# Semiconductor Physics and Devices
# 半导体物理与器件

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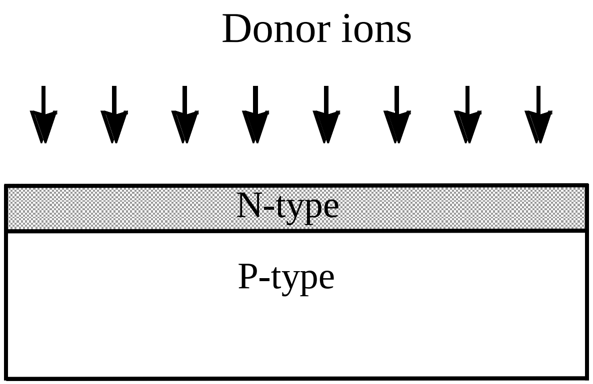
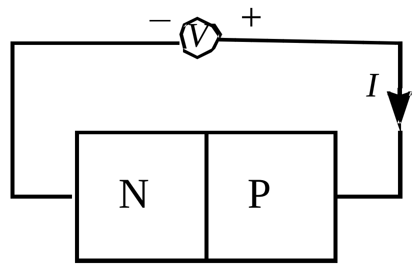
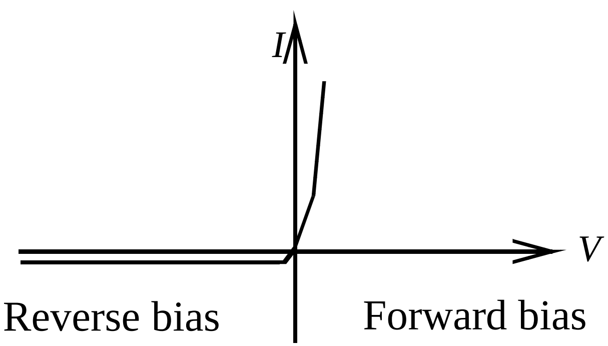
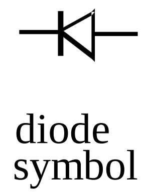
# *Chapter 4 PN and Metal-Semiconductor Junctions*

## *4.1 Building Blocks of the PN Junction Theory*



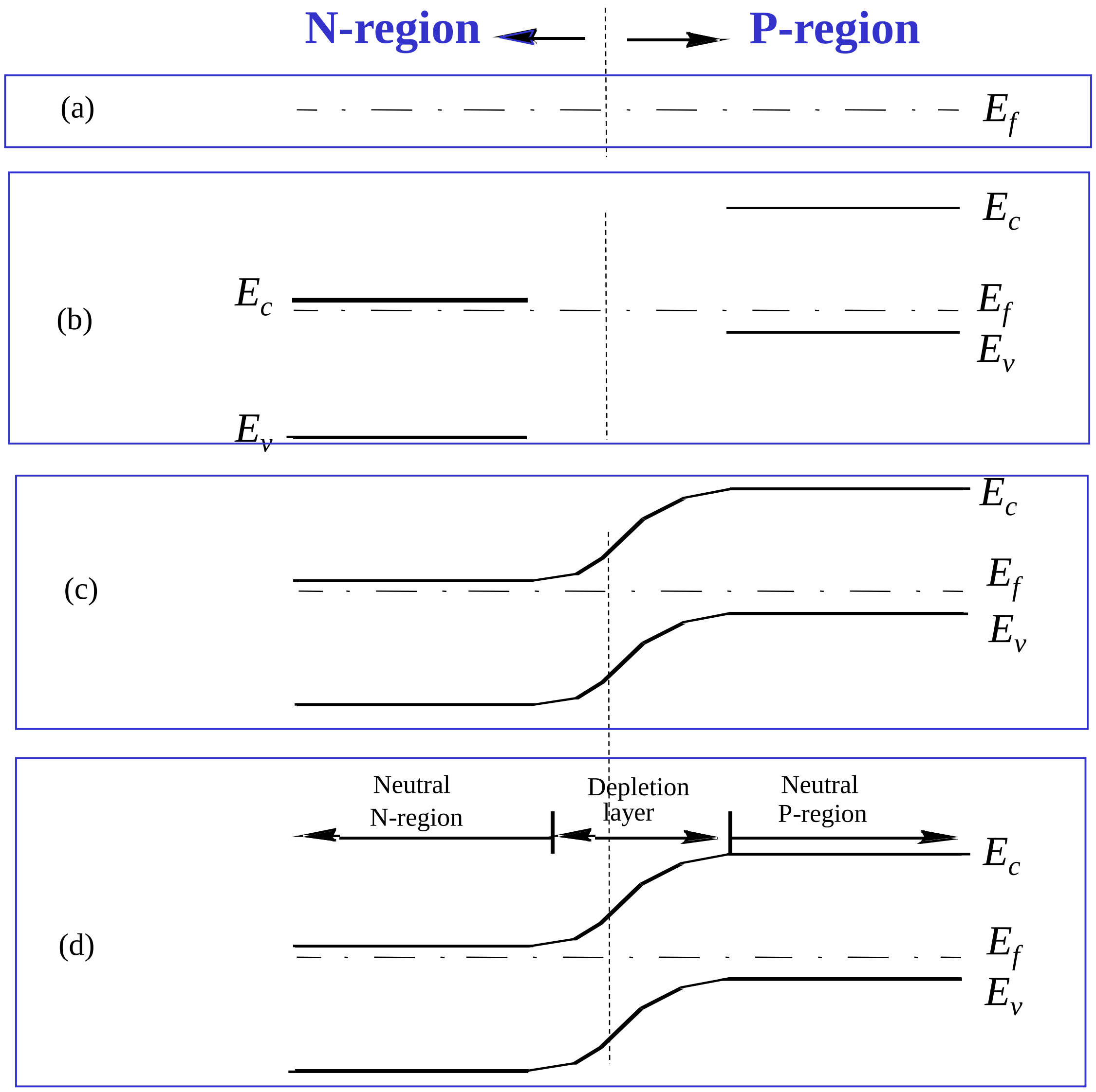




*PN junction is present in perhaps every semiconductor device.*

# 4.1.1 Energy Band Diagram of a PN Junction



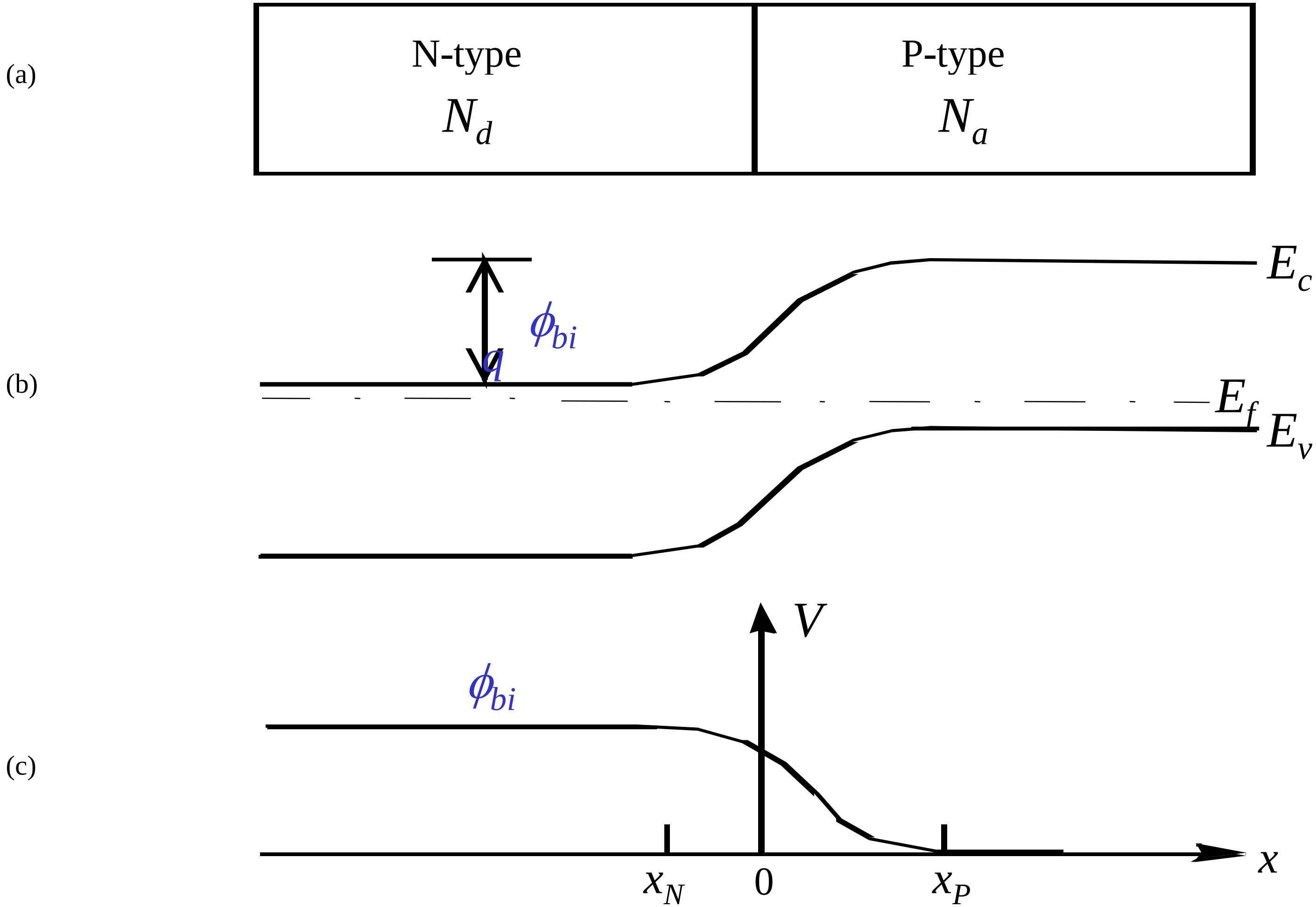

$E_f$ is constant at equilibrium

$E_c$ and $E_v$ are known relative to $E_f$

$E_c$ and $E_v$ are smooth, the exact shape to be determined.

A depletion layer exists at the PN junction where $n \approx 0$ and $p \approx 0$.

# 4.1.2 Built-in Potential




Can the built-in potential be measured with a voltmeter?

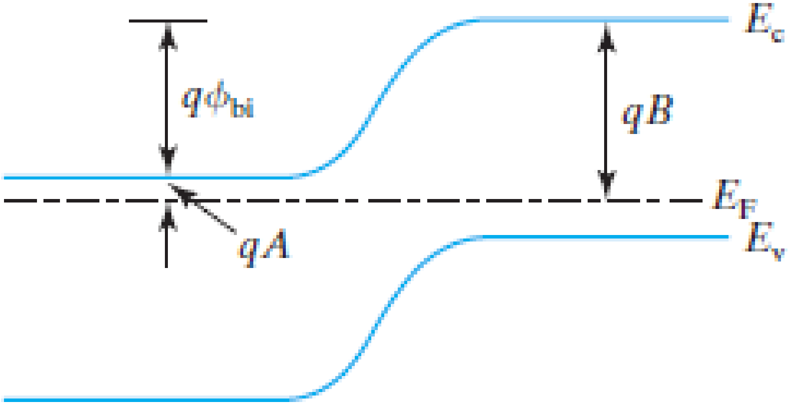
# 4.1.2 Built-in Potential

N-region $\quad n = N_d = N_c e^{-qA/kT} \Rightarrow A = \dfrac{kT}{q} \ln \dfrac{N_c}{N_d}$

P-region $\quad n = \dfrac{n_i^2}{N_a} = N_c e^{-qB/kT} \Rightarrow B = \dfrac{kT}{q} \ln \dfrac{N_c N_a}{n_i^2}$

$$\phi_{bi} = B - A = \frac{kT}{q}\left( \ln \frac{N_c N_a}{n_i^2} - \ln \frac{N_c}{N_d} \right)$$

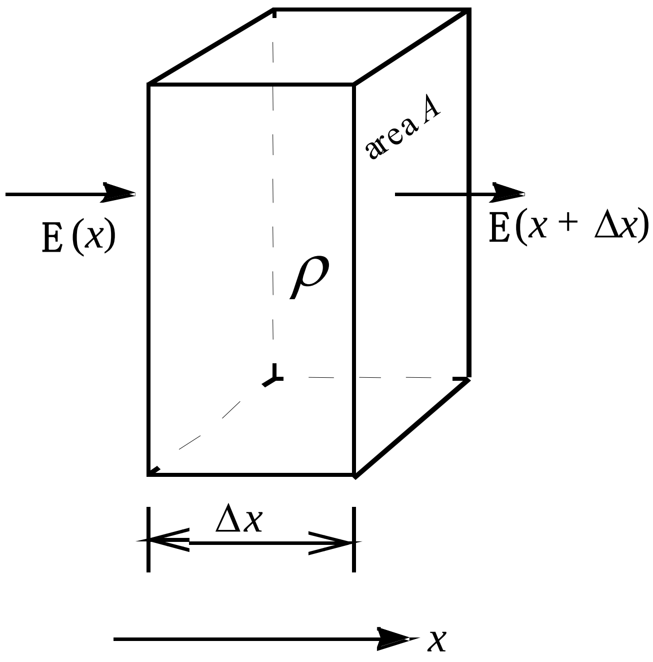
$$\boxed{\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}}$$



## 4.1.3 Poisson's Equation

Gauss's Law:

$$\varepsilon_s \mathcal{E}(x+\Delta x)A - \varepsilon_s \mathcal{E}(x)A = \rho \Delta x A$$




$\varepsilon_s$: permittivity (~12$\varepsilon_o$ for Si)
$\rho$: charge density (C/cm$^3$)

$$\frac{\mathcal{E}(x+\Delta x) - \mathcal{E}(x)}{\Delta x} = \frac{\rho}{\varepsilon_s}$$

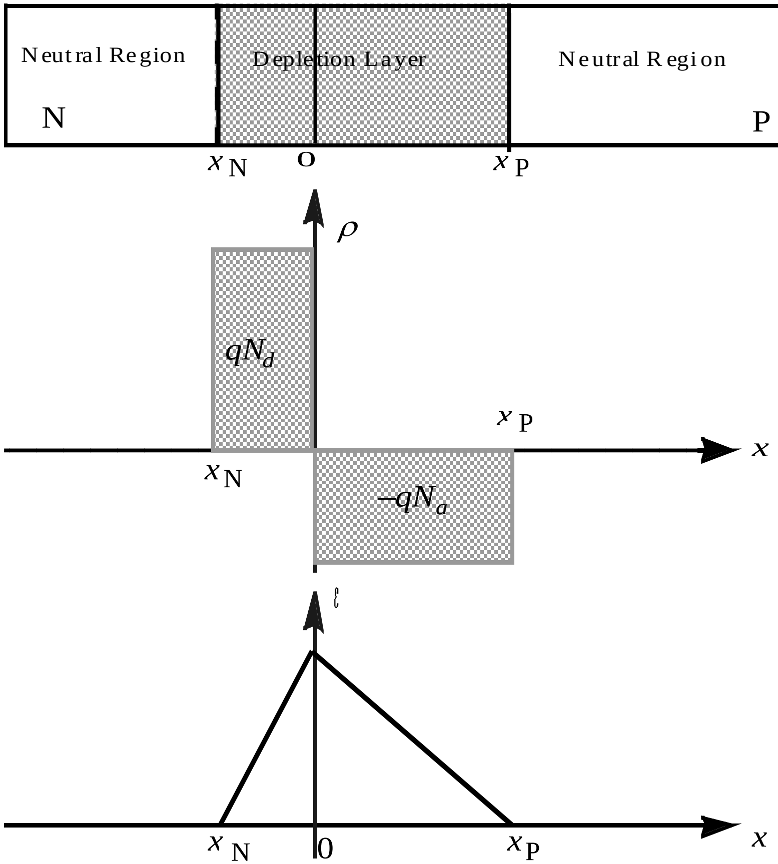
$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_s}$$

$$\boxed{\frac{d^2V}{dx^2} = -\frac{d\mathcal{E}}{dx} = -\frac{\rho}{\varepsilon_s}}$$

***Poisson's equation***

# 4.2 Depletion-Layer Model

## 4.2.1 Field and Potential in the Depletion Layer




On the ***P-side*** of the depletion layer, $\rho = -qN_a$

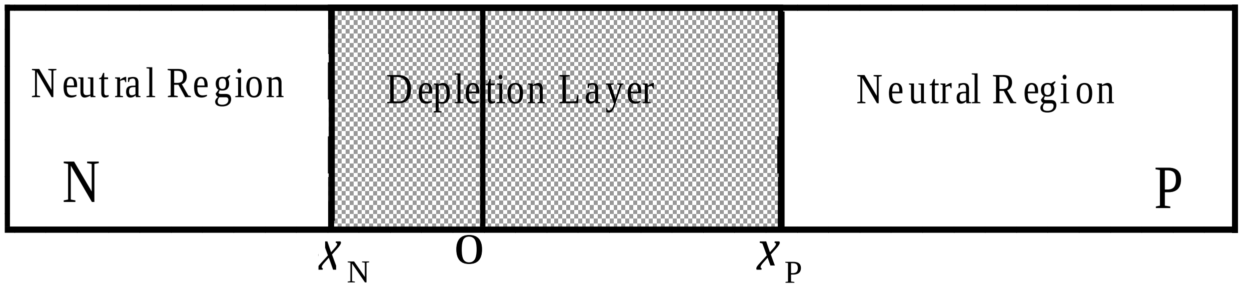
$$\frac{d\mathrm{E}}{dx} = -\frac{qN_a}{\varepsilon_s}$$

$$\mathrm{E}(x) = -\frac{qN_a}{\varepsilon_s}x + C_1 = \boxed{\frac{qN_a}{\varepsilon_s}(x_P - x)}$$

On the ***N-side***, $\rho = qN_d$

$$\boxed{\mathrm{E}(x) = \frac{qN_d}{\varepsilon_s}(x - x_N)}$$

# 4.2.1 Field and Potential in the Depletion Layer



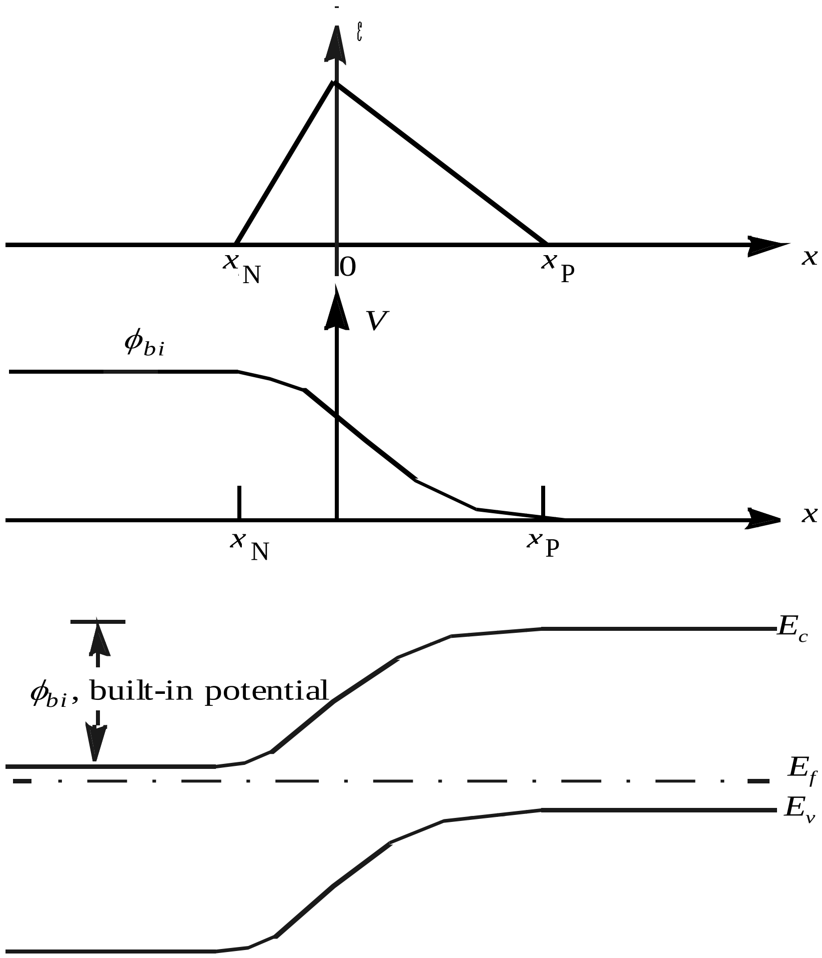

The electric field is continuous at $x = 0$.

$$\boxed{N_a\,|x_P| = N_d|x_P|}$$

Which side of the junction is depleted more?

