Image segmentation with superpixel MRF and shape priors

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Motivation – Gene expression analysis

- Different embryos are used for each gene and stage
- Need a map to a standard template for comparability

$\Rightarrow$ One method: image segmentation

- Challenge 1: Low intensity contrasts, hard to segment even for humans
- Challenge 2: Need detailed information from very high resolution image
- Known: Shape and positions roughly follow a given template

$\Rightarrow$ Approach: Combining superpixels and shape priors using an MRF

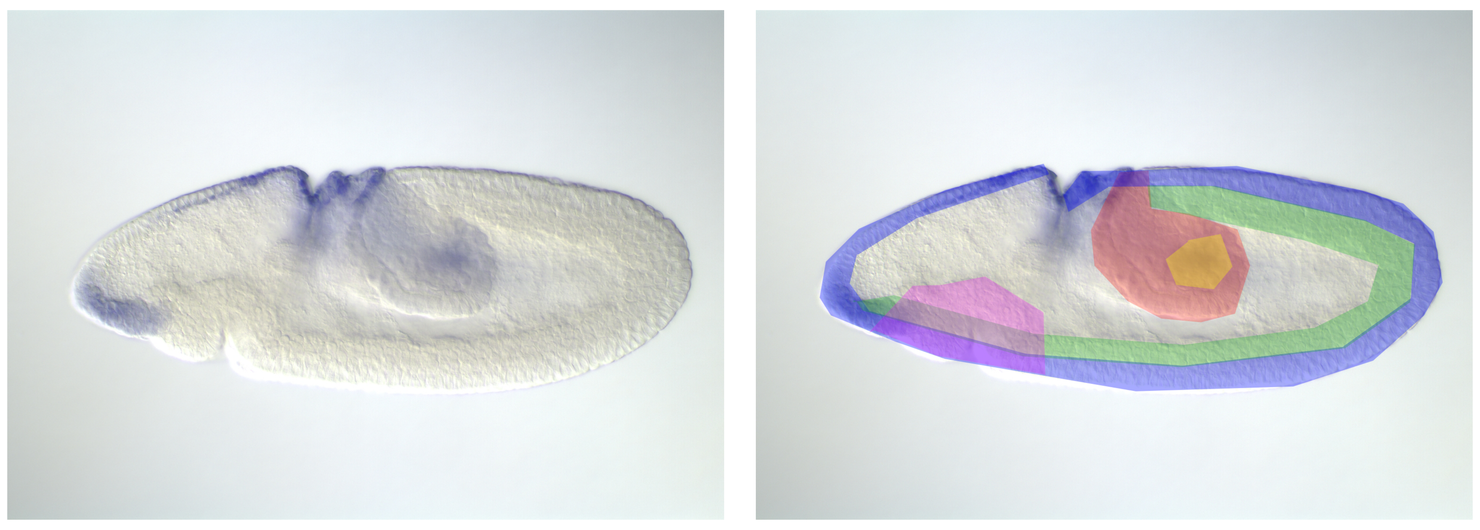


Image segmentation with shape priors

Image segmentation

- Given pixel features x, obtain labels y, pixel coordinates i
- Minimization of energy, equivalent with maximization of the likelihood

$$y = \arg\min_y E(y,x) \iff y = \arg\max_y p(y|x)$$

with $p(y|x) \propto \exp(-E(y,x))$

- Discriminative models (CRF) vs. generative models (MRF) for $p(y|x)$.
- Incorporate priors via generative models $p(y|x) \propto p(x,y)$.

