

# Algorithm Design and Analysis

## Assignment 6

1. You are given a set  $S$  of football teams, where each team  $x \in S$  has already accumulated  $w_x$  wins so far. For every pair of teams  $x, y \in S$  we know that there are  $g_{xy}$  games remaining between  $x$  and  $y$  (that is, if  $x$  wins a remaining games against  $y$ , then  $y$  must win  $g_{xy} - a$  remaining games against  $x$ ). Given a specific team  $z \in S$ , we would like to decide if  $z$  still has a chance to have the maximum number of wins. Alternatively, suppose we have the power to determine who wins each of the remaining games, is there a way such that  $z$  would get as many wins as anybody else by the end of the tournament? Give a polynomial-time algorithm for this problem. Hint: We can assume without loss of generality that  $z$  wins all remaining games. Is there a way to split the wins of the remaining games among the rest of the teams so that none of them get more wins than  $z$ ? Consider a reduction to the network flow problem.
2. Let  $G$  be a flow network in which each edge  $e$  has a capacity  $c(e)$  as well as a lower bound  $d(e)$  on the flow it must carry. Note that  $d(e) \leq c(e)$ . A feasible flow assigns a value within the range of  $[d(e), c(e)]$  to each edge  $e$ . Note that assigning a zero flow to every edge may not give a feasible flow of  $G$  (due to the lower bound requirement of the edges). Show how to make use a maximum flow algorithm to find a feasible flow of  $G$ .
3. How long does it take you to finish the assignment (include thinking and discussing)? Give a score (1,2,3,4,5) to the difficulty.