

A Comprehensive Guide to the CASIO fx-9860GII

THE BETTER GRAPHING CALCULATOR FOR H2 MATHEMATICS

QUICK REFERENCE MATH INCLUDED

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Acknowledgements

Were we to delve deeper into die Welt der Mathematik, we would scarce be able to differentiate the real and the imaginary.

Who would have known, that with functions ranging limitless, and planes stretching to infinity, we could prosper in this World of Mathematics. Just as a plane cannot be fully defined without its normal vector, we thank our tutor Mr Yee Weng Hong for his patient direction, without whom this guide would not have been possible.

We also thank the Hwa Chong Institution Mathematics Department, for it has been the distribution of their awesome and systematic notes, which we have sampled very generously in this friendly guide, that has greatly increased our probability of success.

And to our friends, with whom our paths have crossed, thank you for supporting us the whole way.

This integrated guide is dedicated to everyone who has dared to think different.

Who, against all odds, shall stand firm against the evil Alevelus Examinus – with their Casio fx-9860GII's calculator in hand.

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Preface

This guide is provided as-is and was compiled by a bunch of students in 2016. We tried our best to be mathematically and technically accurate. But software versions (Ver 2.04 now) do change, and this guide may not be valid indefinitely. However, the source of this file is publically available on GitHub. Feel free to contact us.

If there are anything you want to do, but you can't seem to know how to, try Google, or RTFM. Otherwise, just experiment a bit. The CASIO interface should be intuitive enough.

Do note that some notes inside this guide were taken from the H2 Mathematics notes provided by the Hwa Chong Institution (College Section) Math Department.

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Chapter 1

The Basics

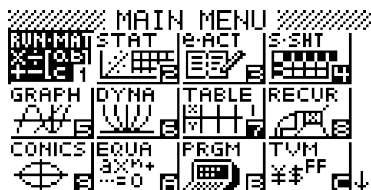
Every command you will ever need is organized neatly in the [OPTN] button on your graphing calculator (GC).

Unfortunately there are some areas where the CASIO calculator isn't very intuitive and we need some documentation, and this section is a primer of the functionalities that we can't really sort into specific H2 Mathematics topics.

Tip: You can just press the button that corresponds to the number/letter at the bottom right of the icons in the menu:

1.1 Display Modes

As a special mention that you probably did not notice, we shall first introduce the different display modes that the calculator is



capable of. In the Set Up menu ($[SHIFT] + [MENU]$), there are different display options, and we summarize their corresponding functions as follows:

Mode	What it does
Fix n	Rounds off answer to n decimal places
Sci n	Rounds off answer to n significant figures
Norm	Converts answer to scientific notation when the value x lies outside of the following range: Norm1: $10^{-3} > x > 10^{10}$ Norm2: $10^{-9} > x > 10^{10}$

In addition to these options, there is also a **Eng** mode that adds on to those above mentioned modes. The $/E$ indicator is on display while the engineering notation is in effect (e.g. Norm1/E).

In the engineering mode, answers are displayed with standard order-of-magnitude prefixes, and the symbol that makes the mantissa a value from 1 to 1000 is automatically chosen:

E (Exa)	$\times 10^{18}$	m (milli)	$\times 10^{-3}$
P (Peta)	$\times 10^{15}$	μ (micro)	$\times 10^{-6}$
T (Tera)	$\times 10^{12}$	n (nano)	$\times 10^{-9}$
G (Giga)	$\times 10^9$	p (pico)	$\times 10^{-12}$
M (Mega)	$\times 10^6$	f (femto)	$\times 10^{-15}$
k (kilo)	$\times 10^3$		

You can also type these prefixes when doing calculations: they are available under [OPTN] > $\triangleright \times 2$ > **ESYM (F1)** . You might find these extremely useful when doing calculations for Physics.

Under that same menu, there is also the **ENG** and $\overleftarrow{\text{ENG}}$ which shifts the decimal place of the displayed value 3 digits to the (left/right), and (decreases/increases) the exponent by 3 respectively.

1.2 Radian/Degree/Gradian

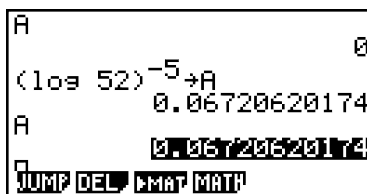
Like the majority of other scientific calculators, this GC is able to accept input and display the magnitude of angles in either degrees, radians, or gradians. To change between the 3, enter the Set Up menu, and scroll down until you see a row labelled **Angle**. The 3 different units are displayed at the bottom of the screen and can be selected by the [F1] to [F3] buttons.

Note that this setting is universal throughout your GC - it will change the units angles are handled in every mode including but not limited to usual calculations, graphing, etc, unless

you specifically specify otherwise (not covered in this section). Therefore, it is vital that you check your units before undergoing any calculation, especially before and during examinations.

1.3 Storing values into variables

You can also store values into variables, just like with your trusty fx-95SG PLUS Calculator. To do this, simply type the value and then use the \rightarrow to store it into a variable:



This also works for vectors and matrices, but the data type must correspond before and after the \rightarrow .

