

# ESPIDAM Exercice 1 – Solution

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## Exercise 1.1 (Final Outbreak Size)

(a) Solve the final size equation. Create a function of  $R_0$  solving for  $\tau$  numerically.

```
#####  
# Function of R_0 solving for tau numerically  
# R_0 - reproduction number  
# tau - final size  
# Return: tau  
#####  
  
tau <- function(R_0) {  
  final_eq <- function(tau) {  
    #####  
    # give the expression here  
    #####  
    1 - tau - exp(-R_0 * tau)  
  }  
  "  
  Note: This final size equation is always a solution tau = 0.  
  When R_0 <=1, it gives only one solution tau = 0;  
  if R_0 > 1, then there are two solutions.  
  "  
  result1 <- uniroot(final_eq, lower = 0, upper = 1)$root  
  result2 <- uniroot(final_eq, lower = 1e-12, upper = 1, extendInt = "yes")$root  
  
  # fill in here (minimum or maximum of result1 and 2 or others?)  
  result <- max(result1, result2)  
  
  return(result)  
}
```

Compute and plot  $\tau$  against  $R_0 \in [0, 5]$ :

```
#####  
# Compute tau for R_0 in [0,5]  
# Plot tau against R_0  
#####  
  
# Create a vector of 10000 R0 values between 0.00001 and 5
```

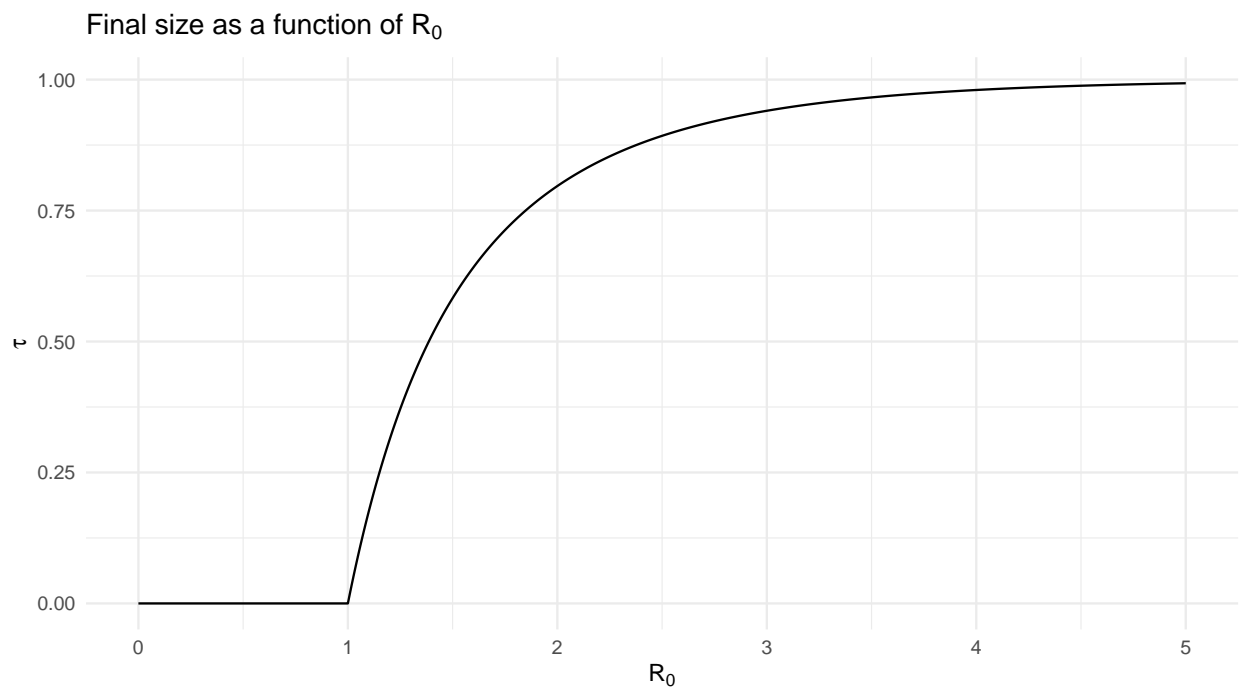
```

R0_vec <- seq(from = 0, to = 5, length.out = 10000)

# Create a vector of the corresponding values of tau
tau_vec <- as.vector(10000)
for (i in 1:10000) {
  R0 <- R0_vec[i]
  #####
  # fill in here (use the previous function tau(...))
  tau_vec[i] <- tau(R_0 = R0)
  #####
}

# Create a plot of tau against R0
ggplot(mapping = aes(x = R0_vec, y = tau_vec)) +
  geom_line() +
  xlab(expression("R"[0])) +
  ylab(expression(tau)) +
  ggtitle(expression("Final size as a function of R"[0])) +
  theme_minimal()

```



- (b) Now suppose there is a fraction  $r$  of initially immune, then the final fraction infected among the initially susceptible. Create a function of  $R_0$  and  $r$  solving for  $\tau$  numerically

```

#####
# Function of R_0 solving for tau numerically, when there is initial immune
# R_0 - reproduction number
# r - fraction of initially immune
# tau - overall final size
# Return: tau
#####

```

```

tau_overall <- function(R_0, r) {
  final_eq <- function(tau) {
    #####
    # YOUR CODE HERE
    #####
    # give the equation
    1 - tau - exp(-R_0 * (1 - r) * tau)
  }
  "
  Note: This final size equation only gives the fraction infected among
  those who are initially susceptible.
  "
  result1 <- uniroot(final_eq, lower = 0, upper = 1)$root
  result2 <- uniroot(final_eq, lower = 1e-12, upper = 1, extendInt = "yes")$root

  #####
  # fill in here (minimum or maximum or others?)
  #####
  result <- max(result1, result2)

  "
  Note: this result is the final size among initially susceptibles.
  then what is the overall fraction infected?
  "
  #####
  # fill in here (here we want the OVERALL fraction infected!)
  #####

  return(result * (1 - r))
}

```

Compute and plot  $\tau$  against  $R_0 \in [0, 5]$  for  $r = 0.3, 0.5, 0.7$ .

```

#####
# Plot the overall final size of  $R_0$  in  $[0, 5]$ 
# in three cases:  $r = 0.3, 0.5, 0.7$ 
#####
# Create a vector of 10000  $R_0$  values between 0.00001 and 5
R0_vec <- seq(from = 0, to = 5, length.out = 10000)
df_tau_r <- data.frame(R0 = R0_vec)

# Create a vector of the corresponding values of tau for  $r = 0.3$ 
tau_vec <- as.vector(10000)
r <- 0.3
for (i in 1:10000) {
  R0 <- R0_vec[i]
  tau_vec[i] <- tau_overall(R_0 = R0, r = r)
}
df_tau_r <- cbind(df_tau_r, overall_infected.3 = tau_vec)

# Create a vector of the corresponding values of tau for  $r = 0.5$ 
tau_vec <- as.vector(10000)
r <- 0.5

