The University of New South Wales

Final Exam

Session 2, 2015

COMP9020

Foundations of Computer Science

Time allowed: **2 hours**
Total number of questions: **10**
Maximum number of marks: **50** (to be scaled to 100)

Not all questions are worth the same.

Answer all questions.

Textbooks, lecture notes, etc. are not permitted, except for up to 2 double-sided A4 sheets containing handwritten notes.

Calculators may not be used.

Answers must be written in ink. Use a pencil or the back of the booklet for rough work. Your rough work will not be marked.

You can answer the questions in any order.

You may take this question paper out of the exam.

Write your answers into the answer booklet provided.

Question 1 (5 marks)

Prove or disprove $(A \Rightarrow B) \wedge B \Rightarrow A$.

Question 2 (5 marks)

Prove or disprove soundness of the following case analysis scheme as a proof technique.

$$(A \Rightarrow C) \wedge (B \Rightarrow C) \wedge (\neg A \wedge \neg B \Rightarrow C) \Leftrightarrow C \tag{1}$$

Question 3 (5 marks)

Prove or disprove that the sequential composition operator ";" distributes over union, that is, for all binary relations $r, s \subseteq X \times Y$ and $t \subseteq Y \times Z$ we have that $(r \cup s); t = (r; t) \cup (s; t)$.

Question 4 (5 marks)

Let $P(n)$ be a predicate on natural numbers such that

$$\exists k \in \mathbb{N}\,(P(k)) \tag{2}$$
$$\forall k \in \mathbb{N}\,(P(k) \Rightarrow P(k+2)) \tag{3}$$

For P's that satisfy (2) and (3), some of the assertions below **C**an hold for some, but not all, such P, other assertions **A**lways hold no matter what the P may be, and some **N**ever hold for any such P. Indicate which case applies for each of the assertions and briefly justify why.

(a) $\forall k \in \mathbb{N}\,(P(k+2))$

(b) $\forall k \in \mathbb{N}\,(\neg P(k+1) \vee P(k+3))$

(c) $P(0) \Rightarrow \forall k \in \mathbb{N}\,(P(k+2))$

(d) $P(0) \wedge \neg P(1) \wedge P(3) \Rightarrow \forall k \in \mathbb{N}\,(P(k) \Leftrightarrow \neg P(k+1))$

(e) $\exists k \in \mathbb{N}\,(P(2k) \wedge \forall \ell \in \mathbb{N}\,(P(2(k+\ell))))$

Question 5 (4 marks)

Define the sequence of numbers

$$a_n = \begin{cases} 1 & \text{for } n \leq 3 \\ a_{n-1} + a_{n-2} + a_{n-3} + a_{n-4} & \text{for } n > 3 \end{cases}.$$

Prove that, for all $n \in \mathbb{N}$, dividing a_n by 3 leaves the remainder 1.

Question 6 (5 marks)

Prove that every connected graph $G = (V, E)$ must satisfy $|E| + 1 \geq |V|$.

Question 7 (6 marks)

For each of the following recurrences, what is $T(n)$'s order of growth?

$$T(n) = 27\,T\left(\frac{n}{3}\right) + 3n^3 + 3 \tag{4}$$
$$T(n) = T(n-1) + 3n^3 + 3n + 3 \tag{5}$$
$$T(n) = 3\,T(n-1) + 3n^3 + 3n + 3 \tag{6}$$

Question 8 (5 marks)

How many 5-letter words over the alphabet $\Sigma = \{\mathtt{b}, \mathtt{d}, \mathtt{u}, \mathtt{x}, \mathtt{z}\}$

(a) include the substring `bud`?

(b) contain all letters of Σ and have `d` occur before `u`?

Question 9 (5 marks)

Two dice are rolled.

(a) What is the probability that the sum of the values is even?

(b) What is the probability that the sum of the values is prime?

Justify your answers briefly.

Question 10 (5 marks)

Suppose student X is taking an exam consisting of 100 multiple choice questions. Each question has five possible answers, exactly one of which is correct. A correct answer scores 3, an incorrect answer scores $-\frac{1}{2}$ and blank scores 0. X did not study at all, and decides to randomly guess all the answers and leave no blanks. What should X expect to score in the exam? Derive the correct answer to this question mathematically.