Special Continuous Random Variables

Yifan 7hu

Iowa State University

Special Continuous Random Variables

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Overview

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Normal Quantiles

The Student *t* Distribution

The Chi-square Distribution

The F Distribution

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The normal (Gaussian) distribution

▶ A random variable X is Normal(μ , σ^2) if its pdf is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

- Using calculus, one can verify that:
 - \triangleright $E(X) = \mu$
 - $ightharpoonup Var(X) = \sigma^2$
- $\stackrel{X-\mu}{\sigma} \sim N(0,1)$, where N(0,1) is the *standard* normal distribution (mean 0, variance 1).

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The standard normal distribution

► A standard normal random variable, usually called Z, has the pdf:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

- ▶ The standard normal pdf is usually denoted $\phi(z)$.
- ▶ The standard normal cdf is usually denoted $\Phi(z)$.

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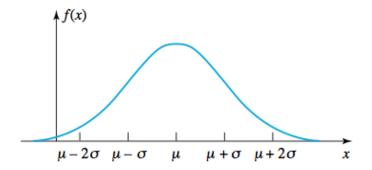
The Student *t* Distribution

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- A normal random variable is (often) a finite average of many repeated, independent, identical trials.
- Examples:
 - Mean width of the next 50 hexamine pellets.
 - ▶ Mean height of the next 30 students.
 - Your SAT score.
 - ▶ Total % yield of the next 40 runs of a chemical process.
 - ▶ The next blood pressure reading.
 - Several kinds of measurement error.
 - Corrosion resistance of carbon/carbon composites.

A look at the normal density: a bell curve



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Normal Probabilities

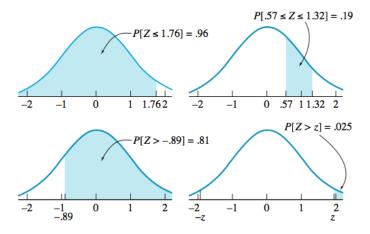
Normal Quantiles

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As usual, areas denote probabilities



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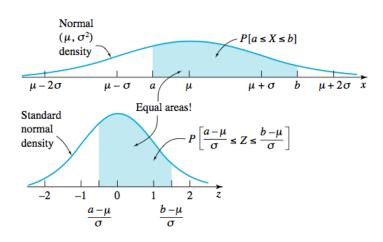
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The relationship between normal probabilities and standard normal probabilities.



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Special Notation of Quantiles

► Since $Z = \frac{X - \mu}{\sigma}$ is *standard* normal probability values from X can be expressed as:

$$P(a \le X \le b) = P\left(\frac{a - \mu}{\sigma} \le Z \le \frac{b - \mu}{\sigma}\right)$$
$$= \int_{(a - \mu)/\sigma}^{(b - \mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

- Unfortunately, the integral cannot be evaluated analytically. Instead, we use either:
 - A computer.
 - A standard normal probability table like the one in Table B.3 in Vardeman and Jobe.

Example: baby food

- ▶ J. Fisher, in his article *Computer Assisted Net Weight Control* (Quality Progress, June 1983), discusses the filling of food containers with strained plums with tapioca by weight. The mean of the values portrayed is about 137.2 g, the standard deviation is about 1.6 g, and data look bell-shaped.
- Let W = the next fill weight. Then, $W \sim N(\mu = 137.2, \sigma^2 = (1.6)^2)$.
- ▶ Let's find the probability that the next jar contains less food by mass than it's supposed to (declared weight = 135.05 g).

$$P(W < 135.0) = P\left(\frac{W - 137.2}{1.6} < \frac{135.05 - 137.2}{1.6}\right)$$
$$= P(Z < -1.34)$$
$$= \Phi(-1.34)$$

▶ The approximate value of $\Phi(-1.34)$ is found to be 0.0901 in Table B.3.

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$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$$