```

```

for (i in 1:10000) {
  R0 <- R0_vec[i]
  tau_vec[i] <- tau_overall(R_0 = R0, r = r)
}
df_tau_r <- cbind(df_tau_r, overall_infected.5 = tau_vec)

#####
# YOUR CODE HERE
#####
# As in the codes for r = 0.3 and 0.5, create a vector of the corresponding
# values of tau for r = 0.7, and consider it as the column in the data frame
# "df_tau_r" , with name "overall_infected.7".

# Create a vector of the corresponding values of tau for r = 0.7
tau_vec <- as.vector(10000)
r <- 0.7
for (i in 1:10000) {
  R0 <- R0_vec[i]
  tau_vec[i] <- tau_overall(R_0 = R0, r = r)
}
df_tau_r <- cbind(df_tau_r, overall_infected.7 = tau_vec)

# Create a plot of tau against R0 in three cases
ggplot(df_tau_r) +
  geom_line(aes(x = R0, y = overall_infected.3, color = "r = 0.3")) +
  geom_line(aes(x = R0, y = overall_infected.5, color = "r = 0.5")) +
  geom_line(aes(x = R0, y = overall_infected.7, color = "r = 0.7")) +
  scale_color_discrete(name = "immune fraction") +
  labs(title = " Assuming there is a fraction r of initially immune") +
  ylab("Overall fraction infected") +
  xlab(expression("R"[0])) +
  ylim(0, 1) + theme_minimal()

```



## Exercise 1.2 (Deterministic and stochastic SIR Model)

- (a) Consider a continuous-time deterministic SIR model with parameter values. Create a function to compute the derivative of the ODE system for SIR Model.

```
#####
# Function to compute the derivative of the ODE system for SIR Model:
# S(t): number of susceptible
# I(t): number of infectives
# ODE:
# dS(t) / dt = -beta / N * S(t) * I(t)
# dI(t) / dt = beta / N * S(t) * I(t) - gamma * I(t)
#
# t - time
# y - current state vector of the ODE at time t
# parms - Parameter vector used by the ODE system
#
# Returns:
# list containing dS(t)/dt and dI(t)/dt
#####
# Parameter values
gamma <- 0.25
beta <- 0.75
# We assume a closed population of size
times <- seq(0, 100, length = 1000)

deter_sir <- function(t, y, parms) {
  beta <- parms[1]
  gamma <- parms[2]
```

```

N <- parms[3]
S <- y[1]
I <- y[2]
return(list(c(
  S = -(beta * S * I) / N, # dS(t)/dt
  #####
  # YOUR CODE HERE
  #####
  # same for dI(t)/dt
  I = beta / N * S * I - gamma * I
)))
}

```

Solve the SIR differential equation system with initial conditions:  $S(0) = N - 1, I(0) = 1$ :

```

#####
# YOUR CODE HERE, fill in the blank space
#####
# using lsoda(..) function

sim <- lsoda(
  y = c(10000 - 1, 1), # initial conditions for S and I
  times = times, # times at which explicit estimates for y are desired
  func = deter_sir, # function computing values of derivatives in the ODE
  parms = c(beta, gamma, N = 10000) # vector/list of parameters used in function
)

```

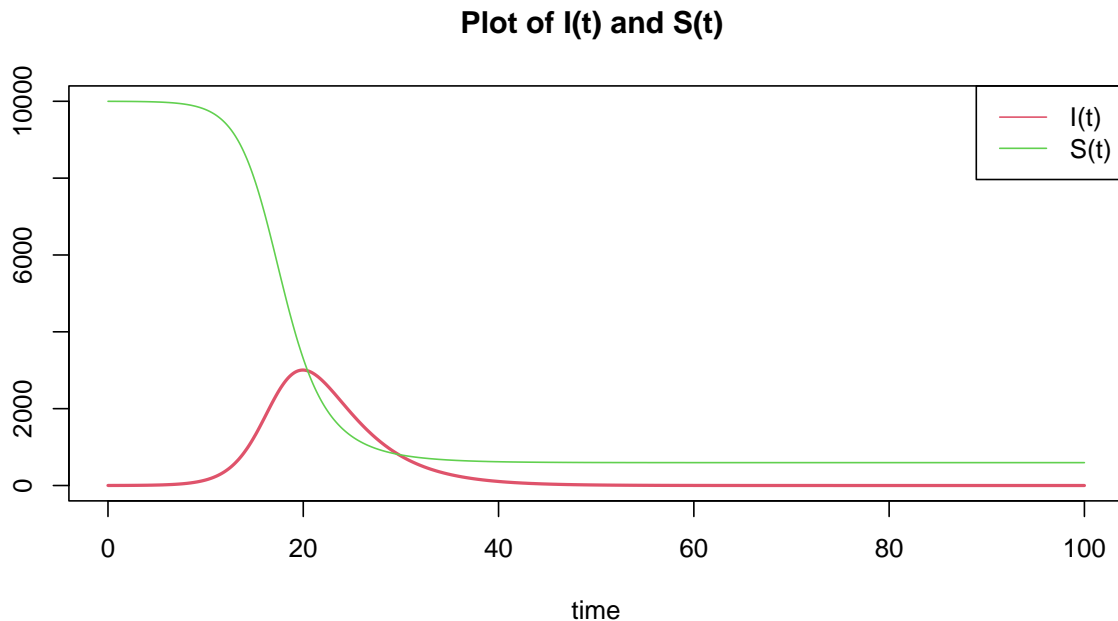
Plot the curves of  $I(t)$  and  $S(t)$  over time:

```

#####
# YOUR CODE HERE
#####

# simply use the function plot(...)
plot(times, sim[, 3], type = "l", lwd = 2,
      col = 2, xlab = "time", ylab = "",
      xlim = c(0, 100), ylim = c(0, 10000))
lines(times, sim[, 2], col = 3)
legend(x = "topright", c("I(t)", "S(t)"), col = c(2, 3), lty = 1)
title("Plot of I(t) and S(t) ")

```



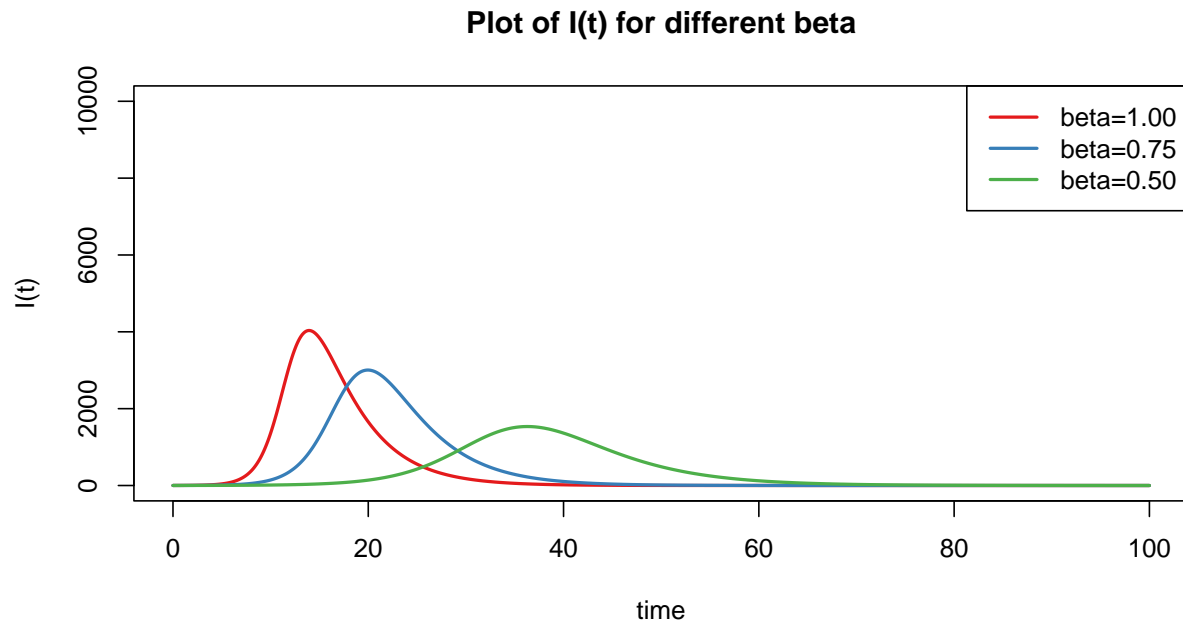
- (b) Now fix  $\gamma = 0.25$ , but choose different values of  $\beta = 1, 0.75$  and  $0.25$ . In each case, solve the SIR differential equation system with initial condition ( $S(0) = N - 1, I(0) = 1$ ). Plot the curves of  $I(t)$  and compare them.

```
# set a vector of different values of beta
beta.grid <- c(1, 0.75, 0.5)

# summarize the I(t) for each case of beta
I <- sapply(
  beta.grid,
  function(beta) {
    #####
    # YOUR CODE HERE, fill in the blank space
    #####
    # again, using lsoda(.) function solve the ODE, compute the I(t)
    lsoda(y = c(10000 - 1, 1), times = times, func = deter_sir,
          parms = c(beta, gamma, N = 10000))[, 3]
  }
)

# plot the three curves of I(t)

pal <- brewer.pal(length(beta.grid), "Set1")
matplot(times, I, type = "l", lwd = 2, lty = 1, col = pal,
        xlab = "time", ylab = "I(t)", ylim = c(0, 10000))
legend(x = "topright", paste("beta=", sprintf("%.2f", beta.grid), sep = ""),
       lty = 1, lwd = 2, col = pal)
title("Plot of I(t) for different beta")
```



