STAT 305 D Exam 2

Show all your work.

1. (25 points) Suppose 80% of all students taking a beginning programming course fail to get their first program to run on first submission. Use a binomial distribution and assign probabilities to the possibilities that among a group of six such students,

   (a) (5 points) all fail on their first submissions.

   (b) (5 points) at least four fail on their first submissions

   (c) (5 points) less than four fail on their first submissions.

   Continuing to use this binomial model:

   (d) (5 points) What is the mean number who will fail?

   (e) (5 points) What are the variance and standard deviation of the number who will fail?

2. (24 points, each part worth 6 points)

   Suppose that X is a continuous random variable with probability density of the form

$$f(x) = \begin{cases} k\left(x^2(1-x)\right) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Evaluate k and sketch a graph of $f(x)$.

   (b) Evaluate $P[X \leq .25]$, $P[X \leq .75]$, $P[.25 < X \leq .75]$, and $P[|X - .5| > .1]$.

   (c) Compute EX and $\sqrt{\text{Var}\, X}$.

   (d) Compute and graph $F(x)$, the cumulative distribution function for X. Read from your graph the .6 quantile of the distribution of X.

3. (26 points) A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf X and Y appears in the accompanying tabulation.

p(x,y)	y = 0	y = 1	y = 2
x = 0	0.10	0.04	0.02
x = 1	0.08	0.20	0.06
x = 2	0.06	0.14	0.30

(a) (6 points) What is $P(X = 1 \text{ and } Y = 1)$?

(b) (6 points) Compute $P(X \leq 1 \text{ and } Y \leq 1)$.

(c) (6 points) Compute the marginal pmf of X and of Y. Using $f_X(x)$, calculate $P(X \leq 1)$.

(d) (8 points) Are X and Y independent random variables? Explain.

4. (25 points) A type of nominal $\frac{3}{4}$ inch plywood is made of five layers. These layers can be thought of as having thicknesses roughly describable as independent random variables with means and standard deviations as follows:

Layer	Mean (in.)	Standard Deviation (in.)
1	.094	.001
2	.156	.002
3	.234	.002
4	.172	.002
5	.094	.001

Find the mean and standard deviation of total thickness associated with the combination of these individual values.