STAT 305 D Homework 9

Due Apr 11, 2013 at 12:40 PM in class

Show all 6 steps in your hypothesis tests.

1.

Let the test statistic Z have a standard normal distribution when H_0 is true. Give the significance level for each of the following situations:

a. H_a: $\mu > \mu_0$, rejection region $z \geq 1.88$
b. H_a: $\mu < \mu_0$, rejection region $z \leq -2.75$
c. H_a: $\mu \neq \mu_0$, rejection region $z \geq 2.88$ *or* $z \leq -2.88$

a. $\alpha = P(z \geq 1.88$ when z has a standard normal distribution$) = 1 - \Phi(1.88) = .0301$

b. $\alpha = P(z \leq -2.75$ when z ~ N(0, 1)$) = \Phi(-2.75) = .003$

c. $\alpha = \Phi(-2.88) + (1 - \Phi(2.88)) = .004$

2. Use the method of critical values (not p-values) in the following problem. You may assume that the test statistic has a N(0,1) distribution.

The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in

$\bar{x} = 94.32$. Assume that the distribution of melting point is normal with $\sigma = 1.20$.

Reject H_0 if either $z \geq 2.58$ or $z \leq -2.58$; $\dfrac{\sigma}{\sqrt{n}} = 0.3$, so

$z = \dfrac{94.32 - 95}{0.3} = -2.27$. Since –2.27 is not < -2.58, don't reject H_0.

3. Use the method of critical values (not p-values) in the following problem.

   Lightbulbs of a certain type are advertised as having an average lifetime of 750 hours. The price of these bulbs is very favorable, so a potential customer has decided to go ahead with a purchase arrangement unless it can be conclusively demonstrated that the true average lifetime is smaller than what is advertised. A random sample of 50 bulbs was selected, the lifetime of each bulb determined, and the appropriate hypotheses were tested using MINITAB, resulting in the accompanying output.

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Variable  N    Mean StDev  SEMean        ZP-Value
lifetime 50  738.44 38.20    5.40 -2.14     0.016
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   What conclusion would be appropriate for a significance level of .05? A significance level of .01? What significance level and conclusion would you recommend?

   With H_0: $\mu = 750$, and H_a: $\mu < 750$ and a significance level of .05, we reject H_0 if $z < -1.645$; $z = -2.14 < -1.645$, so we reject the null hypothesis and do not continue with the purchase. At a significance level of .01, we reject H_0 if $z < -2.33$; $z = -2.14 > -2.33$, so we don't reject the null hypothesis and thus continue with the purchase.

4. Use the method of p-values in the following problem.

A random sample of soil specimens was obtained, and the amount of organic matter (%) in the soil was determined for each specimen, resulting in the accompanying data (from "Engineering Properties of Soil," *Soil Science,* 1998: 93–102).

1.10	5.09	0.97	1.59	4.60	0.32	0.55	1.45
0.14	4.47	1.20	3.50	5.02	4.67	5.22	2.69
3.98	3.17	3.03	2.21	0.69	4.47	3.31	1.17
0.76	1.17	1.57	2.62	1.66	2.05		

The values of the sample mean, sample standard deviation, and (estimated) standard error of the mean are 2.481, 1.616, and .295, respectively. Does this data suggest that the true average percentage of organic matter in such soil is something other than 3%? Carry out a test of the appropriate hypotheses at significance level .10 by first determining the *P*-value. Would your conclusion be different if $\alpha = .05$ had been used? [*Note:* A normal probability plot of the data shows an acceptable pattern in light of the reasonably large sample size.]

μ = the true average percentage of organic matter in this type of soil, and the hypotheses are H_0: $\mu = 3$ v. H_a: $\mu \neq 3$. With n = 30, and assuming normality, we use the t test:

$t = \dfrac{\bar{x} - 3}{s / \sqrt{n}} = \dfrac{2.481 - 3}{.295} = \dfrac{-.519}{.295} = -1.759$. The p-value = 2[P(t > 1.759)] = 2(.041) = .082. At significance level .10, since $.082 \le .10$, we would reject H_0 and conclude that the true average percentage of organic matter in this type of soil is something other than 3. At significance level .05, we would not have rejected H_0.

Do a hypothesis test at significance level $\alpha = 0.01$ to test the hypothesis that the true mean melting point is different from 95.

5. Vardeman and Jobe chapter 6 section 3 problem 3 (page 385). Solve using the method of p-values at significance level $\alpha = 0.05$.