Incorporating shape priors using level set functions

- Model contour of the set $S = \{i \in V : y_i = 1\}$ as the zero level set of a function $\phi(i)$, so $S = \{i \in V : \phi(i) = 0\}$
- Use as additional random variable to obtain $p(y,\phi|x) \propto p(x,y|\phi)p(\phi)$
- Make $p(\phi)$ dependent on prior information

$$p(\phi|\tilde{\phi}) = \exp(-(E_{int}(\phi) + E_{prior}(\phi,\tilde{\phi})))$$

with E_{int} e.g. penalizing length of the contour and a nonparametric kernel density estimate for

$$\exp(-E_{prior}(\phi,\tilde{\phi})) = \sum_{i=1}^{N} \exp\left(-\frac{1}{2\sigma^2} d^2(H_\epsilon(\phi), H_\epsilon(\phi_i))\right)$$

where d is similar to the Hamming distance between two functions

- Minimization using update equation

$$\frac{\partial\phi}{\partial t} = -\frac{\partial E_{int}(\phi)}{\partial\phi} - \frac{\partial E_{prior}(\phi)}{\partial\phi}$$

- Cremers, Daniel, Stanley J. Osher, and Stefano Soatto. "Kernel density estimation and intrinsic alignment for shape priors in level set segmentation" *International Journal of Computer Vision* 69.3 (2006): 335-351.
- Chan, Tony F., and Luminita A. Vese. "Active contours without edges." *Image processing, IEEE* transactions on 10.2 (2001): 266-277..

Combining superpixel MRF and shape priors

Variables

- Graph: $G(V,E)$ with V set of nodes, E set of edges
- Observed superpixel features (intensity, texture): $x_i \in \mathbb{R}^d$, $i \in V$
- Latent variables: Labels $y_i \in 0,\ldots,k, i \in V$, level set function ϕ
- Training data: Segmented shapes $\tilde{\phi}$
- Joint probability distribution:

$$\begin{aligned} p(x,y,\phi) &= \prod_{i\in V}\psi(x_i,y_i)\prod_{(i,j)\in E}\psi(y_i,y_j)\prod_{i\in V}p(y_i|\phi)p(\phi) \\ &=: \prod_{i\in V}p(x_i|y_i)\prod_{(i,j)\in E}p(y_i,y_j)\prod_{i\in V}p(y_i|\phi)p(\phi|\tilde{\phi}) \end{aligned}$$

Probability model

- Gaussian mixture for the conditional probability

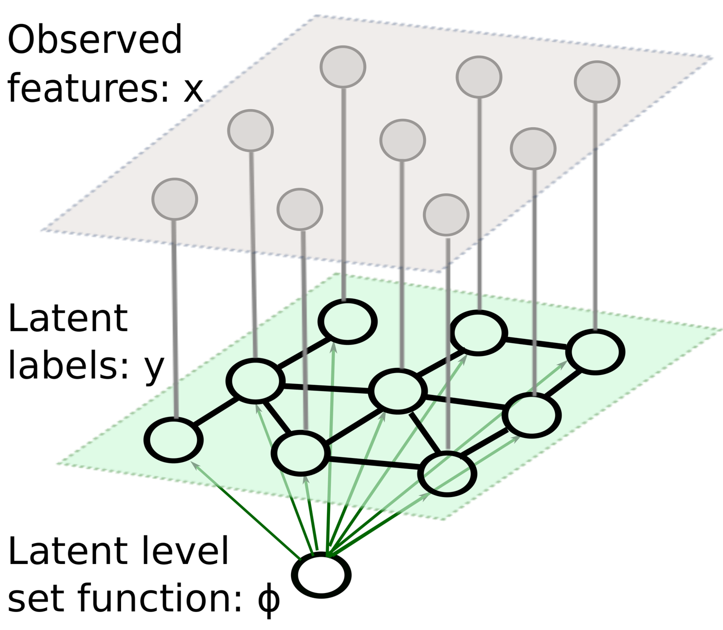
$$p(x_i|y_i = k) = \mathcal{N}(\mu_k, \Sigma_k)$$

- Pairwise (edge) potentials

$$p(y_i,y_j) \propto \theta_1 1_{y_i \neq y_j} + \theta_2 1_{y_i = y_j}$$

- Conditional probability of y given ϕ

$$p(y_i|\phi) = \left(\frac{1}{1+e^{-\lambda\phi(i)}}\right)^{y_i}\left(\frac{1}{1+e^{\lambda\phi(i)}}\right)^{1-y_i}$$



