第四章　　微波传输线理论

4.1传输线方程的解及传输线的特性参数

- 传输线---约束电磁波沿规定方向传输能量和信息的系统。
- 均匀传输线：横向结构和尺寸沿纵轴不变。

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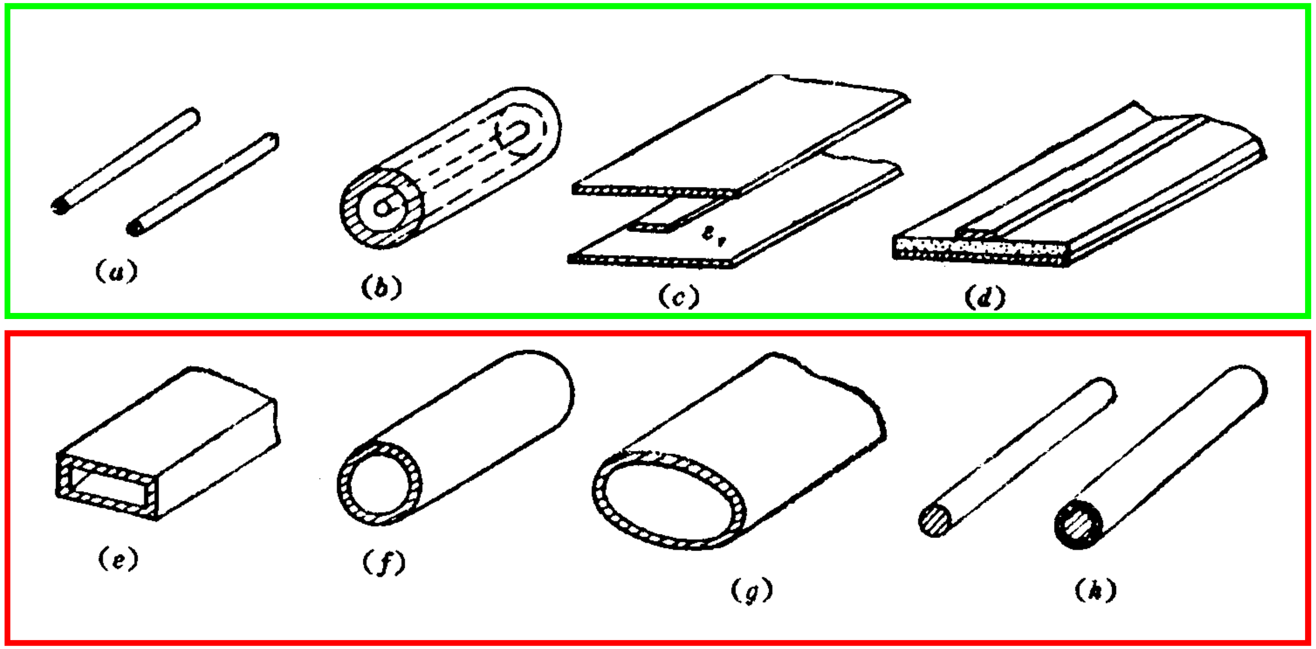
传输线　Transmission lines

(a) 平行双线 Parallel line　(b) 同轴线 Coaxial line (c) 带状线 Strip line
(d) 微带线 Microstrip line
(e) 矩形波导Rectangular waveguide (f) 圆波导 Circular waveguide (g) 椭圆波导Elliptical waveguide (h) 介质波导 Dielectric waveguide

双导体

单导体



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Cage line

A type of transmission line called a cage line, used for high power, low frequency applications. It functions similarly to a large coaxial cable. This example is the antenna feedline for a longwave radio transmitter in Poland, which operates at a frequency of 225 kHz and a power of 1200 kW.