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |

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Some facts about $\Phi(z)$

$$\Phi(z) + \Phi(-z) = 1$$

$$\Phi(z) - \Phi(-z) = 2\Phi(z) - 1$$

$$\Phi(1.96) = 0.975$$

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pecial Notation

| -3.3 .0005 .0005 .0004 .0004 .0004 .0004 .0004 .0004 .0004 .0003 .0005 .0007 .0008 .0003 .0023 .0022 .0011 .0014 .0014 .0014 .0024 .0023 .0022 .0021 .0023 .0022 .0021 .0023 .0022 .0021 .0023 .0022 .0021 .0023 .0022 .0021 .0023 <td< th=""><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th></td<> | | | | | | | | | | | |
|--|------|-------|---------|-------|-------|-------|-------|-------|---------|--------|---------|
| -3.3 .0005 .0005 .0005 .0004 .0004 .0004 .0004 .0004 .0004 .0004 .0004 .0004 .0005 .0006 .0008 .0008 .0008 .0008 .0008 .0007 .0010 .0011 .0011 .0011 .0011 .0011 .0011 .0011 .0011 .0011 .0011 .0015 .0015 .0014 .0024 .0023 .0022 .0021 .0022 .0021 .0022 .0021 .0022 .0021 .0022 .0021 .0022 .0021 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0023 .0022 .0024 .0033 .0034 .0033 .0032 .0022 .0024 .0034 | z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| -3.2 .0007 .0007 .0006 .0006 .0006 .0006 .0005 .0005 .0005 -3.1 .0010 .0009 .0009 .0009 .0008 .0008 .0008 .0008 .0008 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0007 .0001 .0014 .0014 .0022 .0021 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0022 .0037 .0036 .0064 .0039 .0038 .0065 .0065 .0064 .0091 | -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.1 .0010 .0009 .0009 .0009 .0008 .0008 .0008 .0007 .0007 -3.0 .0013 .0013 .0013 .0012 .0012 .0011 .0011 .0011 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0010 .0011 .0011 .0011 .0011 .0011 .0012 .0023 .0022 .0021 .0021 .0020 .0012 .0023 .0022 .0021 .0027 .0025 .0027 .0025 .0024 .0033 .0033 .0033 .0033 .0033 .0033 .0033 .0034 .0033 .0034 .0033 .0034 .0033 .0034 .0033 .0034 .0033 .0034 .0033 .0034 .0033 .0027 .0026 .0026 .0062 .0064 .0043 .0044 .0044 .0043 .0052 .0051 .0068 .0066 .0640 | -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.0 .0013 .0013 .0013 .0012 .0012 .0011 .0011 .0010 .0010 -2.9 .0019 .0018 .0018 .0017 .0016 .0015 .0015 .0014 .0014 -2.8 .0026 .0025 .0024 .0023 .0023 .0023 .0021 .0021 .0022 .0021 -2.7 .0035 .0034 .0033 .0032 .0031 .0030 .0029 .0028 .0027 .0026 -2.5 .0062 .0069 .0059 .0057 .0055 .0054 .0052 .0055 .0052 .0051 .0049 .0048 -2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .0066 .0064 -2.3 .0107 .0104 .0102 .0099 .096 .0094 .091 .0089 .0087 -2.2 .0139 .0136 .0132 .0129 .0125 .0122 | -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| 2-2.9 .0019 .0018 .0018 .0017 .0016 .0016 .0015 .0015 .0014 .0014 -2.8 .0026 .0025 .0024 .0023 .0023 .0022 .0021 .0021 .0020 .0019 -2.7 .0035 .0034 .0033 .0032 .0031 .0030 .0029 .0028 .0027 .0026 -2.6 .0047 .0043 .0044 .0043 .0041 .0040 .0039 .0031 .0032 .0031 .0032 .0031 .0032 .0031 .0030 .0038 .0037 .0035 .0052 .0052 .0051 .0049 .0041 .0049 .0041 .0049 .0041 .0049 .0041 | -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | 8000. | .0008 | 8000. | .0007 | .0007 |
| -2.8 .0026 .0025 .0024 .0023 .0023 .0022 .0021 .0021 .0020 .0019 -2.7 .0035 .0034 .0033 .0032 .0031 .0033 .0037 .0020 .0020 .0027 .0026 -2.6 .0047 .0044 .0044 .0044 .0043 .0037 .0036 -2.5 .0062 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .0049 .0084 -2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .0066 .0064 -2.3 .0107 .0104 .0102 .0099 .0096 .0094 .0091 .0089 .0087 .0084 -2.1 .0139 .0136 .0132 .0122 .0122 .0119 .0116 .0113 .0112 -1.0 .0219 .0174 .0170 .0166 .0162 .0256 .0250 .0244 | -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.7 .0035 .0034 .0033 .0032 .0031 .0030 .0029 .0028 .0027 .0026 −2.6 .0047 .0045 .0044 .0043 .0041 .0040 .0039 .0033 .0037 .0036 −2.4 .0062 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .0049 .0049 −2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .0066 .0064 −2.2 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .0116 .0143 −2.1 .0179 .0174 .0170 .0166 .0162 .0158 .0154 .0150 .0148 .0143 −2.0 .0228 .0221 .0271 .0212 .0207 .0202 .0197 .0192 .0188 .0183 −1.8 .0359 .0351 .0344 .0336 .0327 .0322 | | | .0018 | .0018 | | | | .0015 | | .0014 | |