1.4 Solving for the roots of a polynomial

There are mainly 2 ways to solve for the roots of a polynomial using the GC:

- Using the EQUA > Poly (F2) app
- Plot a graph and solve for roots

The **EQUA > Poly** is quite intuitive, and similar to the standard issue CASIO fx-95SG scientific calculator, so we will just note some limitations/features of this GC:

- The polynomial solver only accepts real coefficients
- You can change the **Set Up > Complex Mode** setting to $a + bi$ for imaginary roots (but not imaginary coefficients)

To plot a graph, you go to **Graph**. Refer to Section 2 (Graphing) for more information.

1.5 Solving a 1-variable Equation

There are mainly 2 ways to solve a 1-variable equation (e.g. $e^x + 5x = 1$) using the GC:

- Using the **EQUA > Solver (F3)** app
- Using **Solve** or **SolveN**
- Plotting a graph
 - Move everything to one side and solve for root
 - Solve for the intersections of 2 or more graphs

1.5.1 Solving using EQUA > Solver

To use the built-in equation solver:

1. Go to **EQUA > Solver** (F3)
2. Enter the equation you would like to solve under **Eq.**. Remember to put the = sign.
 - Alternatively, you can use **RCL** (F1) to recall functions entered into **GRAPH** or **TABLE**
3. Press **[EXE]** and put your initial guess under **X**. An example can be as such:

X	0
Lower	-9E+99
Upper	9E+99

4. Press **Solv** (F6)

Note: The initial guess isn't very important for equations with unique solutions. However, if the equation has multiple solutions, it will give the answer closer to the original guess.

For example, given the equation $(x - 1)(x - 3) = 0$, entering the following will give the corresponding answers:

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X	Answer
0	1
1.5	1
2	1
2.5	3
4	3

After solving the equation, you can always access the value again by going to RUN·MAT and typing X .

1.5.2 Solving using Solve or SolveN

Alternatively, you can use the `Solve` or `SolveN` functions in RUN·MAT to obtain the solution. They can be found under [OPTN] > CALC(F4) .

Their usage is as follows, as far as we have discovered:

```
Solve(<equation>,<initial guess>)  
SolveN(<equation>)
```

Both `Solve` and `SolveN` will give you only 1 answer, and works similar to how the `EQUA` > `Solver` works.

1.6 Solving a Multi-variable Linear System

There are 3 types of linear systems, namely systems with:

- No solution
- 1 unique solution
- Infinitely many solutions

To solve a multi-variable system of linear equations using your GC, there are 2 main ways:

1. Using the EQUA > Siml (F1) app
2. Using matrices

1.6.1 Using the EQUA > Siml app

While you can use matrices to solve any kind of linear system, you can use the EQUA > Siml app *only if the linear system has a unique solution*.

Note: Should you try to use the EQUA > Siml app to solve for a linear system with no solution, or infinitely many solutions, the calculator would just throw a **Ma Error** at you while you stare, bemused, wondering why the calculator is so bad to you.

Anyhow, the app is intuitive enough, so I shall skip it.

1.6.2 Using Matrices

To solve a linear system the matrix way, let's first consider the following linear system:

$$\begin{cases} x + y + 2z = 9 \\ 2x + 4y - 3z = 1 \\ 3x + 6y - 5z = 0 \end{cases}$$

By taking the coefficients of the variables x , y and z , we can form the following augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array} \right)$$

To solve for x , y and z :¹

1. Go to `RUN·MAT > ▸MAT`
2. Select `Mat A` and enter the dimensions
 - m is the number of rows, in this case 3
 - n is the number of columns, in this case 4
3. Enter the augmented matrix above accordingly, putting the augmented column into the 4th column:

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

¹ Fn Keys have been left out to save space.

4. Press [EXIT] then [EXIT] to go back to the main RUN-MAT screen
5. Go to [OPTN] > MAT > ▸ > Rref
6. Then ▸ × 3 > Mat, then [ALPHA] + A, giving the following:

Rref Mat A

7. Press [EXE], and then you should be able to get the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

This can then be rewritten as an augmented matrix:

$$\begin{array}{ccc|c} x & y & z & \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}$$

which can then be rewritten as:

$$\begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$$

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Notice that the left side of the final augmented matrix is the identity matrix. This is what you would typically see for a linear system with a unique solution.

This same procedure can be used to find the solutions of any linear system with any number of variables.

The following are possible matrices you can obtain after the `Rref` command for the other types of linear systems:

- **No solution**

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{array} \right) \rightarrow \begin{cases} x = 2 - 2t \\ y = 3 - 3t \\ 0z = 1 \end{cases}$$

where $t \in \mathbb{R}$

Since $0 = 1$ is not possible, the above linear system is inconsistent and has no solutions.

