Game-theoretic Foundations of Multi-agent Systems

Lecture 1: Introduction

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UNIVERSITY OF
WATERLOO

Outline

1. Course Mechanics

2. Course Outline

3. Overview of Game Theory and Mechanism Design

Course Mechanics

- Course website
  - https://ece.uwaterloo.ca/~smzahedi/crs/ece750/
  - All course information, lecture notes, assignments, etc.

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- Prerequisites
  - None
  - Helpful knowledge: algorithms, probability, comp. complexity, and optimization

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- Anti-requisites
  - ECON.412 (Topics in Game Theory), CO.456 (Introduction to Game Theory),
    CS.886 (Multi-agent Systems), and MSCI.724 (Game Theory and Recent App)

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- Introduction to multi-agent systems
  - Overview of game theory and mechanism design, rationality and self-interest, utility theorem, risk attitudes

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- Games in extensive form
  - Perfect and imperfect-information games, finite and infinite-horizon games, subgame-perfect equilibrium, backward induction, one-shot deviation principle

Tentative Topics (cont.)

- Beyond normal and extensive-form games
  - Repeated games, stochastic games, Bayesian games, congestion games, trigger strategies, folk theorems, Bayes-Nash equilibrium, auctions

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- Learning in multi-agent systems
  - Multi-agent reinforcement learning, fictitious play, Bayesian learning, regret-minimization learning

Textbook and References

- Y. Shoham and K. Leyton-Brown, **Multi-agent Systems: Algorithmic, Game-theoretic, and Logical Foundations** (available online)

- N. Nisan, et al. **Algorithmic Game Theory** (available online)

- T. Roughgarden, **Twenty Lectures on Algorithmic Game Theory** (notes available online)

- D. Fudenberg and D. Levine, **The Theory of Learning in Games**

- D. Fudenberg and J. Tirole, **Game Theory**

- M. J. Osborne and A. Rubinstein, **A Course in Game Theory** (available online)

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- Graduate offering

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  - Final exam 50%

Survey Project (Undergraduate Offering)

- Goal is to review existing literature in sub-area of multi-agent systems and possibly explore open research questions in that sub-area

- Two milestones: proposal (15% - about three weeks into the term) and final written report (85% - end of the term)

Research Project (Graduate Offering)

- Goal is to try to do something novel, rather than merely surveying existing work

- Only real constraint is that it has to have something to do with covered material

- Projects may be theoretical, experimental (based on simulations), experimental (based on real-world data), a useful software artifact, or any combination thereof

- Three milestones: proposal (10%), oral progress report (10%), and final written report (80%)

- Creativity is encouraged

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- What types of modeling problems do we face when addressing real-world games?

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- Optimization theory: Optimize a single objective over a decision variable

$$\text{min.} \quad u(x)$$
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  - n agents, each chooses some $x_i \in X_i$, and has a utility function

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  - How do you get there (learning dynamics, computation of equilibrium)?

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- Example:
  - A says 50 - B says 10 - C says 90
  - 2/3 of average(50, 10, 90) = 2/3 × 50 = 33.33
  - A is the closest ($|50 - 33.33| = 16.67$), so A wins

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- In Economics, MD is all about designing the right incentives
  - Mechanisms map signals from independent agents into allocations and payments
  - Optimal Mechanisms (Myerson): Design a mechanism that maximizes profits
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- In CS/Engineering, focus is more on the design of efficient decentralized protocols that take into account incentives
  - Mechanisms in networks, distributed and online mechanisms, mechanisms that operate with limited information

Example: Single-item Auction

- Sealed-bid auction: every bidder submits bid in sealed envelope

- First-price sealed-bid auction: highest bid wins, pays amount of own bid

- Second-price sealed-bid auction: highest bid wins, pays amount of 2nd-highest bid

Which Auction Generates More Revenue?

- Each bid depends on
  - bidder's true valuation for the item (utility = valuation - payment)
  - bidder's beliefs over what others will bid
  - and ... auction mechanism used

- In first-price auction, it does not make sense to bid your true valuation
  - Even if you win, your utility will be 0

- In second-price auction, it always makes sense to bid your true valuation (we will see this later)

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- Can we solve for the optimal mechanism?

Acknowledgment

- This lecture is a slightly modified version of ones prepared by

    - Asu Ozdaglar [MIT 6.254]

    - Vincent Conitzer [Duke CPS 590.4]