Exact inference for $p(y,\phi|x)$ is not computationally tractable $\Rightarrow$ Variational inference

Segmentation Algorithm

Structured variational inference

- Approximate $p(y,\phi|x)$ by an easier probability distribution

$$Q(y,\phi|x,a,b,\tilde{\phi}) = Q_M(y|x,a)Q_D(\phi|b,\tilde{\phi})$$

with $Q_M(y|x,a) \propto \prod_{i\in V} p(x_i|y_i)p(y_i|a_i)$ and $Q_D(\phi|b) \propto \prod_{i\in V} p(b_i|\phi)p(\phi|\tilde{\phi})$

- Minimize the Kullback Leibler distance $KL(Q||P)$ which yields the fixed point equation

$$\begin{aligned} \log p(y_i|a_i) &= \mathbb{E}_{Q_D} \log p(y_i|\phi) \\ \log p(b_i|\phi) &= \mathbb{E}_{Q_M^i} \log p(y_i|\phi) \end{aligned}$$

- This yields $-E(\phi) = \log Q_D(\phi|b) \propto \log p(\phi|\tilde{\phi}) + \mathbb{E}_{Q_M^i} \log p(y_i|\phi)$ and thus the update equation for finding $\phi^* = \arg\max Q_D(\phi|b)$.

$$\frac{\partial\phi}{\partial t} = -\frac{\partial E_{int}(\phi)}{\partial\phi} - \frac{\partial E_{prior}(\phi)}{\partial\phi} - \mathbb{E}_{Q_M^i}\frac{\partial}{\partial\phi}\log p(y_i|\phi) \qquad (1)$$

- Then choose $y = \arg\max_y Q_M(y|x,a)$.

Algorithm

- Initialize $p(y_i|a_i)$ by a uniform distribution.
- Step k.
  1. In order to calculate $Q_M(y_i|x_i,a)$ estimate the parameters of $p(x_i|y_i)$ (which are μ_k, Σ_k) using the EM algorithm
  2. Given approximate $Q_M(y_i|x_i,a)$ using loopy belief propagation/mean field inference.
  3. Calculate $\log p(b_i|\phi) = \mathbb{E}_{Q_M^i} \log p(y_i|\phi)$.
  4. Calculate $\phi^* = \arg\max_\phi Q_D(\phi|b)$ using (1)
  5. Approximate $p(y_i|a_i) = \exp(\mathbb{E}_{Q_D^i} \log p(y_i|a_i)) = p(y_i|\phi^*)$.
- Upon termination: $y_i = \max_k Q_M(y_i = k|x,a)$.

Example – Touching embryos

Problem

Separate the embryo of interest in the "middle" using shape prior.

Segmentation results

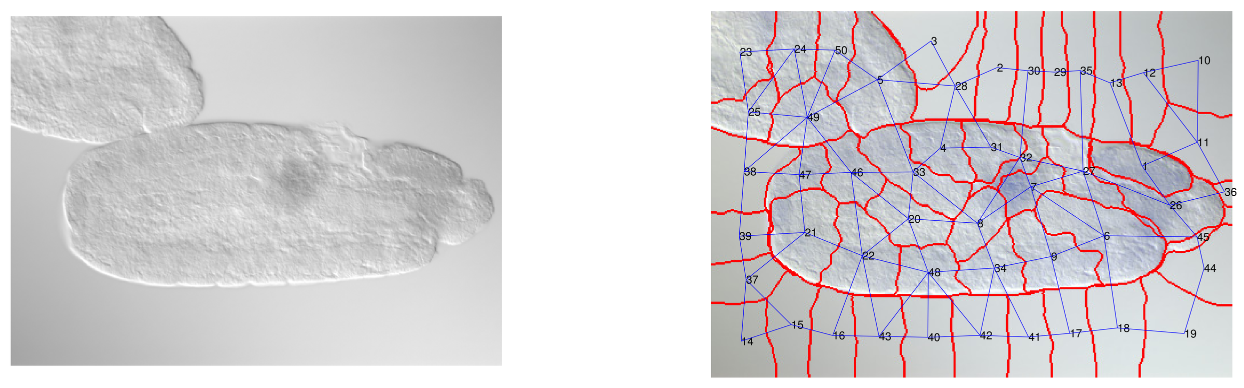


Figure: Left: Original picture. Right: Superpixel segmentation with edge structure for MRF (on a resized image)

