Electric Fields

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Expectations

- Please pay attention to this electrifying presentation
- If not you'll miss out on a lot of important content, such as this joke:
  Q: What is the name of the first electricity detective?
  A: Sherlock Ohms

Analogy with Gravity

- Please flip to pg. 26 of your notes
- Notice how similar the laws governing electric fields and gravitational fields are?
- Please keep that in mind as we go through the presentation!

Charges

- Two kinds of charges: Positive charge and negative charge
- Like charges repel, unlike charges attract
- To charge a body negatively, we can add electrons or *(rarely)* remove protons
- To charge a body positively, we can remove protons or *(rarely)* add electrons

	Charge	Mass
Electron	$-e = 1.60x10^{-19}C$	$9.11x10^{-31}kg$
Proton	$+e = 1.60x10^{-19}C$	$1.67x10^{-27}kg$
Neutron	No charge (0 C)	$1.68x10^{-27}kg$

Table : Let's see who can memorize this table the fastest

Principle of Conservation of Charges

Principle of Conservation of Charges

The principle of conservation of charges states that charges cannot be created nor destroyed. Hence, for any closed system, the sum of all electric charges must be constant.

Principle of Conservation of Charges

If a system starts out with an equal number of positive and negative charges, theres nothing we can do to create an excess of one kind of charge in that system unless we bring in charge from outside the system (or remove some charge from the system). Likewise, if something starts out with a certain net charge, say $+100$ e, it will always have $+100$ e unless it is allowed to interact with something external to it.

Principle of Quantization of Charges

The charge on a single electron is $q_e = 1.60 \cdot 10^{-19} C$ (remember that $1C = 6.242 \cdot 10^{18} e$). All other charges in the universe consist of an integer multiple of this charge. This is known as charge quantisation:

$$Q = nq_e$$

Electrons and protons are not the only things that carry charge. Other particles (positrons, for example) also carry charge in multiples of the electronic charge.

Electric Field

Definition

An electric field is a region of space such that when a charge is placed at a point in the region, it would experience an electrical force acting on it.

Electric Field Strength

Definition

The electric field strength at a given point is the force per unit positive charge that acts on a small test charge placed at that point.

$$E = \frac{F}{q}$$

Rearranging, we get

$$F = q \cdot E$$

Direction of force on charge is dependent on the sign of the charge!
Let's work out Example 1 together.

Drawing Electric Field Lines

- Electric field lines always extend from a positively charged object to a negatively charged object, from a positively charged object to infinity, or from infinity to a negatively charged object.

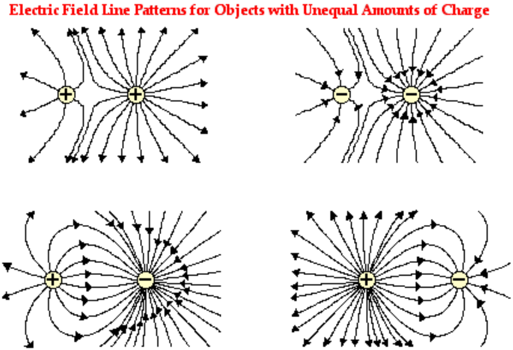

Figure : Representing Electric Field Lines

Drawing Electric Field Lines

- Number of lines drawn is proportional to magnitude of the charge

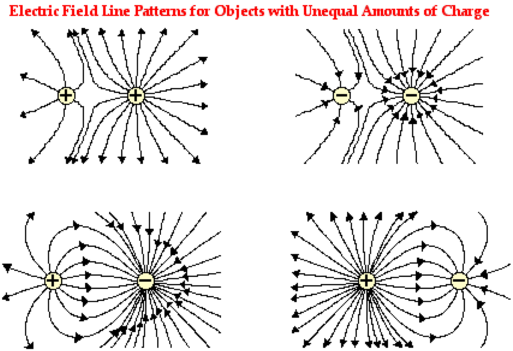

Figure : Representing Electric Field Lines

Drawing Electric Field Lines

- Field lines do not cross because the direction of E at a point is unique.