| -2.6 .0047 .0045 .0044 .0043 .0041 .0040 .0039 .0038 .0037 .0036 -2.5 .0062 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .0049 .0048 -2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .0066 .0064 -2.3 .0107 .0104 .0102 .0099 .0095 .0094 .0091 .0089 .0087 .0084 -2.2 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .0116 .0113 .0114 -2.2 .0228 .0222 .0217 .0212 .0207 .0202 .0197 .0122 .0183 -1.9 .0287 .0281 .0274 .0268 .0262 .0256 .0250 .0244 .0329 .0233 -1.8 .0359 .0351 .0344 .0336 .0329 .0322 | | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.5 .0062 .0060 .0059 .0057 .0055 .0054 .0052 .0051 .0049 .0048 -2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .0066 .0064 -2.3 .0107 .0104 .0102 .0099 .0096 .0091 .0089 .0087 .0084 -2.1 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .0116 .0113 .0110 -2.1 .0179 .0174 .0170 .0166 .0162 .0158 .0154 .0150 .0146 .0143 -1.9 .0287 .0281 .0274 .0268 .0262 .0256 .0250 .0244 .0239 .0231 -1.1 .0287 .0281 .0274 .0288 .0262 .0256 .0250 .0244 .0239 .0231 -1.7 .0446 .0436 .0427 .0418 .0409 .0411 | | | .0034 | | .0032 | | | .0029 | | .0027 | .0026 |
| -2.4 .0082 .0080 .0078 .0075 .0073 .0071 .0069 .0068 .0066 .0064 -2.3 .0107 .0104 .0102 .0099 .0096 .0094 .0091 .0089 .0087 .0087 -2.2 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .0116 .0113 .0114 -2.0 .0228 .0222 .0217 .0212 .0207 .0202 .0197 .0182 .0188 .0183 -1.9 .0287 .0281 .0247 .0268 .0262 .0256 .0250 .0244 .0393 .0331 .0244 .0363 .0329 .0322 .0314 .0307 .0322 .0314 .0307 .0322 .0314 .0307 .0322 .0314 .0307 .0324 .0314 .0307 .0322 .0314 .0307 .0324 .0314 .0307 .0326 .0322 .0314 .0307 .0366 .0516 < | -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.3 .0107 .0104 .0102 .0099 .0096 .0094 .0091 .0089 .0087 .0084 -2.2 .0139 .0136 .0132 .0129 .0125 .0122 .0119 .016 .0113 .0113 .0114 .014 .0164 .0154 .0155 .0154 .0150 .0146 .0143 .0146 .0143 .0202 .0270 .0202 .0197 .0192 .0188 .0183 .0282 .0262 .0256 .0256 .0256 .0256 .0256 .0256 .0256 .0256 .0256 .0256 .0256 .0256 .0261 .0394 .0394 .0361 .0606 .0594 .0585 .0575 .0556 .0556 .0505 .0495 .0485 .0475 </th <th>-2.5</th> <th>.0062</th> <th>.0060</th> <th>.0059</th> <th>.0057</th> <th>.0055</th> <th>.0054</th> <th>.0052</th> <th>.0051</th> <th>.0049</th> <th>.0048</th> | -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.2 0.139 0.136 0.132 0.129 0.125 0.122 0.112 0.112 0.119 0.116 0.113 0.110 −2.1 0.079 0.174 0.170 0.166 0.162 0.158 0.154 0.150 0.164 0.143 0.110 −2.1 0.028 0.0221 0.217 0.021 0.027 0.020 0.027 0.029 0.029 0.029 0.029 0.028 0.233 0.233 −1.8 0.359 0.351 0.344 0.336 0.329 0.322 0.314 0.307 0.301 0.294 −1.6 0.548 0.537 0.562 0.516 0.605 0.495 0.485 0.475 0.465 0.455 0.452 0.554 0.527 0.465 0.455 0.465 0.455 0.465 0.455 0.465 0.452 0.571 0.569 0.451 0.452 0.557 0.465 0.455 0.465 0.452 0.452 0.452 0.452 | -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.1 .0.179 .0.174 .0.170 .0.166 .0.162 .0.158 .0.154 .0.150 .0.146 .0.143 -2.0 .0228 .0222 .0217 .0212 .0207 .0202 .0197 .0192 .0188 .0183 -1.8 .0287 .0281 .0244 .0238 .0229 .0322 .0314 .0330 .0329 .0322 .0314 .0307 .0301 .0244 -1.7 .0446 .0436 .0427 .0418 .0409 .0401 .0392 .0384 .0337 .0366 .0515 .0495 .0485 .0475 .0468 .0475 .0468 .0555 .0495 .0485 .0475 .0468 .0475 .0565 .0565 .0495 .0485 .0475 .0468 .0457 .0569 .0495 .0485 .0475 .0468 .0457 .0468 .0560 .0598 .0582 .0571 .0559 .045 .0478 .0582 .0571 .0559 .045 </th <th></th> <th></th> <th>.0104</th> <th></th> <th>.0099</th> <th></th> <th>.0094</th> <th></th> <th>.0089</th> <th>.0087</th> <th></th> | | | .0104 | | .0099 | | .0094 | | .0089 | .0087 | |
