My solutions to

Deep Learning: Foundations and Concepts

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19 Variational Autoencoders

19.2

Assume $\sigma > 0$ and define $g(\epsilon) := \sigma\epsilon + \mu$. Then $g^{-1}(z) := \frac{z-\mu}{\sigma}$ and $\left(g^{-1}\right)'(z) = \frac{1}{\sigma}$. It follows that

$$\begin{aligned} p_Z(z) &= p_{\sigma\mathcal{E}+\mu}(z) \\ &= p_{\mathcal{E}}\left(\frac{z-\mu}{\sigma}\right)\left|\frac{1}{\sigma}\right| && \text{change of variables via } g \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\frac{z-\mu}{\sigma}\right)^2}{2}} \frac{1}{\sigma} && \mathcal{E} \sim \mathcal{U}(0,1);\ \sigma > 0 \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \end{aligned}$$

is density for $\mathcal{N}(\mu, \sigma^2)$.