PRE-SESSION 2020.

MATHEMATICS PROBLEMS

1.1

$$\frac{y^{58}}{y^4 \cdot y^{12}} = \frac{y^{58}}{y^{16}} = \underline{\underline{y^{42}}}$$

1.2

$$8^2 \cdot 2^x = 2^9$$
$$(2^3)^2 \cdot 2^x = 2^9$$
$$2^6 \cdot 2^x = 2^9 \quad /:2^6$$
$$2^x = 2^3$$
$$\underline{\underline{x = 3}}$$

1.3

$$\frac{x}{y} = 3 \qquad x^{-2}y^2 = \frac{y^2}{x^2} = \left(\frac{y}{x}\right)^2 = \left(\frac{x}{y}\right)^{-2} = 3^{-2} = \frac{1}{3^2} = \underline{\underline{\frac{1}{9}}}$$

1.4

$$\frac{\sqrt{2^{13}}}{\sqrt{8^3}} = \frac{\sqrt{2^{13}}}{\sqrt{2^9}} = \frac{2^{\frac{13}{2}}}{2^{\frac{9}{2}}} = \frac{\cancel{2^{\frac{9}{2}}} \cdot 2^{\frac{4}{2}}}{\cancel{2^{\frac{9}{2}}}} = 2^{\frac{4}{2}} = 2^2 = \underline{\underline{4}}$$

1.5 TRUE

1.6

$$\frac{x^2 - 25}{x - 5} = 3$$

$$\frac{(x+5)\cancel{(x-5)}}{\cancel{x-5}} = 3$$

$$x + 5 = 3 \quad /-5$$
$$\underline{\underline{x = -2}}$$

2.1

$0\,K = -460°F$

$1000\,K = 1340°F$

$$\left.\begin{aligned} 0 &= a + b\cdot(-460) \\ 1000 &= a + b\cdot 1340 \end{aligned}\right\}$$

$1000 - 0 = a + 1340b - (a - 460b)$

$1000 = 1340b + 460b$

$1000 = 1800b \quad /:1800$

$\underline{\frac{5}{9} = b}$

$0 = a + \overbrace{\frac{5}{9}\cdot(-460)}^{-255.556} \quad /+255.56$

$\underline{255.556 = a}$

$K = a + bF$

$K = 255.556 + \frac{5}{9}F$

$K - 255.56 = \frac{5}{9}F$

$\frac{9}{5}K - 460 = F$

$K = F$

$255.56 + \frac{5}{9}F = F \quad /-\frac{5}{9}F$

$255.56 = \frac{4}{9}F$

$F = 575$

$K = 255.56 + \frac{5}{9}\cdot 575 = \underline{\underline{575}}$

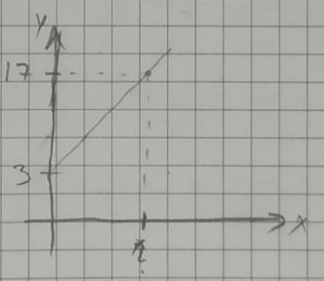
2.2

$f(x) = 2x + 3$

$17 = 2x + 3 \quad /-3$

$14 = 2x \quad /:2$

$\underline{\underline{7 = x}}$



2.3

$3^{2x^2-4x+3} = 27$

$3^{2x^2-4x+3} = 3^3$

$2x^2 - 4x + 3 = 3 \quad /-3$

$2x^2 - 4x = 0$

$x_{1,2} = \frac{4 \pm \sqrt{16 - 4\cdot 2\cdot 0}}{2\cdot 2} = \frac{4+4}{4} = \underline{\underline{2}}$

$\frac{4-4}{4} = \underline{\underline{0}}$

2.4 x: number of years

$$1.01^x = 2$$
$$\ln(1.01^x) = \ln(2)$$
$$x \cdot \ln(1.01) = \ln(2)$$
$$x = \frac{\ln(2)}{\ln(1.01)}$$

$$\underline{\underline{x = 69.667}}$$

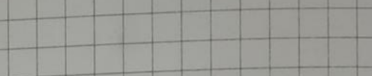
2.5 $\ln\left(\frac{e^2}{e^3}\right) = \ln(e^2) - \ln(e^3) = 2 \cdot \ln e - 3 \cdot \ln e =$