| | | | .0136 | | | | | | .0116 | .0113 | .0110 |
| | | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | | .0150 | .0146 | .0143 |
| −1.8 .0359 .0351 .0344 .0336 .0329 .0322 .0314 .0307 .0301 .0294 −1.7 .0446 .0436 .0427 .0418 .0409 .0401 .0392 .0384 .0375 .0357 .0357 .0357 .0357 .0357 .0357 .0356 .0516 .0505 .0495 .0485 .0475 .0465 .0455 .0450 .0559 .0524 .0550 .0554 .0550 .0554 .0582 .0571 .0559 −1.4 .0608 .0693 .0678 .0764 .0749 .0735 .0721 .0708 .0664 .0681 −1.3 .0968 .0951 .0934 .0918 .0901 .0885 .0869 .0853 .0838 .0823 −1.1 .1151 .1131 .1112 .1093 .1075 .1056 .1038 .1020 .1100 .1170 −1.0 .1587 .1356 .1531 .1292 .1271 . | -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| | | | | | | | | | | | |
| -1.6 .0548 .0537 .0526 .0516 .0505 .0495 .0485 .0475 .0465 .0455 -1.5 .0668 .0655 .0643 .0630 .0618 .0606 .0594 .0582 .0571 .0559 -1.4 .0808 .0793 .0778 .0764 .0749 .0721 .0708 .0694 .0681 -1.3 .0968 .0991 .0934 .0918 .0901 .0885 .0869 .0853 .0838 .0823 -1.1 .1357 .1335 .1314 .1292 .1271 .1251 .1230 .1210 .1190 .1170 .1190 .1170 .1190 .1170 .1190 .1170 .1191 .1190 .1170 .1190 .1170 .1191 .1190 .1170 .1181 .1781 .1762 .1736 .1711 .1685 .1660 .1635 .1611 .1690 .1446 .1423 .1401 .1379 .2402 .2389 .2352 | | | | | | | | | | | |
| -1.5 .0668 .0655 .0643 .0630 .0618 .0606 .0594 .0582 .0571 .0559 -1.4 .0808 .0793 .0778 .0764 .0749 .0735 .0721 .0708 .0664 .0681 -1.3 .0968 .0951 .0934 .0918 .0901 .0885 .0869 .0853 .0838 .0823 -1.2 .1151 .1131 .1112 .1093 .1075 .1056 .1038 .1020 .1003 .0958 -1.1 .1357 .1335 .1314 .1292 .1271 .1251 .1230 .120 .1100 .170 -1.0 .1587 .1352 .1512 .1292 .1469 .1446 .1423 .1401 .1379 -0.9 .1841 .1814 .1788 .1762 .1736 .1711 .1685 .1660 .1635 .1611 -0.8 .2119 .2090 .2061 .2332 .2297 .2266 | | | | | | | | | | | |
| | | | | | | | | | | | |
| -1.3 .0968 .0951 .0934 .0918 .0901 .0885 .0889 .0853 .0828 .0823 -1.2 .1151 .1131 .1103 .1056 .1058 .1020 .1030 .0905 -1.1 .1357 .1335 .1334 .1292 .1271 .1251 .1230 .1210 .1190 .1170 -1.0 .1587 .1562 .1539 .1515 .1492 .1469 .1446 .1423 .1401 .1379 -0.9 .1841 .1848 .1762 .1736 .1711 .1685 .1663 .1613 .1612 .1909 .0401 .1849 .1461 .1483 .1662 .005 .1977 .1949 .1922 .1884 .1867 -0.0 .2420 .2389 .2358 .2327 .2297 .2266 .2246 .2216 .2248 .2412 .2408 .2418 .451 .2476 .2012 .2877 .2843 .2451 .2483 .245 | -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.2 .1151 .1131 .1112 .1093 .1075 .1056 .1038 .1020 .1003 .0985 -1.1 .1357 .1335 .1314 .1292 .1271 .1251 .1230 .1210 .1190 .1170 -1.0 .1587 .1559 .15159 .1515 .1612 .1499 .1446 .1423 .1401 .1379 -0.9 .1841 .1814 .1788 .1762 .1736 .1711 .1685 .1660 .1635 .1611 -0.8 .2119 .2090 .2061 .2033 .2005 .1977 .1494 .1922 .1884 .1867 -0.7 .2420 .2398 .2358 .2327 .2297 .2666 .2236 .2206 .2177 .2148 .1867 -0.6 .2743 .2709 .2676 .2643 .2611 .2578 .2546 .2514 .2483 .2451 .2483 .2451 .2483 .2451 .2483 .2451< | | | | | | | | | | | |
| -1.1 .1357 1335 .1314 1292 .1271 .1251 .1230 .1210 .1190 .1170 -1.0 .1587 .1562 .1539 .1515 .1492 .1469 .1446 .1423 .1401 .1379 -0.9 .1841 .1814 .1788 .1762 .1736 .1711 .1685 .1660 .1635 .1611 -0.8 .2119 .2090 .2061 .2033 .2005 .1977 .1949 .1922 .1884 .1867 -0.6 .2743 .2790 .2267 .2267 .2268 .2362 .2206 .2127 .2148 -0.5 .3085 .3050 .3015 .2981 .2946 .2912 .2877 .2843 .2810 .2776 -0.4 .3446 .3499 .3372 .3336 .3300 .3624 .3228 .3152 .3152 .3461 .578 .3594 .3552 .3483 .3521 .3433 .3452 .4061 | | | | | | | | | | | |
| -1.0 .1587 .1562 .1539 .1515 .1492 .1469 .1446 .1423 .1401 .1379 -0.9 .1841 .1814 .1788 .1762 .1736 .1711 .1685 .1660 .1635 .1611 -0.8 .2119 .2090 .2061 .2033 .2005 .1977 .1949 .1922 .1894 .1867 -0.7 .2420 .2398 .2328 .2227 .2266 .2236 .2236 .2020 .206 .2276 .266 .2263 .2202 .206 .2236 .2202 .206 .2743 .2810 .2476 -0.5 .3085 .3050 .3015 .2981 .2946 .2912 .2872 .2843 .2810 .2776 -0.4 .3446 .3409 .3372 .3360 .3602 .3624 .3283 .3156 .3121 -0.3 .3821 .3783 .3745 .3707 .3669 .3632 .3594 .3557 | | | | | | | | | | | |