(c) Now the rate  $\beta$  depends on time when different measures take place. Create a function to compute the derivative of the ODE system for the SIR Model with  $\beta(t)$ .

```

sir_change <- function(t, y, parms) {
  beta0 <- parms[1]
  beta1 <- parms[2]
  beta2 <- parms[3]

  t1 <- parms[4]
  t2 <- parms[5]
  gamma <- parms[6]

  S <- y[1]
  I <- y[2]

  #####
  # YOUR CODE HERE, fill in the blank space
  #####
  # create a time-dependent rate \beta(t) as function of t

  beta <- function(t) {
    ifelse(t <= t1, beta0, ifelse(t > t2, beta2,
      beta1
    ))
  }

  return(list(c(
    S = -beta(t) / N * S * I, # list containing dS(t)/dt
    I = beta(t) / N * S * I - gamma * I
  ))) # and dI(t)/dt

```



```

}

# set the parameter values
beta0 <- 0.75
beta1 <- 0.65 * 0.75
beta2 <- 0.75 * 0.75
t1 <- 14
t2 <- 28
gamma <- 0.25
N <- 10000
time <- seq(0, 100, length = 1000)

```

Solve the changed SIR differential equation system with initial conditions:  $S(0) = N - 1, I(0) = 1$ .

```

#####
# YOUR CODE HERE, fill in the blank space
#####

sol_SIRchange <- lsoda(
  y = c(N - 1, 1), # initial conditions for S and I
  times = time, # times at which explicit estimates for y are desired
  func = sir_change, # function computing the values of derivatives in the ODE
  parms = c(beta0, beta1, beta2, t1, t2, gamma) # vector/list of parameters
)

```

Plot the curves of  $I(t)$  for fixed beta and time-dependent beta.

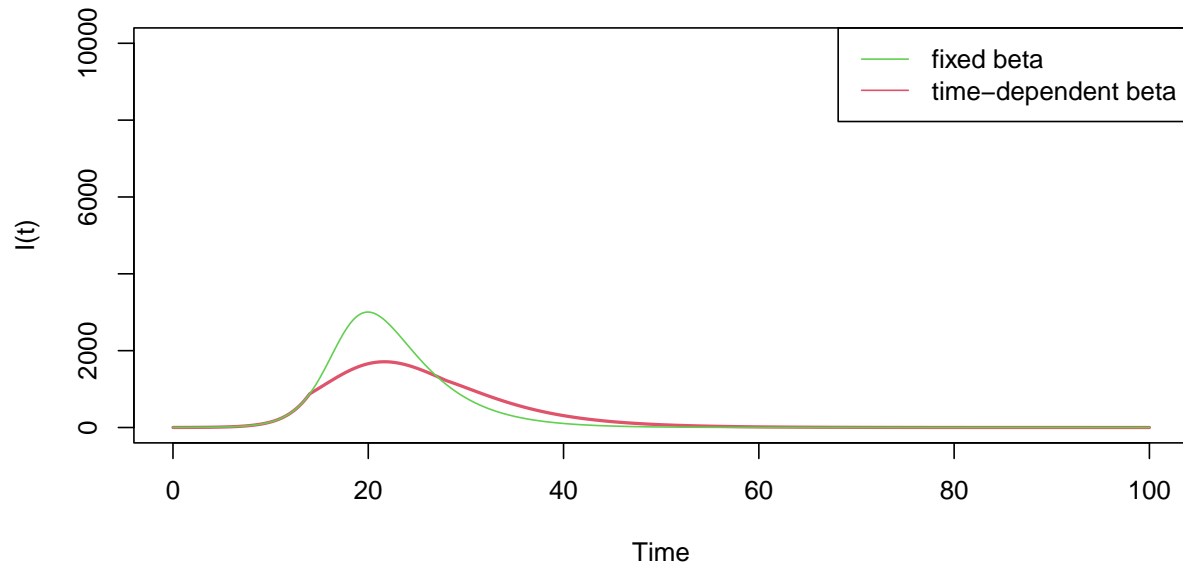
```

#####
# YOUR CODE HERE
#####

# simply use the functions plot(...), lines(...)
plot(time, sol_SIRchange[, 3], type = "l", lwd = 2, col = 2,
      xlab = "Time", ylab = "I(t)", xlim = c(0, 100), ylim = c(0, N))
lines(times, sim[, 3], col = 3)
legend(x = "topright", c("fixed beta",
                        "time-dependent beta"), col = c(3, 2), lty = 1)
title("Plot of I(t) in SIR model with fixed and time-dependent")

```

Plot of  $I(t)$  in SIR model with fixed and time-dependent



(d) Here we turn our focus to stochastic SIR model in continuous time. Always assume that there are fraction  $c = 10\%$  of initial infectives.