$= 2 \cdot 1 - 3 \cdot 1 = \underline{\underline{-1}}$

3.1 $\sum_{i=0}^{\infty} \left(\frac{1}{6^i} + 0.25^i\right)$

$a_0 = \frac{1}{6^0} + 0.25^0 = 2$

$a_1 = \frac{1}{6} + 0.25 = \frac{5}{12} = 0.416$



$a_2 = \frac{1}{36} + 0.625 = \frac{47}{72} = 0.652$

3.2 $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x \to 3} (x+3) = 3 + 3 = \underline{\underline{6}}$

3.3 $f(x) = x^3 - 4 \qquad (-1, -5)$

$f'(-1) = \lim_{x \to -1} \frac{(x^3 - 4) - (-5)}{x - (-1)} = \lim_{x \to -1} \frac{x^3 + 1}{x + 1} =$

$= \lim_{x \to -1} \frac{(x+1)(x^2 - x + 1)}{x + 1} = \lim_{x \to -1} (x^2 - x + 1) = 1 - (-1) + 1 = 1 + 1 + 1 = \underline{\underline{3}}$

$$f'(-5) = \lim_{x \to -5} \frac{(x^3-4)-\overset{-129}{(-125-4)}}{x-(-5)} = \lim_{x\to -5} \frac{x^3+125}{x+5} =$$

$$= \lim_{x \to -5} \frac{\cancel{(x+5)}(x^2-5x+25)}{\cancel{x+5}} = (-5)^2 - 5\cdot(-5) + 25 =$$

$$= 25 + 25 + 25 = \underline{\underline{75}}$$

3.4 $$\left(\frac{x^2+3}{x+2}\right)' = \frac{2x\cdot(x+2) - 1\cdot(x^2+3)}{(x+2)^2} = \frac{2x^2+4x-x^2-3}{(x+2)^2} =$$

$(x^2+3)' = 2x$

$(x+2)' = 1$

$$= \underline{\underline{\frac{x^2+4x-3}{(x+2)^2}}}$$

3.5 $f(x) = x^7 + 4x^2$

$f'(x) = 7x^6 + 4\cdot 2x = 7x^6 + 8x$

$f''(x) = 7\cdot 6x^5 + 8 = \underline{\underline{42x^5+8}}$

3.6 $f(x) = \dfrac{x^4+4x}{\ln(x)}$

$(x^4+4x)' = 4x^3+4$

$(\ln(x))' = \frac{1}{x}$

$$\underline{\underline{f'(x) = \frac{(4x^3+4)\cdot\ln(x) - \frac{1}{x}\cdot(x^4+4x)}{[\ln(x)]^2}}}$$

3.7 $f(x) = 3x^3 - 9x$

$f'(x) = 3\cdot 3x^2 - 9 = 0$

$(3x-3)(3x+3) = 0$

$3x_1 - 3 = 0 \quad /+3$

$3x_1 = 3 \quad /:3$

$\underline{x_1 = 1}$

$3x_2 + 3 = 0 \quad /-3$

$3x_2 = -3 \quad /:3$

$\underline{x_2 = -1}$

$f''(x) = 9 \cdot 2x = 18x$

at x_1: $f''(1) = \underline{\underline{18}}$ → positive ⇒ local minimum point

at x_2 $f''(-1) = \underline{\underline{-18}}$ → negative ⇒ local maximum point

Stationary point of the function is $(1,-1)$.

3.8 $f(x,y) = x^2 + 2y^3$

$f(2,3) = 2^2 + 2 \cdot 3^3 = 4 + \overset{54}{2 \cdot 27} = \underline{\underline{58}}$

3.9 $f(x,y) = \ln(2x - y)$ → Only defined above 0, so

$2x > y$

$\underline{\underline{2 > \frac{y}{x}}}$

3.10 $f(x,y) = x^5 e^y + x^2 y^3$

$$\frac{\partial f(x,y)}{\partial x} = e^y \cdot 5x^4 + y^3 x$$

$$\frac{\partial f(x,y)}{\partial y} = x^5 e^y + x^2 \cdot 3y^2$$

3.11 $f(x,y) = \sqrt{xy} - 0.7x - 0.7y = x^{\frac{1}{2}} \cdot y^{\frac{1}{2}} - 0.7x - 0.7y$