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- 从数学上讲，不同类型的传输线就是给电磁波以不同的边界条件。
- 它们的结构决定了不同频率下电磁波的传输特性：模式（场结构），衰减常数，相位常数，导波波长，特性阻抗等等。
- 传输线上的场分布除了与上述因素有关，还与源（source）和负载 (load) 的情况有关。

基本概念：

电长度：几何长度 $\boldsymbol{l}$ 与电磁波的工作波长 λ 之比 l/λ。

长线效应：$l/\lambda > 0.05$ ---长线，各点的电压和电流均随时间、位置变化。

分布参数效应：传输线的损耗电阻、电感、导线间的互电容沿线分布

分布参数：单位长度的分布电感 $\boldsymbol{L}_0$ 、分布电容 $\boldsymbol{C}_0$ 、分布电阻 $\boldsymbol{R}_0$ 、分布电导 $\boldsymbol{G}_0$ ，由传输线的几何结构和材料性质决定。

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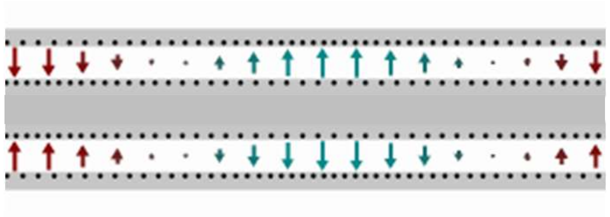
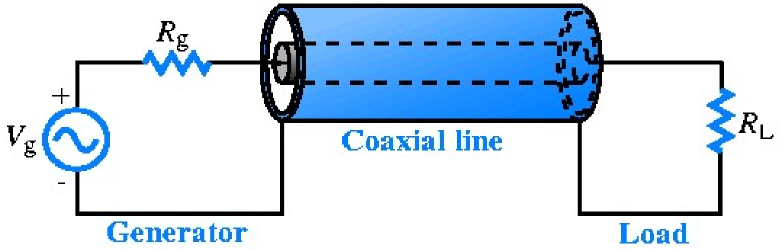
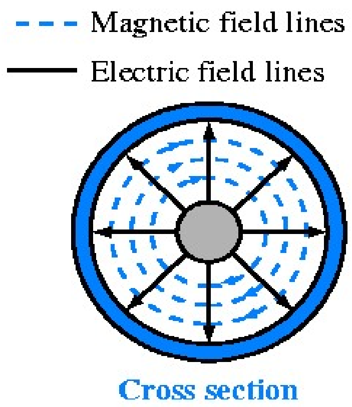
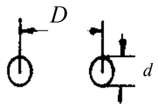
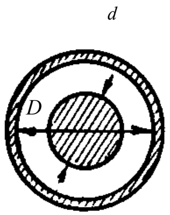






平行双线和同轴线的分布参数计算公式

形式	结构	L_0(H/m)	C_0(F/m)	R_0(Ω/m)	G_0(S/m)
平行双线		$\frac{\mu_1}{\pi} arch\left(\frac{D}{d}\right)$	$\frac{\pi\varepsilon_1}{arch\left(\frac{D}{d}\right)}$	$\frac{2}{\pi d}\sqrt{\frac{\omega\mu_2}{2\sigma_2}}$	$\frac{\pi\sigma_1}{arch\left(\frac{D}{d}\right)}$
同轴线		$\frac{\mu_1}{2\pi}\ln\frac{D}{d}$	$\frac{2\pi\varepsilon_1}{\ln\frac{D}{d}}$	$\frac{1}{\pi}\sqrt{\frac{\omega\mu_2}{2\sigma_2}}\left(\frac{1}{d}+\frac{1}{D}\right)$	$\frac{2\pi\sigma_1}{\ln\frac{D}{d}}$