A one-sided junction is called a ***$N^+P$ junction*** or ***$P^+N$ junction***

# 4.2.1 Field and Potential in the Depletion Layer




On the P-side,

$$V(x) = \frac{qN_a}{2\varepsilon_s}(x_P - x)^2$$

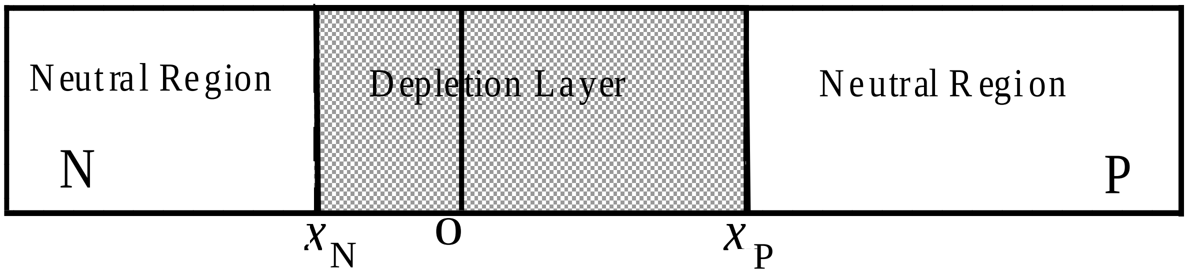
Arbitrarily choose the voltage at $x = x_P$ as $V = 0$.

On the N-side,

$$V(x) = D - \frac{qN_d}{2\varepsilon_s}(x - x_N)^2$$

$$= \phi_{bi} - \frac{qN_d}{2\varepsilon_s}(x - x_N)^2$$

# 4.2.2 Depletion-Layer Width




$V$ is continuous at $x = 0$ →

$$x_P - x_N = W_{dep} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

If $N_a >> N_d$, as in a $P^+N$ junction,

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d}} \approx |x_N|}$$

$$\boxed{|x_P| = |x_N| N_d / N_a \cong 0}$$

What about a $N^+P$ junction?

$$W_{dep} = \sqrt{2\varepsilon_s \phi_{bi} / qN} \quad \text{where} \quad \frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{\textit{lighter dopant density}}$$

***EXAMPLE***: *A $P^+N$ junction has $N_a = 10^{20}$ cm$^{-3}$ and $N_d = 10^{17}$cm$^{-3}$. What is a) its built in potential, b)$W_{dep}$ , c)$x_N$ , and d) $x_P$ ?*

***Solution***:

*a)*

$$\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2} = 0.026\,\mathrm{V} \ln \frac{10^{20} \times 10^{17}\,\mathrm{cm}^{-6}}{10^{20}\,\mathrm{cm}^{-6}} \approx 1\,\mathrm{V}$$

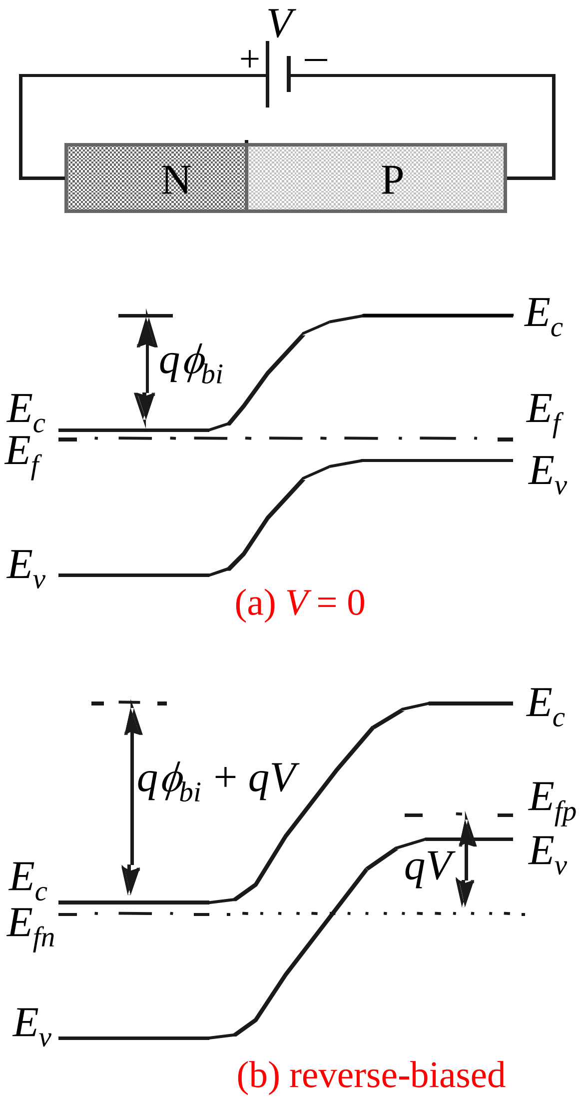
*b)*

$$W_{dep} \approx \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d}} = \left( \frac{2 \times 12 \times 8.85 \times 10^{-14} \times 1}{1.6 \times 10^{-19} \times 10^{17}} \right)^{1/2} = 0.12\,\mu\mathrm{m}$$

*c)* $|x_N| \approx W_{dep} = 0.12\,\mu\mathrm{m}$

*d)* $|x_P| = |x_N| N_d / N_a = 1.2 \times 10^{-4} \mu\mathrm{m} = 1.2\,\text{Å} \approx 0$

# 4.3 Reverse-Biased PN Junction

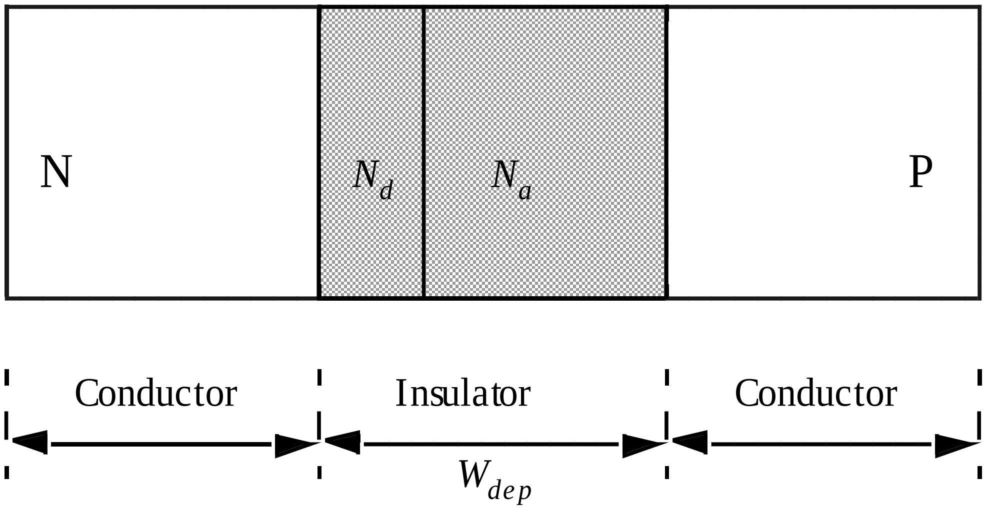



$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s(\phi_{bi} + |V_r|)}{qN}} = \sqrt{\frac{2\varepsilon_s \cdot potential\ barrier}{qN}}}$$

$$\frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{lighter\ \ dopant\ \ density}$$

- ***Does the depletion layer widen or shrink with increasing reverse bias?***

# 4.4 Capacitance-Voltage Characteristics



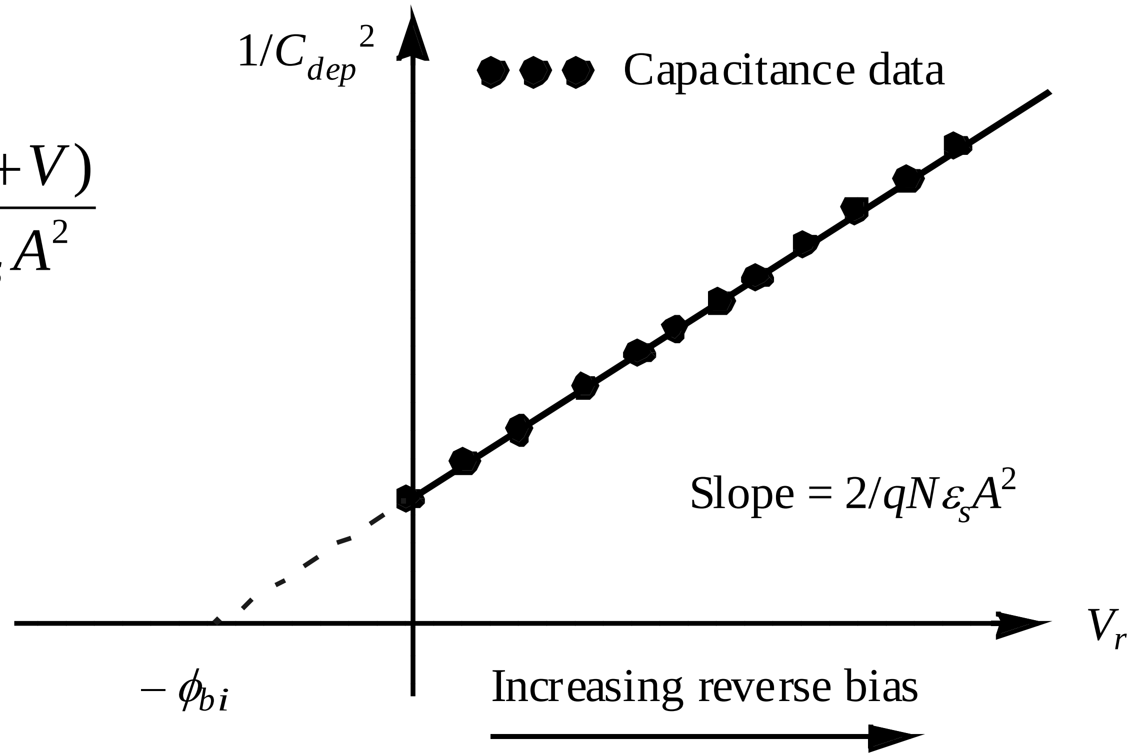

Reverse biased PN junction is a capacitor.

$$\boxed{C_{dep} = A\frac{\varepsilon_s}{W_{dep}}}$$

- Is $C_{dep}$ a good thing?
- How to minimize junction capacitance?

# 4.4 Capacitance-Voltage Characteristics

$$\frac{1}{C_{dep}^{\ 2}} = \frac{W_{dep}^{\ 2}}{A^2 \varepsilon_s^{\ 2}} = \frac{2(\phi_{bi} + V)}{qN\varepsilon_S A^2}$$




- From this C-V data can $N_a$ and $N_d$ be determined?

***EXAMPLE:*** *If the slope of the line in the previous slide is $2x10^{23}$ $F^{-2}$ $V^{-1}$, the intercept is 0.84V, and A is 1 $\mu m^2$, find the lighter and heavier doping concentrations $N_l$ and $N_h$ .*

***Solution:***

$$N_l = 2/(slope \times q\varepsilon_s A^2)$$

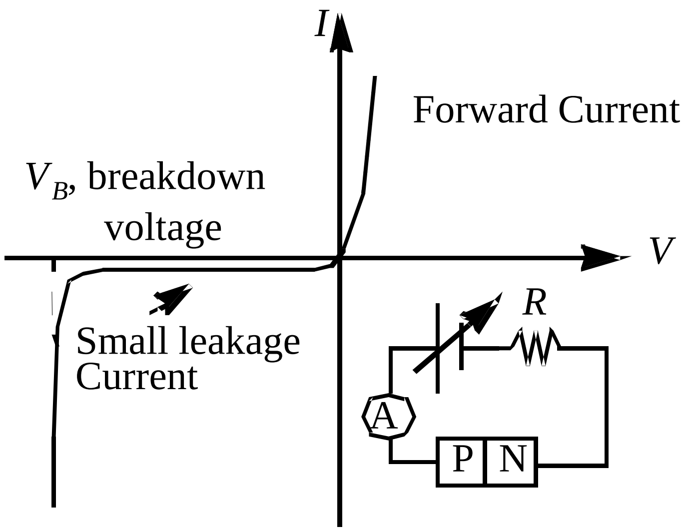
$$= 2/(2\times10^{23}\times1.6\times10^{-19}\times12\times8.85\times10^{-14}\times10^{-8}\,\mathrm{cm}^2)$$

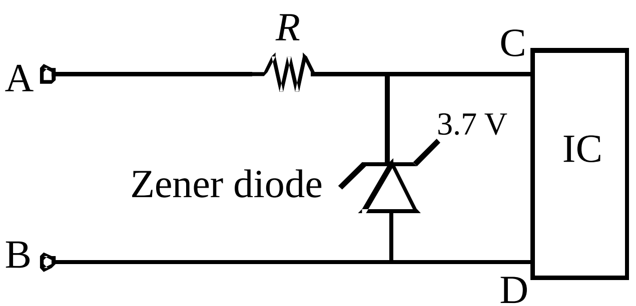
$$= 6\times10^{15}\ \mathrm{cm}^{-3}$$

$$\phi_{bi} = \frac{kT}{q}\ln\frac{N_h N_l}{n_i^{\,2}} \;\Rightarrow\; N_h = \frac{n_i^{\,2}}{N_l}e^{\frac{q\phi_{bi}}{kT}} = \frac{10^{20}}{6\times10^{15}}e^{\frac{0.84}{0.026}} = 1.8\times10^{18}\ \mathrm{cm}^{-3}$$

- *Is this an accurate way to determine $N_l$? $N_h$?*

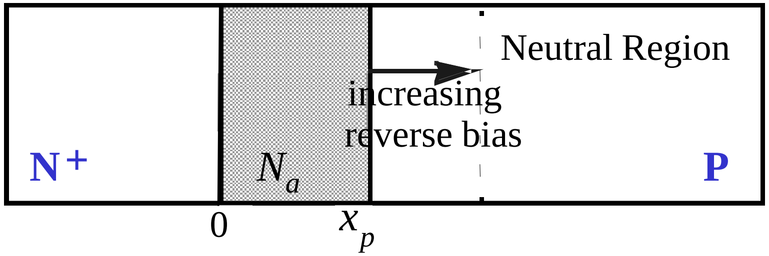
# *4.5 Junction Breakdown*

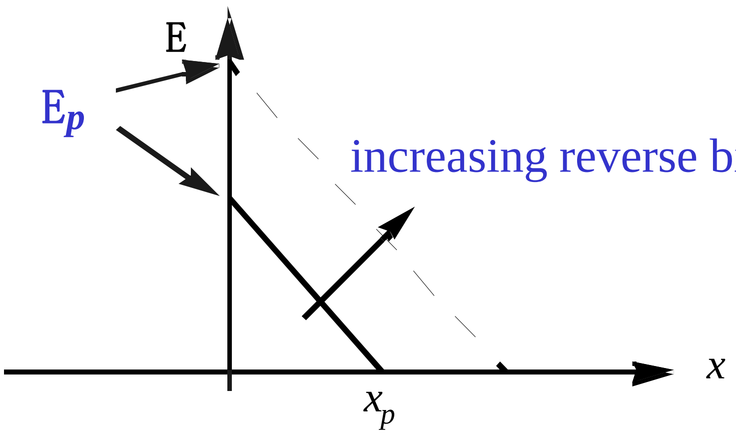



A ***Zener diode*** is designed to operate in the breakdown mode.

# 4.5.1 Peak Electric Field

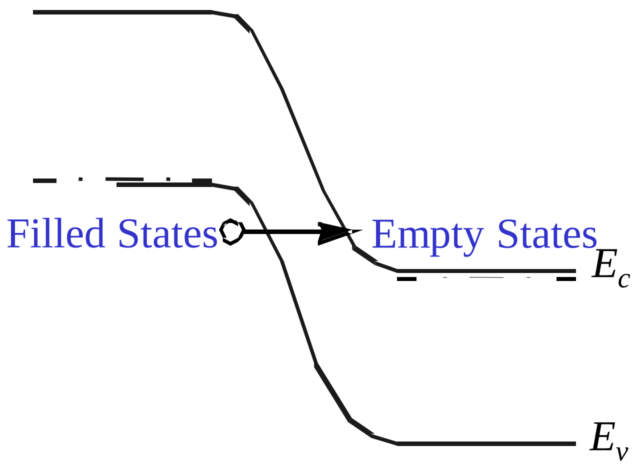



$$\mathcal{E}_p = \mathcal{E}(0) = \left[ \frac{2qN}{\varepsilon_s} (\phi_{bi} + |V_r|) \right]^{1/2}$$

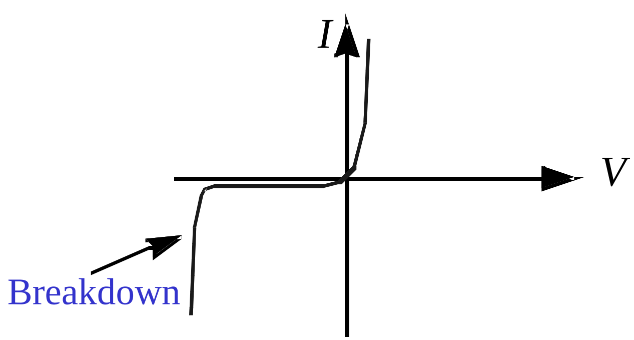
$$V_B = \frac{\varepsilon_s \mathcal{E}_{crit}^{\ 2}}{2qN} - \phi_{bi}$$

# 4.5.2 Tunneling Breakdown



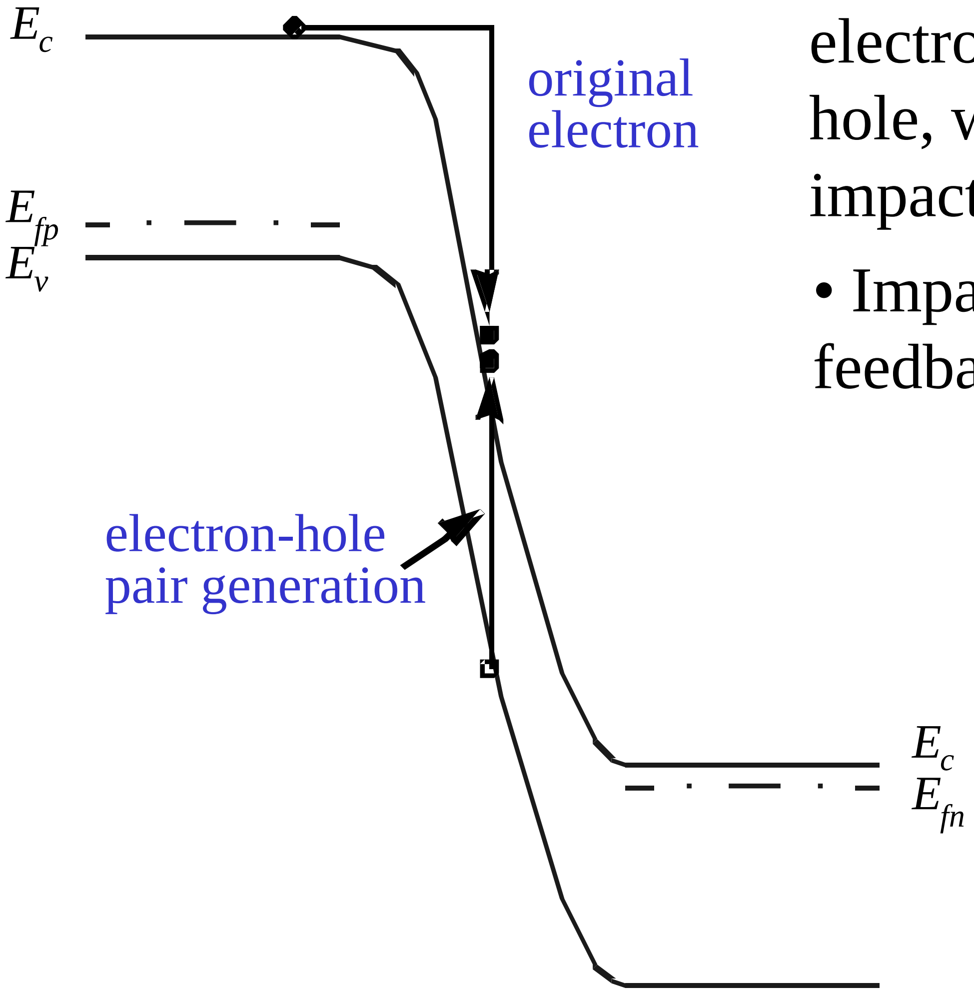

Dominant if both sides of a junction are very heavily doped.

$$J = G\,e^{-H/\mathcal{E}_p}$$




$$\mathcal{E}_p = \mathcal{E}_{crit} \approx 10^6 \text{ V/cm}$$

## 4.5.3 Avalanche Breakdown




• *impact ionization*: an energetic electron generating electron and hole, which can also cause impact ionization.