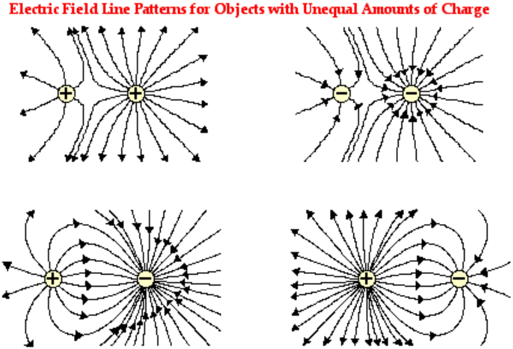

Figure : Representing Electric Field Lines

Drawing Electric Field Lines

- At locations where electric field lines meet the surface of an object, the lines are perpendicular to the surface

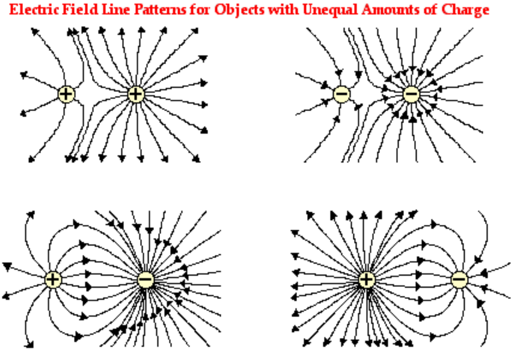

Figure : Representing Electric Field Lines

Electric Field Lines as an Invisible Reality

- Electric field lines are not real!
- The concept of an electric field arose as scientists attempted to explain the action-at-a-distance that occurs between charged objects
- First introduced by 19th century physicist Michael Faraday
- Rather than thinking in terms of one charge affecting another charge, Faraday used the concept of a field to propose that a charged object (or a massive object in the case of a gravitational field) affects the space that surrounds it
- As another object enters that space, it becomes affected by the field established in that space
- Viewed in this manner, a charge is seen to interact with an electric field as opposed to with another charge

Are you still with us?

Where do electrons play football? On an electric field!

Force between Two Point Charges

Couloumb's Law

The magnitude of the electrical force acting between two point charges is proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{QQ\prime}{r^2}$$

Permittivity

- Permittivity is the measure of the resistance that is encountered when forming an electric field in a medium. In other words, permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium.
- ϵ_0 is equal to approximately $8.85 \cdot 10^{-12}$ farad per meter Fm^{-1} in free space (a vacuum)

Principle of Superposition

Principle of Superposition (for electrical forces)

The resultant force on any one of them equals to the vector sum of the forces exerted by the other individual charges.

Let's work out Example 2 together.

Electric Field around a Point Charge

- Recall Coulomb's Law

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qq}{r^2}$$

Electric Field Strength

The magnitude of the electric field strength of a point charge Q at a distance r away from the field is

$$E = \frac{F}{q}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

Electric Field around a Point Charge

- Analogous to gravitational field strength
- Instead of $g = \frac{F_g}{m}$, we now have $E = \frac{F_E}{q}$!

Electric Field around a Point Charge

- E is a vector quantity, and its direction is at a point is given by the direction of the force experienced by a positive charge if it is placed at that point.
- The field is radial of a point charge. It is directed uniformly in all directions outward from the centre if Q is a positive charge and inward toward the centre if Q is a negative charge. At all points that are equal distance away from Q the magnitude of E is the same.

Principle of Superposition (for Electric Field)

The resultant electric field E at a point P in an electric field is the vector sum of the fields at P due to each point charge in the system.

- Very intuitive, right?
- Let's now work on Example 3 together.

Electric Potential

Definition of Electric Potential

The electric potential V, at a point in an electric field, is defined as the work done per unit positive charge, by an external force, in moving a small test charge from infinity to that point in the electric field.

$$V = \frac{W}{q}$$

- SI unit of electric potential is JC^{-1} but it is more common to use the volt, V

Electric Potential

Potential energy is the capacity for doing work which arises from position or configuration. In the electrical case, a charge will exert a force on any other charge and potential energy arises from any collection of charges. For example, if a positive charge Q is fixed at some point in space, any other positive charge which is brought close to it will experience a repulsive force and will therefore have potential energy.

Potential due to a Point charge

Electric potential in a field can arise due to a point charge Q, which is given by

Electric Potential

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$$

- The potential at a point can always be determined by

$$V_{resultant} = V_1 + V_2 + V_3 ...$$

- Let's work on example 4 together.

Electric Potential Energy

Electric Potential Energy

The electric potential energy, U of a charge at a point in an electric field is defined as the work done by an external agent in moving the charge from infinity to that point.