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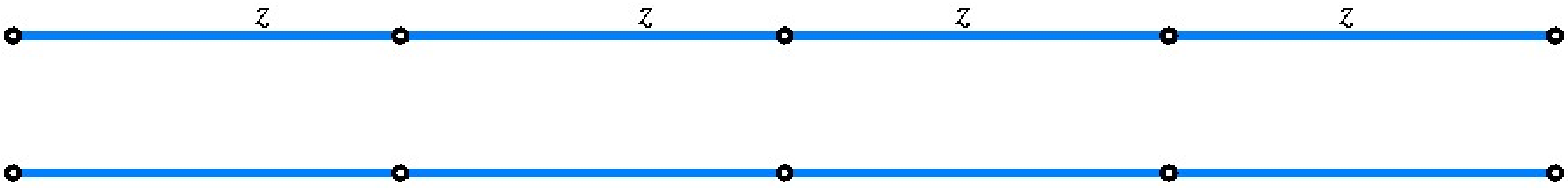
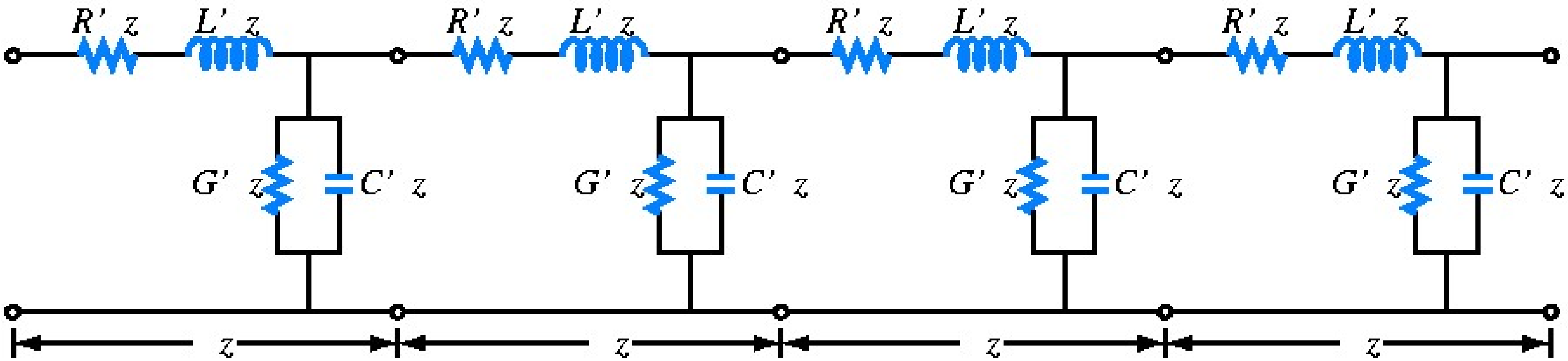
(a) Parallel-wire representation



(b) Differential sections each z long



(c) Each section is represented by an equivalent circuit

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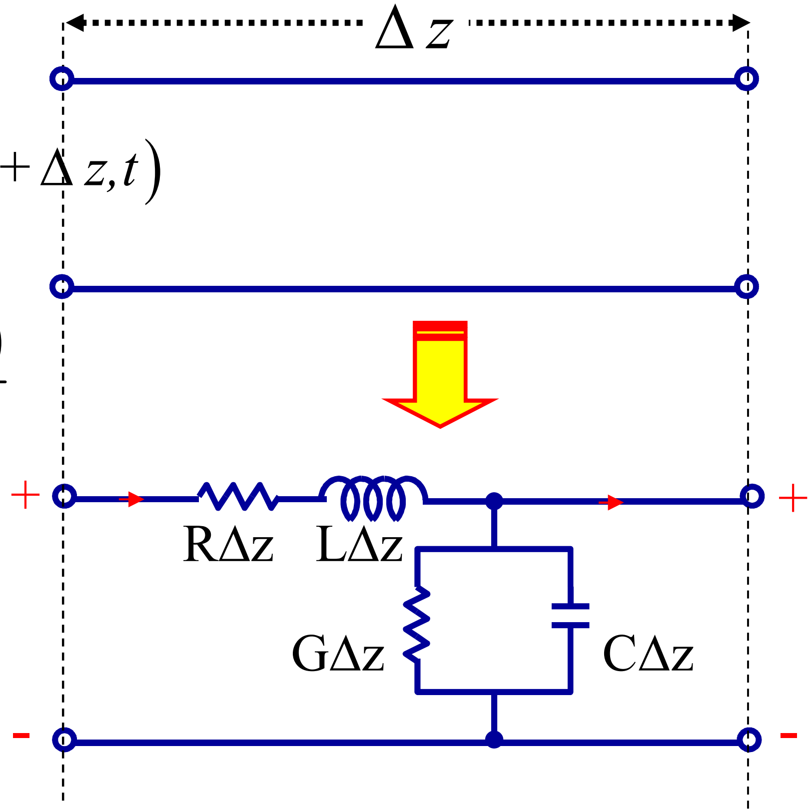
考虑无限小的线元Δz（$\Delta z << \lambda$）

