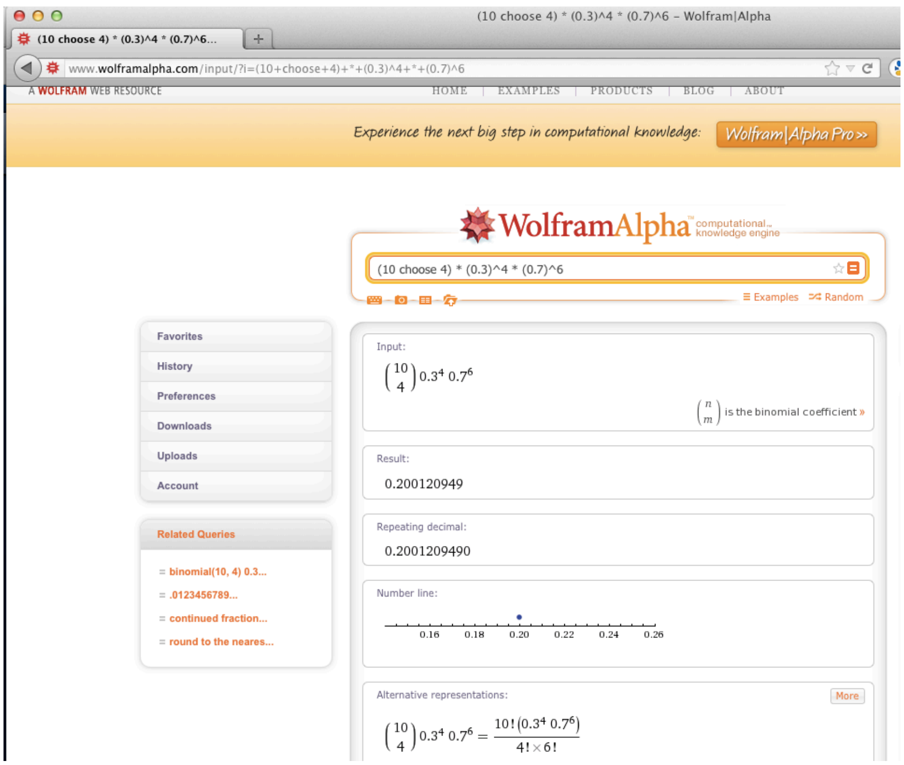
STAT 305 D Homework 5

Due February 28, 2012 at 12:40 PM in class

Note: Wolfram Alpha (www.wolframalpha.com) is really useful for calculating things like binomial coefficients. It's an online calculator with flexible syntax. Here's an example of how I can calculate $\binom{10}{4}(0.3)^4(0.7)^6$, which is the pmf of a Binomial$(10, 0.3)$ random variable:



1. Suppose that an eddy current nondestructive evaluation technique for identifying cracks in critical metal parts has a probability of around $p = 0.20$ of detecting a single crack of length 0.003 inches in a certain material. Suppose further that $n = 8$ specimens of this material, each containing one (and only one) single crack of length 0.003 inches, are inspected using this technique. Let W be the number of specimens out of the total 8 for which the crack was actually detected. Using the appropriate pmf for W, calculate:

   a. $P(W = 3)$

   b. $P(W \leq 2)$

c. E(W)

d. Var (W)

e. The standard deviation, SD(W).

Use the binomial distribution, equation (5-3), with $n = 8$ and $p = .20$.

(a) $P(W = 3) = .1468$.

(b) $P(W \leq 2) = P(W = 0) + P(W = 1) + P(W = 2) = .7969$.

(c) Using equation (5-4), $EW = np = 1.6$.

(d) Using equation (5-5), $\mathrm{Var}W = np(1-p) = 1.28$.

(e) $\sqrt{1.28} = 1.1314$.

2. Take the situation described in the previous exercise. Suppose that some indefinite number of specimens is inspected, one specimen after the other, each containing a single crack of length 0.003 inches or no crack at all. Let Y be the number of specimens inspected in order to obtain the first crack detection. (In other words, Y is the "time index" of the first detection.) Use the appropriate pmf for Y, calculate:

a. $P(Y = 5)$

b. $P(Y \leq 4)$

c. $E(Y)$

d. Var(Y)

e. The standard deviation, SD(Y)

Use the geometric distribution, equation (5-6), with $p = .20$.

(a) $P(Y = 5) = .08192$.

(b) $P(Y \leq 4) = P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = .5904$.

(c) Using equation (5-8), $EY = \frac{1}{p} = 5$.

(d) Using equation (5-9), $\mathrm{Var}Y = \frac{1-p}{p^2} = 20$.

(e) $\sqrt{20} = 4.4721$.