• Impact ionization + positive feedback→*avalanche breakdown*

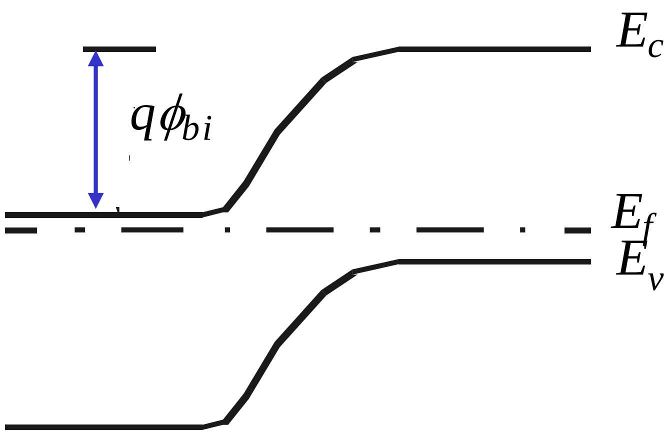
$$V_B = \frac{\varepsilon_s \mathrm{E}_{crit}^{\;2}}{2qN}$$

$$\boxed{V_B \propto \frac{1}{N}} = \frac{1}{N_a} + \frac{1}{N_d}$$

# 4.6 Forward Bias – Carrier Injection

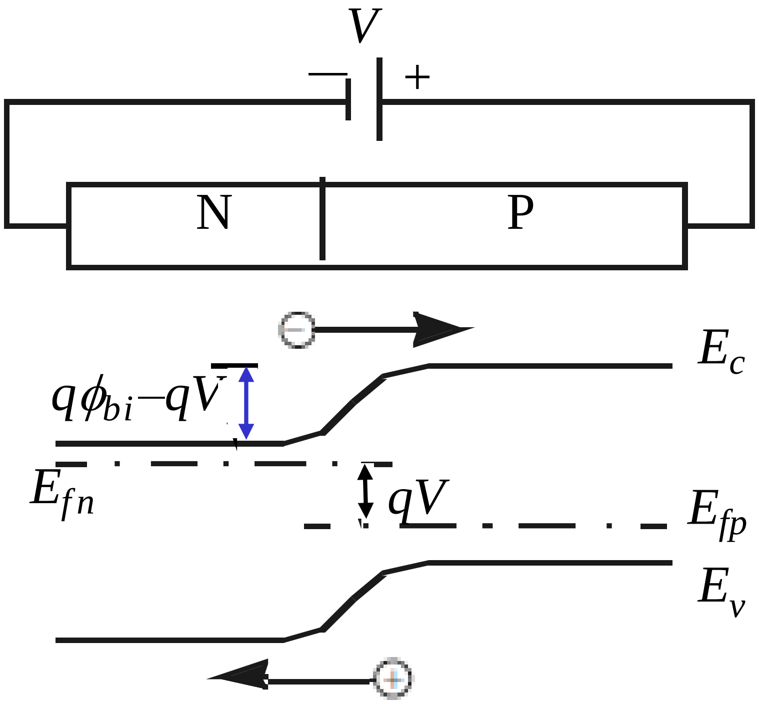
**V=0**

**I=0**




**Drift and diffusion cancel out**

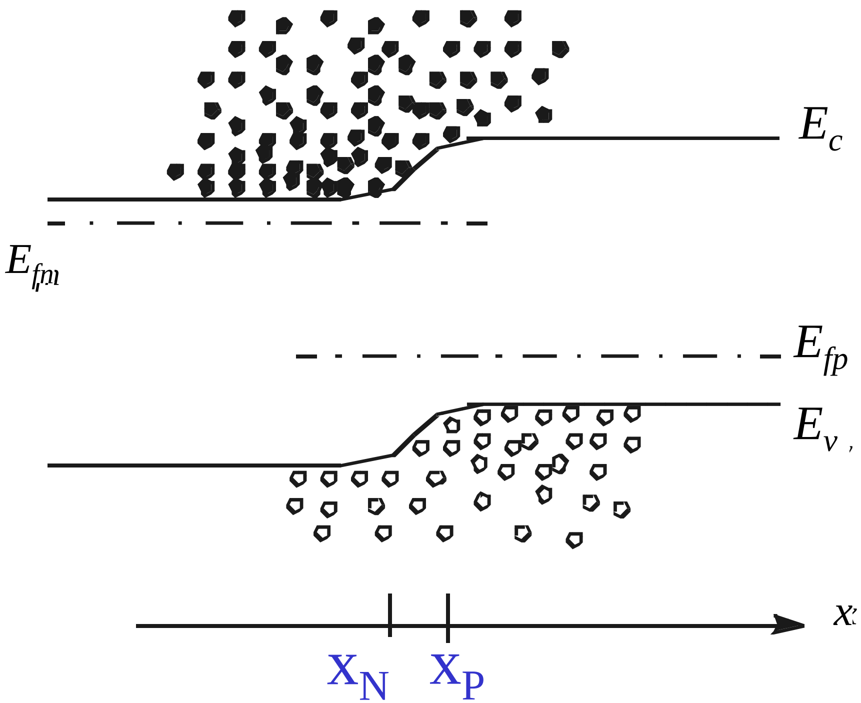
**Forward biased**




**Minority carrier injection**

## 4.6 Forward Bias – Quasi-equilibrium Boundary Condition

$$n(x_{\mathrm{P}}) = N_c e^{-(E_c - E_{fn})/kT} = N_c e^{-(E_c - E_{fp})/kT} e^{(E_{fn} - E_{fp})/kT}$$




$$= n_{P0} e^{(E_{fn} - E_{fp})/kT} = n_{P0} e^{qV/kT}$$

- The minority carrier densities are raised by $e^{qV/kT}$
- Which side gets more carrier injection?

# 4.6 Carrier Injection Under Forward Bias– Quasi-equilibrium Boundary Condition

$$n(x_P) = n_{P0}e^{qV/kT} = \frac{n_i^2}{N_a}e^{qV/kT}$$

$$p(x_P) = p_{N0}e^{qV/kT} = \frac{n_i^2}{N_d}e^{qV/kT}$$

$$n'(x_P) \equiv n(x_P) - n_{P0} = n_{P0}(e^{qV/kT} - 1)$$

$$p'(x_N) \equiv p(x_N) - p_{N0} = p_{N0}(e^{qV/kT} - 1)$$

# *EXAMPLE: Carrier Injection*

*A PN junction has $N_a=10^{19}$cm$^{-3}$ and $N_d=10^{16}$cm$^{-3}$. The applied voltage is 0.6 V.*

***Question**: What are the minority carrier concentrations at the depletion-region edges?*

***Solution**:*

$$n(x_P) = n_{P0}e^{qV/kT} = 10 \times e^{0.6/0.026} = 10^{11}\ \text{cm}^{-3}$$

$$p(x_N) = p_{N0}e^{qV/kT} = 10^4 \times e^{0.6/0.026} = 10^{14}\ \text{cm}^{-3}$$

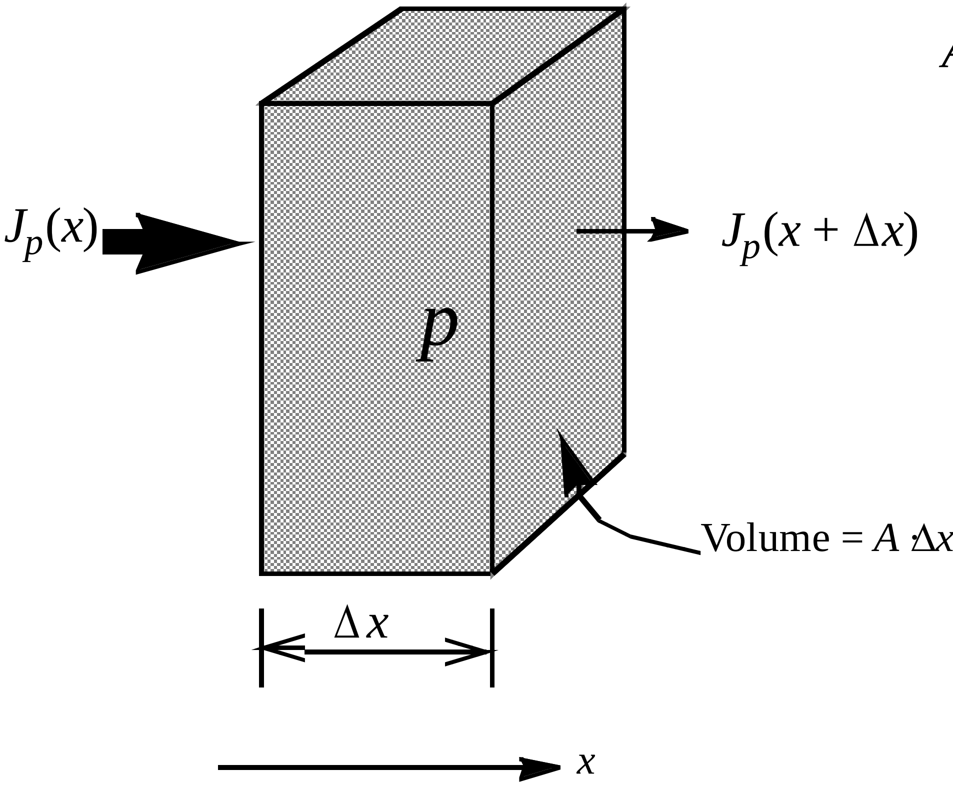
***Question**: What are the excess minority carrier concentrations?*

***Solution**:*

$$n'(x_P) = n(x_P) - n_{P0} = 10^{11} - 10 = 10^{11}\ \text{cm}^{-3}$$

$$p'(x_N) = p(x_N) - p_{N0} = 10^{14} - 10^4 = 10^{14}\ \text{cm}^{-3}$$

# 4.7 Current Continuity Equation




$$A \cdot \frac{J_p(x)}{q} = A \cdot \frac{J_p(x+\Delta x)}{q} + A \cdot \Delta x \cdot \frac{p'}{\tau}$$

$$-\frac{J_p(x+\Delta x) - J_p(x)}{\Delta x} = q\frac{p'}{\tau}$$

$$-\frac{dJ_p}{dx} = q\frac{p'}{\tau}$$

# 4.7 Current Continuity Equation

$$-\frac{dJ_p}{dx} = q\frac{p'}{\tau}$$

*Minority drift current is negligible;*

$\therefore \quad J_p = -qD_p dp/dx$

$$qD_p\frac{d^2p}{dx^2} = q\frac{p'}{\tau_p}$$

$$\boxed{\frac{d^2p'}{dx^2} = \frac{p'}{D_p\tau_p} = \frac{p'}{L_p^{\,2}}}$$

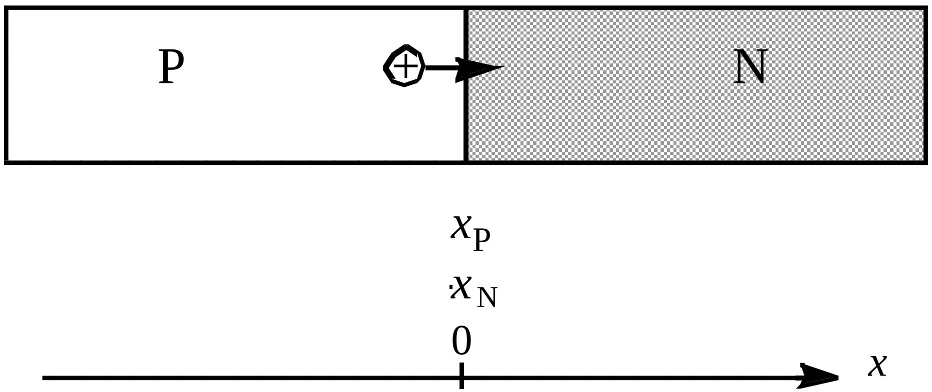
$$\boxed{\frac{d^2n'}{dx^2} = \frac{n'}{L_n^{\,2}}}$$

*$L_p$ and $L_n$ are the diffusion lengths*

$$\boxed{L_p \equiv \sqrt{D_p\tau_p}}$$

$$\boxed{L_n \equiv \sqrt{D_n\tau_n}}$$

# 4.8 Forward Biased Junction-- Excess Carriers




$$\frac{d^2 p'}{dx^2} = \frac{p'}{L_p{}^2}$$

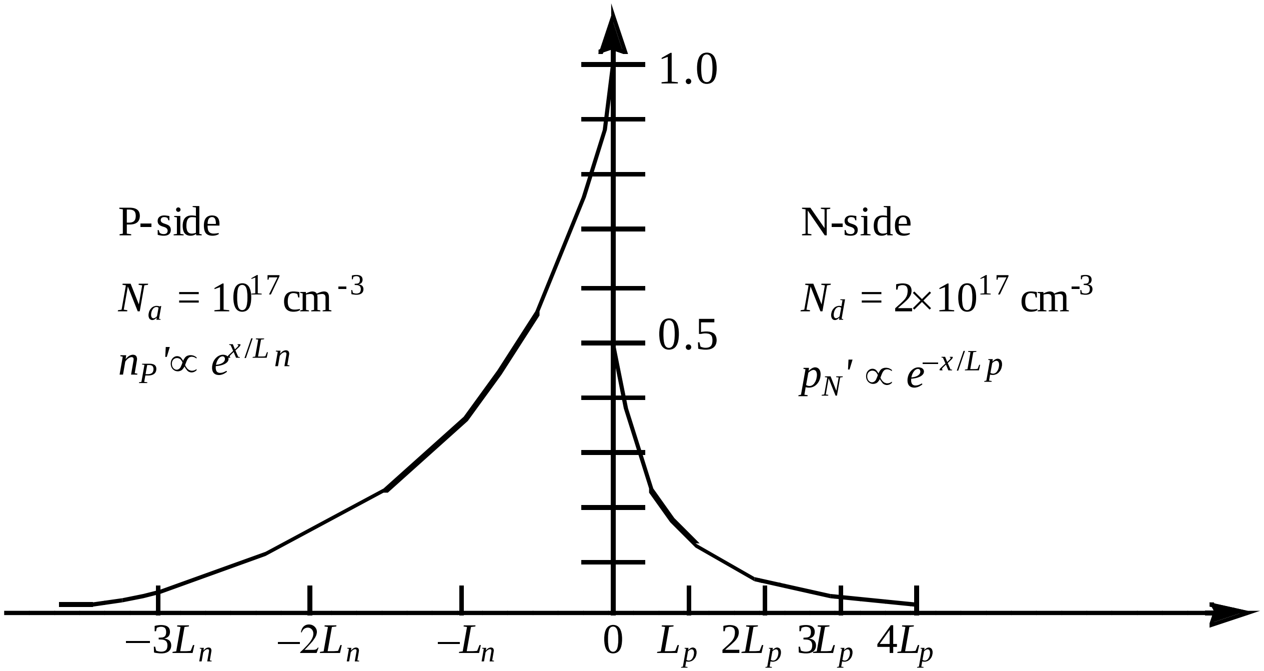
$$p'(\infty) = 0$$

$$p'(x_N) = p_{N0}(e^{qV/kT} - 1)$$

$$p'(x) = Ae^{x/L_p} + Be^{-x/L_p}$$

$$p'(x) = p_{N0}(e^{qV/kT} - 1)e^{-(x - x_N)/L_p}, \;\; x > x_N$$

# 4.8 Excess Carrier Distributions




$$p'(x) = p_{N0}(e^{qV/kT} - 1)e^{-(x-x_N)/L_p}, \ x > x_N$$

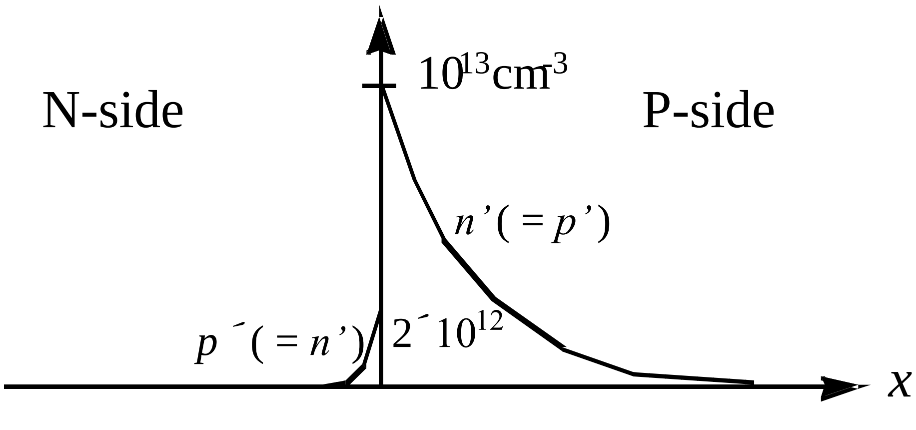
$$n'(x) = n_{P0}(e^{qV/kT} - 1)e^{(x-x_P)/L_n}, \ x < x_P$$

## *EXAMPLE: Carrier Distribution in Forward-biased PN Diode*

| N-type | P-type |
|---|---|
| $N_d = 5\times10^{17}$ cm$^{-3}$ | $N_a = 10^{17}$ cm$^{-3}$ |
| $D_p = 12$ cm$^2$/s | $D_n = 36.4$ cm$^2$/s |
| $\tau_p = 1$ μs | $\tau_n = 2$ μs |

- *Sketch n'(x) on the P-side.*

$$n'(x_P) = n_{P0}(e^{qV/kT}-1) = \frac{n_i^2}{N_a}(e^{qV/kT}-1) = \frac{10^{20}}{10^{17}}e^{0.6/0.026} = 10^{13}\ \text{cm}^{-3}$$



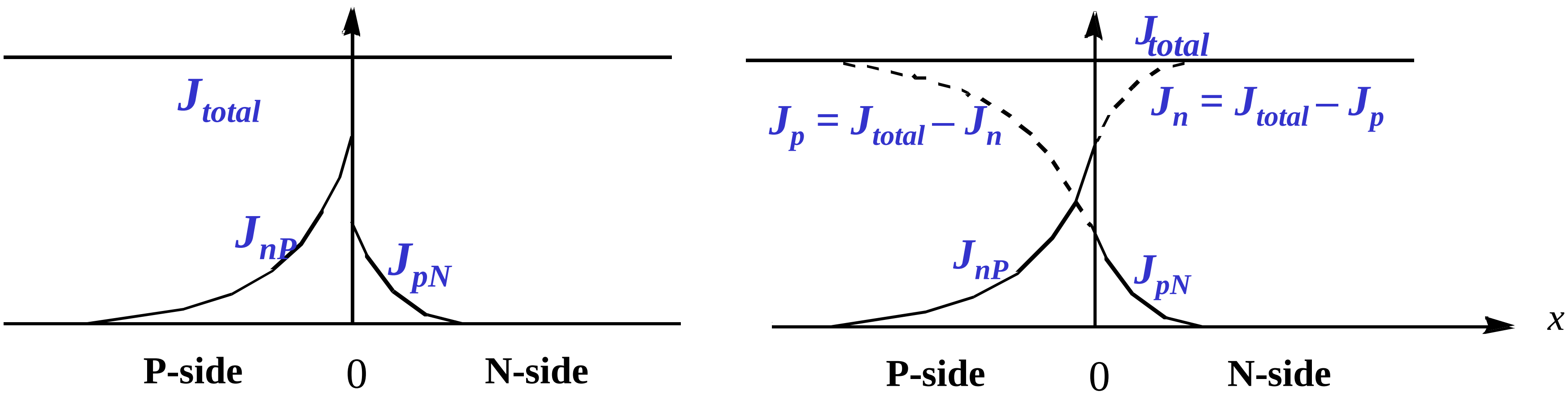
## *EXAMPLE: Carrier Distribution in Forward-biased PN Diode*

- *How does $L_n$ compare with a typical device size?*

$$L_n = \sqrt{D_n \tau_n} = \sqrt{36 \times 2 \times 10^{-6}} = 85\ \mu\text{m}$$

