

# 1 Anisotropic Fresnel coefficients

Let us consider an interface between an isotropic and an anisotropic medium, as shown in figure 1. Since we are focusing on the case of uniaxial anisotropy (only the  $\epsilon_{zz}$  component of the permittivity tensor is different) the case of TE polarized waves is trivial, since the incident wave does not interact with the anomalous component of the permittivity tensor. TM polarized light has only the x component of magnetic intensity field  $\mathbf{H} = (H_x, 0, 0)$ . Let us now consider a wave incident on the interface from the isotropic side. Utilizing Maxwell's equations we can connect the magnitudes of components of the electric intensity fields with the x-component of magnetic intensity:

$$\mathbf{k}_1 \times \mathbf{H}_1 = -\omega \epsilon \mathbf{E}_1 \quad (1)$$

$$E_{1y} = \frac{-k_{1z} H_{1x}}{\omega \epsilon_{\text{rel}}} \quad (2)$$

$$E_{1z} = \frac{k_{1y} H_{1x}}{\omega \epsilon_{\text{rel}}} \quad (3)$$

furthermore, since k-vector of the incident wave has the form

$$\mathbf{k}_1^+ = (0, k_{1y}^+, k_{1z}^+) \quad (4)$$

we can immediately get the y-component from the angle of incidence

$$k_{1y}^+ = k_1^+ \sin(\theta_1) = \sqrt{\epsilon_{\text{rel}}} \frac{\omega}{c} \sin \theta_1 \quad (5)$$

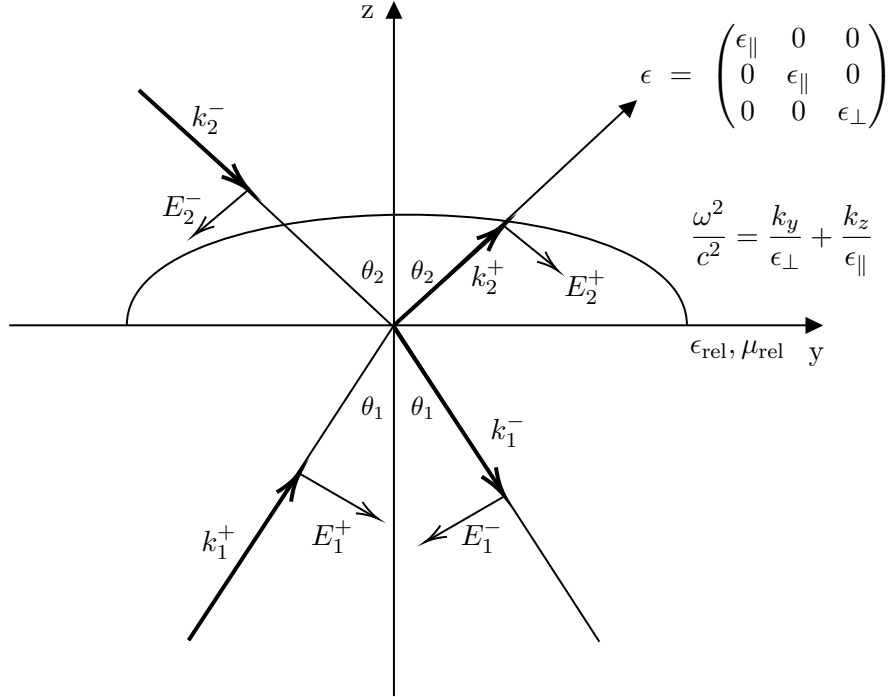


Figure 1: An interface between an isotropic and anisotropic medium. Wavevectors corresponding to incident, reflected and transmitted waves are shown on both sides of the interface. TM polarization.

and utilizing dispersion relation in the anisotropic medium we can get the z-component of transmitted k-vector:

$$k_z = \sqrt{\epsilon_{\parallel}} \frac{\omega}{c} \sqrt{1 - \frac{\epsilon_{\text{rel}}}{\epsilon_{\perp}} \sin^2 \theta_1} \quad (6)$$

because the y component of the wave vector is conserved across the interface. Now we can formulate conservation relations for other fields:

$$\mathbf{H}_1^+ + \mathbf{H}_1^- = \mathbf{H}_2^+ \quad (7)$$

$$E_{1y}^+ + E_{1y}^- = E_{2y}^+ \quad (8)$$

These can be obtained by applying Maxwell's equations to the interface. In equations (7) and (8) we are considering only the incident, reflected and transmitted waves (we are excluding the wave labeled as  $\mathbf{k}_2^-$ ) Now, let us apply equation (1) on the anisotropic side of the interface. Again, carefully observing, that permittivity is a tensor, we get:

$$E_{2y} = \frac{-k_{2z} H_{2x}}{\omega \epsilon_{\parallel}} \quad (9)$$

$$E_{2z} = \frac{k_{2y} H_{2x}}{\omega \epsilon_{\perp}} \quad (10)$$

Since  $\mathbf{k}_1^+ = -\mathbf{k}_1^-$  we can combine equations (2), (8) and (9) to get

$$\frac{-k_{1z}^+ H_{1x}^+}{\omega \epsilon_{\text{rel}}} + \frac{+k_{1z}^+ H_{1x}^-}{\omega \epsilon_{\text{rel}}} = \frac{-k_{2z}^+ H_{2x}^+}{\omega \epsilon_{\parallel}} \quad (11)$$

further applying equation (7) we obtain:

$$\mathbf{H}_{2x}^+ = \frac{\frac{2k_{1z}^+}{\epsilon_{\parallel}}}{\frac{k_{2z}^+}{\epsilon_{\parallel}} + \frac{k_{1z}^+}{\epsilon_{\text{rel}}}} \mathbf{H}_{1x}^+ \quad (12)$$