Simulate simple SIR model using the Gillespie-Algorithm.

```
#####
# Params:
# [0,T] - time horizon
# beta - infection rate
# gamma - recovery rate
# n - initial number of susceptibles.
# m - initial number of infectives
#####

stoch_SIR <- function(T, beta, gamma, n, m, N) {
  # Initialize (x= number of S, y=number of I, t=event time)
  x <- n
  y <- m
  z <- 1
  t <- 0
  N <- n + m
  # Possible events: Infection(S->I) and Removal(I->R)
  eventLevels <- c("S->I", "I->R")
  # Initialize result
  df_SIR <- data.frame(t = t, x = x, y = y, totalinfected = z, event = NA)
  # Loop until time T or the epidemic stops(there is no infectives)
  while (t < T & (y > 0)) {
    #####
    # YOUR CODE HERE, fill in the blank space
    #####
  }
}
```

```

# Draw the waiting type for each possible event

wait <- rexp(2, c(
  "S->I" = beta / N * x * y, # here insert the rate for "S->I"
  "I->R" = gamma * y # the rate for "I->R"
))
"
From those two rates, we draw two exponential
random numbers for each possible event
"

# Determine which event occurs first
# i.e. the event with the smaller or bigger random number?
# which.min(wait) or which.max(wait)?

i <- which.min(wait)

# Record Event Time
t <- t + wait[i]

# Update the number of S and I according to the event type
# if an infection occurs,
if (eventLevels[i] == "S->I") {
  x <- x - 1 # the number of S will decrease by 1.
  # what happen to y, z?
  y <- y + 1

  z <- z + 1
}
# if an recovery occurs,
if (eventLevels[i] == "I->R") {
  # what happen to x, y, z?
  y <- y - 1
}
# Store result
df_SIR <- rbind(df_SIR, c(t, x, y, z, i))
}
# Re-code event type and return
df_SIR$event <- factor(eventLevels[df_SIR$event], levels = eventLevels)

return(df_SIR)
}

```

Plot  $I(t)$  for one simulated stochastic epidemic and deterministic limit for different size of population  $N = 100, 1000$  and  $10000$ .

```

#####
# YOUR CODE HERE, fill in the blank space
#####

# set parameter values:
beta <- 0.75

```

```

gamma <- 0.25
Tmax <- 50 # time limit
c <- 0.1 # initial infectious fraction

compare_DS <- function(N) {

  # do one simulated stochastic epidemic
  traj <- stoch_SIR(T = Tmax, beta = beta, gamma = gamma,
                   n = N * (1 - c), m = N * c, N = N)

  # plot the stochastic curve of I
  plot(traj$t, traj$y,
       type = "s", ylim = c(0, N), xlab = "Time",
       ylab = "Number of Infectious individuals", xlim = c(0, Tmax), col = 1
  )

  # Solve the deterministic ODE

  solution <- lsoda(
    y = c(N * (1 - c), N * c),
    times = seq(0, Tmax, length = 1000),
    func = deter_sir,
    parms = c(beta, gamma, N = N)
  )

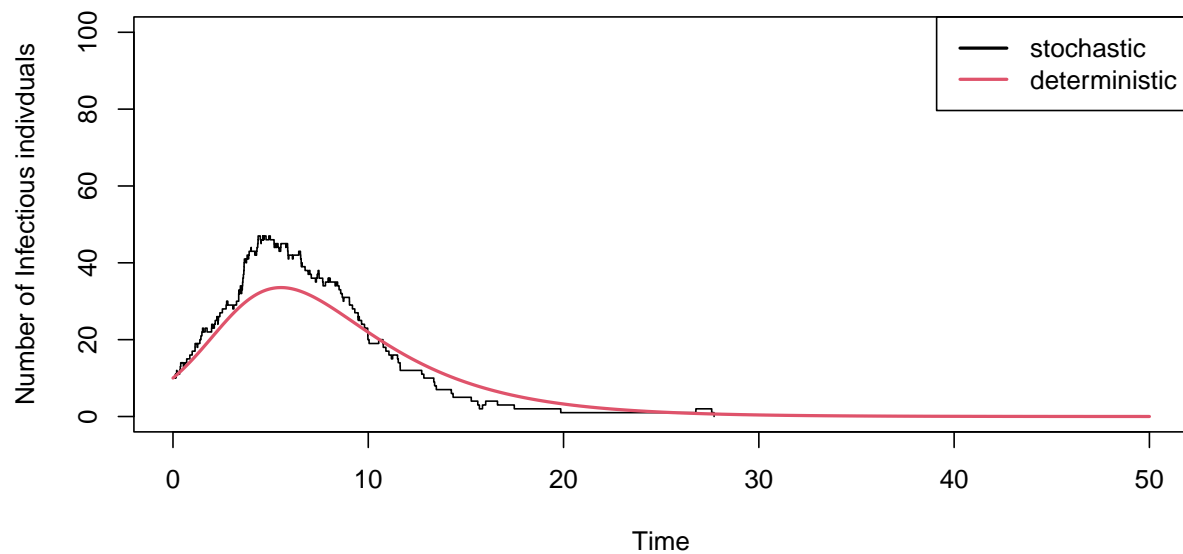
  # add the deterministic curve of I

  lines(solution[, "time"], solution[, 3], col = 2, lwd = 2)
}

## Compare deterministic and stochastic for different N
set.seed(123)
compare_DS(N = 100)
legend(x = "topright", c("stochastic", "deterministic"),
      col = c(1, 2), lty = 1, lwd = 2)
title("Stochastic vs. Deterministic when size of population = 100")

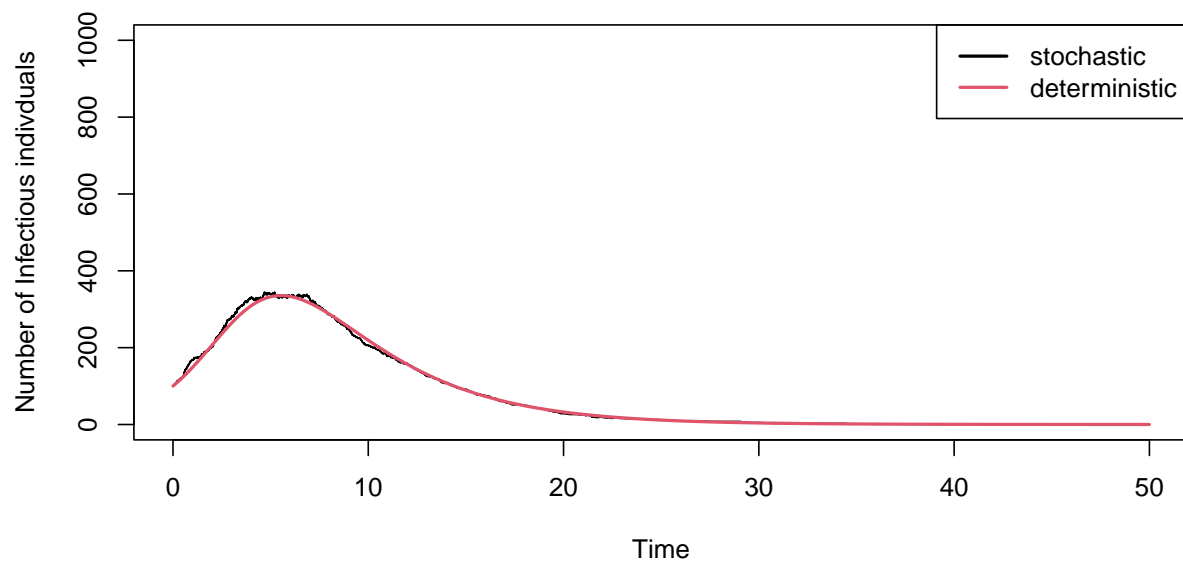
```

