Game-theoretic
Foundations of Multi-agent Systems

Lecture 1: Introduction

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Outline

Course Mechanics

- Course website
  - https://ece.uwaterloo.ca/~smzahedi/crs/ece750/
  - All course information, lecture notes, assignments, etc.

- Office hours
  - Book appointment online
  - Or catch me after class, or send me email to setup meeting

- Prerequisites
  - None
  - Helpful knowledge: algorithms, probability, comp. complexity, and optimization

- Anti-requisites
  - ECON.412 (Topics in Game Theory), CO.456 (Introduction to Game Theory), CS.886 (Multi-agent Systems), and MSCI.724 (Game Theory and Recent App)

Outline

Tentative Topics

- Introduction to multi-agent systems
  - Overview of game theory and mechanism design, rationality and self-interest, utility theorem, risk attitudes

- Games in normal form
  - Pure and mixed-strategy Nash equilibrium, iterative elimination of dominated strategies, price of anarchy, correlated equilibrium, computing solution concepts of normal-form games

- Games in extensive form
  - Perfect and imperfect-information games, finite and infinite-horizon games, subgame-perfect equilibrium, backward induction, one-shot deviation principle

Tentative Topics (cont.)

- Beyond normal and extensive-form games
  - Repeated games, stochastic games, Bayesian games, congestion games, trigger strategies, folk theorems, Bayes-Nash equilibrium, auctions

- Mechanism design
  - Optimal auctions, revenue-equivalence theorem, revelation principle, incentive compatibility, VCG mechanisms

- Learning in multi-agent systems
  - Multi-agent reinforcement learning, fictitious play, Bayesian learning, regret-minimization learning

Textbook and References

- Y. Shoham and K. Leyton-Brown, **Multi-agent Systems: Algorithmic, Game-theoretic, and Logical Foundations** (available online)

- N. Nisan, et al. **Algorithmic Game Theory** (available online)

- T. Roughgarden, **Twenty Lectures on Algorithmic Game Theory** (notes available online)

- D. Fudenberg and D. Levine, **The Theory of Learning in Games**

- D. Fudenberg and J. Tirole, **Game Theory**

- M. J. Osborne and A. Rubinstein, **A Course in Game Theory** (available online)

Course Requirements

- Graduate offering
  - Quizzes 5%
  - Assignments 20%
  - Research project 45%
  - Final exam 30%

- Undergraduate offering
  - Quizzes 5%
  - Assignments 20%
  - Survey project 25%
  - Final exam 50%

Survey Project (Undergraduate Offering)

- Goal is to review existing literature in sub-area of multi-agent systems and possibly explore open research questions in that sub-area

- Two milestones: proposal (15% - about three weeks into the term) and final written report (85% - end of the term)

Research Project (Graduate Offering)

- Goal is to try to do something novel, rather than merely surveying existing work

- Only real constraint is that it has to have something to do with covered material

- Projects may be theoretical, experimental (based on simulations), experimental (based on real-world data), a useful software artifact, or any combination thereof

- Three milestones: proposal (10%), oral progress report (10%), and final written report (80%)

- Creativity is encouraged

Outline

Questions and Problems in (Computational) Game Theory

- How should we represent games (i.e., strategic settings)?
  - Game-theoretic representations not always concise enough

- What does it mean to solve a game?
  - Solution concepts from game theory, e.g., Nash equilibrium

- How computationally hard is it to solve a games?
  - Can we solve them approximately?

- Is there a role for (machine) learning in games?

- What types of modeling problems do we face when addressing real-world games?

Optimization vs. Game Theory

- Optimization theory: Optimize a single objective over a decision variable

$$\min . \quad u(x)$$
$$\text{s.t.} \quad x \in X$$

- Game theory: Study of multi-agent decision problems
  - Model cooperation and competition between intelligent and rational decision makers
  - n agents, each chooses some $x_i \in X_i$, and has a utility function

  $$u_i(x_i, x_{-i}), \quad x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

  - What are the possible outcomes?
  - Steady-state, stable operating point, characteristics?
  - How do you get there (learning dynamics, computation of equilibrium)?

Guess 2/3 of the Average (2/3-Beauty Contest Game)

- Everyone writes down an integer between 0 and 100
- Person closest to 2/3 of the average wins
- Example:
  - A says 50 - B says 10 - C says 90
  - 2/3 of average(50, 10, 90) = 2/3 × 50 = 33.33
  - A is the closest ($|50 - 33.33| = 16.67$), so A wins

Mechanism Design

- Inverse Game Theory: Design of game forms to implement desirable outcomes
  - E.g. to incentivize independent agents to reveal their types truthfully

- In Economics, MD is all about designing the right incentives
  - Mechanisms map signals from independent agents into allocations and payments
  - Optimal Mechanisms (Myerson): Design a mechanism that maximizes profits
  - Efficient Mechanisms (Vickrey-Clarke-Groves (VCG) Mechanisms): Design a mechanism to maximize a social or system-wide objective

- In CS/Engineering, focus is more on the design of efficient decentralized protocols that take into account incentives
  - Mechanisms in networks, distributed and online mechanisms, mechanisms that operate with limited information

Example: Single-item Auction

- **Sealed-bid auction**: every bidder submits bid in sealed envelope

- **First-price** sealed-bid auction: highest bid wins, pays amount of own bid

- **Second-price** sealed-bid auction: highest bid wins, pays amount of 2nd-highest bid

Which Auction Generates More Revenue?

- Each bid depends on
  - bidder's true valuation for the item (utility = valuation - payment)
  - bidder's beliefs over what others will bid
  - and . . . auction mechanism used

- In first-price auction, it does not make sense to bid your true valuation
  - Even if you win, your utility will be 0

- In second-price auction, it always makes sense to bid your true valuation
  (we will see this later)

What is Mechanism Design Again?

- Designing mechanism = designing a game

- We can use game theory to predict what will happen under mechanism
  - If agents act strategically

- When is a mechanism "good"?
  - Should it result in outcomes that are "good" for reported or for true preferences?
  - Should agents ever end up lying about their preferences (in game-theoretic solution)?
  - Should it always generate the best allocation?
  - Should agents ever burn money?(!?)

- Can we solve for the optimal mechanism?

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