- *What is p'(x) on the P- side?*

# 4.9 PN Diode I-V Characteristics



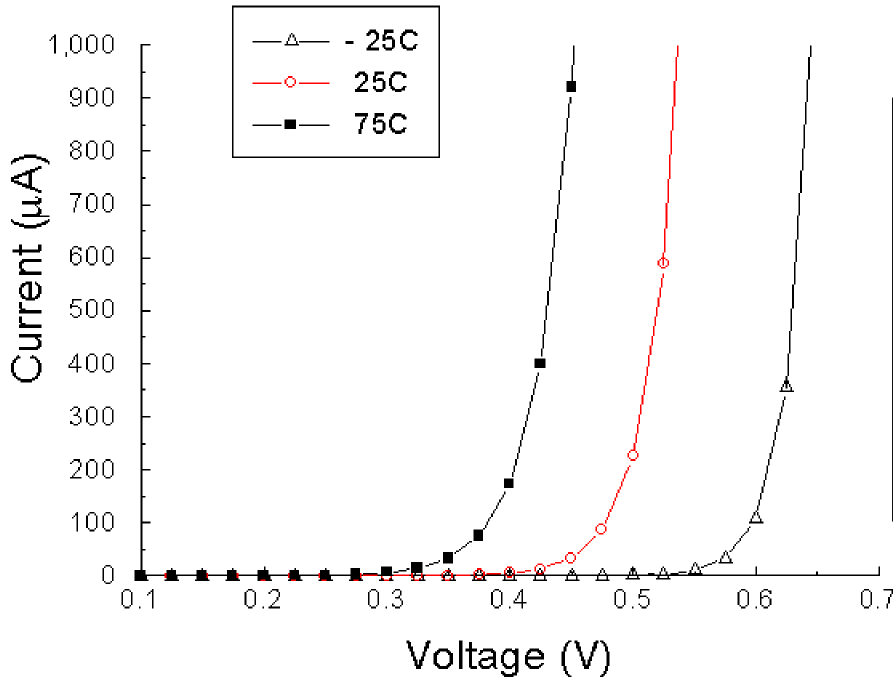

$$J_{pN} = -qD_p \frac{dp'(x)}{dx} = q\frac{D_p}{L_p} p_{N0}(e^{qV/kT} - 1)e^{-(x-x_N)/L_p}$$

$$J_{nP} = qD_n \frac{dn'(x)}{dx} = q\frac{D_n}{L_n} n_{P0}(e^{qV/kT} - 1)e^{(x-x_P)/L_n}$$

$$\text{Total current } = J_{pN}(x_N) + J_{nP}(x_P) = \left( q\frac{D_p}{L_p} p_{N0} + q\frac{D_n}{L_n} n_{P0} \right)(e^{qV/kT} - 1)$$

$$= J \text{ at all } x$$

# *The PN Junction as a Temperature Sensor*

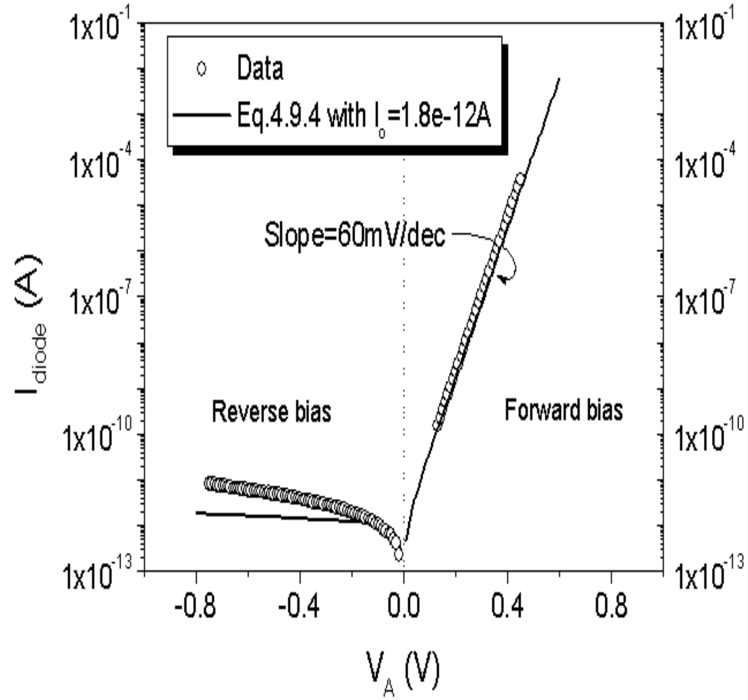



$$I = I_0 (e^{qV/kT} - 1)$$

$$I_0 = Aqn_i^{\,2}\left(\frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a}\right)$$

***What causes the IV curves to shift to lower V at higher T ?***

# 4.9.1 Contributions from the Depletion Region




$$n \approx p \approx n_i e^{qV/2kT}$$

Net recombinat ion (generatio n) rate :

$$\frac{n_i}{\tau_{dep}}(e^{qV/2kT}-1)$$

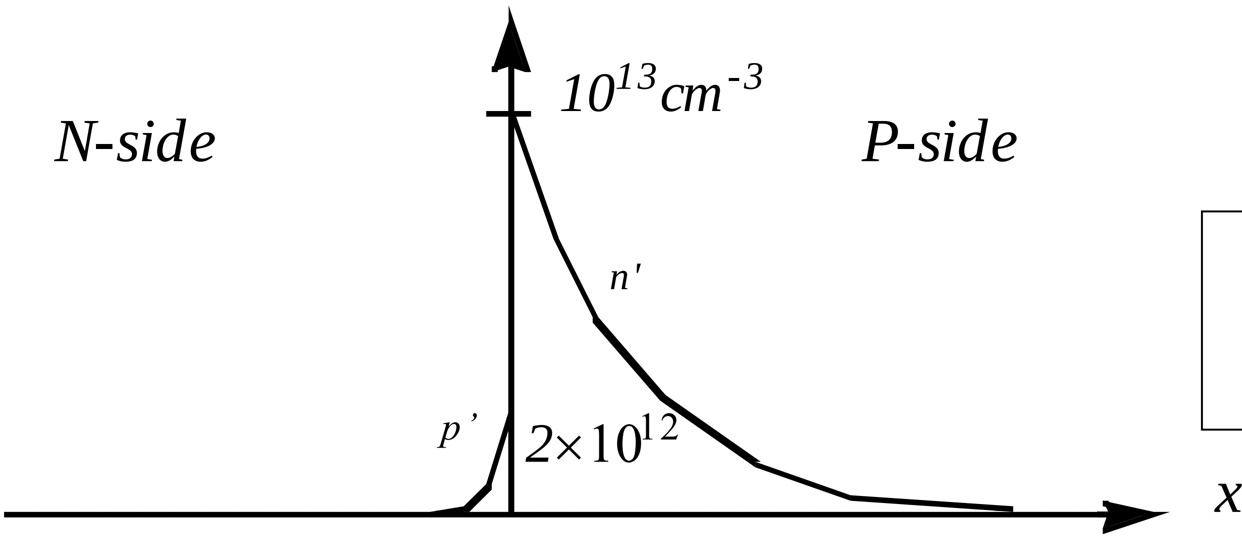
$$I = I_0(e^{qV/kT}-1) + A\frac{qn_iW_{dep}}{\tau_{dep}}(e^{qV/2kT}-1)$$

**Space-Charge Region (SCR) current**

$$I_{leakage} = I_0 + A\frac{qn_iW_{dep}}{\tau_{dep}}$$

**Under forward bias, SCR current is an extra current with a slope 120mV/decade**

# *4.10 Charge Storage*

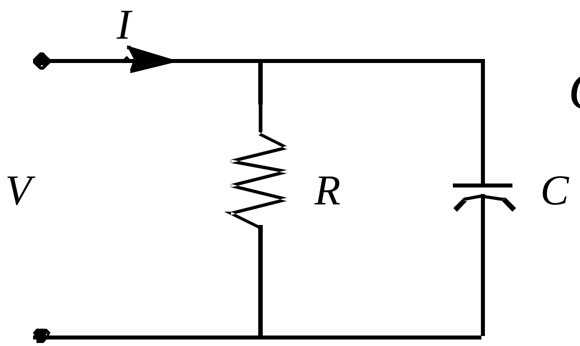



$Q \propto I$

$$I = Q/\tau_s$$
$$Q = I\tau_s$$

What is the relationship between $\tau_s$ (charge-storage time) and $\tau$ (carrier lifetime)?

# *4.11 Small-signal Model of the Diode*




$$G \equiv \frac{1}{R} = \frac{dI}{dV} = \frac{d}{dV} I_0 (e^{qV/kT} - 1) \approx \frac{d}{dV} I_0 e^{qV/kT}$$

$$= \frac{q}{kT} I_0 (e^{qV/kT}) = I_{DC} / \frac{kT}{q}$$

What is $G$ at 300K and $I_{DC}$ = 1 mA?

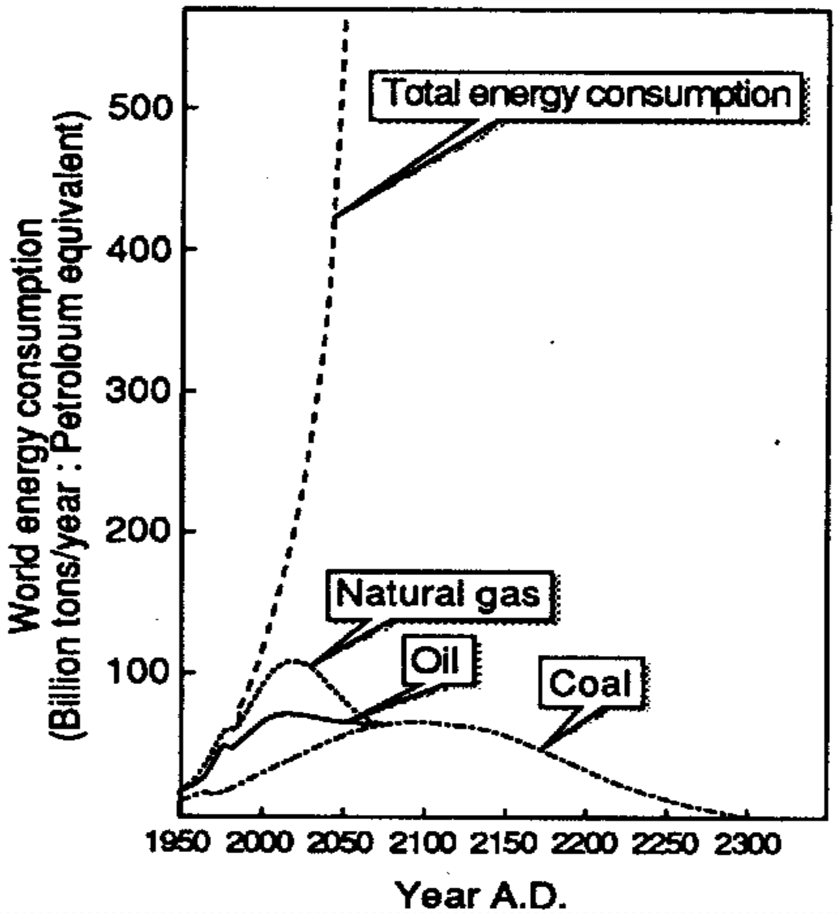
***Diffusion Capacitance:***

$$C = \frac{dQ}{dV} = \tau_s \frac{dI}{dV} = \tau_s G = \tau_s I_{DC} / \frac{kT}{q}$$

Which is larger, diffusion or depletion capacitance?

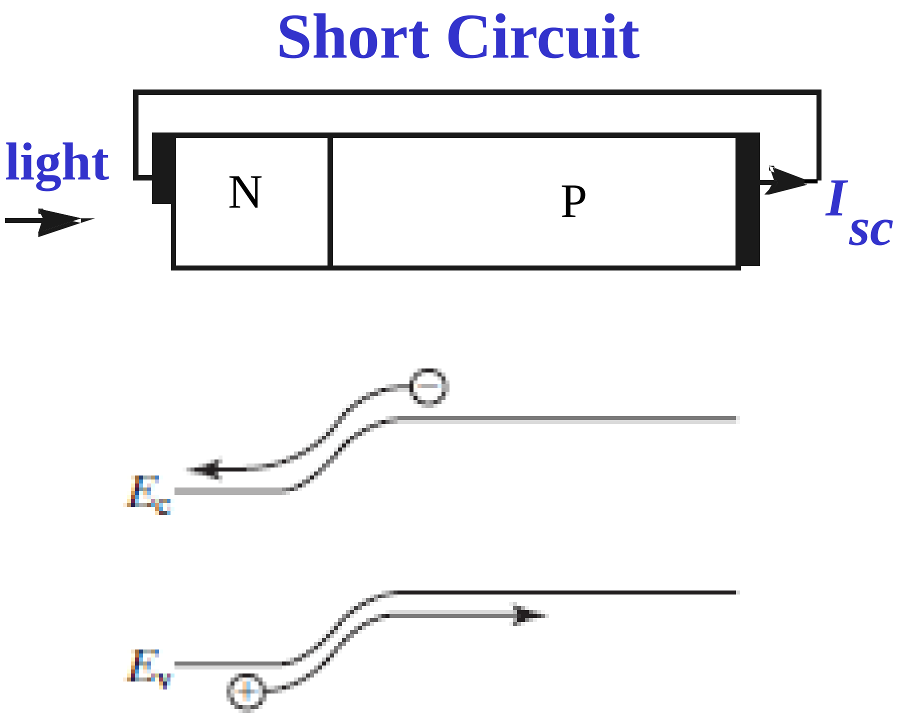
# Part II: Application to Optoelectronic Devices

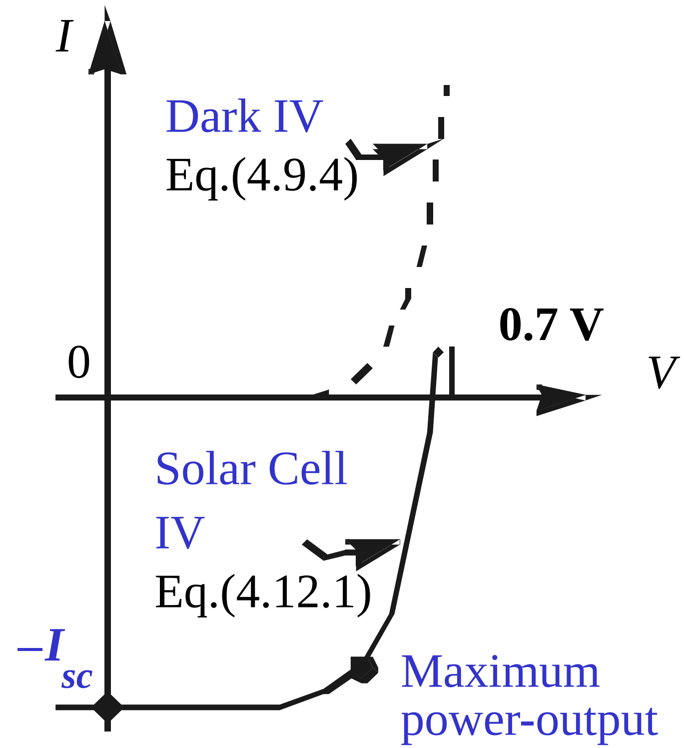
## 4.12 Solar Cells




- ***Solar Cells*** is also known as ***photovoltaic cells***.
- Converts sunlight to electricity with 10-30% conversion efficiency.
- 1 $m^2$ solar cell generate about 150 W peak or 25 W continuous power.
- Low cost and high efficiency are needed for wide deployment.

# 4.12.1 Solar Cell Basics



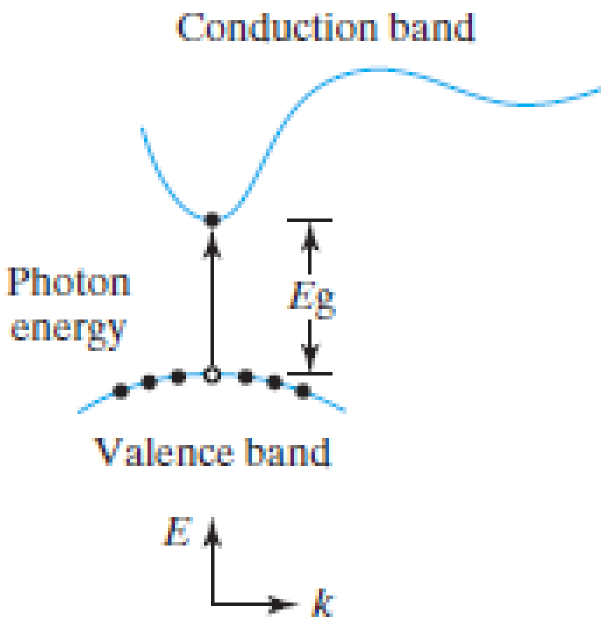




$$I = I_0(e^{qV/kT} - 1) - I_{sc}$$

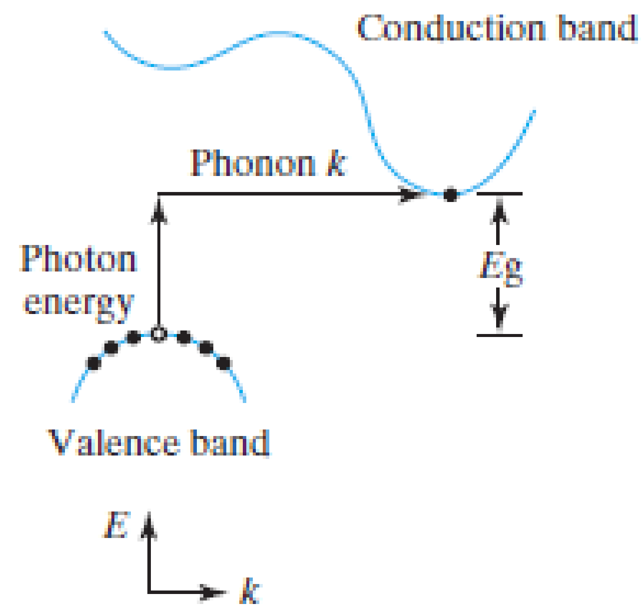
# Direct-Gap and Indirect-Gap Semiconductors

•Electrons have both particle and wave properties.

•An electron has energy E and wave vector k.

