Algorithm Design and Analysis
Assignment 4

1. You are given a set of n jobs, where each job j is associated with a size s_j (how much time it takes to process the job) and a weight w_j (how important the job is). Suppose you have only one machine that can process one unit of jobs per time slot. Assume all jobs are given at time $t_0 = 0$ and are to be processed one by one using this machine. Let $C_j > t_0$ be the time that job j is completed. The goal is to find a schedule (of all the jobs) that minimizes the weighted completion time, i.e.,$\sum_{j=1}^{n} w_j C_j$.

   Greedy algorithms first order the jobs according to some criteria, and then process them one by one. Only one of the following criteria gives an algorithm that minimizes the weighted completion time.

   - Highest Weight First: process jobs in in descending order of their weights.
   - Smallest Size First: process jobs in ascending order of their sizes.
   - Highest Density First: process jobs in descending order of their weight to size ratio, namely in descending order of w_j/s_j.

   (a) Find out which one is correct.

   (b) Reason about its correctness.

   (c) Give counter examples for the other criteria.

2. Let $G = (V, E)$ be a connected undirected graph with positive edge weights. Give an algorithm to find a subset of edges E' with the smallest total weight such that removing E' from G will leave a graph with no cycle. Note that E' must contain at least one edge on every cycle of G. You need to prove the correctness of your algorithm.

3. Alice wants to throw a party and is deciding whom to call. She has n people to choose from, and she has made up a list of which pairs of these people know each other. She wants to pick as many people as possible, subject to two constraints: at the party, each person should have at least five other people whom they know and five other people whom they don't know. Give an efficient algorithm that takes as input the list of n people and the list of pairs who know each other and outputs the best choice of party invitees. Give the running time in terms of n.