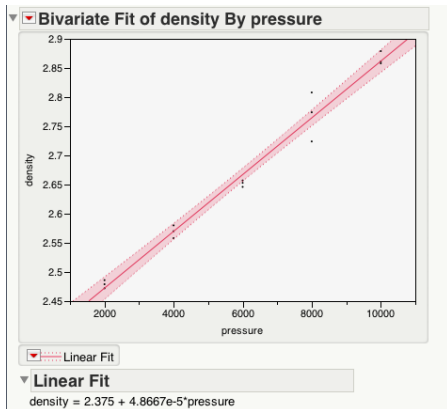
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Ceramics: calculating the margins of error in JMP

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The screenshot shows the JMP software interface. On the left, a dialog box titled 'Simultaneous conf. margin err' is open. It has fields for 'Column Name' (Simultaneous conf. margin err), 'Data Type' (Numeric), 'Modeling Type' (Continuous), and 'Format' (Best, Width 12). There are buttons for 'OK', 'Cancel', 'Apply', and 'Help'. Below the dialog box, the 'ceramics.jmp' data table is visible. The table has columns for 'pressure', 'density', and 'Simultaneous conf. margin'. The data is as follows:

	pressure	density	Simultaneous conf. margin
1	2000	2.486	*
2	2000	2.479	*
3	2000	2.472	*
4	4000	2.558	*
5	4000	2.57	*
6	4000	2.58	*
7	6000	2.646	*
8	6000	2.657	*
9	6000	2.653	*
10	8000	2.724	*
11	8000	2.774	*
12	8000	2.808	*
13	10000	2.861	*
14	10000	2.879	*
15	10000	2.858	*

Ceramics: calculating the margins of error in JMP

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Simultaneous conf. margin err

Table Columns ▼

- pressure
- density
- Simultaneous confidence intervals

Functions (grouped) ▼

- Row
- Numeric
- Transcendental
- Trigonometric
- Character
- Comparison
- Conditional
- Probability
- Discrete Probability

OK

Cancel

Apply

Clear

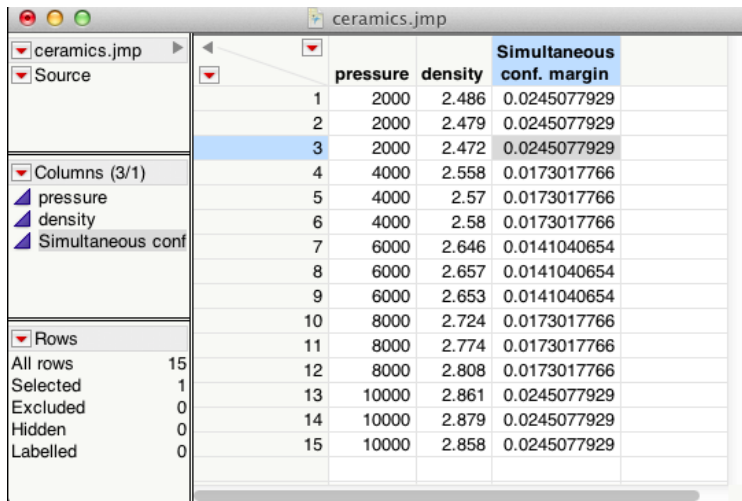
Help

$$0.0549 * \sqrt{0.066 + 0.00000000888 * (pressure - 6000)^2}$$

Ceramics: calculating the margins of error in JMP

More Inference for
Simple Linear
Regression (Ch.
9.1)

Yifan Zhu



ceramics.jmp

	pressure	density	Simultaneous conf. margin
1	2000	2.486	0.0245077929
2	2000	2.479	0.0245077929
3	2000	2.472	0.0245077929
4	4000	2.558	0.0173017766
5	4000	2.57	0.0173017766
6	4000	2.58	0.0173017766
7	6000	2.646	0.0141040654
8	6000	2.657	0.0141040654
9	6000	2.653	0.0141040654
10	8000	2.724	0.0173017766
11	8000	2.774	0.0173017766
12	8000	2.808	0.0173017766
13	10000	2.861	0.0245077929
14	10000	2.879	0.0245077929
15	10000	2.858	0.0245077929

SLR: Inference for
the Mean
Response at some
 x

Prediction interval
for a new y at
some x

Simultaneous
Confidence
Intervals for $\mu_{y|x}$

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SLR: Inference for
the Mean
Response at some
 x