| -0.9 .1841 .1814 .1788 .1762 .1736 .1711 .1685 .1660 .1635 .1611 -0.8 .2119 .2909 .2061 .2033 .2005 .1977 .1494 .1922 .1894 .1867 -0.7 .2420 .2389 .2358 .2237 .2297 .266 .2236 .2206 .2236 .2177 .2148 .2451 .2453 .2451 | | | | | | | | | | | |
| -0.8 2119 2.909 2.061 2.033 2.005 1.977 1.949 1.922 1.894 1.867 -0.7 2.420 2.389 2.258 2.327 2.297 2.266 2.236 2.207 2.148 -0.6 2.743 2.709 2.676 2.643 2.611 2.578 2.546 2.514 2.528 2.514 2.243 2.612 2.766 2.643 2.810 2.776 -0.4 3.446 3.409 3.372 3.336 3.300 3.624 3.228 3.594 3.552 3.534 3.261 2.786 -0.3 3.821 3.783 3.757 3.669 3.632 3.594 3.557 3.520 3.483 -0.2 4.207 4.168 4.129 4.009 4.052 4.013 3.974 3.936 3.897 3.594 3.594 3.594 3.69 3.69 3.69 3.69 3.69 3.69 3.69 3.69 3.69 3.69 3.69 | -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.7 2420 2389 2388 2327 2297 2266 2236 2206 2177 2148 -0.6 2.743 2379 2676 2643 2611 2578 2546 2514 2483 2481 2776 -0.5 3085 3050 3015 2981 2946 2912 2877 2843 2810 2776 -0.4 3446 3409 3372 3336 3300 3264 3228 3192 3150 3121 -0.3 3821 3783 3745 3707 3669 3632 3594 3557 3520 3483 -0.2 4207 4168 4129 4090 4052 4013 3974 3303 3897 3369 -0.1 4602 4562 4522 4483 4443 4404 4364 4325 4266 4247 -0.1 5000 4960 4920 4880 4840 4801 4761 < | -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.6 .2743 .2709 .2676 .2643 .2611 .2578 .2546 .2514 .2483 .2451 -0.5 .3085 .3050 .3015 .2981 .2946 .2912 .2877 .2843 .2810 .2776 -0.4 .3446 .3409 .3725 .3362 .3036 .3036 .3624 .3622 .3557 .3550 .3126 .3125 .352 .3483 .3954 .3557 .3520 .3483 .404 .304 .3936 .3897 .3589 .3652 .3483 .3483 .4443 .4404 .4364 .4325 .4286 .4247 -0.1 .4602 .4562 .4522 .4843 .4443 .4404 .4364 .4252 .4286 .4247 -0.1 .5000 .4960 .4820 .4840 .4801 .4761 .4721 .4681 .4641 | -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.5 .3085 .3050 .3015 .2981 .2946 .2912 .2877 .2843 .2810 .2776 -0.4 .3446 .3499 .3372 .3336 .3300 .3264 .3228 .3192 .3156 .3121 -0.3 .3821 .3783 .375 .3669 .3632 .3594 .3550 .3823 .3852 .3836 -0.2 .4207 .4168 .4129 .4009 .4052 .4013 .3974 .3936 .3897 .3859 -0.1 .4602 .4562 .4522 .4483 .4443 .4404 .4364 .4225 .4286 .4247 -0.1 .5000 .4960 .4820 .4840 .4801 .4761 .4721 .4681 .4641 | -0.7 | .2420 | .2389 | .2358 | .2327 | .2297 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.4 3446 3409 3372 3336 3300 3264 3228 3192 3156 3121 -0.3 3821 3783 3745 3707 3669 3632 3594 3557 3520 3483 -0.1 4207 4168 4129 4009 4052 4013 3974 3936 3897 3859 -0.1 4602 4562 4522 4483 4443 4404 4364 4325 4286 4247 -0.0 5000 4960 4280 4840 4801 4761 4721 4681 4641 | -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.3 .3821 .3783 .3745 .3707 .3669 .3632 .3594 .3557 .3520 .3483 -0.2 .4207 .4168 .4129 .4090 .4052 .4013 .3974 .3936 .3897 .3859 -0.1 .4602 .4562 .4522 .4483 .4443 .4404 .4364 .4325 .4286 .4247 -0.0 .5000 .4960 .4880 .4840 .4801 .4761 .4721 .4681 .4441 | -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.2 .4207 .4168 .4129 .4090 .4052 .4013 .3974 .3936 .3859 .3859 -0.1 .4602 .4562 .4522 .4483 .4443 .4404 .4364 .4325 .4286 .4247 -0.0 .5000 .4960 .4920 .4880 .4840 .4801 .4761 .4721 .4681 .4641 | -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | | .3156 | |
| -0.1 .4662 .4562 .4522 .4483 .4443 .4404 .4364 .4325 .4286 .4247 -0.0 .5000 .4960 .4920 .4880 .4840 .4801 .4761 .4721 .4681 .4641 | -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| | -0.2 | | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| | -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| Vifan Zhu Jowa State University | -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |
| | | , | Vifon 7 | hu | | | | lov | va Stat | e Univ | arcity. |