$$u(z,t) = R\Delta z i(z,t) + L\Delta z\frac{\partial i(z,t)}{\partial t} + u(z+\Delta z,t)$$

$$-\frac{u(z+\Delta z,t) - u(z,t)}{\Delta z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

$$\boxed{-\frac{\partial u(z,t)}{\partial z} = Ri(z,t) + L\frac{\partial i(z,t)}{\partial t}}$$



$$i(z,t) = i(z+\Delta z,t) + \boldsymbol{G}\Delta z u(z+\Delta z,t) + \boldsymbol{C}\Delta z\frac{\partial u(z+\Delta z,t)}{\partial t}$$

$$\boxed{-\frac{\partial i(z,t)}{\partial z} = Gu(z,t) + C\frac{\partial u(z,t)}{\partial t}}$$

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传输线方程（电报方程）：

$$\begin{cases} \dfrac{\partial u(z,t)}{\partial z} = -R_0 i(z,t) - L_0 \dfrac{d\, i(z,t)}{d\, t} \\ \dfrac{\partial i(z,t)}{\partial z} = -G_0 u(z,t) - C_0 \dfrac{du(z,t)}{d\, t} \end{cases}$$

考虑电磁波呈简谐变化

$$\begin{cases} u(z,t) = \mathrm{Re}[U(z)\mathrm{e}^{\mathrm{j}\omega t}] \\ i(z,t) = \mathrm{Re}[I(z)\mathrm{e}^{\mathrm{j}\omega t}] \end{cases} \Rightarrow \begin{cases} \dfrac{\mathrm{d}U(z)}{\mathrm{d}\, z} = -(R_0 + \mathrm{j}\omega L_0)I(\mathrm{z}) = -ZI(\mathrm{z}) \\ \dfrac{\mathrm{d}I(z)}{\mathrm{d}\, z} = -(G_0 + \mathrm{j}\omega C_0)U(\mathrm{z}) = -YU(\mathrm{z}) \end{cases}$$

单位长度的串联阻抗，并联导纳

$$Z = R_0 + \mathrm{j}\omega L_0$$

$$Y = G_0 + \mathrm{j}\omega C_0$$

$$\begin{cases} \dfrac{\mathrm{d}^2U(z)}{\mathrm{d}\, z^2} = -Z\dfrac{\mathrm{d}I(z)}{\mathrm{d}\, z} \\ \dfrac{\mathrm{d}^2I(z)}{\mathrm{d}\, z^2} = -Y\dfrac{\mathrm{d}U(z)}{\mathrm{d}\, z} \end{cases} \qquad \begin{cases} \dfrac{\mathrm{d}^2U(z)}{\mathrm{d}\, z^2} - ZYU(z) = 0 \\ \dfrac{\mathrm{d}^2I(z)}{\mathrm{d}\, z^2} - ZYI(z) = 0 \end{cases}$$

波动方程

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定义传输线的传播常数：

$$\gamma = \alpha + j\beta = \sqrt{ZY} = \sqrt{(R_0 + j\omega L_0)(G_0 + j\omega C_0)}$$

α, β 为传输线的衰减常数（dB/m）和相位常数(rad/m)

$$\begin{cases} \dfrac{\mathrm{d}^2 U(z)}{\mathrm{d}z^2} - \gamma^2 U(z) = 0 \\ \dfrac{\mathrm{d}^2 I(z)}{\mathrm{d}z^2} - \gamma^2 I(z) = 0 \end{cases}$$