Prediction interval
for a new y at
some x

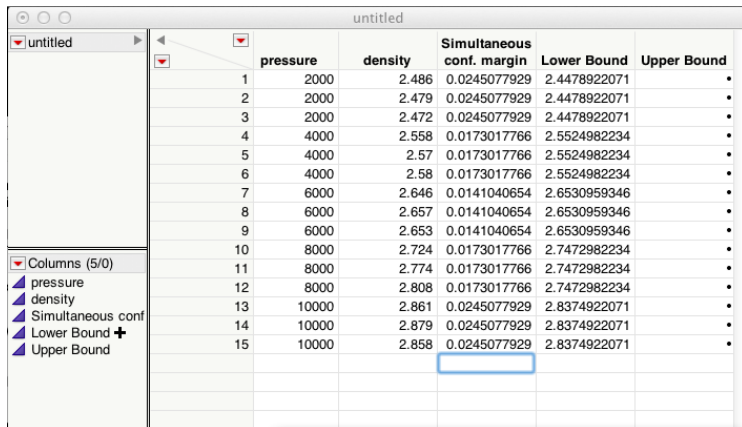
Simultaneous
Confidence
Intervals for $\mu_y|x$

The screenshot shows the JMP software interface. On the left, the 'Column Properties' dialog box for 'Lower Bound' is open, showing it is a Numeric, Continuous variable. On the right, the 'Lower Bound' dialog box is open, showing the 'Table Columns' list with 'Simultaneous confidence interval' selected. The 'Functions (grouped)' list is also visible. At the bottom, the 'Formula' editor shows the formula: $2.375 + 0.0000487 * \text{pressure}$ - Simultaneous conf. margin.

Ceramics: calculating the margins of error in JMP

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The screenshot shows the JMP software interface with a data table titled 'untitled'. The table has 6 columns: an index column (1-15), 'pressure', 'density', 'Simultaneous conf. margin', 'Lower Bound', and 'Upper Bound'. The 'Columns' panel on the left shows the columns are loaded. A blue box highlights the 'Simultaneous conf. margin' column for the last row (index 15).

	pressure	density	Simultaneous conf. margin	Lower Bound	Upper Bound
1	2000	2.486	0.0245077929	2.4478922071	•
2	2000	2.479	0.0245077929	2.4478922071	•
3	2000	2.472	0.0245077929	2.4478922071	•
4	4000	2.558	0.0173017766	2.5524982234	•
5	4000	2.57	0.0173017766	2.5524982234	•
6	4000	2.58	0.0173017766	2.5524982234	•
7	6000	2.646	0.0141040654	2.6530959346	•
8	6000	2.657	0.0141040654	2.6530959346	•
9	6000	2.653	0.0141040654	2.6530959346	•
10	8000	2.724	0.0173017766	2.7472982234	•
11	8000	2.774	0.0173017766	2.7472982234	•
12	8000	2.808	0.0173017766	2.7472982234	•
13	10000	2.861	0.0245077929	2.8374922071	•
14	10000	2.879	0.0245077929	2.8374922071	•
15	10000	2.858	0.0245077929	2.8374922071	•

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The screenshot shows two overlapping windows from the JMP software. The background window is titled "'Upper Bound' in Table 'untitled'" and shows settings for a column named "Upper Bound". The column is set to be "Locked", "Numeric", "Continuous", and in "Best" format. The "Initialize Data" is set to "Missing/Empty". The "Formula" tab is selected, showing a formula editor with the text "optional item" and a "Remove" button. The foreground window is titled "Upper Bound" and contains a "Table Columns" list with "pressure", "density", "Simultaneous confidence interval", "Lower Bound", and "Upper Bound". It also has a "Functions (grouped)" list with categories like Row, Numeric, Transcendental, Trigonometric, Character, Comparison, Conditional, Probability, and Discrete Probability. The "OK", "Cancel", "Apply", "Clear", and "Help" buttons are on the right. The main area of the "Upper Bound" window displays the formula: $2.375 + 0.0000487 * \text{pressure} + \text{Simultaneous conf. margin}$.

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▼ untitled ▶		▼	pressure	density	Simultaneous conf. margin	Lower Bound	Upper Bound
	▼	1	2000	2.486	0.0245077929	2.4478922071	2.4969077929
		2	2000	2.479	0.0245077929	2.4478922071	2.4969077929
		3	2000	2.472	0.0245077929	2.4478922071	2.4969077929
		4	4000	2.558	0.0173017766	2.5524982234	2.5871017766
		5	4000	2.57	0.0173017766	2.5524982234	2.5871017766
		6	4000	2.58	0.0173017766	2.5524982234	2.5871017766
		7	6000	2.646	0.0141040654	2.6530959346	2.6813040654
		8	6000	2.657	0.0141040654	2.6530959346	2.6813040654
		9	6000	2.653	0.0141040654	2.6530959346	2.6813040654
		10	8000	2.724	0.0173017766	2.7472982234	2.7819017766
		11	8000	2.774	0.0173017766	2.7472982234	2.7819017766
		12	8000	2.808	0.0173017766	2.7472982234	2.7819017766
		13	10000	2.861	0.0245077929	2.8374922071	2.8865077929
		14	10000	2.879	0.0245077929	2.8374922071	2.8865077929
		15	10000	2.858	0.0245077929	2.8374922071	2.8865077929

Columns (5/1)
 ▲ pressure
 ▲ density
 ▲ Simultaneous conf
 ▲ Lower Bound +
 ▲ Upper Bound +