- **Infinitely many solutions**

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right) \rightarrow \begin{cases} x = 2 - 2t \\ y = 3 - 3t \\ z = t \end{cases}$$

where $t \in \mathbb{R}$

In this case, the answer is obtained easily through back-substitution. Rewriting the augmented matrix, we observe that:

$$\begin{cases} x + 2z = 2 \\ y + 3z = 3 \end{cases}$$

By letting z be a parameter t ($t \in \mathbb{R}$), we observe that:

$$\begin{cases} x + 2t = 2 \\ y + 3t = 3 \\ z = t \end{cases} \rightarrow \begin{cases} x = 2 - 2t \\ y = 3 - 3t \\ z = t \end{cases}$$

Thus obtaining the general solution.

You will find this useful for both “System of Linear Equations” and “Vectors” (See Chapter 3).

For a more in-depth understanding of what is going on here, you may wish to refer to Section 5.2.

1.7 How to reset your calculator

This is a question that we get asked all the time: 2-8-On DOESN'T WORK, how in the world do I reset this calculator for the exam?

Well, worry not my intrepid mathematician, because we will answer your question in this section. But just to make it clear, we are not *too* sure what the *correct* way to do this is, and these instructions here exists purely as a form of reference.

Okay so, once that is out of the way, there are generally 2 ways of resetting your calculator, and they are both in **SYSTEM > Reset (F5) > ▷ (F6)**:

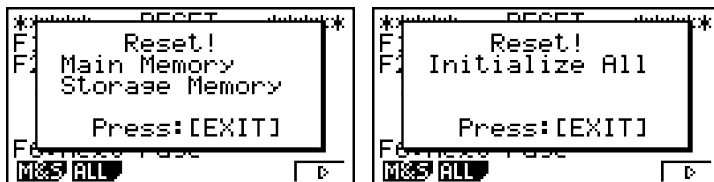
```

*****  RESET  *****
F1:Main&Storage
F2:Initialize All

F6:Next Page
M&S ALL

```

Generally, the `Main&Storage` option will do you well. It deletes any and all calculations made, and programs you have added, and resets your current settings (such as fraction display modes) back to the factory setting. Showing the end result of this screen to the examiner will *usually* suffice.



Alternatively, there is the `Initialize All` option, which initializes your whole calculator to the factory state (aka the state in which you bought your calculator), this will also work.

There is also a button to *restart* the calculator should it act abnormally, or freeze. You will find this at the back of your calculator right above the battery holder.

1.8 Others

Other functionalities, such as copy and pasting, will not be covered here. Please check out the manual provided by Casio on their website. A quick google for “Casio fx9860GII user manual” should suffice.

Chapter 2

Graphing

In this short chapter, we will explore the main functions of a graphing calculator, and that is to graph functions.

2.1 Basic Operations

To graph a function, go to **GRAPH** in the main menu. Enter the functions into **Y1** onwards. Press **DRAW (F6)** or **[EXE]** to graph. Most of these are pretty intuitive, just some things you should take note:

- The **Y** and **X**, etc. at the bottom of the screen is for entering the functions **Y1**, **Y2**, ... and **X1**, **X2**, For the variable x , use the **[X, θ , T]** button instead
- To restrict the domain of the function, type:

$$Y1 = f(x), [\text{start}, \text{end}]$$

Note: Restricting the domain disables the sketching of the inverse of a graph.

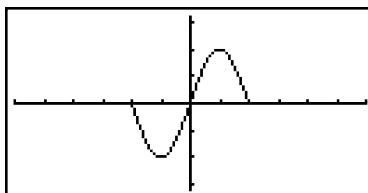


Figure 2.1: Graph of $\sin(x)$, for $-\pi \leq x \leq \pi$

After pressing DRAW (F6 or [EXE]), you can press [AC/on] to break the plotting script.

2.1.1 Solving for values from the plotted graph

To solve for anything, use [G-Solv] (F5) .

==== Add Intersection, and stuff

2.2 Sketching

Once a graph is plotted, you can sketch additional graphs on top of the already plotted graph. This includes sketching the inverse of a function, tangent and normal to a point on the curve.

All these are accessible under the Sketch menu (SHIFT > F4).

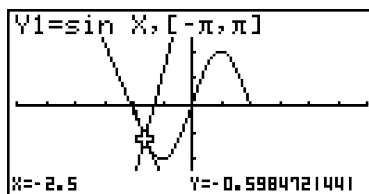


Figure 2.2: Graph of $\sin(x)$, for $-\pi \leq x \leq \pi$, with the normal and tangent drawn at $x = -2.5$

When sketching the normal/tangent, just press the number keys if you would like to enter a specific X -value to plot the normal/tangent at.

2.3 Plotting a table from function

To effectively use your calculator as a function generator, you can use the **TABLE** mode available in the menu:

1. Go to **TABLE** and then enter your function just like how you would in **GRAPH**
2. Go to **SET** (F5)

Once in table mode, you can enter values directly into the X column to generate values using the function.

This is especially useful when you are doing the same operation multiple times on different numbers, or when you are generating

solving a Sequences and Series question using the Method of Differences.

2.4 Taking Integrals

2.4.1 Definite Integrals

Your calculator can be used to solve definite integrals.

One way is to find the area under the graph after plotting the equation.