Special Continuous Random Variables Yifan Zhu

)verview

Normal Probabilities

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e F Distribution

ecial Notation

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 |
|-----|-------|-------|-------|-------|-------|-------|-------|--------|---------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 |
| 8.0 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 |
| 2.0 | .9773 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 |
| 2.9 | .9981 | .9982 | .9983 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 |
| | | Yifan | Zhu | | | | | owa St | ate Uni |
| | | | | | | | | 0 | |

Yifan Zhu

Normal Probabilities

Special Continuous

Random Variables

Normal Quantiles

.09

.5359

.5753 .6141

.6517 .6879 .7224 .7549

.7852

.8133 .8389

.8621 .8830 .9015 .9177 .9319 .9441 .9545 .9633 .9706 .9767 .9817 .9857 .9890 .9916 .9936 .9952 .9964 .9974 .9981 .9986 .9990 .9993 .9995 .9997 .9998 Your turn: using the standard normal table, calculate the following.

- 1. $P(X \le 3), X \sim N(2, 64)$
- 2. $P(X > 7), X \sim N(6, 9)$
- 3. $P(|X-1| > 0.5), X \sim N(2,4)$
- 4. P(X is within 2 standard deviations of its mean.) $X \sim N(\mu, \sigma^2)$

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Answers: normal probabilities

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Special Notation of Quantiles

1. $P(X < 3), X \sim N(2, 64)$

$$P(X \le 3) = P\left(Z \le \frac{3-2}{\sqrt{64}} = 0.125\right)$$
$$= \Phi(0.125)$$
$$= 0.5597 \text{ from the standard normal table}$$

Answers: normal probabilities

2. $P(X > 7), X \sim N(6,9)$

$$P(X > 7) = P\left(Z > \frac{7 - 6}{\sqrt{9}} = 0.33\right)$$
= 1 - P(Z \le 0.33)
= 1 - \Phi(0.33)
= 1 - 0.6293 from the standard normal table
= 0.3707

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3. $P(|X-1| > 0.5), X \sim N(2,4)$

$$\begin{split} P(|X-1|>0.5) &= P(X-1>0.5 \text{ or } X-1<-0.5) \\ &= P(X-1>0.5) + P(X-1<-0.5) \\ &= P(X>1.5) + P(X<0.5) \\ &= P\left(\frac{X-2}{2}>\frac{1.5-2}{2}\right) + P\left(\frac{X-2}{2}<\frac{0.5-2}{2}\right) \\ &= P(Z>-0.25) + P(Z<-0.75) \\ &= 1 - P(Z\leq-0.25) + P(Z\leq-0.75) \\ &= 1 - \Phi(-0.25) + \Phi(-0.75) \\ &= 1 - 0.4013 + 0.2266 \text{ from the standard normal table} \\ &= 0.8253 \end{split}$$

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4. $P(X \text{ is within 2 standard deviations of its mean.}) X \sim N(\mu, \sigma^2)$

$$P(|X - \mu| < 2\sigma) = P(-2\sigma < X - \mu < 2\sigma)$$

$$= P(\mu - 2\sigma < X < \mu + 2\sigma)$$

$$= P\left(\frac{(\mu - 2\sigma) - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{(\mu + 2\sigma) - \mu}{\sigma}\right)$$

$$= P(-2 < Z < 2)$$

$$= P(Z < 2) - P(Z < -2)$$

$$= \Phi(2) - \Phi(-2)$$

$$= 0.9773 - 0.0228$$

$$= 0.9545$$

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Normal quantiles

I can find standard normal quantiles by using the standard normal tabl:e in reverse.

Example: for the jar weights $W \sim (137.2, 1.6^2)$, I will find Q(0.1)

$$0.1 = P(X \le Q(0.1))$$

$$= P\left(Z \le \frac{Q(0.1) - 137.2}{1.6}\right)$$

$$= \Phi\left(\frac{Q(0.1) - 137.2}{1.6}\right)$$

$$\Phi^{-1}(0.1) = \frac{Q(0.1) - 137.2}{1.6}$$

$$Q(0.1) = 137.2 + 1.6 \cdot \Phi^{-1}(0.1)$$

 $\Phi^{-1}(0.1) = -1.28$ from the standard normal table. Hence:

$$Q(0.1) = 137.2 + 1.6(-1.28)$$
$$= 135.152$$

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Table B.3

Standard Normal Cumulative Probabilities

 $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$

| | | | | J | ∞ √ 2π | (2) | | | | |
|------|-------|-------|-------|-------|---------------|-------|-------|-------|-------|-------|
| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| | | | | | | | | | | |

Special Continuous Random Variables

Yifan Zhu

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Your turn: calculate the following:

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- 1. Q(0.95) of $X \sim N(9,3)$
- 2. c such that $P(|X-2|>c)=0.01, X\sim N(2,4)$
- 3. c such that $P(|X \mu| < \sigma c) = 0.95$, $X \sim N(\mu, \sigma^2)$