Now let us calculate the Poynting vector

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \quad (13)$$

$$\mathbf{S} = (0, H_x E_z, -H_x E_y) \quad (14)$$

plugging in equations (2), (3), (9) and (10):

$$\mathbf{S}_1^+ = \frac{\mathbf{H}_{1x}^{+2}}{\omega \epsilon_{\text{rel}}} (0, k_{1y}^+, k_{1z}^+) \quad (15)$$

$$\mathbf{S}_2^+ = \frac{\mathbf{H}_{2x}^{+2}}{\omega} (0, \frac{k_{2y}^+}{\epsilon_{\perp}}, \frac{k_{2z}^+}{\epsilon_{\parallel}}) \quad (16)$$

Coefficient of transmittivity is given as

$$T = \frac{S_{2z}^+}{S_{1z}^+} = \left| \frac{H_{2x}^+}{H_{1x}^+} \right|^2 \frac{k_{2z}^+ \epsilon_{\text{rel}}}{k_{1z}^+ \epsilon_{\parallel}} \quad (17)$$

or, plugging in (12):

$$T = \frac{4k_{1z}^+ k_{2z}^+ \epsilon_{\text{rel}} \epsilon_{\parallel}}{\left( \frac{k_{2z}^+}{\epsilon_{\parallel}} + \frac{k_{1z}^+}{\epsilon_{\text{rel}}} \right)^2} \quad (18)$$

Transmission amplitude is given simply as

$$t = \frac{2k_{1z}^+ / \epsilon_{\text{rel}}}{\frac{k_{2z}^+}{\epsilon_{\parallel}} + \frac{k_{1z}^+}{\epsilon_{\text{rel}}}} \quad (19)$$

From now on, these will be referred to as  $T_1$  and  $t_1$ . Let us now consider the converse problem — a wave traveling from the anisotropic medium and being transmitted into the isotropic medium. We will refer back to figure 1. Incident wave has k-vector  $k_2^-$  and is refracted with angle  $\theta_1$ . Let us now find the angle of incidence  $\theta_2$ . This is given simply as

$$\tan \theta_2 = \frac{k_{2y}^-}{k_{z2}^-} = \frac{k_y}{k_{2z}} = \sqrt{\frac{\epsilon_{\text{rel}}}{\epsilon_{\parallel}}} \frac{\sin \theta_1}{\sqrt{1 - \frac{\epsilon_{\text{rel}}}{\epsilon_{\perp}} \sin^2 \theta_1}} \quad (20)$$

Equivalents of equations (7) and (8) will take the form

$$\mathbf{H}_2^- + \mathbf{H}_2^+ = \mathbf{H}_1^- \quad (21)$$

$$E_{2y}^- + E_{2y}^+ = E_{1y}^- \quad (22)$$

using exactly the same method we arrive at the relation

$$t_2 = \frac{2k_{2z}^- / \epsilon_{\parallel}}{\frac{k_{2z}^-}{\epsilon_{\parallel}} + \frac{k_{1z}^-}{\epsilon_{\text{rel}}}} \quad (23)$$

and using  $t = r + 1$ :

$$r_2 = \frac{\frac{k_{2z}^-}{\epsilon_{\parallel}} - \frac{k_{1z}^-}{\epsilon_{\text{rel}}}}{\frac{k_{2z}^-}{\epsilon_{\parallel}} + \frac{k_{1z}^-}{\epsilon_{\text{rel}}}} \quad (24)$$

## 2 Transmission through an anisotropic slab, Fabry-Perot oscillations

Standard formula for Fabry-Perot oscillations still holds:

$$T = \frac{1}{1 + \frac{4R_0}{(1 - R_0)^2} \sin^2 k_{2z}\ell} \quad (25)$$

where  $\ell$  is the width of the slab and  $R_0 = |r_2|^2$ . Since  $r_2$  for the TE polarization is identical to the isotropic case, entire relationship reduces to its usual, isotropic, form. TM polarization is different, since the  $r_2$  amplitude takes on a different form. In the graph below, we present a depiction of the dependency of the transmission coefficient on the incidence angle and angular frequency of incident electromagnetic radiation. The graph also includes the same plots of the case of a photonic crystal with  $N = 10$  layers; layer  $a$  has  $\epsilon_a = 1$  and  $\ell_a = 1$  and layer  $b$  has  $\epsilon_b = 4$  and  $\ell_b = 0.5$ . Approximation of a photonic crystal by an anisotropic slab should be valid only for small values of angular frequency. Note also the clearly visible Brewster's angle for TM polarization. The value of  $\omega_0$  is  $\frac{\pi}{2\sqrt{\epsilon_b\ell_b}}$ .

