

Please submit your homework with codes (hard copy) in class and upload the corresponding codes to the Blackboard. Problems marked with * will be graded in detail and they are worth 50% of the total score. Remaining problems, worth the remaining 50% of the total score, will be given full mark if reasonable amount of work is shown.

1. * Write a C program that prints a table of binomial probability mass function

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)},$$

similar to this one:

x\p	0.0100	0.0644	0.1189	0.1733	0.2278	0.2822	0.3367	0.3911	0.4456	0.5000
0	0.9510	0.7167	0.5311	0.3861	0.2746	0.1905	0.1284	0.0837	0.0524	0.0312
1	0.0480	0.2469	0.3583	0.4047	0.4050	0.3746	0.3259	0.2688	0.2105	0.1562
2	0.0010	0.0340	0.0967	0.1697	0.2389	0.2945	0.3308	0.3453	0.3384	0.3125
3	0.0000	0.0023	0.0130	0.0356	0.0705	0.1158	0.1679	0.2218	0.2719	0.3125
4	0.0000	0.0001	0.0009	0.0037	0.0104	0.0228	0.0426	0.0712	0.1093	0.1562
5	0.0000	0.0000	0.0000	0.0002	0.0006	0.0018	0.0043	0.0092	0.0176	0.0312

You are only allowed to use the C standard IO (stdio.h) and math (math.h) libraries. Use the following symbolic constants to define the behavior of the table:

```
#define P0 0.01 /* lower limit of the probability (p) */
#define P1 0.5  /* upper limit of the probability (p) */
#define PLEN 10 /* number of columns */
#define N 5     /* number of experiments (n) */
```

2. * Use the inverse transform method to sample from the density

$$f(x) \propto 1/x, \quad 1 < x < 10.$$

- (a) Derive the algorithm.
- (b) Write a C program that draws and prints one observation from this density.
- (c) Write an R program to draw a sample of 5000 observations. Plot the estimated density of the sample.

3. Consider the following probability density function:

$$f(x) \propto q(x) = \frac{e^{-x}}{1+x^2}, \quad x > 0.$$

Use rejection sampling to sample from $f(x)$ with the following density functions:

$$g_1(x) = e^{-x}, \quad g_2(x) = \frac{2}{\pi(1+x^2)}, \quad x > 0.$$

- (a) For each density function ($f(x)$, $g_1(x)$ and $g_2(x)$), write an R program to draw a sample of 5000 random observations and plot the estimated density function for $0 < x < 5$.
 - (b) Comments on the speeds of sampling and the results using $g_1(x)$ and $g_2(x)$.
4. Design a rejection algorithm to sample from the following density on the upper right quarter of the unit disc:

$$f(x, y) \propto x^\alpha y, \quad x > 0, \quad y > 0, \quad x^2 + y^2 \leq 1.$$

You don't need to run simulations for this problem. Just describe your algorithm in detail. Note that $\alpha > -1$. (Finding an algorithm that targets for $\alpha \geq 0$ is simple. However, it is tricky to get an algorithm that works for any $\alpha > -1$.)