1. Plot the graph following the instructions from Section 2.
2. Press **G-Solv** (F5) , followed by **▷** (F6) and **$\int dx$** (F3) .
3. A cursor on the curve will now appear on the graph, along with x and y values. Move the cursor to the lower bound of the definite integral you're trying to find, and press **EXE**, followed by the upper bound, and press **EXE** again.
4. The area under the graph should now be shaded and the value of the area is displayed as $\int dx$.

This method is great if you already have the graph plotted and want to find the integral quickly without entering the equation again. If you do not need to plot the graph and simply want to find the definite integral directly, you may find the integration function in the **RUN-MAT** app more useful.

1. From the main menu, enter the **RUN·MAT** app (1).
2. Press the **OPTN** button and select the **CALC** option (**F4**) .
3. Under the **CALC** submenu, choose the $\int dx$ option (**F4**).
4. Enter the equation as per standard Mathematical notation.

Note: You should change the variable to be integrated from whatever letter it was originally to x since the calculator forces the integral to be integrated with respect to x as can be seen by the dx at the back.

2.4.2 Indefinite Integrals

SEAB bans all calculators capable of direct symbolic integration and differentiation. In other words, you cannot enter $3x^2$ into your graphing calculator and get x^3 or vice versa. To be fair, this feature was never in the international version of the Casio anyway.

Thankfully, even in the Singaporean version of the Casio, you can plot the graph of the integral visually.

For example, we will try to plot the integral of $2x$. Assuming that your GC is working properly and the Laws of Mathematics have not changed in drastic ways, the graph should be identical to that of x^2 .

1. From the main menu, enter the **GRAPH** app (5).

2. Under Y1, enter the function $2x$.
3. Move to Y2. Enter the following function:

$$Y2 = \int_0^x Y1 \, dx$$

Note: Y2 will take a significant amount of time to plot (around 5-10 seconds). It is recommended that you de-select Y1 by pressing **SEL** (F1) to prevent Y1 from being plotted, saving a couple of seconds.

4. Press **DRAW** (F6). A parabola that identical to $y = x^2$ should be yield. You can plot $y = x^2$ on top of the integral to confirm.

While this function is unable to give you the equation of the integral directly, you can use double check your integrals using this method. Plotting the integral using this method can also be useful if you have to find multiple definite integrals (using the trace function in the graph page).

Chapter 3

Vectors

Casio added support for vectors with the 02.04.5301 firmware update. This is added on top of support for Matrices, which will be covered mainly in Chapter 5 (Linear Algebra).¹

Caution: Vector capabilities will be disabled in the new *Exam Mode* that ships with the Ver. 2.09 update.² However, as of present we are not aware of any requirements to activate *Exam Mode* during SEAB's examinations.

¹While vectors are also covered in the Linear Algebra syllabus, this chapter will focus more on what is needed to tackle the H2 syllabus.

²We will not be covering this as we are covering Ver. 2.04.5301, the latest SEAB approved firmware version.

3.1 Entering a Vector

Functionally, vectors in this GC is no different from a 1 by n or a n by 1 matrix. Hence, you can use **Mat** and **Vct** interchangeably, so long as they are of the correct order.

To enter a vector into the GC:

1. Enter the RUN-MAT mode and select \triangleright MAT (F3)
2. Select $M \leftrightarrow V$ (F6). The top row should now say **Vector** and the **Vct A** row should be highlighted by default
3. Press **EXE** and enter the vector's dimensions. For the following example vector, it is a 3×1 vector, so enter $m = 3$ and $n = 1$

$$\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$

4. Enter the values in your vector into the three rows as shown on your textbook/exam paper
5. Press **EXIT** twice to return to the main RUN-MAT screen

Alternatively, if you are in RUN-MAT and your Input/Output is set to **Math** in the settings, you can enter vectors simply by going to **MATH** (F4) \triangleright **MAT** and then choosing the appropriately sized matrix. These entered matrices will replace your **Vct A**/etc. described in the next few sections.

Note that the labelling follows the conventional $m \times n$ format, where m is the number of rows, and n is the number of columns.

3.2 Vector Calculations

3.2.1 Addition and Subtraction

To perform addition or subtraction calculations between two vectors:

1. At the main **RUN·MAT** screen, press **[OPTN]**, followed by **MAT** (**F2**). Press **▷** (**F6**) twice, then select **Vct** (**F1**).
2. Press the **[ALPHA]** button, then press the corresponding letter that represents your first vector. For example, if you entered the vector at the first row previously, press the button that corresponds to **A** (which is **[X,θ,T]**).
3. Press the usual addition or subtraction signs at the bottom right of your calculator depending on which calculation you wish to do.
4. Repeat steps 1 and 2 for your second vector, then press **[EXE]** to calculate the result.

3.2.2 Dot Product

Vector specific calculations are slightly more complicated than addition or subtraction. To find the dot product of two vectors:

1. In the vector section under **OPTN > MAT**, select **DotP** (**F2**).
2. Press **Vct**, then press the corresponding letter that represents the first vector in your dot product expression.

3. Press the [,] button, then repeat step 2 but for the second vector in your dot product expression. It is good practice at this stage to press the [)] button to close the opening bracket that comes together with the **DotP**(function.
4. Press [EXE] to calculate the result.