The machine screw measurement study of DuToit, Hansen, and Osborne referred to in Exercise 4 of Section 6.1 involved measurement of diameters of each of 50 screws with both digital and vernier-scale calipers. For the student referred to in that exercise, the differences in measured diameters (digital minus vernier, with units of mm) had the following frequency distribution:

Difference	−.03	−.02	−.01	.00	.01	.02
Frequency	1	3	11	19	10	6

(a) Make a 90% two-sided confidence interval for the mean difference in digital and vernier readings for this student.

(b) Assess the strength of the evidence provided by these differences to the effect that there is a systematic difference in the readings produced by the two calipers (at least when employed by this student).

(c) Briefly discuss why your answers to parts (a) and (b) of this exercise are compatible. (Discuss how the outcome of part (b) could easily have been anticipated from the outcome of part (a).)

(a) Use equation (6-22). For 90% confidence, the appropriate z is 1.645. The interval is

$$.0004 \pm 1.645 \left(\frac{.01159873}{\sqrt{50}} \right) = .0004 \pm .002698308$$

(b) 1. H_0: $\mu_d = 0$.
2. H_a: $\mu_d \neq 0$.
3. The test statistic is given by equation (6-24), with # = 0. The reference distribution is the standard normal distribution. Observed values of Z far above or below zero will be considered as evidence against H_0.
4. The sample gives

$$z = .24.$$

5. The observed level of significance is

$$\begin{aligned} & 2P(\text{a standard normal random variable} > .24) \\ = \; & 2P(\text{a standard normal random variable} < -.24) \end{aligned}$$

which is equal to $2(.4052) = .8104$, according to Table D-3. There is no evidence of a systematic difference in the readings produced by the two calipers.

(c) The confidence interval in part (a) contains zero; in fact, zero is near the middle of the interval. This means that zero is a very plausible value for the mean difference—there is no evidence that the mean is not equal to zero. This is reflected by the large p-value in part (b).

6. Musculoskeletal neck-and-shoulder disorders are all too common among office staff who perform repetitive tasks using visual display units. The article Upper-Arm Elevation During Office Work (Ergonomics, 1996: 1221-1230) reported on a study to determine whether more varied work conditions would have any impact on arm movement. The accompanying data was obtained from a sample of n = 16 subjects. Each observation is the amount of time, expressed as a proportion of total time observed, during which arm elevation was below 30°. The two measurements from each subject were obtained 18 months apart. During this period, work conditions were changed, and subjects were allowed to engage in a wider variety of work tasks. Does the data suggest that true average time during which elevation is below 30 differs after the change from what it was before the change? Do a hypothesis test

to answer the question at significance level $\alpha = 0.01$.

Subject	1	2	3	4	5	6	7	8
Before	81	87	86	82	90	86	96	73
After	78	91	78	78	84	67	92	70
Subject	9	10	11	12	13	14	15	16
Before	74	75	72	80	66	72	56	82
After	58	62	70	58	66	60	65	73

First, compute the differences:

Subject	1	2	3	4	5	6	7	8
Before	81	87	86	82	90	86	96	73
After	78	91	78	78	84	67	92	70
Difference	3	−4	8	4	6	19	4	3
Subject	9	10	11	12	13	14	15	16
Before	74	75	72	80	66	72	56	82
After	58	62	70	58	66	60	65	73
Difference	16	13	2	22	0	12	−9	9

Let μ_D denote the true average difference between elevation time before the change in work conditions and time after the change.

H_0: $\mu_D = 0$ (there is no difference between true average time before the change and true average time after the change)

H_a: $\mu_D \neq 0$

$$t = \frac{\bar{d} - 0}{s_D/\sqrt{n}} = \frac{\bar{d}}{s_D/\sqrt{n}}$$

$n = 16$, $\Sigma d_i = 108$, $\Sigma d_i^2 = 1746$, from which $\bar{d} = 6.75$, $s_D = 8.234$, and

$$t = \frac{6.75}{8.234/\sqrt{16}} = 3.28 \approx 3.3$$

Since $.004 < .01$, the null hypothesis can be rejected at either significance level .05 or .01. It does appear that the true average difference between times is something other than zero; that is, true average time after the change is different from that before the change.

7. Weekly feedback. You get full credit as long as you write something.

   a. Is there any aspect of the subject matter that you currently struggle with? If so, what specifically do you find difficult or confusing? The more detailed you are, the better I can help you.

You got full credit as long as you wrote something.

b. Do you have any questions or concerns about the material, class logistics, or anything else? If so, fire away. You got full credit as long as you wrote something.