Homework 1 for Design and Analysis of Algorithms (Spring 2021)

Due: March 12th, 2021

Problem 1. (Exercise 0.1 in [DPV08]) In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$).

	$f(n)$	$g(n)$
(a)	$n - 100$	$n - 200$
(b)	$n^{1/2}$	$n^{2/3}$
(c)	$100n + \log n$	$n + (\log n)^2$
(d)	$n \log n$	$10n \log 10n$
(e)	$\log 2n$	$\log 3n$
(f)	$10 \log n$	$\log(n^2)$
(g)	$n^{1.01}$	$n \log^2 n$
(h)	$n^2/\log n$	$n(\log n)^2$
(i)	$n^{0.1}$	$(\log n)^{10}$
(j)	$(\log n)^{\log n}$	$n/\log n$
(k)	$\sqrt{n}$	$(\log n)^3$
(l)	$n^{1/2}$	$5^{\log_2 n}$
(m)	$n2^n$	3^n
(n)	2^n	2^{n+1}
(o)	$n!$	2^n
(p)	$(\log n)^{\log n}$	$2^{(\log_2 n)^2}$
(q)	$\sum_{i=1}^{n} i^k$	n^{k+1}.

Problem 2. (Exercise 0.2 in [DPV08]) Show that, if c is a positive real number, then $g(n) = 1 + c + c^2 + \cdots + c^n$ is:

(a) $\Theta(1)$ if $c < 1$.

(b) $\Theta(n)$ if $c = 1$.

(c) $\Theta(c^n)$ if $c > 1$.

Problem 3. Show that there exists a C++ program P who can generate all pairs of natural numbers (x, y) such that $P_x(y)$ terminates [1]. That is, the program P can keep printing pairs of numbers (x, y) such that $P_x(y)$ terminates and for every (x', y') such that $P_{x'}(y')$ terminates, the program P can print it at some time.

[1] Since the number of such pairs are infinite, the program P must run forever.

References

[DPV08] Sanjoy Dasgupta, Christos H Papadimitriou, and Umesh Virkumar Vazirani. *Algorithms*. McGraw-Hill Higher Education New York, 2008. 1