3.2.3 Cross Product

Cross product calculations are similar to dot products. To find the cross product between two vectors:

1. In the vector section under **OPTN > MAT**, select **CrsP** (F3).
2. Press **Vct**, then press the corresponding letter that represents the first vector in your cross product expression.
3. Press the [,] button, then repeat step 2 but for the second vector in your cross product expression. It is good practice at this stage to press the [)] button to close the opening bracket that comes together with the **CrossP**(function.
4. Press [EXE] to calculate the result.

It is important to note here that the [×] button on your calculator will not work as a cross or dot product operator! Should you attempt to use it as such, your GC will return a **Dimension ERROR**.

3.2.4 Angle Between Two Vectors

Your GC is also capable of finding the angle between two vectors. To do so:

1. In the vector section under **OPTN > MAT**, select **Angle** (F4).
2. Press **Vct**, then press the corresponding letter that represents the first vector.
3. Press the **[,]** button, then repeat step 2 but for the second vector. It is good practice at this stage to press the **[)]** button to close the opening bracket that comes together with the **Angle(** function.
4. Press **[EXE]** to calculate the result. Note that the angle calculated can be in radians, degrees, or gradians depending on your settings at the moment. Refer to Section 1.2 if you are not sure how to change the units.

3.2.5 Unit Vectors

If you ever need to find the unit vector of a particular vector, you can do so directly with your GC. To do so:

1. In the vector section under **OPTN > MAT**, select **UnitV** (F5).
2. Press **Vct**, then press the corresponding letter that represents your vector. It is good practice at this stage to press the **[)]** button to close the opening bracket that comes together with the **UnitV(** function.

3. Press [EXE] to calculate the result.

3.3 Problems involving Vectors

3.3.1 Finding the line of intersection between 2 or more planes

Let's say you are given 2 planes:

$$\pi_1 : \mathbf{r} \cdot \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} = 4 \quad \text{and} \quad \pi_2 : \mathbf{r} \cdot \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = 2$$

First, convert them to linear equations:

$$\begin{cases} 3x + y + 5z = 4 \\ 5x + y + 3z = 2 \end{cases}$$

Then solve for the general solution as described in Section 1.6.2 (Using Matrices to Solve Multi-variable Linear System):

$$\begin{cases} x = t - 1 \\ y = -8t + 7 \\ z = t \end{cases} \quad \text{where } t \in \mathbb{R}$$

Separating out the t , we realize that the solution (the line of intersection) can be written as such:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{r} = \begin{pmatrix} 1 \\ -8 \\ 1 \end{pmatrix} t + \begin{pmatrix} -1 \\ 7 \\ 0 \end{pmatrix} \quad t \in \mathbb{R}$$

Similarly, you can expand this method to find the point of intersection (unique solution).

Chapter 4

Statistics

One major use of a graphing calculator is for use in statistics. In the following chapters, we will outline the methods with which we can use our GC. Calculator functions in this section can generally be found under `[OPTN] > [STAT]`. Permutation and Combinations will not be covered. Calculated variables can be found under `[VARS] > [STAT]`

In this chapter, all random variables are represented by capital letters (usually X or Y). Small letters are reserved to represent values (e.g. x).

Note: However, please do not use Z , as it is reserved for the standard normal distribution.

4.1 Discrete Random Variable

A discrete (countable) random variable $X \in \mathbb{Z}^*$ has expectation (or expected value or mean) μ

$$E(X) = \mu = \sum_{\text{all } x} xP(X = x) \quad (4.1)$$

(You can think of this as the weighted sum of all the possibilities for X)

and variance $\sigma^2 = \text{stddev}^2$

$$\text{Var}(X) = \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2 \quad (4.2)$$

Note: The above does not apply to continuous random variables.

The function that gives the probability that a discrete random variable is exactly equal to some value is called the *Probability Mass Function* (PMF). However, we will write PDF throughout this section instead, so as to be in-line with the curriculum/calculator functions.

4.1.1 Using Data

If given a set of histogram data (i.e. categories and its corresponding frequencies), one can calculate statistical properties of it.

For example, given the following data:

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

You can do 1-Variable Statistical calculations by:

1. Go into **STAT** and then enter the X-values into **List 1** and its frequency into **List 2**
2. Go to **CALC > SET** and adjust the settings accordingly for 1-Var (**XList:List1, Freq:List2**)
3. **[EXIT]** and then choose **1VAR**

If you want to further manipulate these calculated values (such as squaring σ to find variance), you can go to **RUN·MAT** and then **[VARS] > [STAT] > [X]** to finding the value(s) you wanted.

Let's do a simple example to illustrate how to do this:

– *Example:* Expressway accidents –

Over a period of 120 days, the number of serious accidents in a day along a certain expressway are recorded as follows:

0	1	2	3	4	5	6	7 or more
21	36	32	18	9	3	1	0

where the 1st row is the number of serious accidents in a day, and the 2nd is the number of days that number of serious accidents was recorded on.