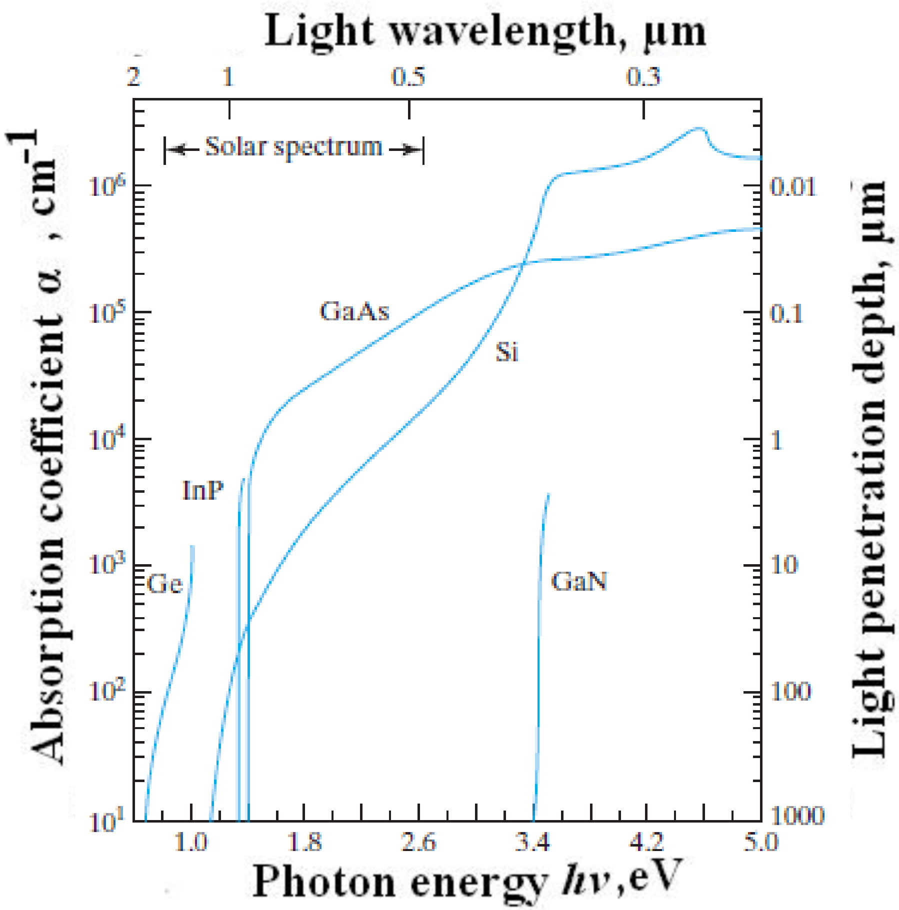



**direct-gap semiconductor**




**indirect-gap semiconductor**

# 4.12.2 Light Absorption




$$\text{Light intensity } (x) \propto e^{-\alpha x}$$

**$\alpha$(1/cm): absorption coefficient**

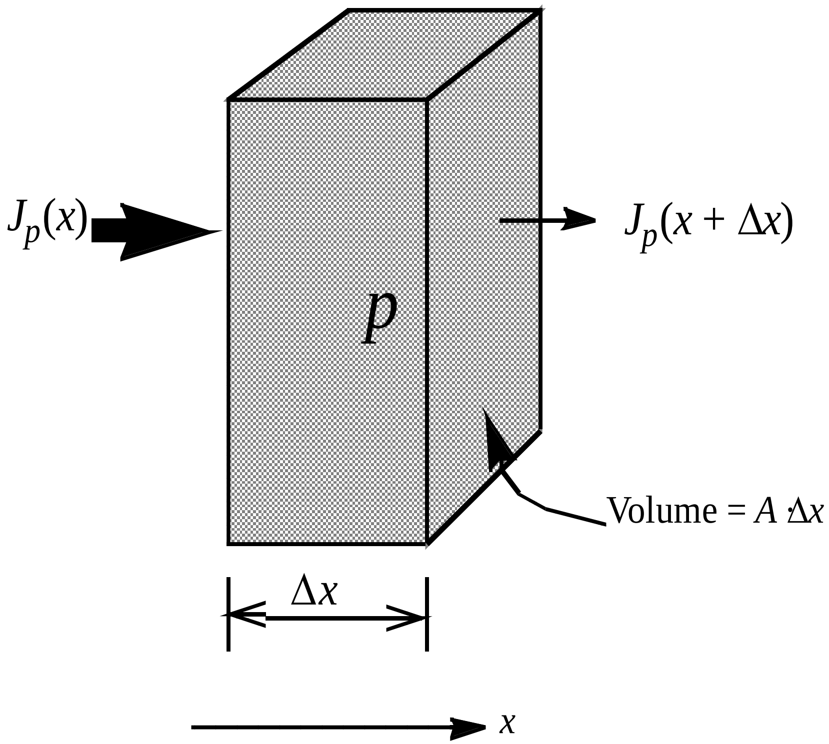
**$1/\alpha$ : light penetration depth**

$$\text{Photon Energy (eV)} = \frac{hc}{\lambda}$$

$$= \frac{1.24}{\lambda}(\mu m)$$

***A thinner layer of direct-gap semiconductor can absorb most of solar radiation than indirect-gap semiconductor. But Si…***

## *4.12.3 Short-Circuit Current and Open-Circuit Voltage*



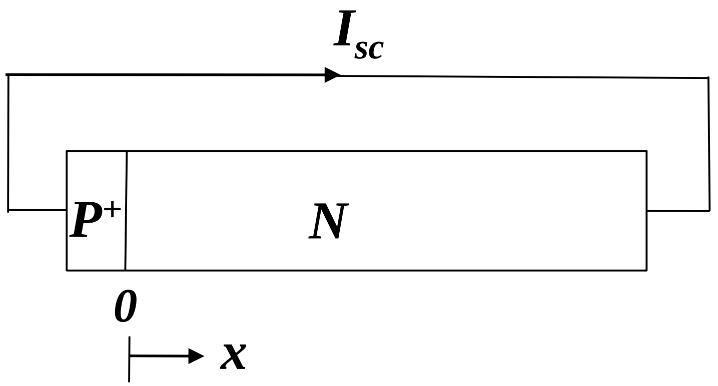

If light shines on the N-type semiconductor and generates holes (and electrons) at the rate of G $s^{-1}cm^{-3}$ ,

$$\frac{d^2 p'}{dx^2} = \frac{p'}{L_p^2} - \frac{G}{D_p}$$

If the sample is uniform (no PN junction),
$d^2p'/dx^2 = 0 \rightarrow p' = GL_p^2/D_p = G\tau_p$

# *Solar Cell Short-Circuit Current, $I_{sc}$*

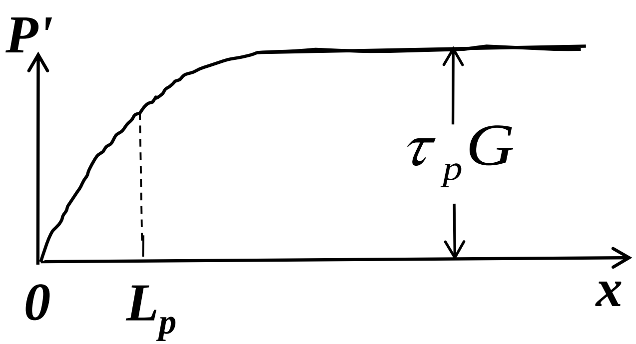
Assume very thin P+ layer and carrier generation in N region only.




$$p'(\infty) = L_p^2 \frac{G}{D_p} = \tau_p G$$

$$p'(0) = 0$$

$$p'(x) = \tau_p G(1 - e^{-x/L_p})$$




$$J_p = -qD_p \frac{dp'(x)}{dx} = q\frac{D_p}{L_p}\tau_p G e^{-x/L_p}$$

$$I_{sc} = AJ_p(0) = AqL_p G$$

G is really not uniform. $L_p$ needs be larger than the light penetration depth to collect most of the generated carriers.

## *Open-Circuit Voltage*

- Total current is $I_{SC}$ plus the PV diode (dark) current:

$$I = Aq\frac{n_i^2}{N_d}\frac{D_p}{L_p}(e^{qV/kT} - 1) - AqL_pG$$

- Solve for the open-circuit voltage ($V_{oc}$) by setting $I=0$

(assuming $e^{qV_{oc}/kT} >> 1$)

$$0 = \frac{n_i^2}{N_d}\frac{D_p}{L_p}e^{qV_{oc}/kT} - L_pG$$

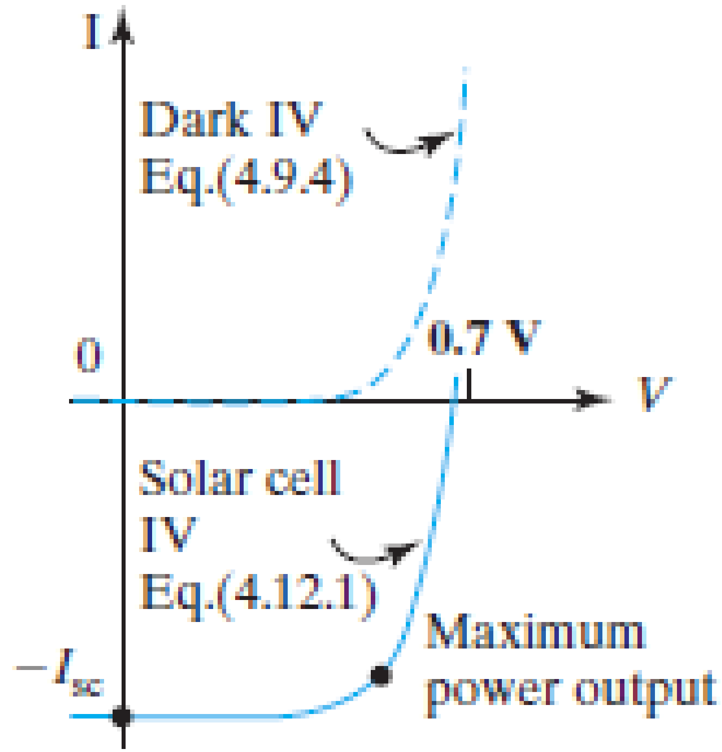
$$V_{oc} = \frac{kT}{q}\ln(\tau_p G N_d / n_i^2)$$

***How to raise $V_{oc}$?***

# 4.12.4 Output Power

A particular operating point on the solar cell I-V curve maximizes the output power (I ×V).

$$\text{Output Power} = I_{sc} \times V_{oc} \times FF$$




•Si solar cell with 15-20% efficiency dominates the market now

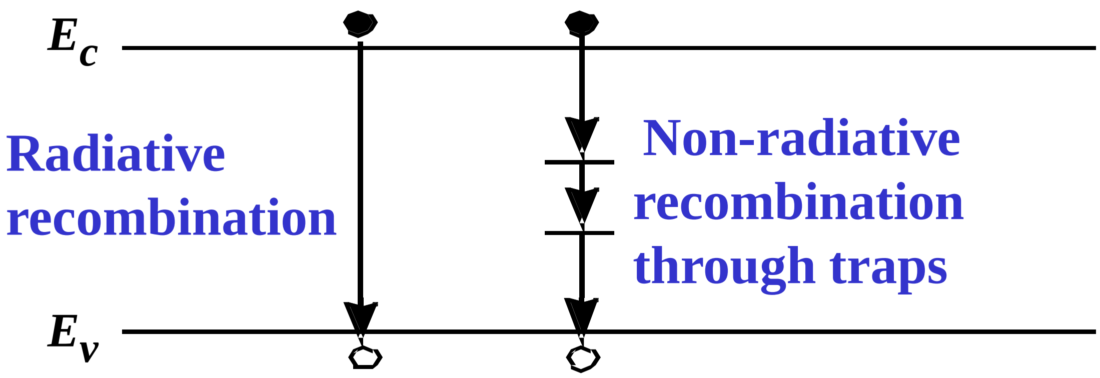
•Theoretically, the highest efficiency (~24%) can be obtained with 1.9eV >Eg>1.2eV. Larger Eg lead to too low Isc (low light absorption); smaller Eg leads to too low Voc.

•***Tandem solar cells*** gets 35% efficiency using large ***and*** small Eg materials tailored to the short and long wavelength solar light.

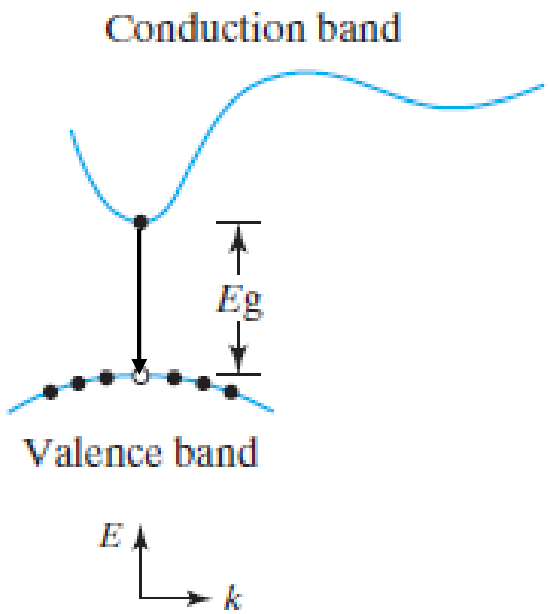
# 4.13 Light Emitting Diodes and Solid-State Lighting

**Light emitting diodes (LEDs)**

- LEDs are made of compound semiconductors such as InP and GaN.
- Light is emitted when electron and hole undergo ***radiative recombination*****.**

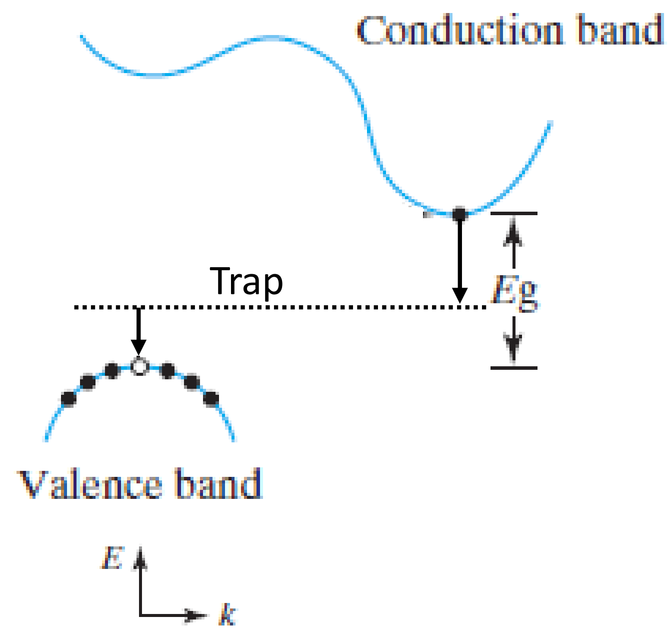


# *Direct and Indirect Band Gap*




Direct band gap
Example: GaAs

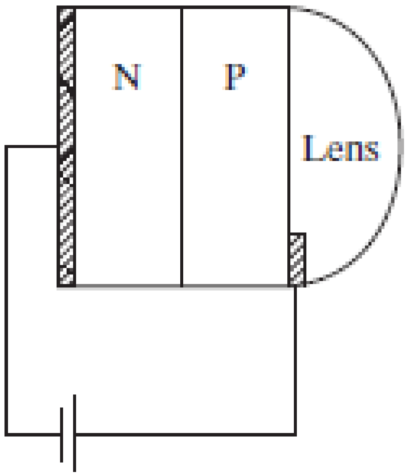
Direct recombination is efficient as k conservation is satisfied.

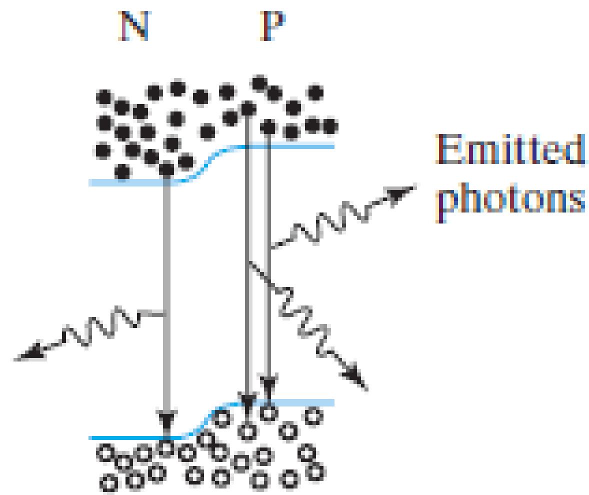



Indirect band gap
Example: Si

Direct recombination is rare as k conservation is not satisfied

# 4.13.1 LED Materials and Structure




$$\text{LED wavelength}\,(\mu\text{m}) = \frac{1.24}{\text{photon energy}} \approx \frac{1.24}{E_g(eV)}$$

# 4.13.1 LED Materials and Structure

| | $E_g(eV)$ | Wavelength (μm) | Color | Lattice constant (Å) |
|---|---|---|---|---|
| InAs | 0.36 | 3.44 | | 6.05 |
| InN | 0.65 | 1.91 | infrared | 3.45 |
| InP | 1.36 | 0.92 | | 5.87 |
| GaAs | 1.42 | 0.87 | Red Yellow | 5.66 |
| GaP | 2.26 | 0.55 | Green Blue | 5.46 |
| AlP | 3.39 | 0.51 | | 5.45 |
| GaN | 2.45 | 0.37 | | 3.19 |
| AlN | 6.20 | 0.20 | UV | 3.11 |

**Light-emitting diode materials**

**compound semiconductors**

binary semiconductors:

**- Ex: GaAs, efficient emitter**

ternary semiconductor :

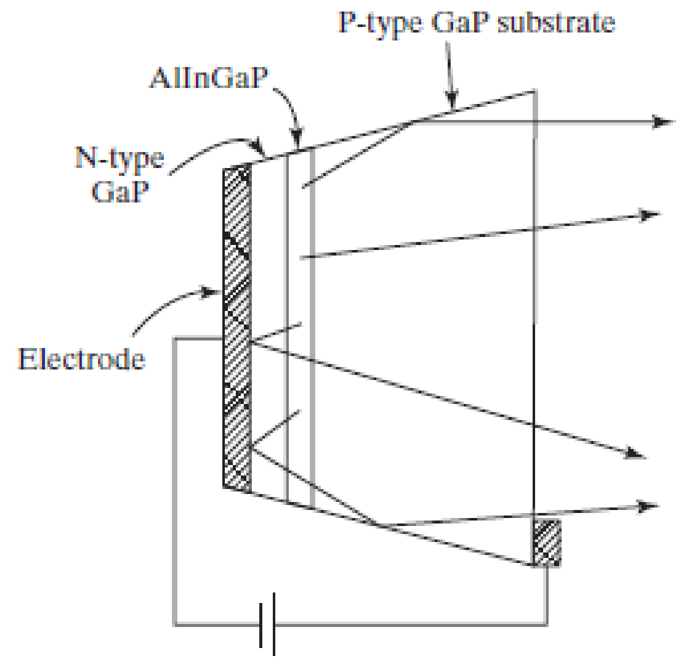
**- Ex: $GaAs_{1-x}P_x$, tunable $E_g$ (to vary the color)**

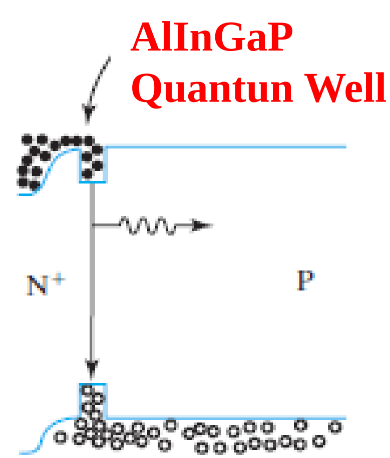
quaternary semiconductors:

**- Ex: AlInGaP , tunable $E_g$ and lattice constant (for growing high quality epitaxial films on inexpensive substrates)**

# Common LEDs

| Spectral range | Material System | Substrate | Example Applications |
|---|---|---|---|
| Infrared | InGaAsP | InP | Optical communication |
| Infrared -Red | GaAsP | GaAs | Indicator lamps. Remote control |
| Red-Yellow | AlInGaP | GaA or GaP | Optical communication. High-brightness traffic signal lights |
| Green-Blue | InGaN | GaN or sapphire | High brightness signal lights. Video billboards |
| Blue-UV | AlInGaN | GaN or sapphire | Solid-state lighting |
| Red-Blue | Organic semicon-ductors | glass | Displays |



# 4.13.2 Solid-State Lighting

**luminosity (lumen, lm)**: a measure of visible light energy normalized to the sensitivity of the human eye at different wavelengths

| Incandescent lamp | Compact fluorescent lamp | Tube fluorescent lamp | **White LED** | Theoretical limit at peak of eye sensitivity ( $\lambda$=555nm) | Theoretical limit (white light) |
|---|---|---|---|---|---|
| **17** | **60** | **50-100** | **90-?** | **683** | **~340** |

**Luminous efficacy of lamps in lumen/watt**

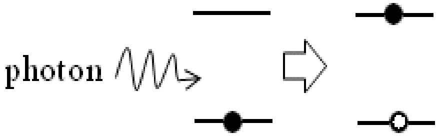
**Organic Light Emitting Diodes (OLED)** :
has lower efficacy than nitride or aluminide based compound semiconductor LEDs.