### Stochastic vs. Deterministic when size of population = 100

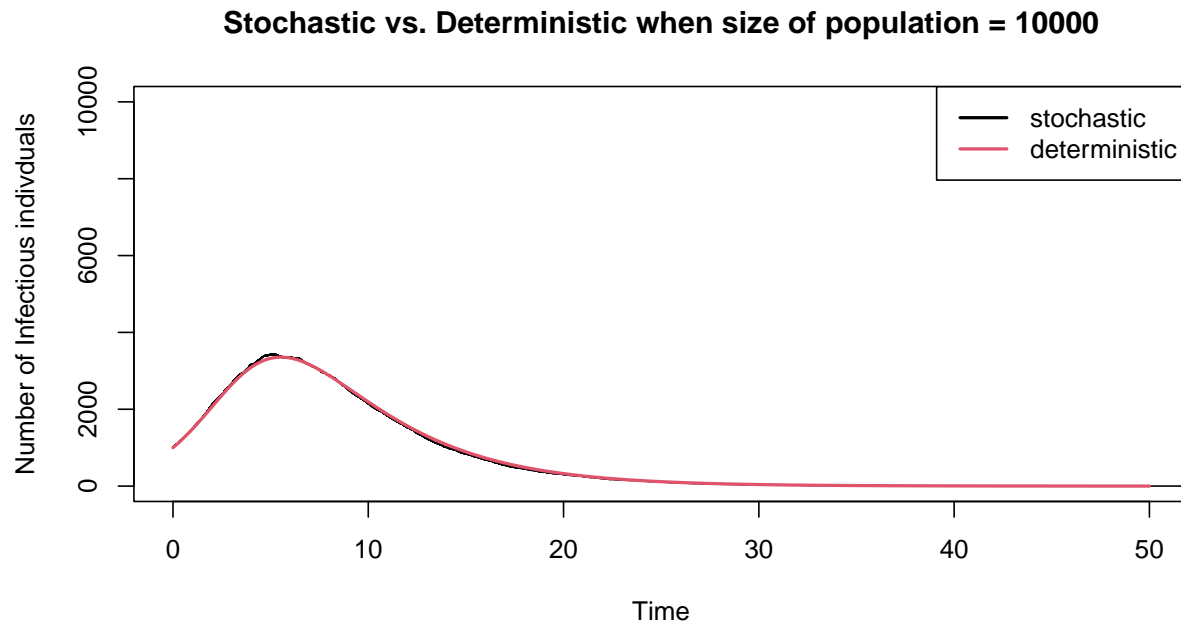


```
#####
# YOUR CODE HERE, do the same for the other two cases N=1000, 10000
#####
set.seed(123)
compare_DS(N = 1000)
legend(x = "topright", c("stochastic", "deterministic"),
      col = c(1, 2), lty = 1, lwd = 2)
title("Stochastic vs. Deterministic when size of population = 1000")
```

### Stochastic vs. Deterministic when size of population = 1000



```
set.seed(123)
compare_DS(N = 10000)
legend(x = "topright", c("stochastic", "deterministic"),
      col = c(1, 2), lty = 1, lwd = 2)
title("Stochastic vs. Deterministic when size of population = 10000")
```



- (e) Let  $\beta = 0.375$  and  $\gamma = 0.25$  (implying that  $R_0 = 1.5$ ), do 5000 simulations in three cases when the size of population  $N = 500, 1000$  and  $5000$  with one initial infective.

Stochastic SIR model create a function final size.

```
# total infected:
fsize_SIR <- function(T, beta, gamma, n, m, N) {
  # Initialize (x= number of S, y=number of I, z= y=number of R, t=event time)
  x <- n
  y <- m
  z <- 0
  t <- 0
  # size of population
  N <- n + m

  # Possible events: Infection(S->I) and Removal(I->R)
  eventLevels <- c("S->I", "I->R")

  # Loop until time T or the epidemic stops(there is no infectives)
  while (t < T & (y > 0)) {
    #####
    # YOUR CODE HERE, follow the codes in part d
    #####
  }
}
```

```

# Draw the waiting type for each possible event
wait <- rexp(2, c("S->I" = beta / N * x * y, "I->R" = gamma * y))
# Determine which event occurs first
i <- which.min(wait)
# Record Event Time
t <- t + wait[i]
# Update the number of S and I according to the event type
if (eventLevels[i] == "S->I") {
  x <- x - 1
  y <- y + 1
  z <- z + 1
} # if infection
if (eventLevels[i] == "I->R") {
  y <- y - 1
} # if recovery
}
return(z) # return the number of recovered
}

```

Do 5000 simulations with different size of community N.

```

#####
# YOUR CODE, fill in the blank space
#####
# set parameter values:
beta <- 0.375
gamma <- 0.25
Tmax <- 500 # time limit [0,500]
IO <- 1
nSim <- 5000 # number of simulations

compare_hist <- function(N) {
  df_fsize <- data.frame(sim = NA, totalinfected = NA)

  for (i in 1:nSim) {
    z <- fsize_SIR(T = Tmax, beta = beta, gamma = gamma,
                  n = N - IO, m = IO, N = N)

    df_fsize <- rbind(df_fsize, c(i, z))
  }
  return(df_fsize$totalinfected)
}

```

Make a histogram of the final size distribution in each case. Give your comments.

```

N <- 500

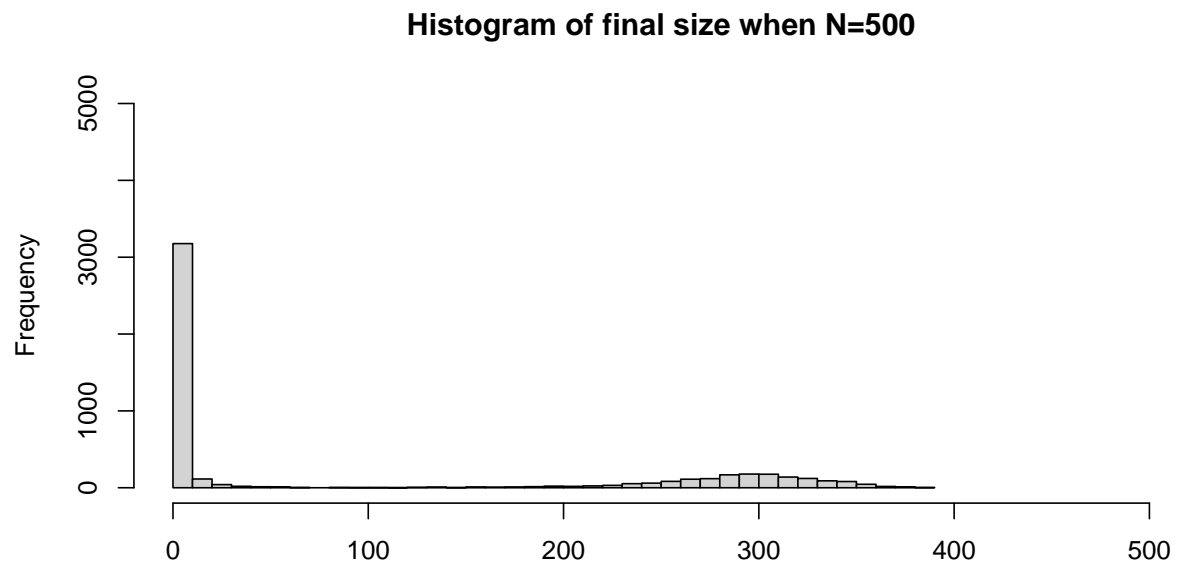
hist(compare_hist(N = 500),
     breaks = 50,
     main = "Histogram of final size when N=500",
     xlab = "",
     xlim = c(0, N),

```

```

ylim = c(0, nSim)
)