To find the mean and variance of this dataset:

1. Go to STAT and enter the data appropriately:

	List 1	List 2	List 3	List 4
SUB				
1	0	21		
2	1	36		
3	2	32		
4	3	18		

GRAPH CALC TEST INTP DIST

2. Go to CALC > SET and set the 1Var XList as List1 and 1Var Freq as List2. You need not care about 2Var for this example
3. Press [EXIT] and then 1VAR (F1) and you should get the following:

1-Variable	
\bar{x}	=1.75833333
Σx	=211
Σx^2	=581
s_x	=1.3228494
s_x	=1.32839597
n	=120

↓

4. You now have the mean \bar{x} and standard deviation σx
5. To find the variance of this data, go to **RUN·MAT** and you will find σx under **VARS > STAT > X > σx** . Enter that and then square it.

You have now found that $\bar{x} = 1.7583$ and $\text{Var}(x) = 1.7499$.

***Sidenote:** Since the variance and the mean is about the same, we can thus conclude that a Poisson model may be valid for this data set.*

Refer to Section 4.4.1 (Plotting Statistics/Visualization) for plotting statistical data.

4.1.2 Binomial Distribution

A binomial random variable X has the following characteristics:

1. The experiment consists of n repeated independent trials
2. Each trial only has 2 outcomes: “success” or “failure”
3. The probability of a ‘success’ p is constant in each trial

The probability $P(X = x)$ of obtaining x successes in n trials is given by:

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \quad x \in \mathbb{Z}^* \quad (4.3)$$

and has $E(X) = np$ and $\text{Var}(X) = np(1 - p)$.

We write $X \sim B(n, p)$.

Note: the probability is only non-zero when $0 \leq x \leq n$ (i.e. there is an upper bound).

Binomial PDF

To obtain the probability $P(X = x)$, we can either use the formula, or use the in-built `BinomialPD` function:

`BinomialPD([X], n, p)`

When `X` is not provided, the output answer will be a `List`.

Alternatively, we can use the `STAT` app to obtain the probability:

1. Go into `STAT` (2 in the menu)
2. Press `DIST(F5) > BINM(F5) > Bpd(F1)`
3. Set the following:

Data	Variable
x	<code>X</code>
Numtrial	n
p	p
Save Res	<code>None</code>

4. Press `[EXE]` or scroll to `Execute` and press `CALC(F1)`
5. You can do further manipulation of the value obtained by:

- a) Going to RUN·MAT
- b) Go to [VARS] > STAT(F3) > RESLT(F6) > DIST(F3)
- c) There you can find the p value by pressing p (F1) and then [EXE]

Binomial CDF

To obtain the probability $P(X \leq x)$ (*cumulative distribution function* of x), we can use the in-built **BinomialCD** function:

$$\text{BinomialCD}([X], n, p)$$

Similarly, if X is not provided, the output answer will be a **List**.

Alternatively, following instructions in the previous section (Section 4.1.2 \Rightarrow Binomial PDF), and choosing **Bcd** instead.

As a general rule of thumb:

Answer in:	Use:
\mathbb{Z}	Table
\mathbb{R}	Graph

Refer to Section 2.3 for plotting results from a table, which might be useful for plotting the PDF/CDF of a binomial distribution, since $P(X = x)$ is undefined when $x \notin \mathbb{Z}^*$

4.1.3 Poisson Distribution

A random variable that follows a Poisson distribution has the following characteristics:

- Events occur randomly in a fixed interval and independently of one another.
- The mean rate of occurrence of the event is **constant** in the given interval.
- The probability of more than 1 event occurring within a short interval is **negligible**.

The probability $P(X = x)$ of obtaining x successes in a fixed interval is given by:

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}, \quad x \in \mathbb{Z}^* \quad (4.4)$$

We write $X \sim \text{Po}(\lambda)$, where λ is the mean number of occurrence of the event in that fixed interval.

This random variable X has $E(X) = \text{Var}(X) = \lambda$.

Note:

- X ranges over an infinite number of integer values (i.e. there is no upper bound)
- $\lambda \propto t$, where t is the length of the interval

Note:

$\lambda \in \mathbb{Z}$	2 modes
$\lambda \in \mathbb{R} \notin \mathbb{Z}$	1 mode

Poisson PDF

To obtain the probability $P(X = x)$, we can either use the formula, or use the in-built **PoissonPD** function:

$$\text{PoissonPD}(x, \lambda)$$

Alternatively, we can use the **STAT** app to obtain the probability:

1. Go into **STAT** (2 in the menu)
2. Press **DIST(F5) > POISN(▷ F1) > Ppd(F1)**
3. Set the following:

Data	Variable
x	X
μ	λ
Save Res	None

4. Press **[EXE]** or scroll to **Execute** and press **CALC(F1)**
5. You can do further manipulation of the value obtained by:
 - a) Going to **RUN·MAT**

- b) Go to [VARS] > STAT(F3) > RESLT(F6) > DIST(F3)
- c) There you can find the p value by pressing p (F1) and then [EXE]

Poisson CDF

The cumulative probability $P(X \leq x)$ can be calculated by using the in-built `PoissonCD` function:

$$\text{PoissonCD}(x, \lambda)$$

Alternatively, following instructions in the previous section (Section 4.1.3 \Rightarrow Poisson PDF), and choosing `Pcd` instead.