波动方程

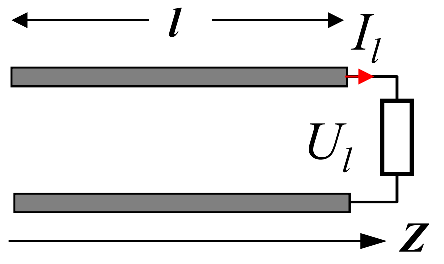
$$U(z) = A\mathrm{e}^{-\gamma z} + B\mathrm{e}^{\gamma z}$$

$$I(z) = -\frac{1}{Z}\frac{\mathrm{d}U(z)}{\mathrm{d}z} = \frac{\gamma}{Z}(A\mathrm{e}^{-\gamma z} - B\mathrm{e}^{\gamma z}) = \frac{1}{Z_0}(A\mathrm{e}^{-\gamma z} - B\mathrm{e}^{\gamma z})$$

正Z方向和负Z方向行波

传输线的特性阻抗

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R_0 + \mathrm{j}\omega L_0}{G_0 + \mathrm{j}\omega C_0}}$$



A, B常数由端接条件决定，对终端：

$$\begin{cases} U_l = Ae^{-\gamma l} + Be^{\gamma l} \\ I_l = \dfrac{1}{Z_0}(Ae^{-\gamma l} - Be^{\gamma l}) \end{cases}$$

$$\begin{cases} A = \dfrac{U_l + Z_0 I_l}{2} e^{\gamma l} \\ B = \dfrac{U_l - Z_0 I_l}{2} e^{-\gamma l} \end{cases}$$

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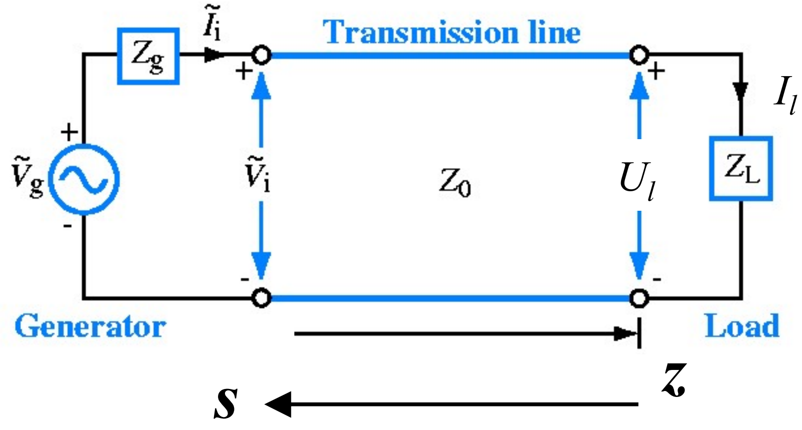
终端条件下的解：



$$
\begin{cases}
U(z)=\dfrac{U_l+Z_0I_l}{2}\mathrm{e}^{\gamma(l-z)}+\dfrac{U_l-Z_0I_l}{2}\mathrm{e}^{-\gamma(l-z)}\\
I(z)=\dfrac{U_l+Z_0I_l}{2Z_0}\mathrm{e}^{\gamma(l-z)}-\dfrac{U_l-Z_0I_l}{2Z_0}\mathrm{e}^{-\gamma(l-z)}
\end{cases}
$$

$s=l-z$

$$
\begin{cases}
U(s)=\dfrac{U_l+Z_0I_l}{2}\mathrm{e}^{\gamma s}+\dfrac{U_l-Z_0I_l}{2}\mathrm{e}^{-\gamma s}\\
I(s)=\dfrac{U_l+Z_0I_l}{2Z_0}\mathrm{e}^{\gamma s}-\dfrac{U_l-Z_0I_l}{2Z_0}\mathrm{e}^{-\gamma s}
\end{cases}
$$

利用双曲函数 $\cosh x=\dfrac{\mathrm{e}^{x}+\mathrm{e}^{-x}}{2}$ 和 $\sinh x=\dfrac{\mathrm{e}^{x}-\mathrm{e}^{-x}}{2}$ ，上式可以简化：

$$
\begin{cases}
U(s)=U_l\cosh\gamma s+Z_0I_l\sinh\gamma s\\
I(s)=\dfrac{U_l}{Z_0}\sinh\gamma s+I_l\cosh\gamma s
\end{cases}
$$

