1. Let $X$, $Y$, and $Z$ be random variables with expected values and standard deviations given below:

| | Expected Value | Standard Deviation |
|---|---|---|
| $X$ | 1.5 | 3.2 |
| $Y$ | 0 | 8.1 |
| $Z$ | 6 | 2.7 |

Find:

- $E(8 + 2X + Y + Z)$
- $SD(8 + 2X + Y + Z)$

- The expected value of the linear combination is:

$$\begin{aligned} E(8 + 2X + Y + Z) &= 8 + 2E(X) + E(Y) + E(Z) \\ &= 8 + 2 \cdot 1.5 + 0 + 6 \\ &= 17 \end{aligned}$$

- Before computing the standard deviation, note:

$$Var(8 + 2X + Y + Z) = 2^2 Var(X) + Var(Y) + Var(Z)$$

Remember that the standard deviation is the square root of the variance:

$$\begin{aligned} [SD(8 + 2X + Y + Z)]^2 &= 2^2[SD(X)]^2 + [SD(Y)]^2 + [SD(Z)]^2 \\ SD(8 + 2X + Y + Z) &= \sqrt{2^2[SD(X)]^2 + [SD(Y)]^2 + [SD(Z)]^2} \\ &= \sqrt{2^2[3.2]^2 + [8.1]^2 + [2.7]^2} \\ &\approx 10.671 \end{aligned}$$

2. Let $X$ be the the number of crankshafts that fail in a given test of a certain type of vehicle ($X = 0, 1, 2$). Let $Y = 1$ if the clutch fails during that same test and $Y = 0$ otherwise. Consider the joint distribution of $X$ and $Y$:

| $Y \backslash X$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0.35 | 0.1 | 0.05 |
| 1 | 0.2 | 0.25 | 0.05 |

Find or answer the following:

- $P(X = 1 \text{ and } Y = 1)$
- $P(X = 0)$
- $P(X > 0 \text{ and } Y = 1)$

- The marginal pmfs of $X$ and $Y$
- Are $X$ and $Y$ independent? Why or why not?

- $P(X = 1 \text{ and } Y = 1) = 0.25$ from the table.
- $P(X = 0)$:

$$\begin{aligned} P(X = 0) &= P(X = 0, Y = 0) + P(X = 0, Y = 1) \\ &= 0.35 + 0.2 \\ &= 0.55 \end{aligned}$$

,

- $P(X > 0 \text{ and } Y = 1)$

$$\begin{aligned} P(X > 0, Y = 1) &= P(X = 1, Y = 1) + P(X = 2, Y = 1) \\ &= 0.25 + 0.05 \\ &= 0.3 \end{aligned}$$

- For the marginal pmf of $X$, take the row sums of the table:

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f_X(x)$ | 0.55 | 0.35 | 0.1 |

For the marginal pmf of $Y$, take the column sums of the table:

| $y$ | 0 | 1 |
|---|---|---|
| $f_Y(y)$ | 0.5 | 0.5 |

- $X$ and $Y$ are independent random variables if and only if $P(X = x, Y = y) = P(X = x) \cdot P(Y = y)$ for all values $x$ and $y$. That is, the joint pmf must always be the product of the two marginals. However, in this case, $P(X = 1, Y = 1) = 0.25$, while $P(X = 1) \cdot P(Y = 1) = f_X(1) \cdot f_Y(1) = 0.35 \cdot 0.5 = 0.175$. Therefore, $X$ and $Y$ are not independent.