Additive Property of Poisson Random Variable

If $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$, where X and Y are **independent**, then:

$$X + Y \sim \text{Po}(\lambda + \mu) \quad (4.5)$$

4.2 Continuous Random Variable

The probability for a continuous variable X to fall within a particular region $[a, b]$ is given by:

$$P(a \leq X \leq b) = \int_a^b f(x) \quad (4.6)$$

where $f(x)$ is the *probability density function* (PDF) of X .

In particular,

$$\int_{-\infty}^{\infty} f(x) = 1$$

Moreover,

$$\begin{aligned} P(a \leq X \leq b) &= P(a \leq X < b) \\ &= P(a < X \leq b) \\ &= P(a < X < b) \end{aligned}$$

4.2.1 Normal Distribution

A (continuous) random variable $X \in \mathbb{R}$ that follows a normal distribution with mean μ and standard deviation σ has a *probability density function* (PDF) given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad (4.7)$$

We write $X \sim N(\mu, \sigma^2)$

Note: Z is reserved to represent the standard normal distribution. Do not use it to represent your variables (that do not follow standard normal distribution).

Normal Distribution PDF

There are one of 2 ways to plot a graph of the normal distribution PDF:

- Plot the actual equation
- Use the in-built `NormPD` function

However, it must be noted that the `NormPD` plots slower than using the actual equation. Using `G-Solv` is also slower.

The usage of `NormPD` is

$$\text{NormPD}(X, [\sigma, \mu])$$

In this case, X can be either a single value, or a `List`. The corresponding p -value will also be adjusted accordingly.

If σ and μ are not provided, the standard normal distribution with $\sigma = 1$ and $\mu = 0$ is assumed.

Note: You should not use this command to calculate the probability of a certain random variable $P(X = x)$ where $x \in \mathbb{R}$. This is because for a continuous random variable, this does not make sense. It should always be a range. (i.e. use the CDF in the next section)

Normal Distribution CDF

CDF stands for *Cumulative Distribution Function*. This can be calculated by taking the integral of the normal PDF from $-\infty$ to x . One can take integral by plotting the graph out (refer to previous section), and then `G-Solv > ∫dx`.

Alternatively, you can use the built-in `NormCD`:

$$\text{NormCD}(\text{<Lower>}, \text{<Upper>}, [\sigma, \mu])$$

If σ and μ are not provided, the standard normal distribution with $\sigma = 1$ and $\mu = 0$ is assumed.

Sidenote: You might realize that the value given by the calculator differs from that given by its Texas Instrument counterpart. This is no error as both calculators probably uses slightly different methods of calculating the probability.

As a yardstick, we have compared the answers given by both calculators to that given by WolframAlpha, and it seems that the Casio calculator gives a more accurate value.

To plot the Normal Distribution CDF, you can use:

$$Y = \text{NormCD}(-1\text{E}99, X, [\sigma, \mu])$$

Note: $-1\text{E}99$ is to simulate $-\infty$.

Alternatively, we can use the STAT app to obtain the probability:

1. Go into STAT (2 in the menu)
2. Press $\text{DIST}(\text{F5}) > \text{NORM}(\text{F1}) > \text{Ncd}(\text{F1})$
3. Set the following:

Data	Variable
Lower	<i>Lower</i>
Upper	<i>Upper</i>
σ	σ
μ	μ
Save Res	None

4. Press [EXE] or scroll to **Execute** and press CALC(F1)
5. You can do further manipulation of the value obtained by:
 - a) Going to RUN-MAT
 - b) Go to [VARS] > STAT(F3) > RESLT(F6) > DIST(F3)
 - c) There you can find the p value by pressing p (F1) and then [EXE]

Note: You cannot use this method for plotting

Finding the value given the probability

One sometimes need to find a given $P(X < a) = b$ where $a, b \in \mathbb{R}$ and b is the probability of X being less than a .

To do this, we need the **InvNormCD** function built into the calculator. The usage of **InvNormCD** is as follows:

$$\text{InvNormCD}(b, [\sigma, \mu])$$

This will give you back a .

4.3 Hypothesis Testing

The calculator has in built apps for hypothesis testing. In the H2 Math syllabus, we are mainly interested in the Z-test and t-test, both of which are included as apps in the calculator.

4.3.1 Z-test

To do a Z-test:

1. Go into **STAT** (2 in the menu)
2. Press **TEST** (F3) > **Z** (F1) > **1-s** (F1)
3. At the row labelled **Data**, ensure that it is set to **Variable** aka **Var** (F2) .
4. Select the type of test to be conducted, i.e. if $\mu \neq \mu_0$ (F1) , $\mu < \mu_0$ (F2) , or $\mu > \mu_0$ (F3) .
5. Enter the value of μ_0 , the mean of the population that is under H_0 to be rejected or not rejected.
6. Enter σ , the standard deviation of the population.
7. Enter \bar{x} , the mean of the sample.
8. Enter n , the number of samples taken.
9. Go to the row labelled **Execute**, and press **EXE**. A page with the Z -value and the p -value displayed accordingly.