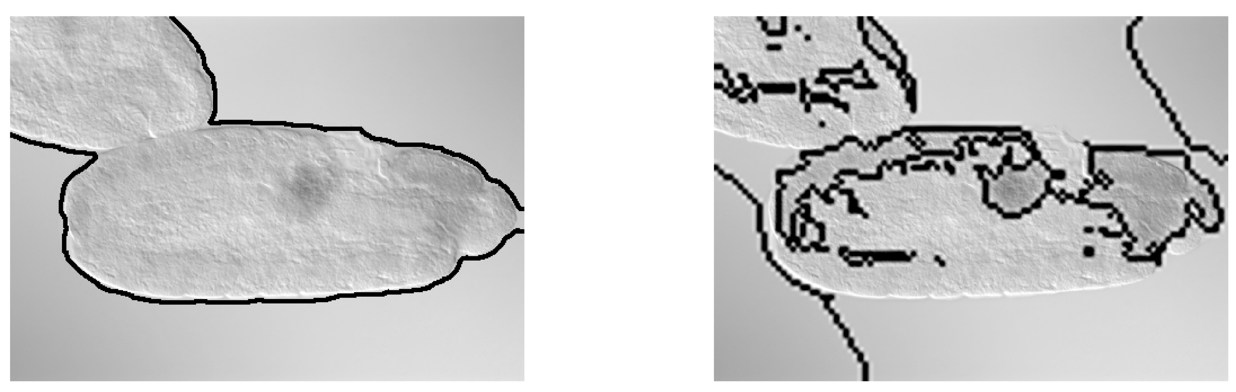


Figure: Left: MRF on superpixels. Right: Level set segmentation

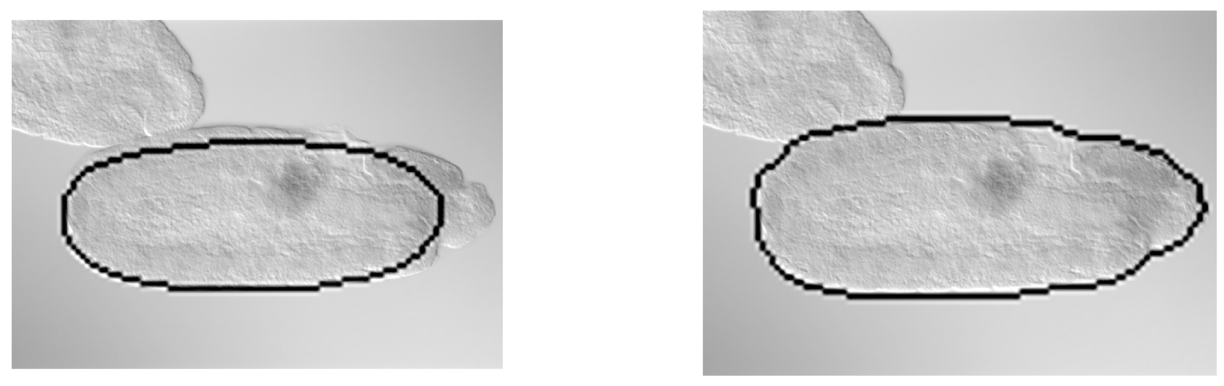


Figure: Left: Level set segmentation and prior. Right: MRF with level set segmentation

Future work

- Multilabel extension
- Application on the real problem: Segmentation of the gut, mouth etc.
- Finding better features for MRF to speed up algorithm