对无耗传输线，有条件 $R_0=0$ 、$G_0=0$

$$\gamma=\mathrm{j}\omega\sqrt{L_0C_0}=\mathrm{j}\beta \qquad Z_0=\sqrt{L_0/C_0}$$

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$$cosh(j\boldsymbol{x}) = cos\boldsymbol{x}$$
$$sinh(j\boldsymbol{x}) = jsin\boldsymbol{x}$$

$$\begin{cases} U(s) = U_l \cos\beta s + jZ_0 I_l \sin\beta s \\ I(s) = j\dfrac{U_l}{Z_0}\sin\beta s + I_l \cos\beta s \end{cases}$$

传输线上的实时电压、电流：

$$\begin{cases} u(z,t) = \mathrm{Re}[Ae^{j\omega t-\gamma z} + Be^{j\omega t+\gamma z}] \\ i(z,t) = \mathrm{Re}[\dfrac{1}{Z_0}(Ae^{j\omega t-\gamma z} - Be^{j\omega t+\gamma z})] \end{cases}$$

一般传输线

$$\gamma = \alpha + \mathrm{j}\beta \Rightarrow \begin{cases} u(z,t) = \boxed{Ae^{-\alpha z}\cos(\omega t-\beta z)} + \boxed{Be^{\alpha z}\cos(\omega t+\beta z)} \\ i(z,t) = \boxed{\dfrac{A}{Z_0}e^{-\alpha z}\cos(\omega t-\beta z)} - \boxed{\dfrac{B}{Z_0}e^{\alpha z}\cos(\omega t+\beta z)} \end{cases}$$

信号源向负载传播的电压（电流）波和由负载向信号源传播的反射波的叠加。

$$\begin{cases} u(z,t) = u^+(z,t) + u^-(z,t) \\ i(z,t) = i^+(z,t) + i^-(z,t) = \dfrac{1}{Z_0}[u^+(z,t) - u^-(z,t)] \end{cases}$$

一般为行波与驻波的混合分布。

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❑ 传输线的特性参数

（1）特性阻抗 Z_0：任意点电压入射波与电流入射波之比。

$$Z_0 = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}}$$

a. 无耗线

因为 $R_0 = 0, G_0 = 0$ ⇒ $Z_0 = \sqrt{\dfrac{L_0}{C_0}}$

b. 微波低耗线

在微波情形下，$R_0 << \omega L_0$ 、$G_0 << \omega C_0$，则

$$Z_0 = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}} = \sqrt{\frac{j\omega L_0\left(1+\dfrac{R_0}{j\omega L_0}\right)}{j\omega C_0\left(1+\dfrac{G_0}{j\omega C_0}\right)}} \cong \sqrt{\frac{L_0}{C_0}}\left(1+\frac{R_0}{2j\omega L_0}\right)\left(1-\frac{G_0}{2j\omega C_0}\right)$$

$$\cong \sqrt{\frac{L_0}{C_0}}\left[1+\frac{1}{2}\left(\frac{R_0}{j\omega L_0}-\frac{G_0}{j\omega C_0}\right)\right] \cong \sqrt{\frac{L_0}{C_0}}$$

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平行双线：$Z_0 = \frac{1}{\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{2D}{d} = \frac{120}{\sqrt{\varepsilon_r}}\ln\frac{2D}{d} = \frac{276}{\sqrt{\varepsilon_r}}\lg\frac{2D}{d}$

同轴线：$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{D}{d} = \frac{60}{\sqrt{\varepsilon_r}}\ln\frac{D}{d} = \frac{138}{\sqrt{\varepsilon_r}}\lg\frac{D}{d}$

（2）传播常数 γ 与导波波长 λ_g

$$\boxed{\gamma = \sqrt{(R_0 + \mathrm{j}\omega L_0)(G_0 + \mathrm{j}\omega C_0)} = \alpha + \mathrm{j}\beta}$$

α, β 为传输线的衰减常数 (dB/m) 或 $(\boldsymbol{NP}/m)$（1NP=8.68dB）和相位常数 (rad/m)