3. A process for making plate glass produces an average of four seeds (small bubbles) per 100 square feet. Use Poisson distributions and assess probabilities that:

   (a) a particular piece of glass 5 ft $\times$ 10 ft will contain more than two seeds

   (b) a particular piece of glass 5 ft $\times$ 5 ft will contain no seeds

8. (a) Use the Poisson distribution, equation (5-10), with $\lambda = 2$.

$$P(X > 2) = 1 - P(X \leq 2) = 1 - .6767 = .3233.$$

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(b) Use the Poisson distribution, equation (5-10), with $\lambda = 1$.

$$P(X = 0) = .3679.$$

4. (a) Transmission line interruptions in a telecommunications network occur at an average rate of one per day. Use a Poisson distribution as a model for:

   $X =$ the number of interruptions in the next five-day work week

   Calculate $P(X = 0)$.

   (b) Now, consider the random variable:

   $Y =$ the number of weeks in the next $n = 4$ weeks in which there are no interruptions.

   What is a reasonable probability distribution for Y? Calculate $P(Y = 2)$.

(a) Use the Poisson distribution, equation (5-10), with $\lambda = 5$.

$$P(X = 0) = .0067.$$

(b) $Y \sim$ Binomial with $n = 4$ and $p = .0067$. Use equation (5-3) with $n = 4$ and $p = .0067$.

$$P(Y = 2) = .00027.$$

5.

The random number generator supplied on a calculator is not terribly well chosen, in that values it generates are not adequately described by a distribution uniform on the interval $(0, 1)$. Suppose instead that a probability density

$$f(x) = \begin{cases} k(5 - x) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a more appropriate model for $X =$ the next value produced by this random number generator.

(a) Find the value of k.

(b) Sketch the probability density involved here.

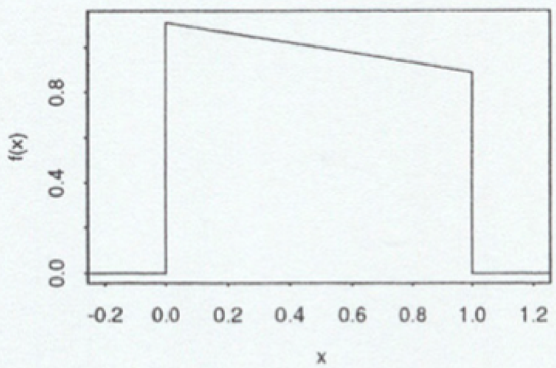
(c) Evaluate $P[.25 < X < .75]$.

(d) Compute and graph the cumulative probability function for X, $F(x)$.

(a) Use equation (5-13) and solve for $k = \frac{2}{9}$.

(b)

$$f(x) = \begin{cases} \frac{2}{9}(5-x) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

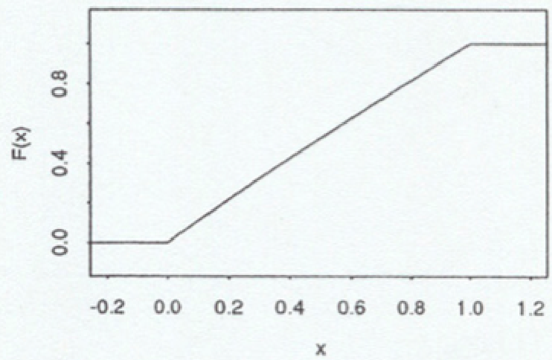


(c)

$$\begin{aligned} P(.25 < X < .75) &= \int_{.25}^{.75} f(x)\,dx \\ &= .5. \end{aligned}$$

(d) Using equation (5-16),

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx = \begin{cases} 0 & \text{for } x \le 0 \\ \frac{2}{9}(5x - \frac{1}{2}x^2) & \text{for } 0 < x < 1 \\ 1 & \text{for } x \ge 1. \end{cases}$$



6. X be the time between two successive arrivals at the drive-up window of a local bank. Suppose $X \sim \text{Exp}(1)$. Calculate:

   a. $P(X \le 4)$

   b. $P(2 \le X \le 5)$

   a.

$$P(X \le 4) = 1 - e^{-(1)(4)} = 1 - e^{-4} = .982$$

   b.

$$P(2 \le X \le 5) = 1 - e^{-(1)(5)} - \left[1 - e^{-(1)(2)}\right] = e^{-2} - e^{-5} = .129$$

7. Weekly feedback. You get full credit as long as you write something.

   a. Is there any aspect of the subject matter that you currently struggle with? If so, what specifically do you find difficult or confusing? The more detailed you are, the better I can help you.
   You got full credit as long as you wrote something.

b. Do you have any questions or concerns about the material, class logistics, or anything else? If so, fire away. You got full credit as long as you wrote something.