Electric Potential Energy

Relationships between U and V

$$U = qV$$

- Recall that potential energy in gravitational field is $U = m\phi$
- Notice any similarities?

Electric Potential Energy

Relationships between U and V

Work done by the external agent in moving a charge Q from point A to point B in an electric field is given by

$$U_{A\to B} = U_B - U_A = Q(V_B - V_A)$$

Similar to gravitational potential energy, the electric potential energy is stored in a system of charges and not possessed by a single charge Q.

Electric Potential Energy

- Work done in moving a charge from point A to point B, and
- The change in potential energy is independent of the path taken

Potential Energy of a System of Two Point Charges

- Given two charges Q_1 and Q_2 separated by a distance r. Q_1 sets up an electric field around the region and Q2 is in the electric field of Q_1. Hence,
- Potential of the electric field by Q1 at where Q2 is

$$V = \frac{Q_1}{4\pi\epsilon_0 r}$$

- Multiplying both sides by Q_2, we get the potential energy of the system, U

Potential Energy Between Two Point Charges

$$U = Q_2 V = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$$

Potential Energy of a System of Two Point Charges

Since any negative sign on the charge represents the direction, the magnitude can be found by ignoring the charge and including it afterwards when the magnitude of potential energy is obtained.

Potential Energy of a System of Two or More Charges

We will demonstrate the potential energy of a system with three charges

- Step One: Bring q_1 from infinity to the point. As there is no electric field initially in the region, work done

$$W_1 = 0J$$

Potential Energy of a System of Two or More Charges

- Step Two: Bring q_2 from infinity to the point. As q_1 is already in place, it would have set up an electric field in the region and an external force will need to do work against the electrical force experienced by q_2 due to its interaction with the field set up by q_1. Hence, work done
$$W_2 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

Potential Energy of a System of Two or More Charges

- Step Three: We bring q_3 from infinity to its position in the electric field. Since, q_1 and q_2 are now in place, the electric field in the region is now due to q_1 and q_2, and the external force will need to do work against the force on q_3 due to its interaction with the field set up by both q_2 and q_3. Hence, the work done in this process is

$$W_3 = \frac{q_1 q_3}{4\pi\epsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}}$$

- For more charges, use the same idea

Potential Energy of a System of Two or More Charges

- Hence, the net potential energy of the system is the net work to assemble the system: Potential energy of this system, $U = W_{12} + W_{13} + W23$

$$= \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\epsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}}$$

- For more charges, use the same concept

Derivation of Electrical Force

$$U = \int_{\infty}^{r} F_{ext} dr$$

$$\frac{dU}{dr} = F_{ext}$$

but $F_E = -F_{ext}$

$$F_E = -\frac{dU}{dr}$$

Electrical Force

Relation between Potential Energy and Electrical Force

From here, we can see that:

$$F = -\frac{dU}{dr}$$

- Magnitude of force given by

$$F = \left|\frac{dU}{dr}\right(\; |$$

- The direction of the force on the charge is given by the negative (-) sign i.e. the force on the charge points towards a decreasing potential energy (towards the point that allows the charge to achieve a lower energy).

Potential Gradient

Relation between Potential Gradient and Electric Field Strength

$$F = -\frac{dU}{dr}$$

$$qE = -\frac{dqV}{dr}$$

$$E = -\frac{dV}{dr}$$

- $\frac{dV}{dr}$ is known as the potential gradient of the field.

Equipotential Lines or Surfaces

Definition

An equipotential surface is a surface in which every point on the surface is at the same potential.

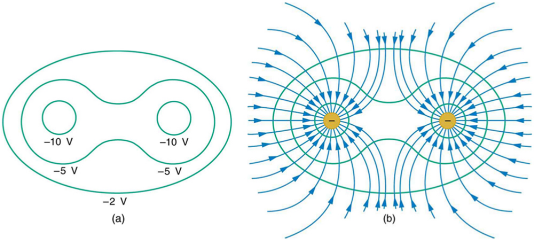

Figure : Equipotential Lines

Equipotential Lines or Surfaces

- Field strength: Spacing of equipotential lines
- Field lines and equipotential lines cut at right angles
- Field direction: Towards lower potential

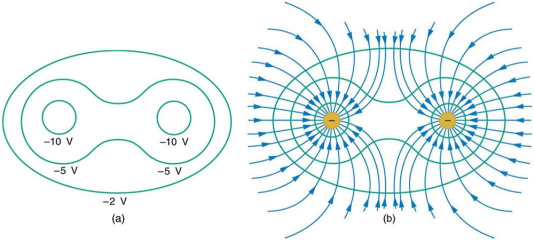

Figure : Equipotential Lines

Charges in Equilibrium on Conductors

- Charged conductors that have reached electrostatic equilibrium share a variety of unusual characteristics.
- One characteristic of a conductor at electrostatic equilibrium is that the electric field anywhere beneath the surface of a charged conductor is zero.
- If an electric field did exist beneath the surface of a conductor (and inside of it), then the electric field would exert a force on all electrons that were present there.

