STAT 305 D Exam 2

Show all your work.

1. (25 points) Suppose 90% of all students taking a beginning programming course fail to get their first program to run on first submission. Use a binomial distribution and assign probabilities to the possibilities that among a group of six such students,

   (a) (5 points) all fail on their first submissions.

   (b) (5 points) at least four fail on their first submissions

   (c) (5 points) less than four fail on their first submissions.

   Continuing to use this binomial model:

   (d) (5 points) What is the mean number who will fail?

   (e) (5 points) What are the variance and standard deviation of the number who will fail?

   Use equation (5-3) with $n = 6$ and $p = .9$.

   (a) $P(X = 6) = .531$.

   (b) $P(X \geq 4) = .984$.

   (c) $P(X < 4) = 1 - P(X \geq 4) = .016$.

   (d) $EX = np = 5.4$.

   (e) $\text{Var}X = np(1-p) = .54$; std. dev. of $X = \sqrt{.54} = .735$.

2. (20 points) Find $E(X)$ and $\text{Var}(X)$ for a continuous distribution with probability density

$$f(x) = \begin{cases} 0.3 & 0 < x \leq 1 \\ 0.7 & 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Using equation (5-18),

$$EX = \int_0^1 .3x\,dx + \int_1^2 .7x\,dx = 1.2.$$

using equation (5-19),

$$\mathrm{Var}X = \int_0^1 .3x^2\,dx + \int_1^2 .7x^2\,dx - (1.2)^2 = .2933.$$

3. Suppose that X is a normal random variable with mean $\mu = 10.2$ and standard deviation $\sigma = 0.7$. Evaluate the following probabilities involving X:

   (a) (3 points) $P(X \leq 10.1)$

   (b) (3 points) $P(X > 10.5)$

   (c) (3 points) $P(9.0 < X < 10.3)$

   (d) (4 points) $P(|X - 10.2| \leq 0.25)$

   (e) (4 points) $P(|X - 10.2| > 1.51)$

   Find numbers # such that the following statements about X are true:

   (f) (4 points) $P(|X - 10.2| < \#) = 0.80$

   (g) (4 points) $P(X < \#) = 0.80$

   (h) (5 points) $P(|X - 10.2| > \#) = 0.04$

Probabilities involving X are just areas under the normal curve with $\mu = 10.2$ and $\sigma = .7$.

Each of these areas has an equal corresponding area under the standard normal curve. Define $Z = \frac{X-10.2}{.7}$. Then Z is a standard normal random variable. Re-express each of the problems below in terms of Z.

(a) $P(X \leq 10.1) = P(Z < -.14) = .4443$.

(b) $P(X > 10.5) = 1 - P(X \leq 10.5) = 1 - P(Z \leq .43) = 1 - .6664 = .3336$.

(c) $P(9.0 < X < 10.3) = P(-1.71 < Z < .14) = P(Z < .14) - P(Z \leq -1.71) =$ $.5557 - .0436 = .5121$.

(d) $P(|X - 10.2| \leq .25) = P(9.95 \leq X \leq 10.45) = P(-.36 \leq Z \leq .36)$ $= P(Z \leq .36) - P(Z < -.36) = .6406 - .3594 = .2812$.

(e) $P(|X - 10.2| > 1.51) = 1 - P(|X - 10.2| \leq 1.51) = 1 - P(8.69 \leq X \leq 11.71) =$ $1 - P(-2.16 \leq Z \leq 2.16) = 1 - (P(Z \leq 2.16) - P(Z < -2.16)) = 1 - (.9846 - .0154) = .0308$.

(f) $P(|X - 10.2| < \#) = .80$ is equivalent to $P(X - 10.2 \leq \#) = .90$ (by symmetry). This is equivalent to $P(Z \leq \frac{\#}{.7}) = .90$. Looking up .90 in the body of the table,

$$\frac{\#}{.7} \approx 1.28$$

so $\# \approx .896$.

(g) $P(X < \#) = .80$ is equivalent to $P(Z < \frac{\#-10.2}{.7}) = .80$. Looking up .80 in the body of the table,

$$\frac{\# - 10.2}{.7} \approx .84$$

so $\# \approx 10.788$.

(h) $P(|X - 10.2| > \#) = .04$ is equivalent to $P(|X - 10.2| \leq \#) = .96$, which is equivalent to $P(X - 10.2 \leq \#) = .98$ (by symmetry). This is equivalent to $P(Z \leq \frac{\#}{.7}) = .98$. Looking up .98 in the body of the table,

$$\frac{\#}{.7} \approx 2.05$$

so $\# \approx 1.435$.

4. (25 points) A 10 ft cable is made of 50 strands. Suppose that, individually, 10 ft strands have breaking strengths with mean 45 lb and standard deviation 4 lb. Suppose further that the breaking strength of a cable is roughly the sum of the strengths of the strands that make it up.

   (a) (13 points) Find a plausible mean and standard deviation for the breaking strength of such 10 ft cables.

(b) (12 points) Evaluate the probability that a 10 ft cable of this type will support a load of 2230 lb. (*Hint*: If $\overline{X}$ is the mean breaking strength of the strands, $\sum$(Strengths) $\geq$ 2230 is the same as $\overline{X} \geq (\frac{2230}{50})$. Now, use the central limit theorem.)

(a) Let $X_1, \ldots, X_{50}$ be the 50 individual strengths. The strength of the cable is then

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$U = \sum X_i$. Using equation (5-55), the mean of the sum is the sum of the means:

$$EU = EX_1 + EX_2 + \cdots + EX_{50} = 50(45) = 2250 \text{ lbs.}$$

Assuming that the individual strengths are independent, you can use equation (5-56) to say that the variance of the sum is the sum of the variances:

$$\text{Var}U = \text{Var}X_1 + \text{Var}X_2 + \cdots + \text{Var}X_{50} = 50(4)^2 = 800 \text{ lbs.}^2,$$

so the standard deviation of U is $\sqrt{800} = 28.28$ lbs.

(b) Since $n = 50$ is large, the central limit theorem says that $\bar{X} = \frac{\sum X_i}{50}$ is approximately normal. The mean of $\bar{X}$ is 45 and the standard deviation of $\bar{X}$ is $\frac{4}{\sqrt{50}}$ (see equations (5-55) and (5-56). Since $\bar{X} = \frac{U}{50}$,

$$\begin{aligned} P(U \geq 2230) = P\left(\bar{X} \geq \frac{2230}{50}\right) &= 1 - P\left(\bar{X} < \frac{2230}{50}\right) \\ &= 1 - P\left(Z < \frac{\frac{2230}{50} - 45}{\frac{4}{\sqrt{50}}}\right) \\ &= 1 - P(Z < -.71) \\ &= 1 - .2389 = .7611. \end{aligned}$$

(Z is a standard normal random variable.)