```



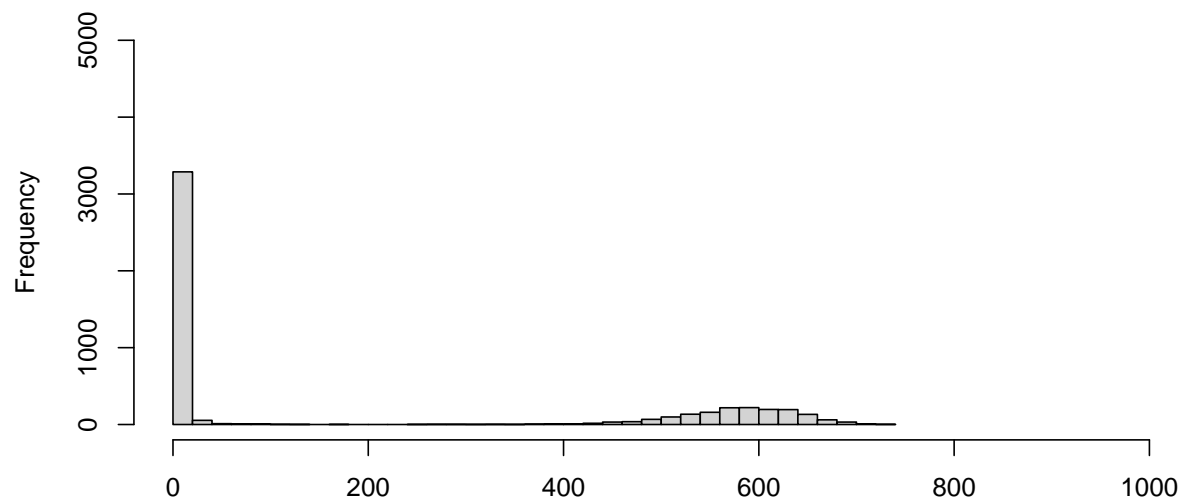
```

#####
# YOUR CODE
#####
# do the other two cases: N = 1000 and 5000
hist(compare_hist(N = 1000),
      breaks = 50, main = "Histogram of final size when N=1000",
      xlab = "", xlim = c(0, 1000), ylim = c(0, 5000)
)

```

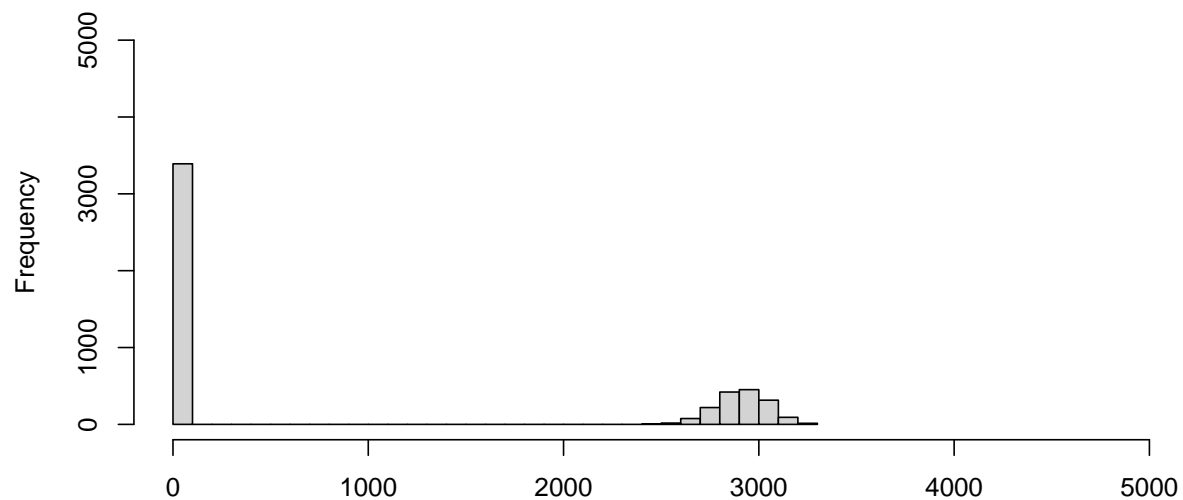


**Histogram of final size when N=1000**



```
hist(compare_hist(N = 5000),  
      breaks = 45, main = "Histogram of final size when N=5000",  
      xlab = "", xlim = c(0, 5000), ylim = c(0, 5000)  
)
```

**Histogram of final size when N=5000**



## Exercise 1.3 (SEIR Model)

(a)

```
#####
# YOUR CODE HERE, fill in the blank space
#####
# do the same way as for SIR model, but with four states: S E I R

#####
# Function to compute the derivative of the ODE system for SEIR Model
#
# t - time
# y - current state vector of the ODE at time t
# parms - Parameter vector used by the ODE system
#
# Returns:
# list containing dS(t)/dt, dE(t)/dt, and dI(t)/dt
#####

seir <- function(t, y, parms, N) {
  beta <- parms[1]
  rho <- parms[2]
  gamma <- parms[3]
  N <- parms[4]

  S <- y[1]
  E <- y[2]
  I <- y[3]
  return(list(c(S = -beta / N * S * I, E = beta / N * S * I - rho * E,
                I = rho * E - gamma * I)))
}

# Parameter Values:
times <- seq(0, 100, length = 1000)
gamma <- 1 / 7
beta <- 0.4
rho <- 1 / 5
I0 <- 1
N <- 100

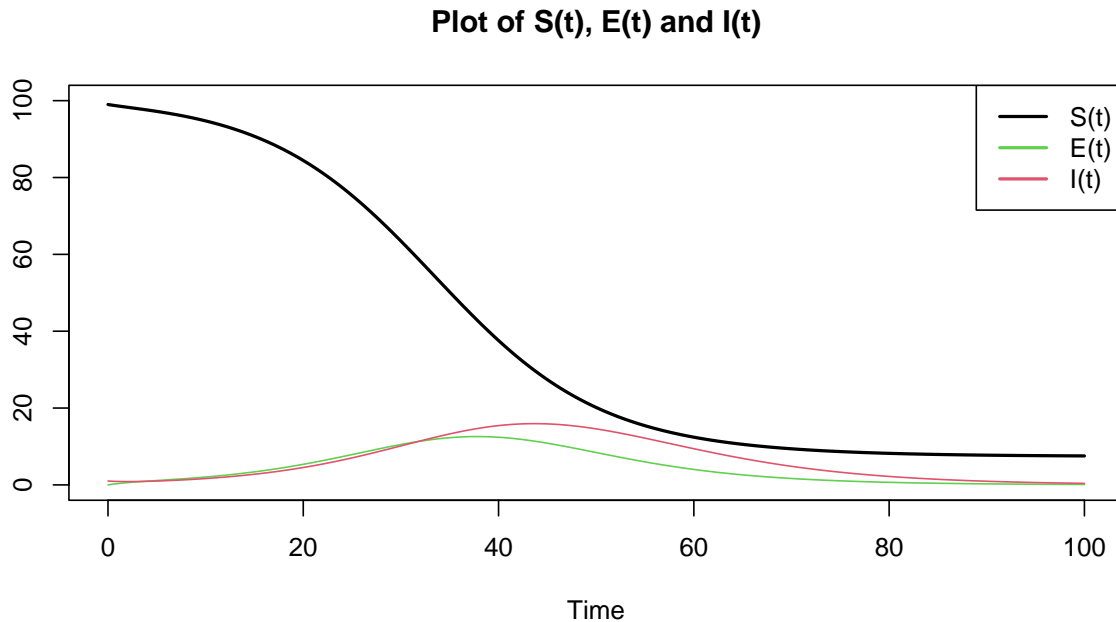
# Solve the ODE and plot
sol_seir <- lsoda(
  y = c(N - I0, 0, I0), # insert the corresponding initial conditions
  times = times,
  func = seir,
  parms = c(beta, rho, gamma, N)
)

plot(times, sol_seir[, 2],
  type = "l", lwd = 2, col = 1, xlab = "Time",
  ylab = "", xlim = c(0, 100), ylim = c(0, N)
)
```

```

lines(times, sol_seir[, 3], col = 3)
lines(times, sol_seir[, 4], col = 2)
legend(x = "topright", c("S(t)", "E(t)", "I(t)"),
      col = c(1, 3, 2), lty = 1, lwd = 2)
title("Plot of S(t), E(t) and I(t)")

