Ex 9.2.

$$P(x=1 \mid y=0) = \frac{P(y=0 \mid x=1)\,P(x=1)}{0.85 \times 0.9 + 0.15 \times 0.1}$$

$$= 0.019$$

Ex 9.4

$$P(x=1 \mid y=0) = \frac{0.15 \times 0.1}{0.15 \times 0.1 + 1 \times 0.9}$$

$$= \frac{0.015}{0.015 + 0.9} = \frac{0.015}{0.915}$$

Ex 9.7 marginal distribution

$P(y=0) = P(y=1) = 0.5$.

$H(Y) = H(0.5)$

$H(Y|X) = H(0.15)$

$I(X;Y) = 1 - 0.61 = 0.39$ bits

Ex 9.8

marginal distribution. $P(y=0) = 0.5 + 0.075 = 0.575$

$P(y=1) = 0.5 \times 0.85 = 0.425$.

$H(Y) = H_2(0.425)$

$H(Y|X=0) = 0$

$H(Y|X=1) = H_2(0.15)$

$H(Y|X) = 0.5\,H_2(0.15)$

$I(X;Y) = H_2(0.425) - \frac{1}{2}H_2(0.15) = 0.679$ bits.

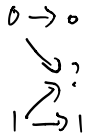
Ex 9.12

$I(X;Y) = H_2((1-f)p_1 + (1-p_1)f) - H_2(f)$

要 $(1-f)p_1 + (1-p_1)f = 0.5$.

$p_1 + f - 2p_1 f = 0.5 \Rightarrow p_1 = \frac{0.5 - f}{1 - 2f} = \frac{1}{2}$

Ex9.13 $H(Y) = H((1-p_1)(1-f), f, p_1(1-f))$



$H(Y|X) = H_2(0.15)$

$I(X;Y) = H((1-p_1)(1-f), f, p_1(1-f)) - H_2(f)$

用 $H(X)$ 等. (只有两次)

$H(X) - H(X|Y) = H(X) - (f(H(X|y=?))$

$= 1-f$

Ex9.14.

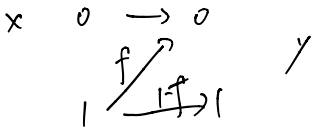
	00	01	10	11
00	0.7225	0	0	0
?0	0.1275	0	0.1275	0
10	0	0	0.7225	0
0?	0.0225	0.1275	0	0
??	0.0025	0.0225	0.0225	0.0225
1?	0	0	0.1275	0.0225
01	0	0.7225	0	0
?1	0	0.1275	0	0.1275
11	0	0	0	0.7225

选两位重复最少的.

10 和 11

Ex9.15

Z channel



$P_X = \{p_0 \quad p_1\}$.

$P(y=1) = (1-f)p_1$

$P(y=0) = p_0 + f p_1$

$H(Y) = H_2(p_1(1-f))$

$H(Y|X) = p_0 \cdot 0 + p_1 H_2(f)$

$I(X;Y) = H(Y) - H(Y|X) = H_2(p_1(1-f)) - p_1 H_2(f)$

$f = 0.15$. $p_1^* = 0.445$

$C(Q_Z) = 0.685$

对概率求导为

$$\frac{d}{dP_1} I(X;Y) = (1-f)\log_2 \frac{1-P_1(1-f)}{P_1(1-f)} - H_2(f)$$

令其为0. 得

$$\frac{1}{P_1(1-f)} = 2^{\frac{H_2(f)}{1-f}} + 1$$

$$P_1^* = \frac{1/1-f}{2^{\frac{H_2(f)}{1-f}} + 1}$$

$$\lim_{f\to 1} P_1^* = 1/e$$

Exq.17

$$H(X) = 5 \times \frac{1}{5}\log_2 5 = \log_2 5$$

$$H(Y|X) = 4 \times \frac{1}{4}\log_2 4 = \log_2 4$$

$$I(X;Y) = \log_2 1.25$$

Exq.18

(1) $$P(x=1|y,\alpha,\sigma) = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\alpha)^2}{2\sigma^2}} \cdot \frac{1}{2}}{\frac{1}{\sqrt{2\pi\sigma^2}}\left(e^{-\frac{(y-\alpha)^2}{2\sigma^2}} + e^{-\frac{(y+\alpha)^2}{2\sigma^2}}\right)\cdot\frac{1}{2}}$$

$$= \frac{1}{1+e^{-\frac{4\alpha y}{2\sigma^2}}} = \frac{1}{1+e^{-\frac{2\alpha y}{\sigma^2}}}$$

(2) $\alpha(y) > 0$ $P(x=1|y,\alpha,\sigma) > \frac{1}{2}$

$\alpha(y) < 0$ $P(x=1|y,\alpha,\sigma) < \frac{1}{2}$

∴ 只需看 $\alpha(y)$ 符号决定 x 是 1 或 1

3×9.20

P(at least two people have the same birthday)

$= 1 - \frac{364}{365} \cdot \frac{363}{365} \cdots$

$= 1 - \frac{364 \cdots 342}{365^{23}}$ $= 1 - \frac{A(A-1)\cdots(A-S+1)}{A^S}$

一共可能的有 $S(S-1)/2$ 对.

某一对生日相同的概率是 $\frac{1}{A}$

$\therefore E(N) = \frac{S(S-1)}{2A}$