导波波长 $\lambda_g = 2\pi/\beta$：传输线上相位相差2π的距离。

a. 无耗线

$R_0 = 0 \quad G_0 = 0 \Rightarrow \gamma = \mathrm{j}\omega\sqrt{L_0 C_0}$

$$\boxed{\begin{cases} \alpha = 0 \\ \beta = \omega\sqrt{L_0 C_0} \end{cases}}$$

b.微波低耗线 $R_0 << \omega L_0 \quad G_0 << \omega C_0$

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$$\gamma = j\omega\sqrt{L_0C_0}\sqrt{\left(1-\frac{jR_0}{\omega L_0}\right)\left(1-\frac{jG_0}{\omega C_0}\right)} \cong j\omega\sqrt{L_0C_0}\left[1-\frac{j}{2\omega}\left(\frac{R_0}{L_0}+\frac{G_0}{C_0}\right)\right]$$

$$=\left(\frac{R_0}{2}\sqrt{\frac{C_0}{L_0}}+\frac{G_0}{2}\sqrt{\frac{L_0}{C_0}}\right)+j\omega\sqrt{L_0C_0}$$

$$\begin{cases}\alpha = \dfrac{R_0}{2Z_0}+\dfrac{G_0Z_0}{2} = \alpha_c + \alpha_d \\ \beta = \omega\sqrt{L_0C_0}\end{cases}$$

介质损耗

欧姆损耗

无耗和低耗情况：
(双线、同轴线)

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L_0C_0}} = \frac{1}{f\sqrt{\mu\varepsilon}} = \frac{c}{f\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

传输线的损耗有时也可用它的品质因数Q来衡量，它定义为：

$$\begin{cases}Q_c = \dfrac{\omega L_0}{R_0} \\ Q_d = \dfrac{\omega C_0}{G_0} = \dfrac{1}{\tan\delta}\end{cases}$$

$$\begin{cases}\alpha_c = \dfrac{R_0}{2Z_0} = \dfrac{\beta}{2Q_c} \\ \alpha_d = \dfrac{G_0Z_0}{2} = \dfrac{\beta}{2Q_d}\end{cases}$$

$\alpha = \dfrac{\beta}{2Q}$

介质损耗角正切

总衰减常数 $\alpha = \alpha_c + \alpha_d$ 对应于总品质因数Q

$$\frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_d}$$

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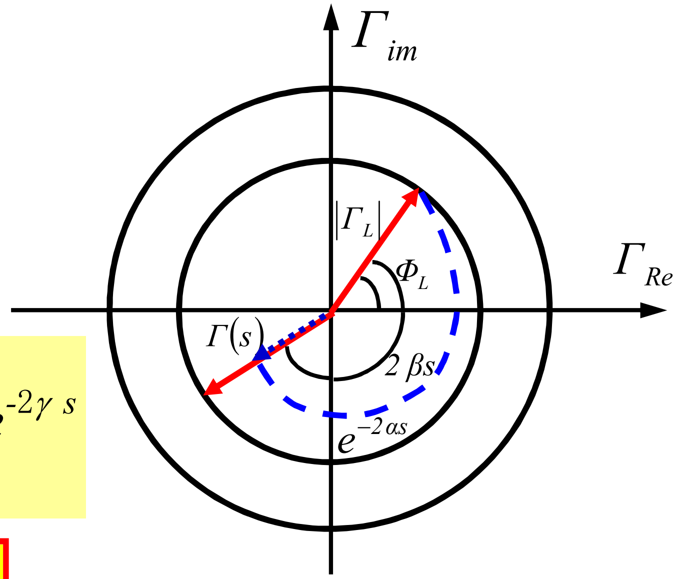
4.2 反射系数、驻波比和输入阻抗

