STAT 305 D Homework 8

Due Apr 4, 2013 at 12:40 PM in class

1. For each of the following random intervals, find the probability that the interval contains the number, 0.

   a. $(Z-2, Z+2)$, $Z \sim N(0,1)$

$$\begin{aligned}
P(0 \text{ in } (Z-2, Z+2)) &= P(Z-2 < 0 < Z+2) \\
&= P(-2 < -Z < 2) \\
&= P(-2 < Z < 2) \\
&= P(Z < 2) - P(Z < -2) \\
&= P(Z \leq 2) - P(Z \leq -2) \\
&= \Phi(2) - \Phi(-2) \\
&= 0.977 - 0.0228 \\
&= 0.9545
\end{aligned}$$

   b. $(X-4, X+4)$, $X \sim N(0,4)$

$$\begin{aligned}
P(0 \text{ in } (X-4, X+4)) &= P(X-4 < 0 < X+4)) \\
&= P(-4 < -X < 4) \\
&= P(-4 < X < 4) \\
&= P\left(\frac{-4-0}{\sqrt{4}} < \frac{X-0}{\sqrt{4}} < \frac{4-0}{\sqrt{4}}\right) \\
&= P(-2 < Z < 2) \\
&= P(Z < 2) - P(Z < -2) \\
&= P(Z \leq 2) - P(Z \leq -2) \\
&= \Phi(2) - \Phi(-2) \\
&= 0.977 - 0.0228 \\
&= 0.9545
\end{aligned}$$

   c. $(X-6, X+6)$, $X \sim N(0,9)$

$$
\begin{aligned}
P(0 \text{ in } (X-6, X+6)) &= P(X-6 < 0 < X+6) \\
&= P(-6 < -X < 6) \\
&= P\left(\frac{-6-0}{\sqrt{9}} < \frac{X-0}{\sqrt{9}} < \frac{6-0}{\sqrt{9}}\right) \\
&= P(-2 < Z < 2) \\
&= P(Z < 2) - P(Z < -2) \\
&= P(Z \leq 2) - P(Z \leq -2) \\
&= \Phi(2) - \Phi(-2) \\
&= 0.977 - 0.0228 \\
&= 0.9545
\end{aligned}
$$

d. $(X - 2\sigma, X + 2\sigma)$, $X \sim N(0, \sigma^2)$ for some $\sigma > 0$.

$$
\begin{aligned}
P(0 \text{ in } (X-2\sigma, X+2\sigma)) &= P(X-2\sigma < 0 < X+2\sigma) \\
&= P(-2\sigma < -X < 2\sigma) \\
&= P(-2\sigma < X < 2\sigma) \\
&= P\left(\frac{-2\sigma-0}{\sigma} < \frac{X-0}{\sigma} < \frac{2\sigma-0}{\sigma}\right) \\
&= P(-2 < Z < 2) \\
&= P(Z < 2) - P(Z < -2) \\
&= P(Z \leq 2) - P(Z \leq -2) \\
&= \Phi(2) - \Phi(-2) \\
&= 0.977 - 0.0228 \\
&= 0.9545
\end{aligned}
$$

2. Vardeman and Jobe chapter 6 section 1 exercise 1 (page 344).

Interpret the statement, "The interval from 6.3 to 7.9 is a 95% confidence interval for the mean μ."

[6.3, 7.9] ppm is a set of plausible values for the mean. The method used to construct this interval correctly contains means in 95% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 95% of the time, we might say that we have 95% confidence that it was correct this time.

3. Vardeman and Jobe chapter 6 section 1 exercise 2 (page 344).

In Chapter Exercise 2 of Chapter 3, there is a data set consisting of the aluminum contents of 26 bihourly samples of recycled PET plastic from a recycling facility. Those 26 measurements have $\bar{y} = 142.7$ ppm and $s \approx 98.2$ ppm. Use these facts to respond to the following. (Assume that $n = 26$ is large enough to permit the use of large-sample formulas in this case.)

(a) Make a 90% two-sided confidence interval for the mean aluminum content of such specimens over the 52-hour study period.

(b) Make a 95% two-sided confidence interval for the mean aluminum content of such specimens over the 52-hour study period. How does this compare to your answer to part (a)?

(c) Make a 90% upper confidence bound for the mean aluminum content of such samples over the 52-hour study period. (Find # such that $(-\infty, \#)$ is a 90% confidence interval.) How does this value compare to the upper endpoint of your interval from part (a)?

(d) Make a 95% upper confidence bound for the mean aluminum content of such samples over the 52-hour study period. How does this value compare to your answer to part (c)?

(e) Interpret your interval from (a) for someone with little statistical background. (Speak in the context of the recycling study and use Definition 2 as your guide.)

(a) You can use equation (6-9), since this is a large sample. The appropriate z for 90% confidence is 1.645. The interval is

$$\begin{aligned} 142.7 \pm 1.645\left(\frac{98.2}{\sqrt{26}}\right) &= 142.7 \pm 31.68 \\ &= [111.02, 174.38]. \end{aligned}$$

(b) Now $z = 1.96$, and the interval is

$$\begin{aligned} 142.7 \pm 1.96\left(\frac{98.2}{\sqrt{26}}\right) &= 142.7 \pm 37.75 \\ &= [104.95, 180.45]. \end{aligned}$$

This interval is wider than the one from (a). In order to have more confidence of containing the mean, the interval must be wider.