```



- (b) Modify the SEIR model such that  $\beta(t)$  becomes a time-dependent function. Create a function to compute the derivative of the ODE system for SEIR Model with changing point.

```

#####
# YOUR CODE HERE, fill in the blank space
#####

#####
#
# t - time
# y - current state vector of the ODE at time t
# parms - Parameter vector used by the ODE system
#
# Returns:
# list containing dS(t)/dt, dE(t)/dt, and dI(t)/dt
#####
seir_change <- function(t, y, parms, N) {
  beta0 <- parms[1]
  beta1 <- parms[2]
  t1 <- parms[3]
  w <- parms[4]
  rho <- 1 / 5
  gamma <- parms[5]
  N <- parms[6]

```

```

S <- y[1]
E <- y[2]
I <- y[3]

# write the time-dependent rate \beta(t) using ifelse(,)

beta <- function(t) {
  ifelse(t <= t1 - w, beta0, ifelse(t > t1 + w, beta1,
    beta0 + (beta1 - beta0) / (2 * w) * (t - (t1 - w))
  ))
}

# write the return list containing dS(t)/dt, dE(t)/dt, and dI(t)/dt
return(list(c(S = -beta(t) / N * S * I, E = beta(t) / N * S * I - rho * E,
  I = rho * E - gamma * I)))
}

# plot \beta(t):
# plot(time, b, type="l", xlab="Time", ylab="beta(t)")
# title("Plot of beta(t)")

```

Take  $t_1 = 30$ ,  $w = 5$ ,  $\beta_0 = 0.4$  and  $\beta_1 = 0.12$ , solve the ODE system for the SEIR model with time-varying transmission rate in part (b) numerically using R command `lsoda` and plot  $S(t)$ ,  $E(t)$  and  $I(t)$  for  $t \in [0, 100]$ .

```

#####
# YOUR CODE HERE, fill in the blank space
#####
# set the parameter values: such as
beta0 <- 0.4
beta1 <- 0.12
t1 <- 30
w <- 5
gamma <- 1 / 7
N <- 100
I0 <- 1
time <- seq(0, 100, length = 1000)

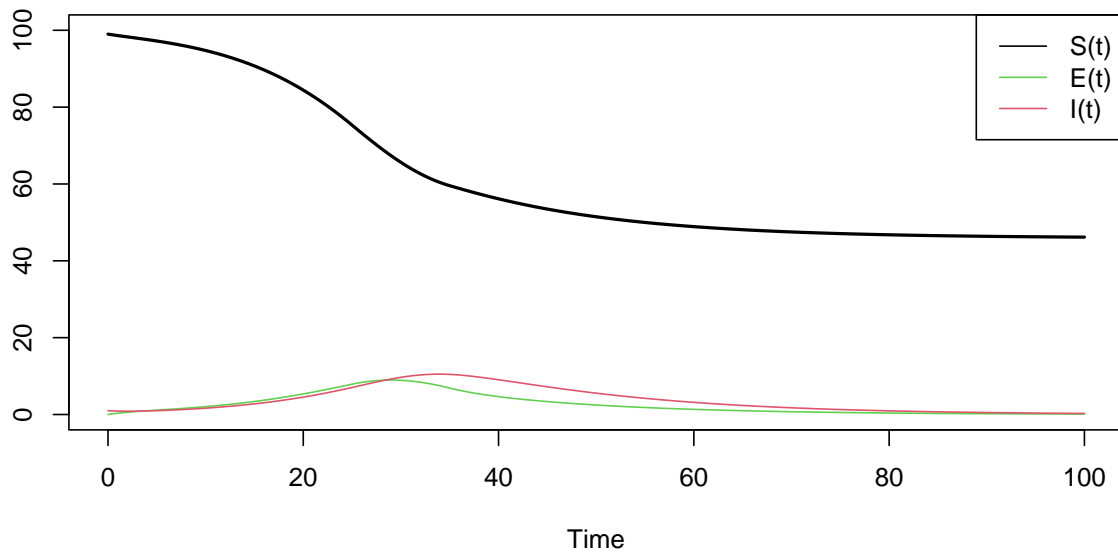
# solve the ODE system for the changed SEIR model

sol_SEIRchange <- lsoda(
  y = c(N - I0, 0, I0),
  times = time, func = seir_change,
  parms = c(beta0, beta1, t1, w, gamma, N)
)

# plot curves of $S(t), E(t)$ and $I(t)$
plot(times, sol_SEIRchange[, 2],
  type = "l", lwd = 2, col = 1, xlab = "Time",
  ylab = "", xlim = c(0, 100), ylim = c(0, 100)
)
lines(times, sol_SEIRchange[, 3], col = 3)
lines(times, sol_SEIRchange[, 4], col = 2)
legend(x = "topright", c("S(t)", "E(t)", "I(t)"), col = c(1, 3, 2), lty = 1)
title("Plot of S(t), E(t) and I(t) in SEIR model with time-dependent beta(t)")

```

**Plot of  $S(t)$ ,  $E(t)$  and  $I(t)$  in SEIR model with time-dependent  $\beta(t)$**



- (d) Now for  $N = 100, 1000$  and  $10000$ , do one simulation of the stochastic SEIR epidemic starting from fraction infected 10% with exponentially distributed incubation period with mean 5 days and the above time-changing  $\beta(t)$ . Overlay it on the plot of the deterministic curve as done in Exercise 2.

Simulate a stochastic SEIR epidemic model.

```
#####
# YOUR CODE HERE, fill in the blank space
#####

#####
#
# Params:
# [0,T] - time horizon
# beta - infection rate
# gamma - recovery rate
# rho - rate of (E -> I)
# n - initial number of susceptibles.
# m - initial number of infectives, here m=1
# three events: "S->E", "E->I", "I->R"
#####

stoch_SEIR <- function(T, n, m, parms, N) {
  beta0 <- parms[1]
  beta1 <- parms[2]
  t1 <- parms[3]
  w <- parms[4]
  gamma <- parms[5]
  rho <- parms[6]
```

```

N <- parms[7]

# write the function for beta(t)
beta <- function(t) {
  ifelse(t <= t1 - w, beta0, ifelse(t > t1 + w, beta1,
    beta0 + (beta1 - beta0) / (2 * w) * (t - (t1 - w))
  ))
}

# Initialize (x= number of S, y=number of I, z=number of E, t=event time)
x <- n
y <- m
z <- 0
time <- 0
# Possible events:
eventLevels <- c("S->E", "E->I", "I->R")
# Initialize result
df_SEIR <- data.frame(time = time, x = x, y = y, z = z, event = NA)
# Loop until time T or the epidemic stops(there is no infectives)
while (time < T & (y > 0)) {
  # Draw the waiting type for each possible event
  b <- beta(time)
  wait <- rexp(3, c("S->E" = b / N * x * y,
    "E->I" = rho * z, "I->R" = gamma * y))
  # Determine which event occurs first
  i <- which.min(wait)
  # Record Event Time
  time <- time + wait[i]
  # Update the number of S, I, E according to the event type
  if (eventLevels[i] == "S->E") {
    x <- x - 1
    z <- z + 1
  }
  if (eventLevels[i] == "E->I") {
    z <- z - 1
    y <- y + 1
  }
  if (eventLevels[i] == "I->R") {
    y <- y - 1
  }
  # Store result
  df_SEIR <- rbind(df_SEIR, c(time, x, y, z, i))
}
# Re-code event type and return
df_SEIR$event <- factor(eventLevels[df_SEIR$event], levels = eventLevels)

return(df_SEIR)
}