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Special Notation of Quantiles

1. Q(0.95) for $X \sim N(9,3)$

$$0.95 = P(X \le Q(0.95))$$

$$= P\left(\frac{X - 9}{\sqrt{3}} < \frac{Q(0.95) - 9}{\sqrt{3}}\right)$$

$$= P\left(Z < \frac{Q(0.95) - 9}{\sqrt{3}}\right)$$

$$0.95 = \Phi\left(\frac{Q(0.95) - 9}{\sqrt{3}}\right)$$

$$\Phi^{-1}(0.95) = \frac{Q(0.95) - 9}{\sqrt{3}}$$

$$Q(0.95) = \sqrt{3} \cdot \Phi^{-1}(0.95) + 9$$

$$= \sqrt{3} \cdot (1.645) + 9 \quad \text{(from the std. normal table)}$$

$$= 11.85$$

The Student *t* Distribution

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2. c such that $P(|X-2|>c)=0.01, X\sim N(2.1,4)$

$$0.01 = P(|X - 2| > c)$$

$$= P(X - 2 > c \text{ or } X - 2 < -c)$$

$$= P(X - 2 > c) + P(X - 2 < -c)$$

$$= P\left(\frac{X - 2}{2} > \frac{c}{2}\right) + P\left(\frac{X - 2}{2} < -\frac{c}{2}\right)$$

$$= P\left(Z > \frac{c}{2}\right) + P\left(Z < -\frac{c}{2}\right)$$

$$= P\left(Z < -\frac{c}{2}\right) + P\left(Z < -\frac{c}{2}\right) \quad (\phi(z) \text{ is symmetric about } 0)$$

$$= 2P\left(Z < -\frac{c}{2}\right)$$

$$0.01 = 2\Phi(-c/2)$$

$$0.005 = \Phi(-c/2)$$

$$0.005 = \Phi(-c/2)$$

$$c = -2\Phi^{-1}(0.005)$$

$$= -2 \cdot (-2.575) \quad (\text{using the standard normal table})$$

$$= 5.15$$

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3. c such that $P(|X - \mu| < \sigma c) = 0.95$, $X \sim N(\mu, \sigma^2)$

$$\begin{array}{l} 0.95 = P(|X - \mu| < \sigma c) \\ = P(-\sigma c < X - \mu < \sigma c) \\ = P\left(-c < \frac{X - \mu}{\sigma} < c\right) \\ = P\left(-c < Z < c\right) \\ = P(Z < c) - P(Z < -c) \\ = (1 - P(Z > c)) - P(Z < -c) \\ = (1 - P(Z < -c)) - P(Z < -c) \\ \text{ (since } \phi(z) \text{ is symmetric about 0)} \\ = 1 - 2P(Z < -c) \\ 0.95 = 1 - 2\Phi(-c) \\ 0.05 = 2\Phi(-c) \\ c = -\Phi^{-1}(0.025) \\ = -(-1.96) \quad \text{from the standard normal table} \\ = 1.96 \end{array}$$

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▶ A random variable T has a t_{ν} distribution — that is, a t distribution with ν degrees of freedom — if its pdf is:

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{(\nu\pi)^{\frac{1}{2}}} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \quad -\infty < t < \infty$$

► Gamma function:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt, \, x > 0$$

- We use the t table (Table B.4 in Vardeman and Jobe) to calculate quantiles and probabilities.
- ► Like the standard normal distribution, the *t* distribution is mound-shaped and symmetric about 0.
- ▶ The t distribution has fatter tails than the normal, but approaches the shape of the normal as $\nu \to \infty$

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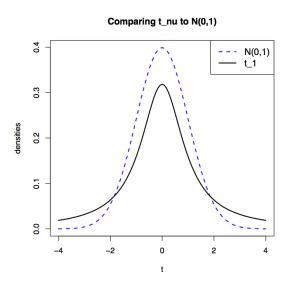
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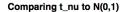
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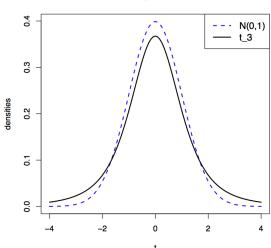
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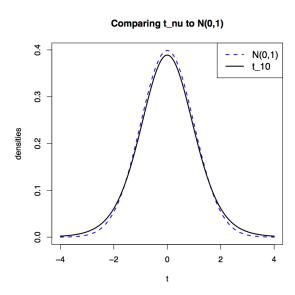
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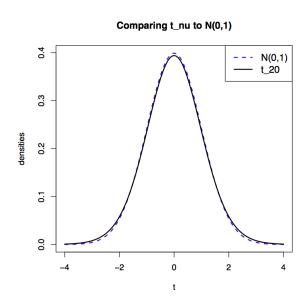
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Find probabilities and quantiles of t_{ν} with the t table.