Terms: **luminosity** measured in **lumens**. **luminous efficacy,**

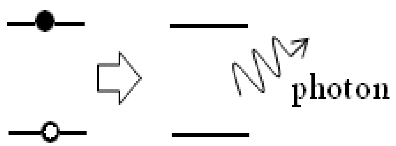
# 4.14 Diode Lasers

## 4.14.1 Light Amplification

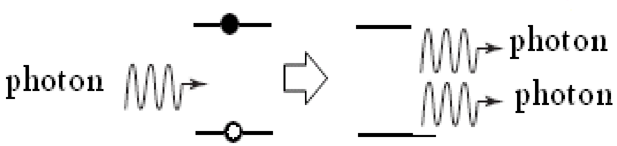
(a) Absorption



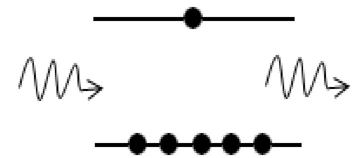

(b) Spontaneous Emission



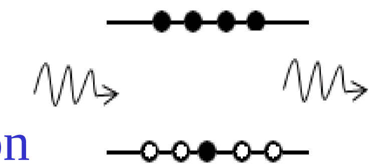

(c) Stimulated Emission




(d) Net Light Absorption



(e) Net Light Amplification



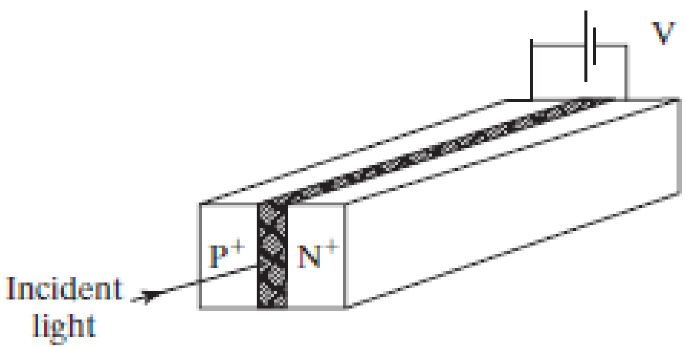
**Light amplification** requires ***population inversion*: electron occupation probability is larger for higher E states than lower E states.**

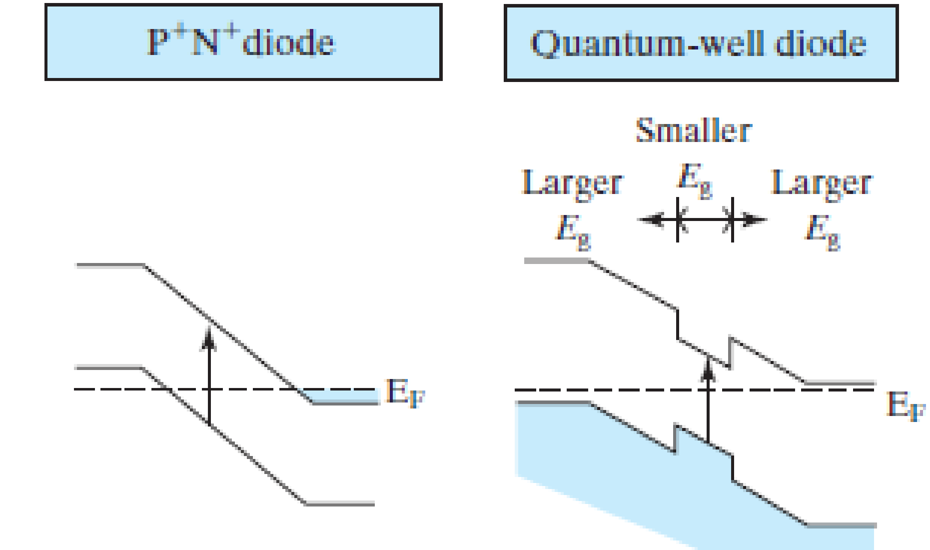
**Stimulated emission: emitted photon has identical frequency and directionality as the stimulating photon; light wave is amplified.**

# 4.14.1 Light Amplification in PN Diode

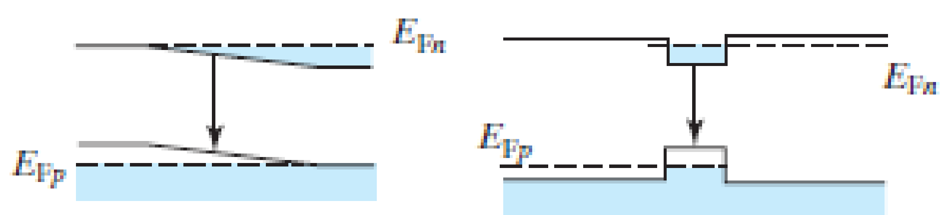
*Population inversion is achieved when*

$$qV = E_{fn} - E_{fp} > E_g$$



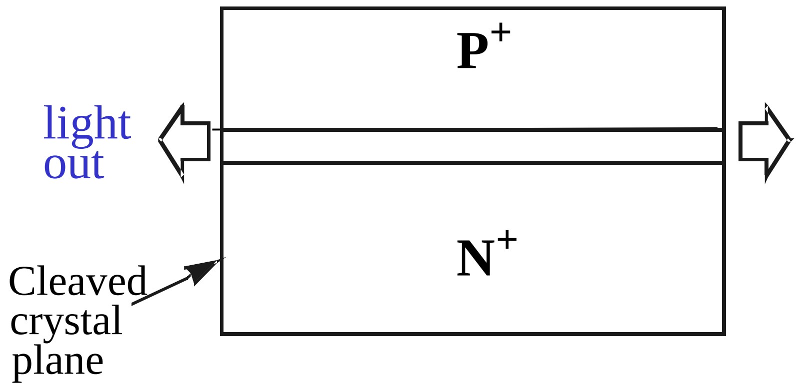




**Equilibrium, V=0**




**Population inversion, qV > Eg**

# 4.14.2 Optical Feedback and Laser



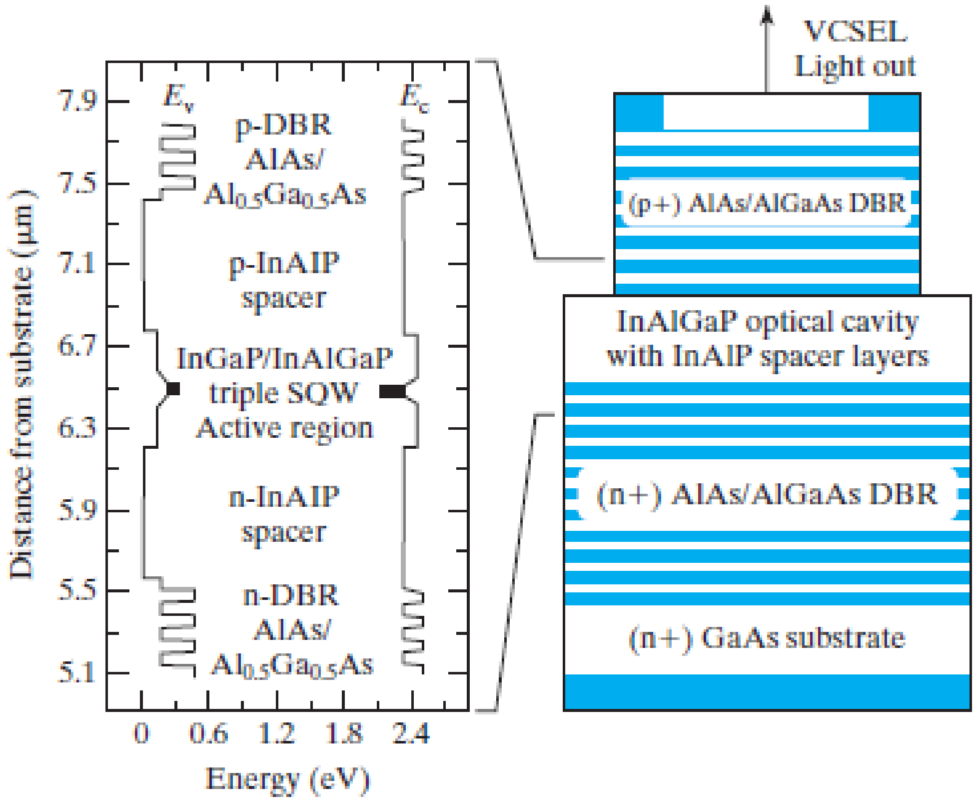

***Laser threshold* is reached (light intensity grows by feedback) when**

$$R_1 \times R_2 \times G \geq 1$$

•**R1, R2**: reflectivities of the two ends
•**G** : light amplification factor (gain) for a round-trip travel of the light through the diode

Light intensity grows until $R_1 \times R_2 \times G = 1$ , when the light intensity is just large enough to stimulate carrier recombinations at the same rate the carriers are injected by the diode current.

# 4.14.2 Optical Feedback and Laser Diode




- ***Distributed Bragg reflector (DBR)*** **reflects light with multi-layers of semiconductors.**
- ***Vertical-cavity surface-emitting laser (VCSEL)*** **is shown on the left.**
- **Quantum-well laser has smaller threshold current because fewer carriers are needed to achieve population inversion in the small volume of the thin small-*Eg* well.**

## *4.14.3 Laser Applications*

***Red diodelasers****: CD, DVD reader/writer*

***Blue diode lasers****: Blu-ray DVD (higher storage density)*

***1.55 μm infrared* diode lasers:** *Fiber-optic communication*

## *4.15 Photodiodes*

***Photodiodes****:* Reverse biased PN diode. Detects photo-generated current (similar to Isc of solar cell) for optical communication, DVD reader, etc.
***Avalanche photodiodes:*** Photodiodes operating near avalanche breakdown amplifies photocurrent by impact ionization.

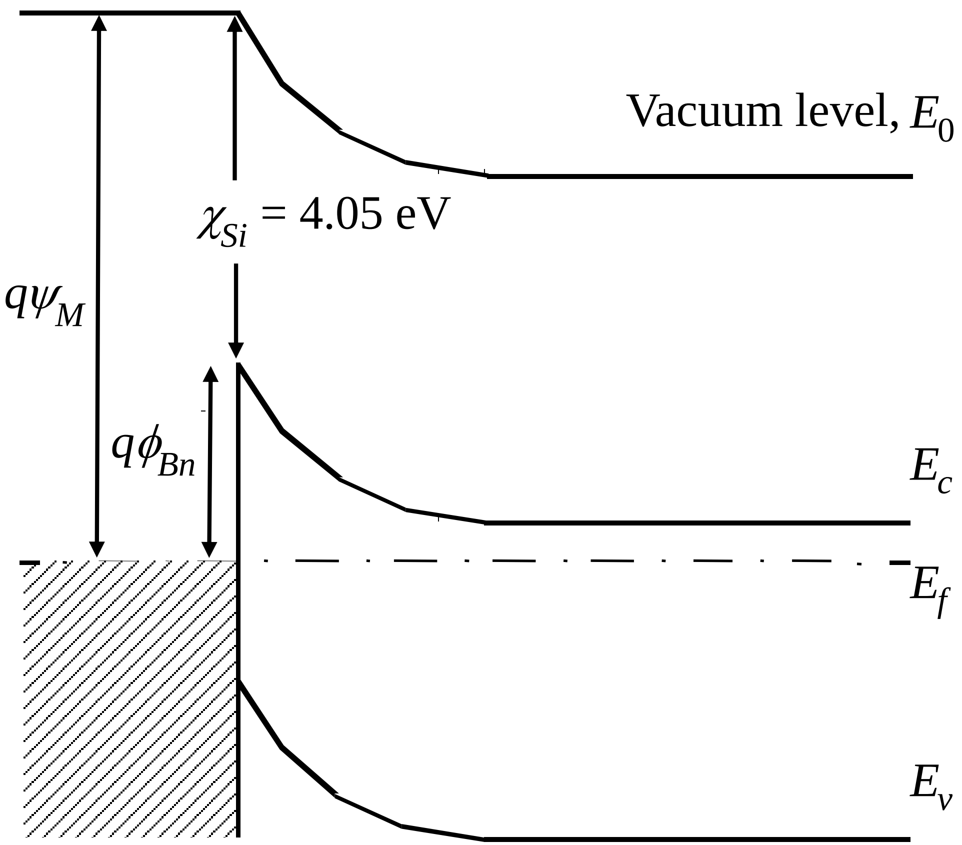
# ***Part III: Metal-Semiconductor Junction***

Two kinds of metal-semiconductor contacts:

- *Rectifying* ***Schottky diodes****: metal on lightly doped silicon*

- *Low-resistance* ***ohmic contacts****: metal on heavily doped silicon*

# $\phi_{Bn}$ *Increases with Increasing Metal Work Function*




$\psi_M$ : Work Function of metal

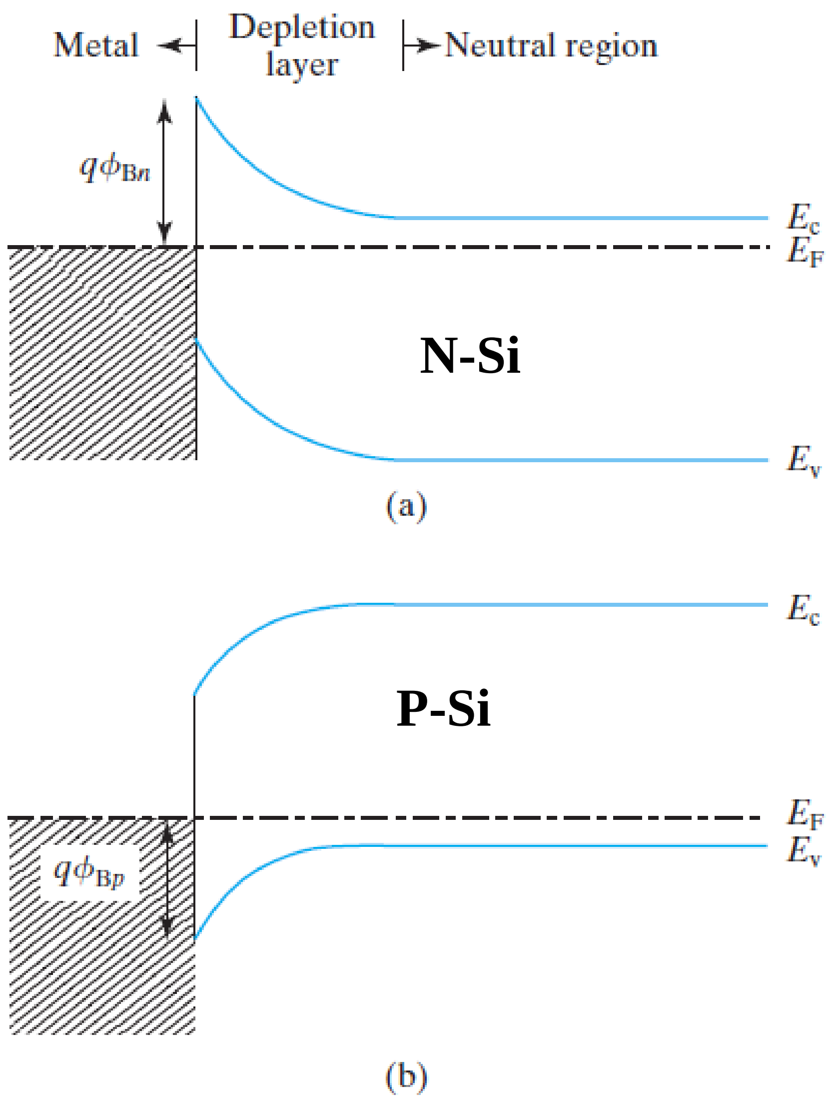
$\chi_{Si}$ : Electron Affinity of Si

Theoretically,

$$\phi_{Bn} = \psi_M - \chi_{Si}$$

# 4.16 Schottky Barriers

*Energy Band Diagram of Schottky Contact*




- Schottky barrier height, $\phi_B$ , is a function of the metal material.

- $\phi_B$ is the most important parameter. The sum of $q\phi_{Bn}$ and $q\phi_{Bp}$ is equal to $E_g$ .

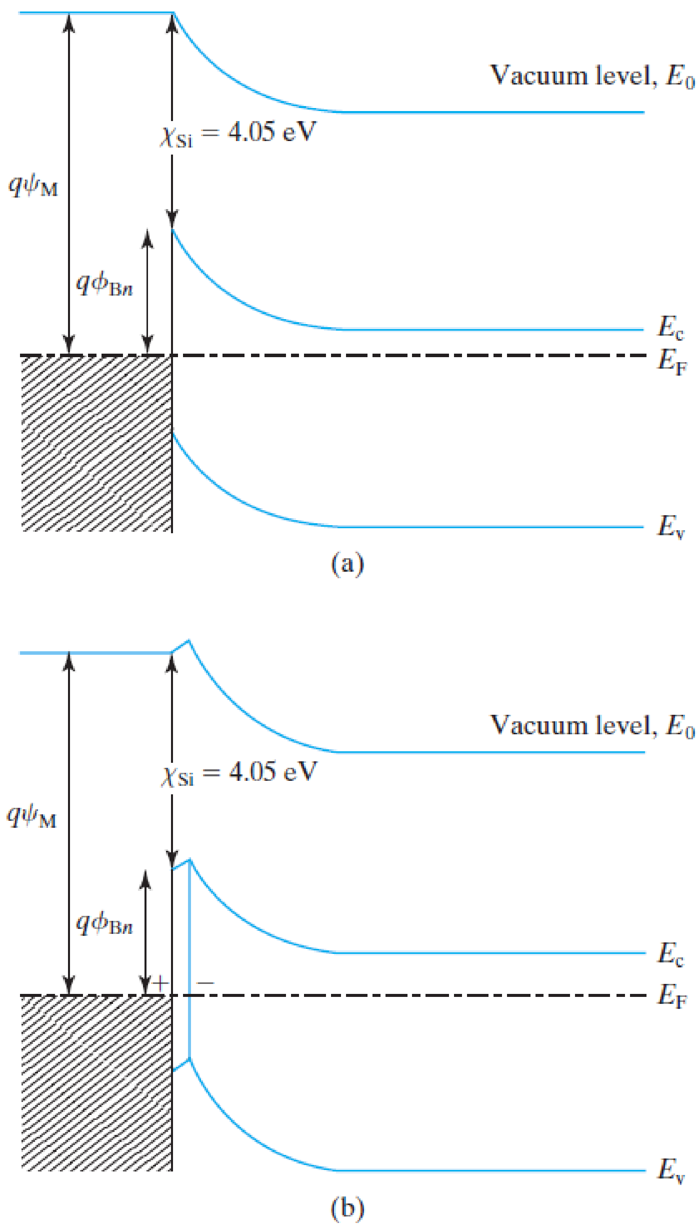
## *Schottky barrier heights for electrons and holes*

| Metal | Mg | Ti | Cr | W | Mo | Pd | Au | Pt |
|---|---|---|---|---|---|---|---|---|
| $\phi_{Bn}$ (V) | 0.4 | 0.5 | 0.61 | 0.67 | 0.68 | 0.77 | 0.8 | 0.9 |
| $\phi_{Bp}$ (V) | | 0.61 | 0.5 | | 0.42 | | 0.3 | |
| Work Function $\psi_m$ (V) | 3.7 | 4.3 | 4.5 | 4.6 | 4.6 | 5.1 | 5.1 | 5.7 |

$$\phi_{Bn} + \phi_{Bp} \approx E_g$$

$\phi_{Bn}$ increases with increasing metal work function

# *Fermi Level Pinning*




- A high density of energy states in the bandgap at the metal-semiconductor interface pins $E_f$ to a narrow range and $\boldsymbol{\phi_{Bn}}$ ***is typically 0.4 to 0.9 V***
- ***Question***: What is the typical range of $\phi_{Bp}$?