Charges in Equilibrium on Conductors

- This net force would begin to accelerate and move these electrons. But objects at electrostatic equilibrium have no further motion of charge about the surface. So if this were to occur, then the original claim that the object was at electrostatic equilibrium would be a false claim.
- If the electrons within a conductor have assumed an equilibrium state, then the net force upon those electrons is zero.

Charges in Equilibrium on Conductors

- The electric field lines either begin or end upon a charge and in the case of a conductor, the charge exists solely upon its outer surface.
- The lines extend from this surface outward, not inward. This of course presumes that our conductor does not surround a region of space where there was another charge.

Additional rules on drawing Electric Field/Equipotential Lines

1. Electric field along the surface is zero
2. The surface of a conductor is an equipotential surface
3. The potential in the conductor is constant everywhere inside the conductor and equal to its value at the surface
4. Electric field in the conductor is therefore zero. (No potential gradient) Hence, no field lines are drawn.

Electric Field between Two Charged Parallel Plates

- The charges on each plate are spread uniformly over the inside surface of each plate because of their mutual repulsion and the attraction by the opposite charges on the other plate.
- The lines of forces are straight, parallel to each other and equally spaced.

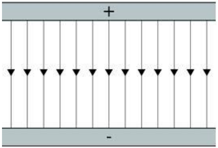

Figure : Electric Field between two parallel oppositely charged plates

Electric Field between Two Charged Parallel Plates

- The electric field in the space between the two plates is said to be uniform.

Magnitude of Electric Field Strength

$$E = \frac{V}{d}$$

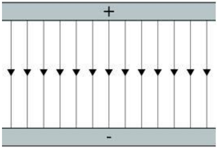

Figure : Electric Field between two parallel oppositely charged plates

Motion of Charged Particles in a Uniform Electric Field

- Consider a particle of mass m, carrying charge q and placed in an uniform electric field E.
- The field exerts a force qE on it, giving it an acceleration a in the direction of the force.
- By N2L, $ma = qE$
- Acceleration of the charge is therefore given by

$$a = \frac{qE}{m}$$

- Note that in most questions, the mass is so small that the electric force overwhelms the gravitational force on the mass (weight). Hence the weight can be ignored.

Gauss's Law

https://www.youtube.com/watch?v=QNIJC1emss8

Gauss's Law

The net electric flux through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the net electric charge enclosed within that closed surface.

- It was formulated by Carl Friedrich Gauss in 1835, and finally published in 1867.
- It is one of the four Maxwells equations, forming the basis of classical electrodynamics.
- Gausss Law can be used to derive Coulombs Law, and vice versa.

Gauss's Law

Integral form:

$$\oint_S E_n dA = \frac{1}{\varepsilon_0} Q_{inside}$$

Differential form:

$$\nabla E = \frac{\rho}{\epsilon_0} = 4\pi k \rho$$

- Its integral form is useful for calculating electric fields around charged objects.
- Its differential form is mathematically equivalent to its integral form.
- ∇E is the divergence of the electric field, ϵ_0 is the electric constant, and ρ is the total electric charge density (charge per unit volume).

Summary

- The electric field is a field of force
- $E = \frac{F}{q}$
- Rules for drawing electric fields
- Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{QQ\prime}{r^2}$$

- Electric Field
- Electric Potential
- Electric Potential Energy
- Potential Energy of a System of Two or More Point Charges

Summary

- Potential Energy of a System of Two or More Point Charges
- Electrical Force
- Potential Gradient
- Equipotential Lines
- Charges in Equilibrium on Conductors
- Electric field between Parallel Charged Plates
- Motion of Charged Particles in an Uniform Electric Field
- Gauss's Law

End

We have come to the end of our presentation. We hope you have enjoyed it.