```

```

#####
# YOUR CODE HERE, fill in the blank space
#####

```

```

# set paramter values:

```

```

beta0 <- 0.4
beta1 <- 0.12
t1 <- 30
w <- 5
gamma <- 1 / 7
rho <- 1 / 5
c <- 0.1

compare_ds_seir <- function(N) {
  beta <- function(t) {
    ifelse(t <= t1 - w, beta0, ifelse(t > t1 + w, beta1,
      beta0 + (beta1 - beta0) / (2 * w) * (t - (t1 - w))
    ))
  }
  # Do stochastic simulations
  set.seed(123)
  traj <- stoch_SEIR(T = 100, n = N * (1 - c), m = N * c,
    parms = c(beta0, beta1, t1, w, gamma, rho, N))

  # plot stochastic I(t)

  plot(traj$time, traj$y,
    type = "s", ylim = c(0, N), xlab = "Time",
    ylab = "Number of Infectious individuals", xlim = c(0, 100), col = 1
  )

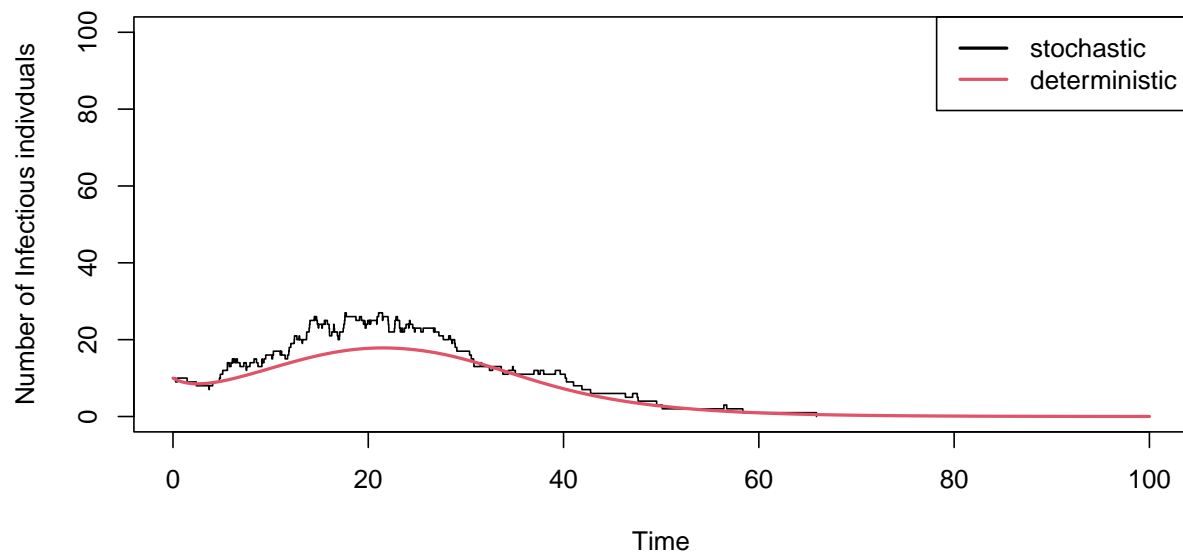
  # Solve the deterministic ODE

  solution <- lsoda(
    y = c(N * (1 - c), 0, N * c), times = seq(0, 100, length = 1000),
    func = seir_change, parms = c(beta0, beta1, t1, w, gamma, N)
  )
  lines(solution[, "time"], solution[, 4], col = 2, lwd = 2)
  invisible()
}

# plot stochastic and deterministic for different N
compare_ds_seir(N = 100)
title("stochastic vs deterministic for SEIR model with beta(t) when N = 100")
legend(x = "topright", c("stochastic", "deterministic"),
  col = c(1, 2), lty = 1, lwd = 2)

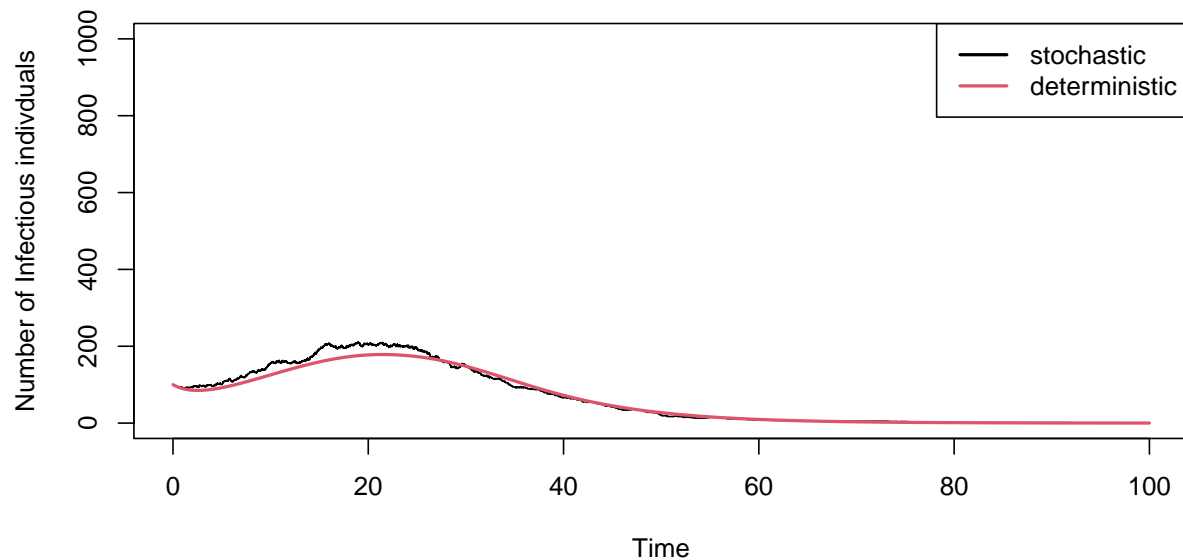
```

### stochastic vs deterministic for SEIR model with $\beta(t)$ when $N = 100$



```
# do the same for other two cases
compare_ds_seir(N = 1000)
title("stochastic vs deterministic for SEIR model with  $\beta(t)$  when  $N = 1000$ ")
legend(x = "topright", c("stochastic", "deterministic"),
      col = c(1, 2), lty = 1, lwd = 2)
```

### stochastic vs deterministic for SEIR model with $\beta(t)$ when $N = 1000$





```
compare_ds_seir(N = 10000)
title("stochastic vs deterministic for SEIR model with beta(t) when N = 10000")
legend(x = "topright", c("stochastic", "deterministic"),
      col = c(1, 2), lty = 1, lwd = 2)
```