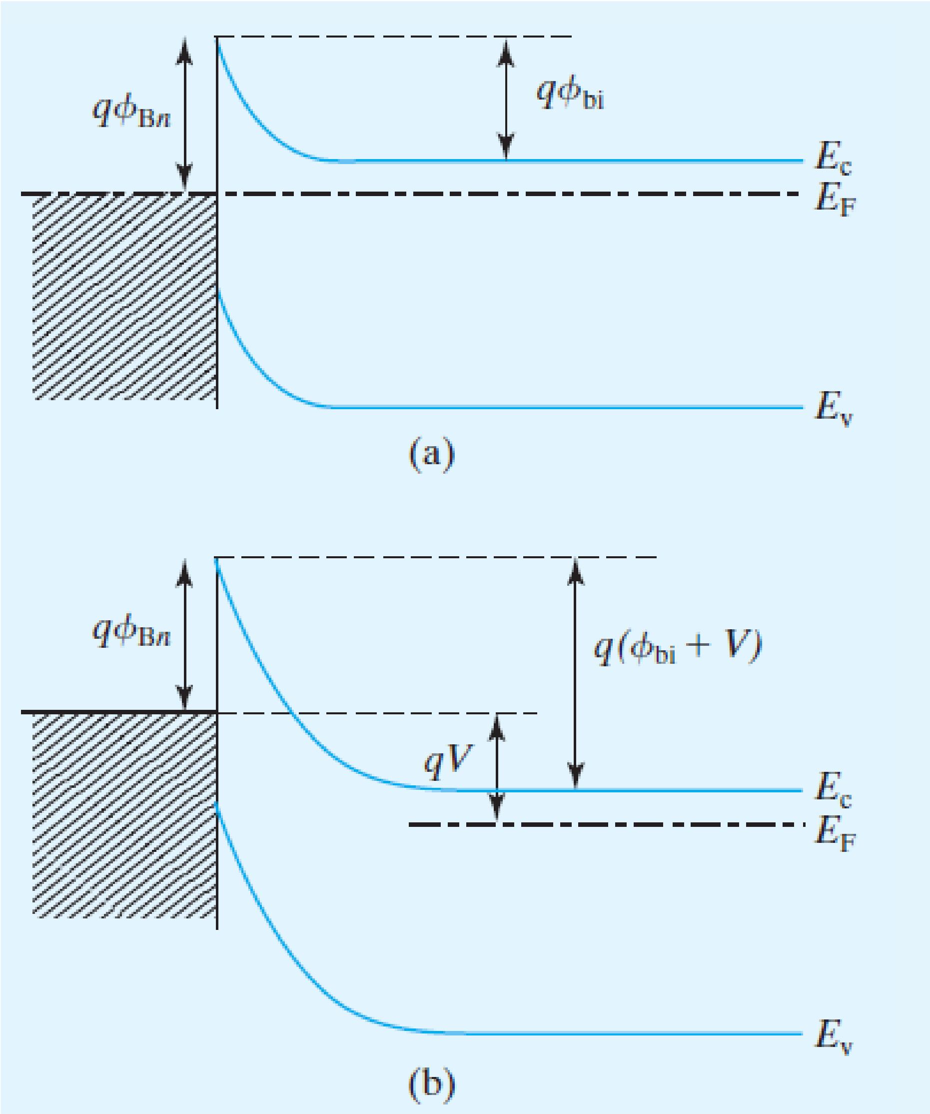
## *Schottky Contacts of Metal Silicide on Si*

***Silicide*: A silicon and metal compound. It is conductive similar to a metal.**

Silicide-Si interfaces are more stable than metal-silicon interfaces. After metal is deposited on Si, an annealing step is applied to form a silicide-Si contact. The term ***metal-silicon contact*** includes and almost always means silicide-Si contacts.

| Silicide | $ErSi_{1.7}$ | HfSi | $MoSi_2$ | $ZrSi_2$ | $TiSi_2$ | $CoSi_2$ | $WSi_2$ | $NiSi_2$ | $Pd_2Si$ | PtSi |
|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_{Bn}$ (V) | 0.28 | 0.45 | 0.55 | 0.55 | 0.61 | 0.65 | 0.67 | 0.67 | 0.75 | 0.87 |
| $\phi_{Bp}$ (V) | | | 0.55 | 0.49 | 0.45 | 0.45 | 0.43 | 0.43 | 0.35 | 0.23 |

# *Using C-V Data to Determine $\phi_B$*




$$q\phi_{bi} = q\phi_{Bn} - (E_c - E_f)$$

$$= q\phi_{Bn} - kT \ln \frac{N_c}{N_d}$$

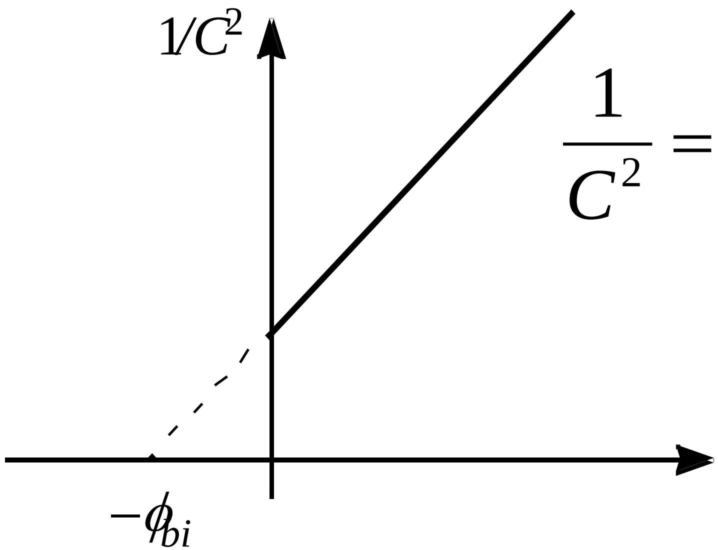
$$W_{dep} = \sqrt{\frac{2\varepsilon_s(\phi_{bi} + V)}{qN_d}}$$

$$C = \frac{\varepsilon_s}{W_{dep}} A$$

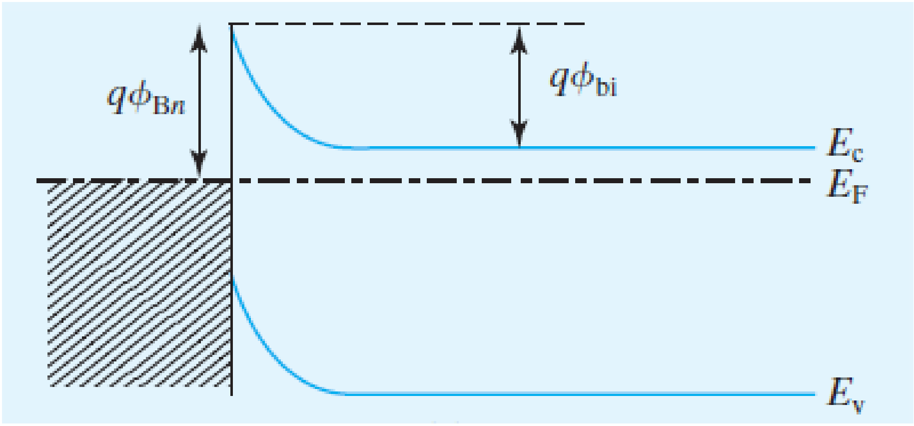
***Question:***

*How should we plot the CV data to extract $\phi_{bi}$?*

# *Using CV Data to Determine $\phi_B$*



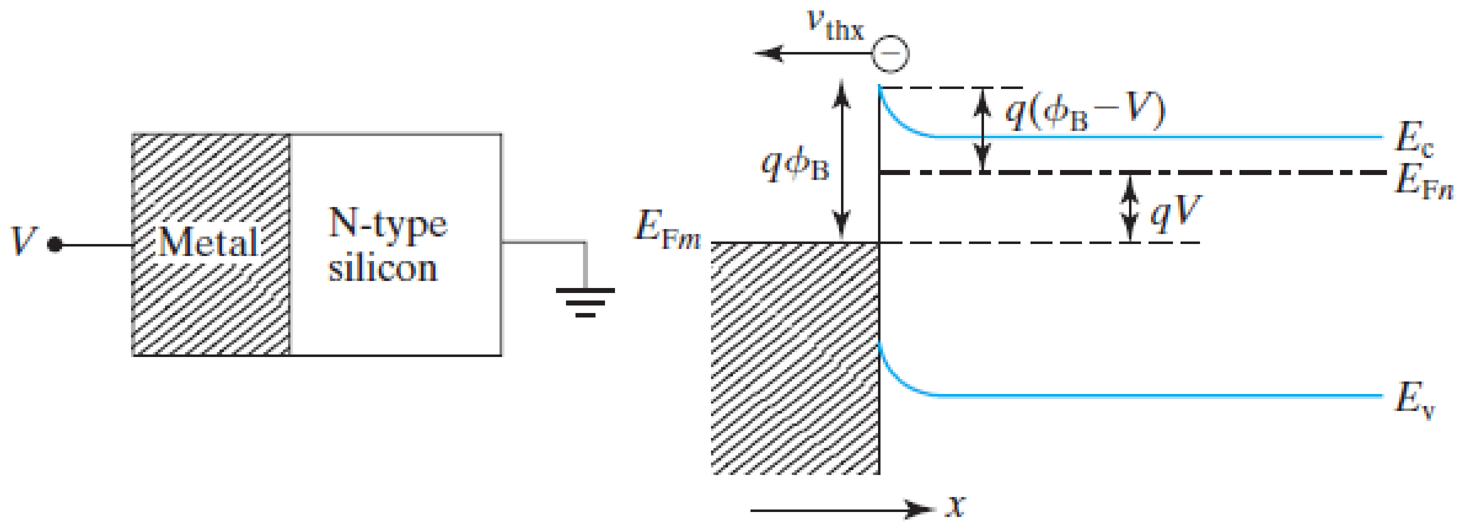

$$\frac{1}{C^2} = \frac{2(\phi_{bi} + V)}{qN_d \varepsilon_s A^2}$$




Once $\phi_{bi}$ is known, $\phi_B$ can be determined using

$$q\phi_{bi} = q\phi_{Bn} - (E_c - E_f) = q\phi_{Bn} - kT\ln\frac{N_c}{N_d}$$

# 4.17 Thermionic Emission Theory



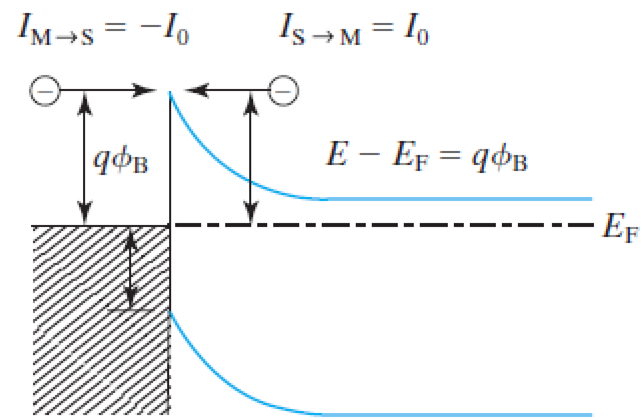

$$n = N_c e^{-q(\phi_B - V)/kT} = 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2} e^{-q(\phi_B - V)/kT}$$

$$v_{th} = \sqrt{3kT / m_n} \qquad v_{thx} = -\sqrt{2kT / \pi m_n}$$

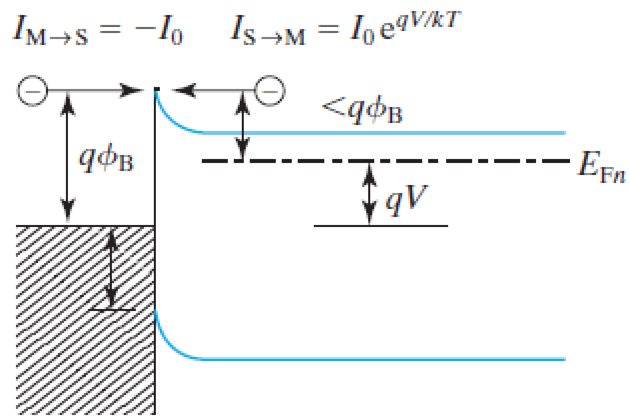
$$J_{S \to M} = -\frac{1}{2} q n v_{thx} = \frac{4\pi q m_n k^2}{h^3} T^2 e^{-q\phi_B / kT} e^{qV/kT}$$

$$= J_0 e^{qV/kT}, \text{where } J_o \approx 100 e^{-q\phi_B / kT} \text{ A/cm}^2$$

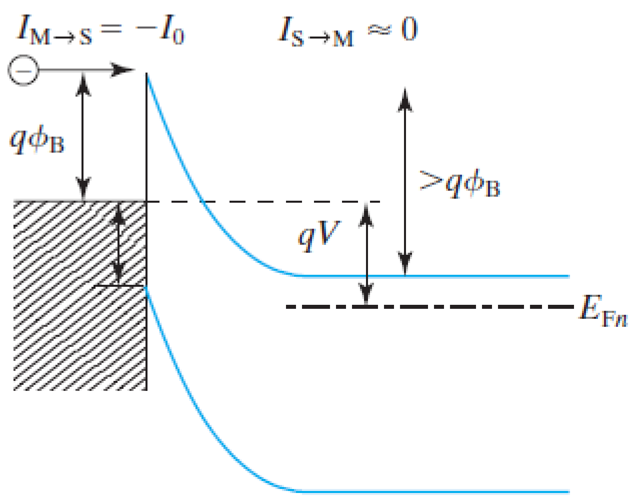
# 4.18 Schottky Diodes



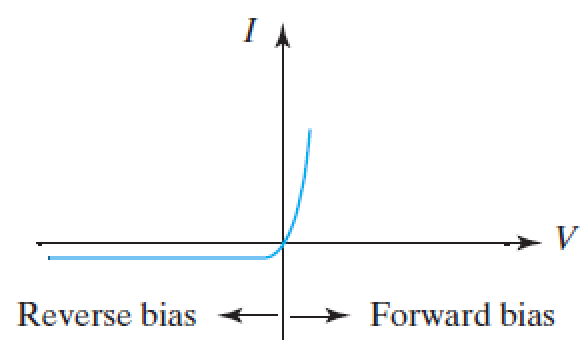

(a) $V = 0$. $I_{S\to M} = |I_{M\to S}| = I_0$




(b) Forward bias. Metal is positive wrt Si. $I_{S\to M} \gg |I_{M\to S}| = I_0$



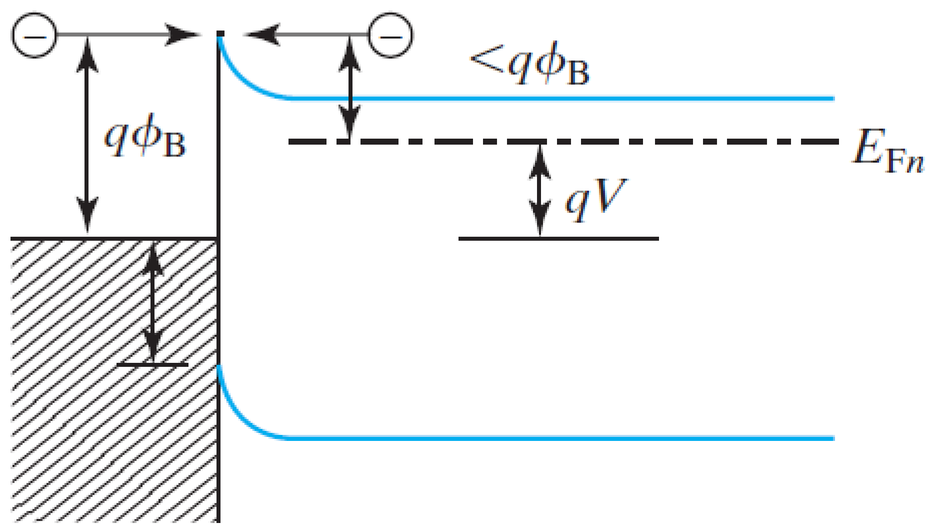

(c) Reverse bias. Metal is negative wrt Si. $I_{S\to M} \ll |I_{M\to S}| = I_0$




(d) Schottky diode $IV$.

# 4.18 Schottky Diodes

$I_{M\to S} = -I_0 \qquad I_{S\to M} = I_0\, e^{qV/kT}$



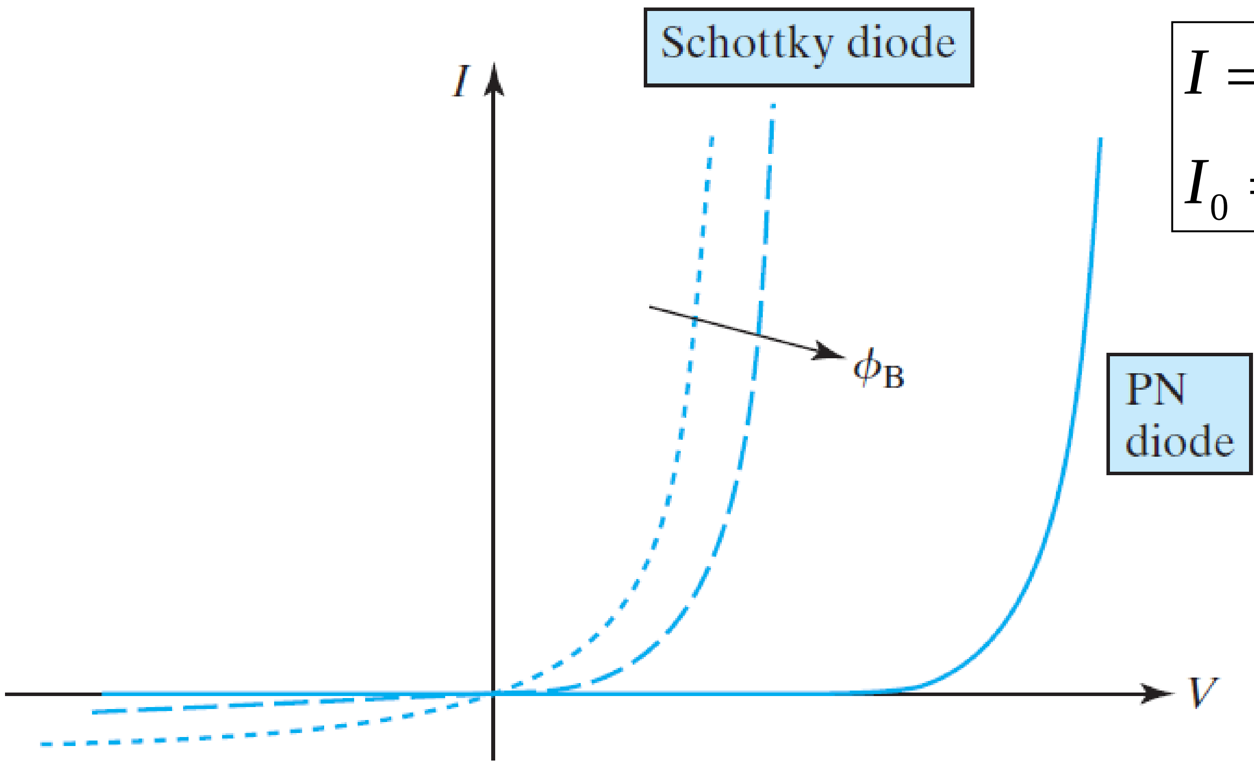

(b) Forward bias. Metal is positive wrt Si. $I_{S\to M} \gg |I_{M\to S}| = I_0$

$$I_0 = AKT^2 e^{-q\phi_B/kT}$$

$$K = \frac{4\pi q m_n k^2}{h^3} \approx 100\ \mathrm{A/(cm^2 \cdot K^2)}$$

$$I = I_{S\to M} + I_{M\to S} = I_0 e^{qV/kT} - I_0 = I_0(e^{qV/kT} - 1)$$

# 4.19 Applications of Schottly Diodes




$$I = I_0(e^{qV/kT} - 1)$$

$$I_0 = AKT^2 e^{-q\phi_B/kT}$$

- $I_0$ of a Schottky diode is $10^3$ to $10^8$ times larger than a PN junction diode, depending on $\phi_B$. A larger $I_0$ means a smaller forward drop $V$.
- A Schottky diode is the preferred rectifier in low voltage, high current applications.

# *Switching Power Supply*

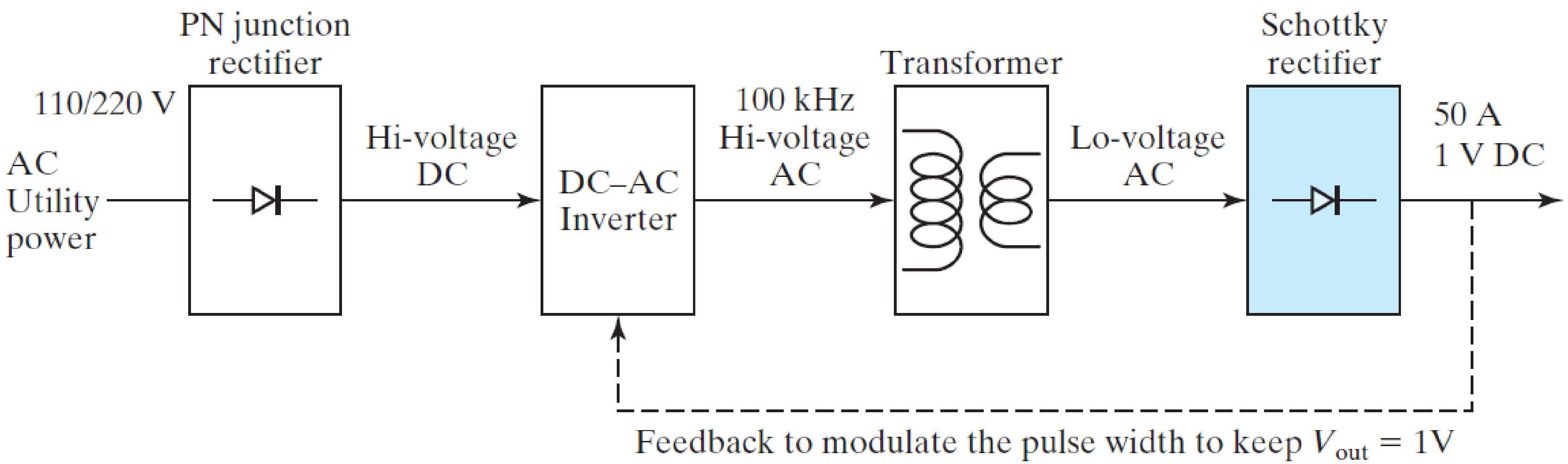



**FIGURE 4–41** Block diagram of a switching power supply for electronic equipment such as PCs.

## *4.19 Applications of Schottky diodes*

***Question****:* What sets the lower limit in a Schottky diode's forward drop?

• ***Synchronous Rectifier***: For an even lower forward drop, replace the diode with a wide-W MOSFET which is not bound by the tradeoff between diode *V* and leakage current.

• There is no minority carrier injection at the Schottky junction. Therefore, Schottky diodes can operate at higher frequencies than PN junction diodes.

