STAT 305 D Exam 2

Show all your work.

1. (25 points) Suppose 80% of all students taking a beginning programming course fail to get their first program to run on first submission. Use a binomial distribution and assign probabilities to the possibilities that among a group of six such students,

   (a) (5 points) all fail on their first submissions.

   $$P(\text{all fail}) = P(X=6) = \frac{6!}{(6-6)!6!}.8^6.2^{6-6} = .8^6 = 0.262$$

   (b) (5 points) at least four fail on their first submissions

   $$\begin{aligned} P(X \geq 4) &= P(X=4) + P(X=5) + P(X=6) \\ &= \frac{6!}{(6-4)!4!}.8^4.2^{6-4} + \frac{6!}{(6-5)!5!}.8^5.2^{6-5} + \frac{6!}{(6-6)!6!}.8^6.2^{6-6} \\ &= 0.245 + 0.393 + 0.262 \\ &= 0.9 \end{aligned}$$

   (c) (5 points) less than four fail on their first submissions.
   $P(X < 4) = 1 - P(X \geq 4) = 1 - .9 = 0.1$

   Continuing to use this binomial model:

   (d) (5 points) What is the mean number who will fail?
   $E(X) = np = 6 \cdot 0.8 = 4.8$

   (e) (5 points) What are the variance and standard deviation of the number who will fail?

   $$Var(X) = n \cdot p \cdot (1-p) = 6 \cdot 0.8 \cdot 0.2 = 0.96$$
   $$SD(X) = \sqrt{0.96} = 0.98$$

2. (24 points, each part worth 6 points)

Suppose that X is a continuous random variable with probability density of the form

$$f(x) = \begin{cases} k\left(x^2(1-x)\right) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Evaluate k and sketch a graph of $f(x)$.

(b) Evaluate $P[X \leq .25]$, $P[X \leq .75]$, $P[.25 < X \leq .75]$, and $P[|X - .5| > .1]$.

(c) Compute EX and $\sqrt{\mathrm{Var}\, X}$.

(d) Compute and graph $F(x)$, the cumulative distribution function for X. Read from your graph the .6 quantile of the distribution of X.

(a) Use equation (5-13) and solve for $k = .12$.

$$f(x) = \begin{cases} 12(x^2(1-x)) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

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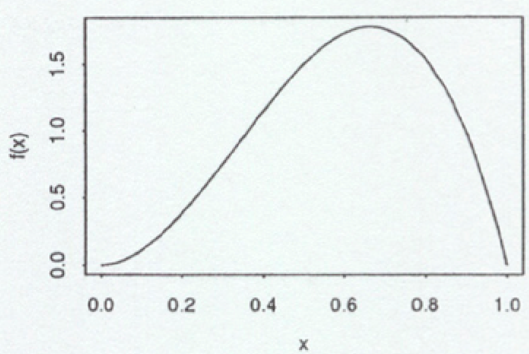


(b) All of the following probabilities correspond to areas under $f(x)$.

$$P(X \le .25) = \int_0^{.25} f(x)\,dx = .0508.$$

$$P(X \le .75) = \int_0^{.75} f(x)\,dx = .7383.$$

$$P(.25 < X \le .75) = P(X \le .75) - P(X < .25) = .7383 - .0508 = .6875$$

$$\begin{aligned} P(|X-.5| > .1)) &= 1 - P(|X-.5| \le .1)) \\ &= 1 - P(-.1 < X - .5 < .1)) \\ &= 1 - P(.4 < X < .6)) \\ &= 1 - \int_{.4}^{.6} f(x)\,dx \\ &= 1 - .2960 = .7040. \end{aligned}$$

(c) Using equation (5-18),

$$EX = .6.$$

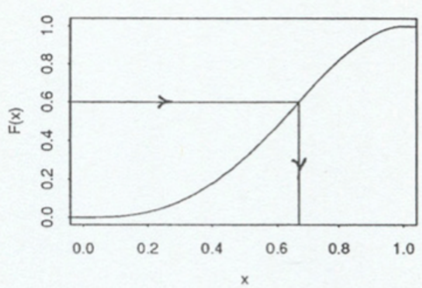
Using equation (5-19),

$$\sqrt{\mathrm{Var}\,X} = \sqrt{.04} = .2.$$

(d) Using equation (5-16),

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \begin{cases} 0 & \text{for } x \le 0 \\ 12\left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right) & \text{for } 0 < x < 1 \\ 1 & \text{for } x \ge 1. \end{cases}$$



The .6 quantile of the distribution is the x such that $P(X \le x) = .6$. This is just $F^{-1}(.6)$, so you need to find .6 on the vertical axis, and find the x that produces this value for $F(x)$. (By trial and error, the exact value is $x = .67082$.)

3. (26 points) A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf X and Y appears in the accompanying tabulation.

p(x,y)	y = 0	y = 1	y = 2
x = 0	0.10	0.04	0.02
x = 1	0.08	0.20	0.06
x = 2	0.06	0.14	0.30

(a) (6 points) What is $P(X = 1 \text{ and } Y = 1)$?

(b) (6 points) Compute $P(X \le 1 \text{ and } Y \le 1)$.

(c) (6 points) Compute the marginal pmf of X and of Y. Using $f_X(x)$, calculate $P(X \le 1)$.

(d) (8 points) Are X and Y independent random variables? Explain.

a. $P(X = 1, Y = 1) = p(1,1) = .20$

b. $P(X \le 1 \text{ and } Y \le 1) = p(0,0) + p(0,1) + p(1,0) + p(1,1) = .42$

c By summing row probabilities, $p_x(x) = .16, .34, .50$ for $x = 0, 1, 2$, and by summing column probabilities, $p_y(y) = .24, .38, .38$ for $y = 0, 1, 2$. $P(X \le 1) = p_x(0) + p_x(1) = .50$

d $P(0,0) = .10$, but $p_x(0) \cdot p_y(0) = (.16)(.24) = .0384 \ne .10$, so X and Y are not independent.

4. (25 points) A type of nominal $\frac{3}{4}$ inch plywood is made of five layers. These layers can be thought of as having thicknesses roughly describable as independent random variables with means and standard deviations as follows:

Layer	Mean (in.)	Standard Deviation (in.)
1	.094	.001
2	.156	.002
3	.234	.002
4	.172	.002
5	.094	.001

Find the mean and standard deviation
of total thickness associated with the combination of these individual values.

The total thickness is the sum of the thicknesses of the layers. Define $X_1, \ldots, X_5$ to be the thicknesses of the layers; then $U = X_1 + \cdots + X_5$ is the total thickness. Using equation (5-58),

$$EU = EX_1 + \cdots + EX_5 = .750 \text{ in.}$$

To use equation (5-59), you have to square the given standard deviations first.

$$\text{Var}U = \text{Var}X_1 + \cdots + \text{Var}X_5 = .000014,$$

so the standard deviation of U is $\sqrt{.000014} = .00374$ in.