$f_x'(x,y) = y^{\frac{1}{2}} \cdot \frac{1}{2} x^{-\frac{1}{2}} - 0.7 \quad = 0$

$f_y'(x,y) = x^{\frac{1}{2}} \cdot \frac{1}{2} y^{-\frac{1}{2}} - 0.7 \quad = 0$

$$y^{\frac{1}{2}} \cdot \frac{1}{2} x^{-\frac{1}{2}} - 0.7 = x^{\frac{1}{2}} \cdot \frac{1}{2} y^{-\frac{1}{2}} - 0.7 \quad / +0.7$$

$$\sqrt{y} \cdot \cancel{\frac{1}{2}} \cdot \frac{1}{\sqrt{x}} = \sqrt{x} \cdot \cancel{\frac{1}{2}} \cdot \frac{1}{\sqrt{y}} \quad / : \sqrt{x}, : \sqrt{y}$$

$$\frac{1}{\sqrt{x} \cdot \sqrt{x}} = \frac{1}{\sqrt{y} \cdot \sqrt{y}}$$

$$\frac{1}{x} = \frac{1}{y}$$

$\underline{x = y}$ → symmetric function

$f_y(x,y) = \sqrt{x} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}} - 0.7 = 0$

$$\frac{1}{2}\cdot\frac{\sqrt{x}}{\sqrt{x}} - 0.7 = 0$$

$$\frac{1}{2} = 0.7$$

$$\Downarrow$$

3.12

$$f(x,y) = x^2y^2 \rightarrow max \quad s.t. \quad x+y=10$$

$$g(x,y) = x+y-10$$

$$\mathcal{L} = x^2y^2 - \lambda\cdot(x+y-10)$$

$$\left.\begin{aligned}\frac{\partial \mathcal{L}}{\partial x} &= y^2\cdot 2x - \lambda = 0\\ \frac{\partial \mathcal{L}}{\partial y} &= x^2\cdot 2y - \lambda = 0\end{aligned}\right\}$$

$$y^2\cdot 2x - \lambda = x^2\cdot 2y - \lambda$$

$$y^2\cdot 2x = x^2\cdot 2y \quad /:x^2$$

$$\frac{y^2\cdot 2x}{x^2} = 2y \quad /:y^2$$

$$\frac{2x}{x^2} = \frac{2y}{y^2}$$

$$\frac{2}{x} = \frac{2}{y}$$

$$\underline{x=y}$$

$$x+y-10=0$$

$$x+x-10=0$$

$$2x=10$$

$$\underline{\underline{x=5}} \rightarrow \underline{\underline{y=5}}$$

4.1

$$\underline{A} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{cc|ccc} & & 1 & 4 & 1 \\ & & 2 & 1 & 2 \\ \hline 2 & 3 & 8 & 11 & 8 \\ 4 & 1 & 6 & 17 & 6 \\ 1 & 2 & 5 & 6 & 5 \end{array}$$

$$\underline{A} \cdot \underline{B} = \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$$

4.2

$$\underline{A} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{ccc|cc} & & & 2 & 3 \\ & & & 4 & 1 \\ & & & 1 & 2 \\ \hline 1 & 4 & 1 & 19 & 9 \\ 2 & 1 & 2 & 10 & 11 \end{array}$$

$$\underline{B} \cdot \underline{A} = \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix}$$

4.3

$$\underline{A} = \begin{bmatrix} 3.3 & 5.1 \\ 6.1 & 1.23 \\ 45.76 & 0 \end{bmatrix} \qquad \underline{A}^T = \begin{bmatrix} 3.3 & 6.1 & 45.76 \\ 5.1 & 1.23 & 0 \end{bmatrix}$$

4.4

$$\underline{A} = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 2 \\ 2 & 5 & 3 \end{bmatrix}$$

$$\det(A) = 2 \cdot 5 \cdot 3 + 3 \cdot 2 \cdot 2 + 0 \cdot 4 \cdot 5 -$$
$$- 0 \cdot 5 \cdot 2 - 2 \cdot 2 \cdot 5 - 3 \cdot 4 \cdot 3 =$$
$$= 30 + 12 - 20 - 36 = \underline{\underline{-14}}$$

$$\begin{matrix} 2 & 3 & 0 & 2 & 3 \\ 4 & 5 & 2 & 4 & 5 \\ 2 & 5 & 3 & 2 & 5 \end{matrix}$$

5.1 $\Omega = \{(H,H),(H,T),(T,T),(T,H)\}$

5.2 $n = 30$

$k = 3$

no replacement, order matters $\Rightarrow$ permutation

$\underline{30}$ $\underline{29}$ $\underline{28}$

$\# P_k^n = 30 \cdot 29 \cdot 28 = \underline{\underline{24360}}$

5.3 $\# \Omega = 6 \cdot 6 \cdot 2 - 6 = 66$

$\# A = 3 \cdot 6 \cdot 2 - 3 = 33$

$A = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1) \dots ,(6,1)$

$P(A) = \frac{33}{66} = \underline{\underline{\frac{1}{2}}}$