My solutions to Deep Learning: Foundations and Concepts

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14 Sampling

14.4

$$F(y) := \int_{-\infty}^{y} \frac{1}{\pi} \frac{1}{1 + \hat{y}^2} d\hat{y}$$

$$= \frac{1}{\pi} \left[\arctan y \right]_{-\infty}^{y}$$

$$= \frac{1}{\pi} \left(\arctan y - \left(-\frac{\pi}{2} \right) \right)$$

$$= \frac{\arctan y}{\pi} + \frac{1}{2}$$

with

$$F^{-1}(z)=\tan\left(\pi\left(z-0.5\right)\right):(0,1)\longrightarrow\mathbb{R}$$
 So $Y:=F^{-1}(Z)=\tan(\pi(Z-0.5))\sim\mathrm{Cauchy}(0,1).$