Alternatively, with a set of data entered in the **STAT**

4.4 Use for Experimental Data/SPA

The calculator can be used to determine the equation of a best fit line/parabola etc. from a set of data from experiments. From the main menu, enter the **S-SHT** app (4).

In the app, simply enter each set of data into each row. Enter the data on the same axis under the same column.

For example, consider the following experimental data for an experiment involving electrical current (I) and potential difference (V):

V/V	I/A
0.7148	0.0588
0.7372	0.0633
0.7780	0.0752
1.058	0.143
1.200	0.170
1.340	0.210

Enter this set of data into the GC as follows:

SHEET	A	B
1	0.7148	0.0588
2	0.7372	0.0633
3	0.7780	0.0752
4	1.058	0.143
5	1.200	0.170
6	1.340	0.210

To plot the points onto a graph, first press the right arrow key (\triangleright , **F6**) and select the **GRPH** option (**F1**) .

Once in the **GRPH** menu, we will first configure the graph options.

1. Press **SET** (**F6**) and ensure that **StatGraph1** is shown on the first line and the **Graph Type** is **Scatter**.
2. Move to **XCellRange** and enter **B1:B6**. This means that the values from cells **B1** to **B6** inclusive are that for the horizontal x -axis.
3. Move to **YCellRange** and enter **A1:A6**.
4. Finally, ensure that the **Frequency** option is **1**.
 - You may select any **Mark Type** that suits your personal preferences.
5. Press **EXIT** to leave the configuration menu and return to the spreadsheet.

Now, press **GPH1 (F1)** . A graph with markings corresponding to the points on the spreadsheet should be yield.

Your calculator can find the equation of a best fit line or curve. In this case, the data is expected to be linearly related (and it is linearly related).

1. Press **CALC (F1)** .
2. There are now a number of options corresponding to different types of graphs and relationships such as x^2 , \ln , and e^x . In this case, our data is linear, so select **X (F2)** .
3. Select **ax+b (F1)** .
4. A table with various values is yield. The value a corresponds to the gradient, and b is the y -intercept. r^2 represents how closely the data fit to the line, with the value of 1 corresponding to a perfect, ideal fit.
5. To view the graph visually, select **DRAW (F6)** . To generate y -values using the equation, select **COPY (F5)** , and select one of the **Y1/Y2...** to copy to.

Note: You will need to re-enter the formula before graphing/using the table, or it will cause a condition error. This re-entering could simply be going to the function in **Table**, pressing the right arrow and then pressing **[EXE]**.

The linear regression result can also be obtained using the concept of best approximations, covered in Section 5.3 under Linear Algebra.

4.4.1 Plotting Statistics/Visualization

In this section, we would explore ways we can use the calculator to help us visualize statistics, just like how it would appear in our lecture notes. The steps here are purely provided as an example, and to expose you to these functions, and you should adapt accordingly for your own use.

Binomial/Poisson Distribution

To visualize a binomial distribution, let's first generate a list of values for a certain binomial distribution. For example, let's say that $X \sim B(50, \frac{1}{3})$:

We can generate a list of probabilities easily in RUN-MAT :

$$\text{BinomialPD}(50, \frac{1}{3}) \rightarrow \text{List 1}$$

Thereafter, go into STAT and you'll see the probabilities all listed out for you in the List 1:

	List 1	List 2	List 3	List 4
SUB				
1	1.5E-9			
2	3.9E-8			
3	4.8E-7			
4	3.8E-6			
	1.568328545E-09			
GRAPH CALC TEST INTR DIST				

We can then easily graph this result:

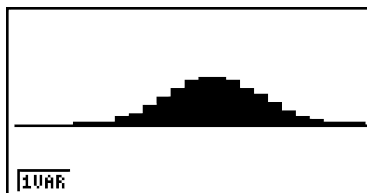
1. Go to GRPH (F1) > SET (F6)
2. Scroll to Graph Type and choose Bar (\triangleright > F3)

3. Select:

Data1	List1
Stick Type	Length

4. Press EXIT and then GPH1 (F1)

The graph should have been plotted. You can scroll, zoom in, or out to see more:



This should look like those graphs they have in your lecture notes.

Note: the x-coordinate of 1 corresponds to $P(X = 0)$

A similar procedure can be followed to plot a graph of a variable that follows a Poisson distribution.

Chapter 5

Linear Algebra

Some of you might find yourself doing linear algebra, and needing to do matrix manipulations. In this chapter, I will outline some basic concepts covered in MA1101R (NUS H3 Course), and how you can use your GC to find the answer.

You may find certain concepts here useful for H2 Mathematics as well, especially in terms of matrix manipulations. I suggest you read this chapter as a complement of Chapter 3 (Vectors).

5.1 Storing Matrices

accessing individual matrix elements

5.2 Solving Linear Systems

introduce augmented matrix how to augment if 2

5.2.1 Elementary Row Operations and Row-Echelon Forms

row equiv

5.3 Best Approximations

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