(c) To make a 90% one-sided confidence interval, construct a 80% two-sided confidence interval, and use the upper endpoint. The appropriate z for a 80% two-sided confidence interval is 1.28, so the 90% one-sided confidence interval is

$$\begin{aligned} 142.7 + 1.28\left(\frac{98.2}{\sqrt{26}}\right) &= 142.7 + 24.65 \\ &= 167.35. \end{aligned}$$

This value is smaller than the upper endpoint from part (a). Setting the lower endpoint equal to $-\infty$ requires you to move the upper endpoint in so that the confidence remains at 90%.

(d) To make a 95% one-sided confidence interval, construct a 90% two-sided confidence interval, and use the upper endpoint. This was done in part (a), so the 90% one-sided confidence interval is 174.38. This is larger than the answer to (c); in order to achieve higher confidence, you must make the interval "wider".

(e) [111.02, 174.38] ppm is a set of plausible values for the mean aluminum content of samples of recycled PET plastic from the recycling pilot plant at Rutgers University. The method used to construct this interval correctly contains means in 90% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 90% of the time, we might say that we have 90% confidence that it was correct this time.

4. Vardeman and Jobe chapter 6 section 1 exercise 3 (page 344).

Return to the context of Exercise 2. Suppose that in order to monitor for possible process changes, future samples of PET will be taken. If it is desirable to estimate the mean aluminum content with ± 20 ppm precision and 90% confidence, what future sample size do you recommend?

$n = [Z\ s/\ B]^2 = [\ (1.645)(98.2)/\ 20\ ]^2 = 65.24$ or about 66.

5. Vardeman and Jobe chapter 6 section 1 exercise 4 (page 344).

DuToit, Hansen, and Osborne measured the diameters of some no. 10 machine screws with two different calipers (digital and vernier scale). Part of their data are recorded here. Given in the small frequency table are the measurements obtained on 50 screws by one of the students using the digital calipers.

Diameter (mm)	Frequency
4.52	1
4.66	4
4.67	7
4.68	7
4.69	14
4.70	9
4.71	4
4.72	4

(a) Compute the sample mean and standard deviation for these data.

(b) Use your sample values from (a) and make a 98% two-sided confidence interval for the mean diameter of such screws as measured by this student with these calipers.

(c) Repeat part (b) using 99% confidence. How does this interval compare with the one from (b)?

(d) Use your values from (a) and find a 98% lower confidence bound for the mean diameter. (Find a number # such that (#, ∞) is a 98% confidence interval.) How does this value compare to the lower endpoint of your interval from (b)?

(e) Repeat (d) using 99% confidence. How does the value computed here compare to your answer to (d)?

(f) Interpret your interval from (b) for someone with little statistical background. (Speak in the context of the diameter measurement study and use Definition 2 as your guide.)

(a) $\bar{x} = 4.6858$ and $s = .02900317$.

(b) Since this is a large sample, you can use equation (6-9), with $z = 2.33$ for 98% confidence. The two-sided confidence interval is

$$\begin{aligned} 4.6858 \pm 2.33\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 \pm .009556884 \\ &= [4.676, 4.695] \text{ mm.} \end{aligned}$$

(c) $z = 2.58$ for 98% confidence. The two-sided confidence interval is

$$\begin{aligned} 4.6858 \pm 2.58\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 \pm .0105823 \\ &= [4.675, 4.696] \text{ mm.} \end{aligned}$$

This interval is wider than the one in (b). To increase the confidence that μ is in the interval, you need to make the interval wider.

(d) To make a 98% one-sided interval, construct a 96% two-sided interval and use the lower endpoint. For a 96% two-sided interval, the appropriate z is $Q_{SN}(.98) = 2.05$. The resulting 98% one-sided interval is

$$\begin{aligned} 4.6858 - 2.05\left(\frac{.02900317}{\sqrt{50}}\right) &= 4.6858 - .008408418 \\ &= 4.677 \text{ mm.} \end{aligned}$$

This is larger than the lower endpoint of the interval in (b). Since the upper endpoint here is set to ∞, the lower endpoint must be increased to keep the confidence level the same.

(e) To make a 99% one-sided interval, construct a 98% two-sided interval and use the lower endpoint. This was done in part (a), and the resulting lower bound is 4.676. This is smaller than the value in (d); to increase the confidence, the interval must be made "wider".

(f) $[4.676, 4.695]$ ppm is a set of plausible values for the mean diameter of this type of screw as measured by this student with these calipers. The method used to construct this interval correctly contains means in 98% of repeated applications. This particular interval either contains the mean or it doesn't (there is no probability involved). However, because the *method* is correct 98% of the time, we might say that we have 98% confidence that it was correct this time.

6. Weekly feedback. You get full credit as long as you write something.

   a. Is there any aspect of the subject matter that you currently struggle with? If so, what specifically do you find difficult or confusing? The more detailed you are, the better I can help you.

      You got full credit as long as you wrote something.

   b. Do you have any questions or concerns about the material, class logistics, or anything else? If so, fire away. You got full credit as long as you wrote something.