# 4.20 Quantum Mechanical Tunneling

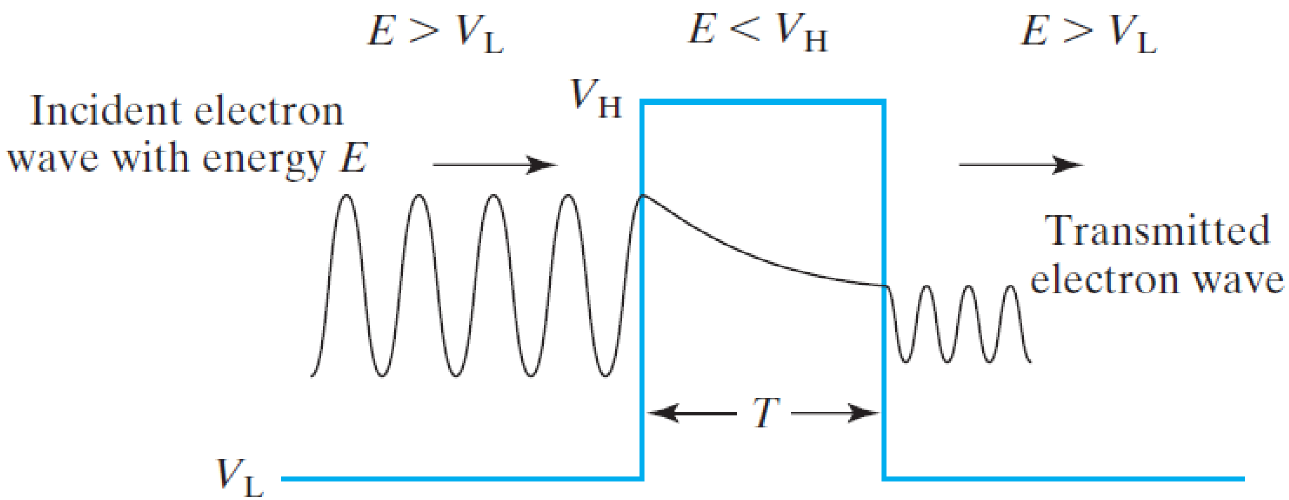
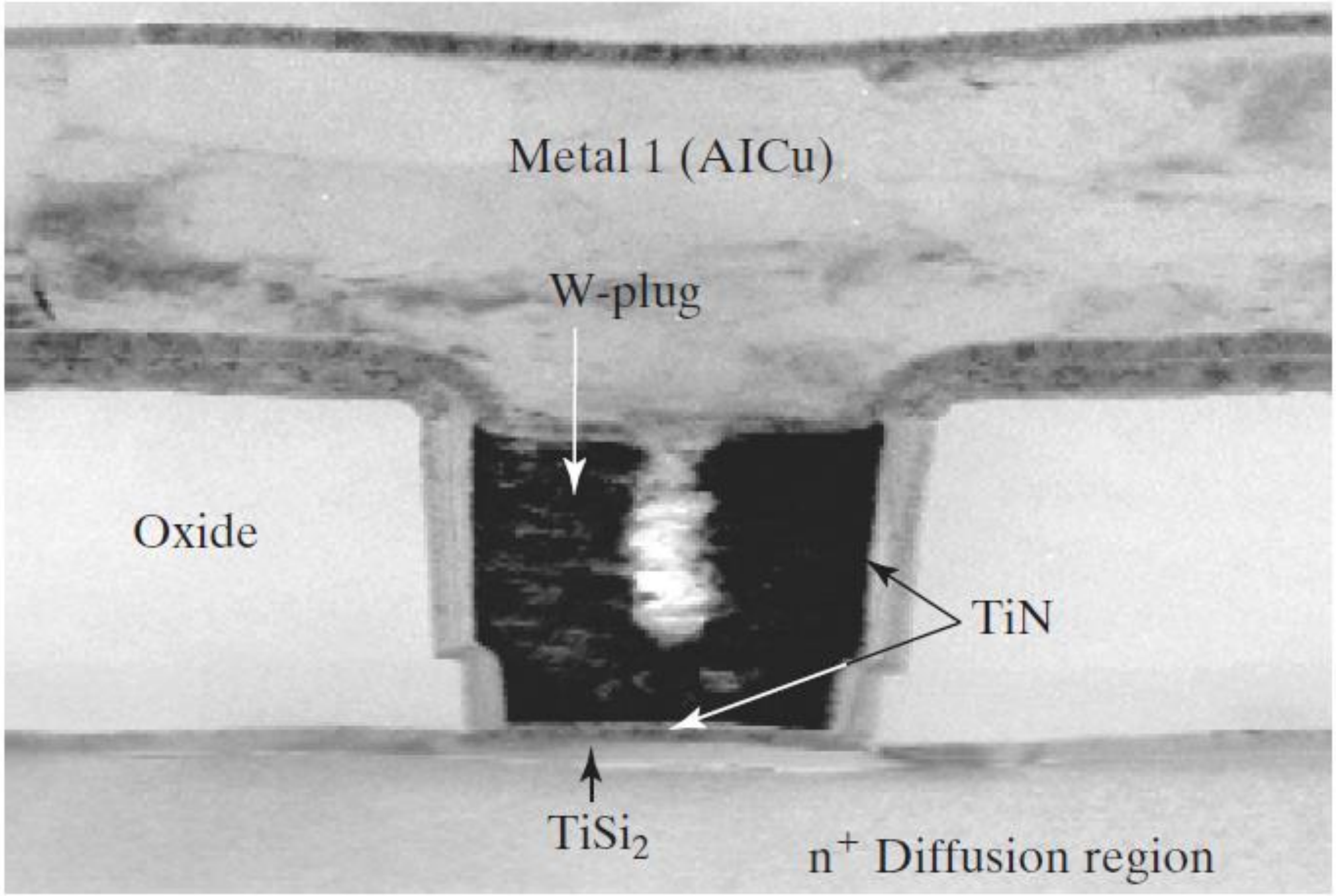



**FIGURE 4–42** Illustration of quantum mechanical tunneling.

***Tunneling probability*:**

$$P \approx \exp\left(-2T\sqrt{\frac{8\pi^2 m}{h^2}(V_H - E)}\right)$$

# *4.21 Ohmic Contacts*

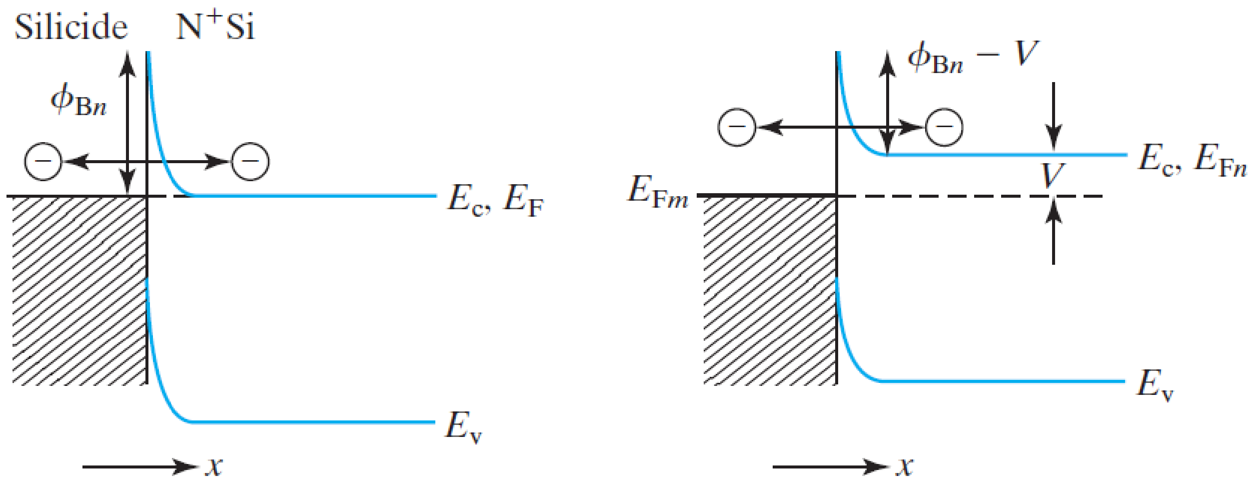


# 4.21 Ohmic Contacts

$$W_{dep} = \sqrt{\frac{2\varepsilon_s \phi_{Bn}}{qN_d}}$$

Tunneling probability:

$$P \approx e^{-H\phi_{Bn}/\sqrt{N_d}}$$



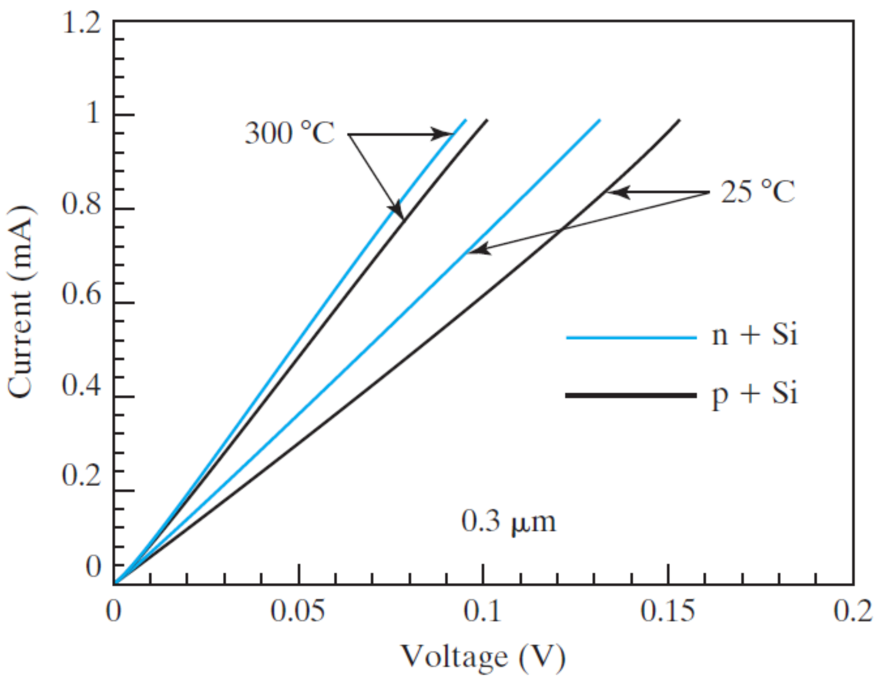

(a) (b)

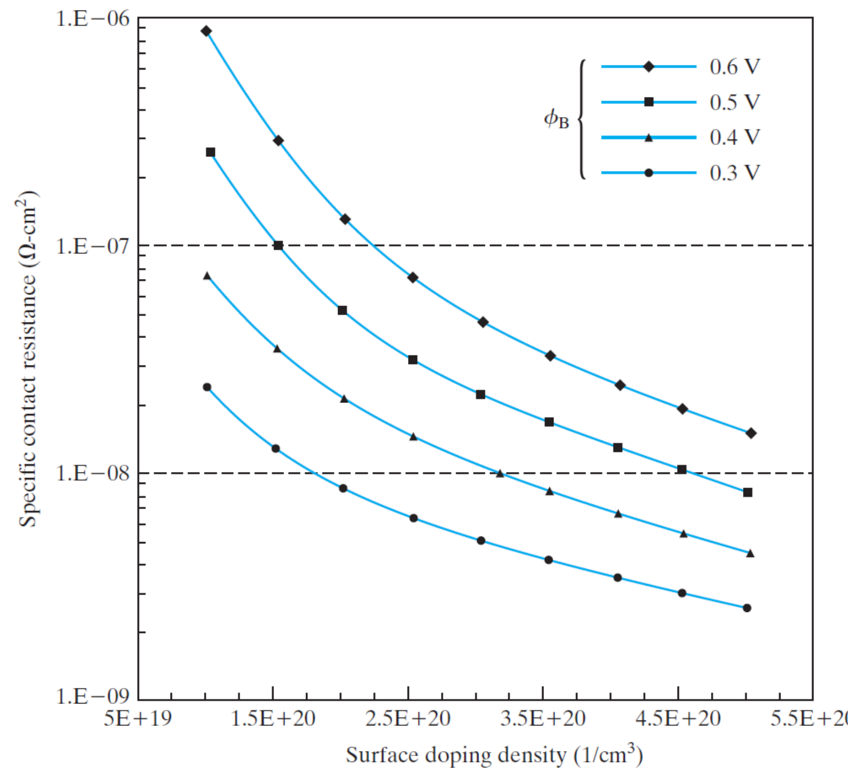
$$T \approx W_{dep}/2 = \sqrt{\varepsilon_s \phi_{Bn}/2qN_d}$$

$$H = \frac{4\pi}{h}\sqrt{\varepsilon_s m_n/q}$$

$$J_{S\rightarrow M} \approx \frac{1}{2} qN_d v_{thx} P = qN_d \sqrt{kT/2\pi m_n}\, e^{-H(\phi_{Bn}-V)/\sqrt{N_d}}$$

# 4.21 Ohmic Contacts




$$R_c \equiv \left(\frac{dJ_{S\to M}}{dV}\right)^{-1} = \frac{2e^{H\phi_{Bn}/\sqrt{N_d}}}{qv_{thx}H\sqrt{N_d}} \propto e^{H\phi_{Bn}/\sqrt{N_d}} \ \ \Omega\cdot\mathrm{cm}^2$$

# 4.22 Chapter Summary

## Part I: PN Junction

$$\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}$$

The potential barrier increases by 1 V if a 1 V reverse bias is applied

depletion width

$$W_{dep} = \sqrt{\frac{2\varepsilon_s \cdot potential\ barrier}{qN}}$$

junction capacitance

$$C_{dep} = A\frac{\varepsilon_s}{W_{dep}}$$

# *4.22 Chapter Summary*

- Under forward bias, minority carriers are injected across the jucntion.

- The quasi-equilibrium boundary condition of minority carrier densities is:

$$n(x_p) = n_{P0}e^{qV/kT}$$
$$p(x_N) = p_{N0}e^{qV/kT}$$

- Most of the minority carriers are injected into the more lightly doped side.

# *4.22 Chapter Summary*

- Steady-state continuity equation:

$$\boxed{\frac{d^2 p'}{dx^2} = \frac{p'}{D_p \tau_p} = \frac{p'}{L_p{}^2}}$$

$$\boxed{L_p \equiv \sqrt{D_p \tau_p}}$$

- Minority carriers diffuse outward $\propto e^{-|x|/L_p}$ and $e^{-|x|/L_n}$
- $L_p$ and $L_n$ are the diffusion lengths

$$\boxed{\begin{aligned} I &= I_0 (e^{qV/kT} - 1) \\ I_0 &= Aqn_i{}^2 \left( \frac{D_p}{L_p N_d} + \frac{D_n}{L_n N_a} \right) \end{aligned}}$$

# *4.22 Chapter Summary*

Charge storage: $\boxed{Q = I\tau_s}$

Diffusion capacitance: $\boxed{C = \tau_s G}$

Diode conductance: $\boxed{G = I_{DC} / \dfrac{kT}{q}}$

# *4.22 Chapter Summary*

## *Part II: Optoelectronic Applications*

$$\text{Solar cell power} = I_{sc} \times V_{oc} \times FF$$

•~100um Si or <1um direct–gap semiconductor can absorb most of solar photons with energy larger than $E_g$.

•Carriers generated within diffusion length from the junction can be collected and contribute to the Short Circuit Current $I_{sc}$.

•Theoretically, the highest efficiency (~24%) can be obtained with 1.9eV >$E_g$>1.2eV. Larger $E_g$ lead to too low $I_{sc}$ (low light absorption); smaller $E_g$ leads to too low Open Circuit VoltageVoc.

•Si cells with ~15% efficiency dominate the market. >2x cost reduction (including package and installation) is required to achieve cost parity with base-load non-renewable electricity.

# 4.22 Chapter Summary

***LED and Solid-State Lighting***

•Electron-hole recombination in direct-gap semiconductors such as GaAs produce light.

•Tenary semiconductors such as GaAsP provide tunable $E_g$ and LED color.

•Quatenary semiconductors such as AlInGaP provide tunable $E_g$ and lattice constants for high quality epitaxial growth on inexpensive substrates.

•Beyond displays, communication, and traffic lights, a new application is space lighting with luminous efficacy >5x higher than incandescent lamps. White light can be obtained with UV LED and phosphors. Cost still an issue.

•Organic semiconductor is an important low-cost LED material class.

# 4.22 Chapter Summary

## *Laser Diodes*

•Light is amplified under the condition of population inversion – states at higher E have higher probability of occupation than states at lower E.

•Population inversion occurs when diode forward bias $qV > E_g$.

•Optical feedback is provided with cleaved surfaces or distributed Bragg reflectors.

•When the round-trip gain (including loss at reflector) exceeds unity, laser threshold is reached.

•Quantum-well structures significantly reduce the threshold currents.

•Purity of laser light frequency enables long-distance fiber-optic communication. Purity of light direction allows focusing to tiny spots and enables DVD writer/reader and other application.

# *4.22 Chapter Summary*

## *Part III: Metal-Semiconductor Junction*

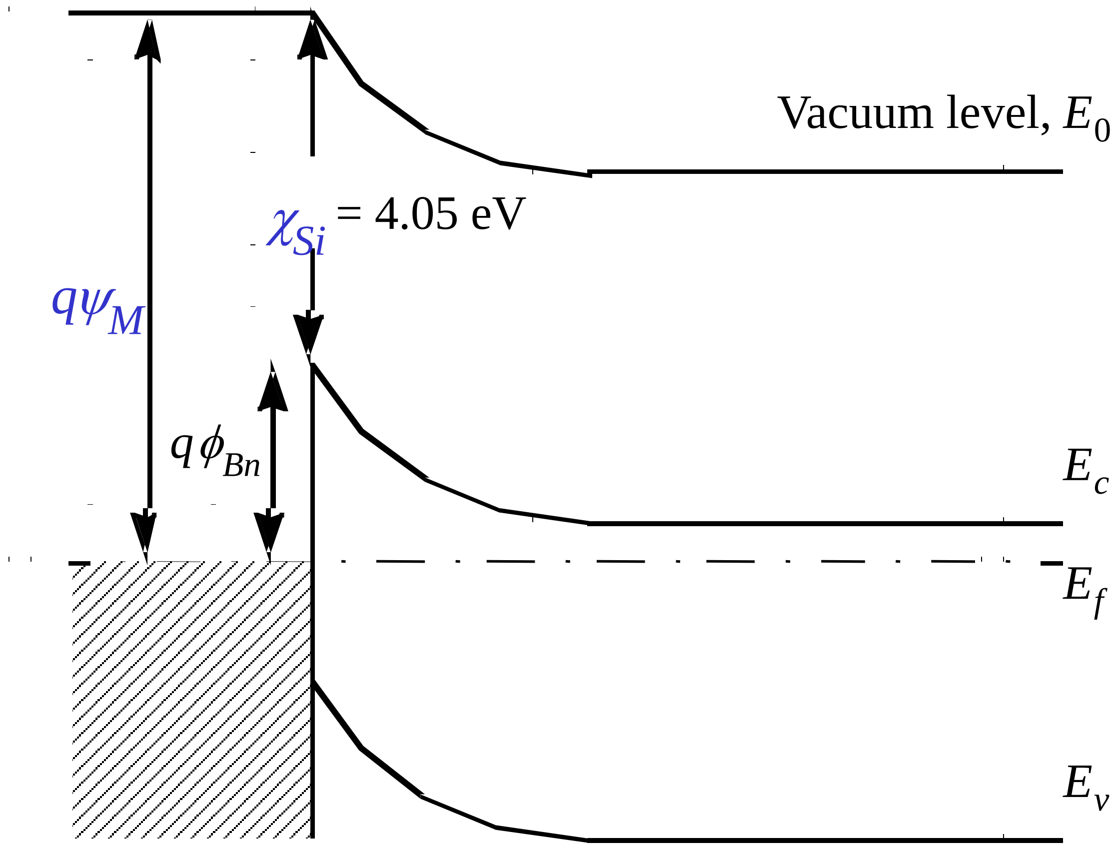
$$I_0 = AKT^2 e^{-q\phi_B/kT}$$

•Schottky diodes have large reverse saturation current, determined by the Schottky barrier height $\phi_B$, and therefore lower forward voltage at a given current density.

•Ohmic contacts relies on tunneling. Low resistance contact requires low $\phi_B$ and higher doping concentration.

$$R_c \propto e^{-(\frac{4\pi}{h}\phi_B\sqrt{\varepsilon_s m_n / qN_d})}\ \Omega\cdot\mathrm{cm}^2$$

# *$\phi_{Bn}$ Increases with Increasing Metal Work Function*




Ideally,

$$\phi_{Bn} = \psi_M - \chi_{Si}$$

***Homework:***

***4.18***

***4.19***

***4.21***

***4.22***

***4.23***

***4.24***

***4.26***