► Say $T \sim t_5$. $P(T \le 1.476) = 0.9$

Table B.4
t Distribution Quantiles

| ν | Q(.9 | Q(.95) | Q(.975) | Q(.99) | Q(.995) | Q(.999) | Q(.9995) |
|---|------|---------|---------|--------|---------|---------|----------|
| 1 | 3.07 | 8 6.314 | 12.706 | 31.821 | 63.657 | 318.317 | 636.607 |
| 2 | 1.88 | | 4.303 | 6.965 | 9.925 | 22.327 | 31.598 |
| 3 | 1.63 | | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | 1.53 | 3 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | 1.47 | 6 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |

You can find quantiles labeled in the top row.

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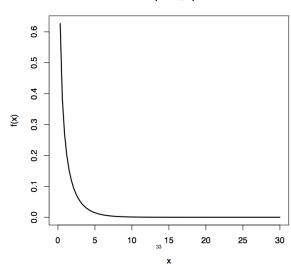
A random variable $S \sim \chi^2_{\nu}$ (is chi-square with ν degrees of freedom) if its pdf is:

$$f(x) = \begin{cases} 0 & : x \le 0\\ \frac{1}{\Gamma(\nu/2)2^{\nu/2}} \cdot x^{\nu/2 - 1} \cdot e^{-x/2} : 0 < x < \infty \end{cases}$$

- ▶ Use Table B.5 in Vardeman and Jobe to find chi-square probabilities and quantiles.
- ightharpoonup A chi-square random variable is the sum of squares of ν independent standard normal random variables.
- ► A chi-sugare distribution is not symmetric.

A look at the chi-square density





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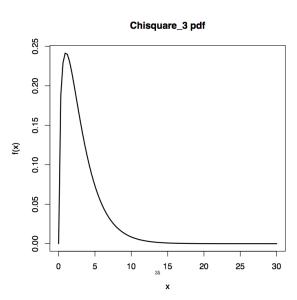
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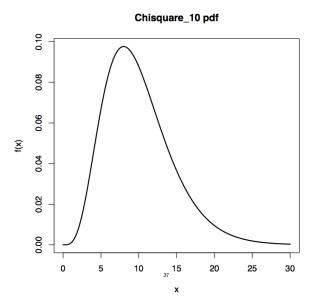
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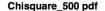
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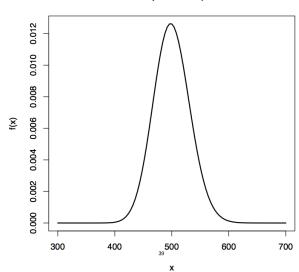
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Use Table B.5 to find chi-square probabilities and quantiles.

• Q(0.9) of χ_6^2 is 10.645.

Table B.5
Chi-Square Distribution Quantiles

| ν | Q(.005) | Q(.01) | Q(.025) | Q(.05) | Q(.1) | Q(.9) | Q(.95) | Q(.975) | Q(.99) | Q(.995) |
|---|---------|--------|---------|--------|-------|--------|--------|---------|--------|---------|
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12,592 | 14.449 | 16.812 | 18,548 |

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 \triangleright X has an F_{ν_1,ν_2} distribution if it has pdf:

: x < 0 $f(x) = \begin{cases} \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \cdot \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2} \frac{x^{\nu_1/2 - 1}}{[1 + (\nu_1/\nu_2)x]^{(\nu_1 + \nu_2)/2}} : 0 < x < \infty \end{cases}$

- An F_{ν_1,ν_2} random variable is a $\chi^2_{\nu_1}$ RV divided by an independent $\chi^2_{\nu_2}$ RV. That's why ν_1 is the **numerator** degrees of freedom and ν_2 is the denominator degrees of freedom.
- Use Tables B.6A-B.6E to find probabilities and quantiles.



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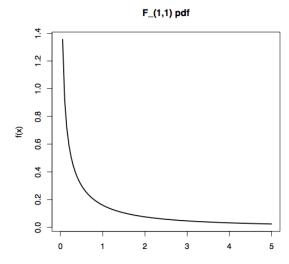
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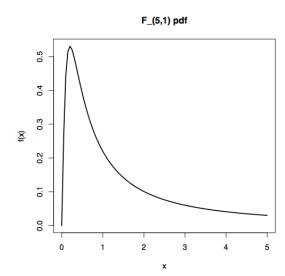
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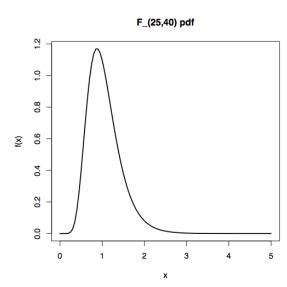
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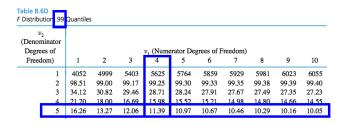
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Find probabilities and quantiles of the F distribution with Tables B.6A-B.6E

▶ The 0.99 quantile of the $F_{4.5}$ distribution is 11.39.



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- 1. Q(p) for N(0,1) is often denoted z_p .
- 2. Q(p) for t_{ν} is often denoted $t_{\nu,p}$.
- 3. Q(p) for χ^2_{ν} is often denoted $\chi^2_{\nu,p}$.
- 4. Q(p) for F_{ν_1,ν_2} is often denoted $F_{\nu_1,\nu_2,p}$.