（1）反射系数

某点反射波电压（电流）与入射波电压（电流）之比。

$$\begin{cases} \Gamma_U(z) = \dfrac{U^-(z)}{U^+(z)} = \dfrac{B}{A}\mathrm{e}^{2\gamma z} \\ \Gamma_I(z) = \dfrac{I^-(z)}{I^+(z)} = -\dfrac{B}{A}\mathrm{e}^{2\gamma z} \end{cases}$$



$$\Gamma = \frac{U_l - Z_0 I_l}{U_l + Z_0 I_l} e^{-2\gamma(l-z)} = \frac{Z_l - Z_0}{Z_l + Z_0} e^{-2\gamma s} = \Gamma_l e^{-2\gamma s}$$

$$Z_l = \frac{U_l}{I_l}$$

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0} = \left|\frac{Z_l - Z_0}{Z_l + Z_0}\right| \mathrm{e}^{\mathrm{j}\varphi_l} = |\Gamma_l| \mathrm{e}^{\mathrm{j}\varphi_l}$$

$$\Gamma(s) = |\Gamma_l| \mathrm{e}^{-2\alpha s} \mathrm{e}^{\mathrm{j}(\varphi_l - 2\beta s)}$$

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Γ_l 为负载的反射系数 $\Gamma_l = \dfrac{Z_l - Z_0}{Z_l + Z_0} = \left|\dfrac{Z_l - Z_0}{Z_l + Z_0}\right| \mathrm{e}^{\mathrm{j}\varphi_l} = |\Gamma_l| \mathrm{e}^{\mathrm{j}\varphi_l}$

终端匹配， $\boldsymbol{Z}_l = \boldsymbol{Z}_0$ 无反射，为向负载的行波。

终端短路， $\boldsymbol{Z}_l = 0, \Gamma_l = -1$

终端开路， $\boldsymbol{Z}_l = \infty, \Gamma_l = 1$

终端为纯电抗， $|\Gamma_l| = 1$

全反射、驻波

其他情况为行波和驻波的混合状态。

传输线演示

（2）电压驻波比ρ

电压驻波比（VSWR）定义为传输线上电压最大值与最小值之比：

$$\boxed{\rho = \frac{|U(s)|_{max}}{|U(s)|_{min}}}$$

取s=0, 令： $U_l^+ = \dfrac{U_l + Z_0 I_l}{2},\ U_l^- = \dfrac{U_l - Z_0 I_l}{2},\ \Gamma_l = \dfrac{U_l^-}{U_l^+} = |\Gamma_l| \mathrm{e}^{\mathrm{j}\varphi_l}$

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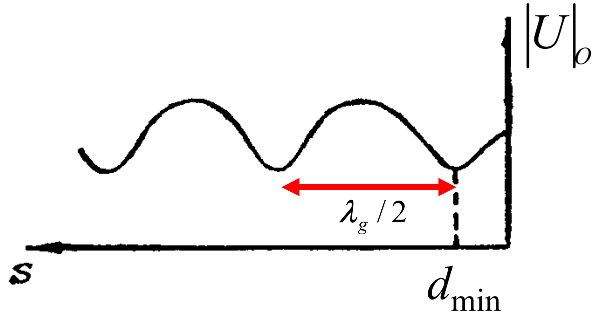
对无耗线，$\gamma = \mathrm{j}\beta$　$U(s) = U_l^+ \mathrm{e}^{\mathrm{j}\beta s} + U_l^- \mathrm{e}^{-\mathrm{j}\beta s} = U_l^+ \mathrm{e}^{\mathrm{j}\beta s}[1 + |\Gamma_l| \mathrm{e}^{\mathrm{j}(\varphi_l - 2\beta s)}]$

当$\varphi_l - 2\beta s = \pm 2n\pi$时，反射波和入射波相位相同，合成波的幅值最大：$|U(s)|_{max} = |U_l^+|(1 + |\Gamma_l|)$

当$\varphi_l - 2\beta s = \pm(2n+1)\pi$时，反射波和入射波相位相反，合成波的幅值小：$|U(s)|_{min} = |U_l^+|(1 - |\Gamma_l|)$

驻波比：$\rho = \dfrac{1 + |\Gamma_l|}{1 - |\Gamma_l|}$　$|\Gamma_l| = \dfrac{\rho - 1}{\rho + 1}$



为了求出反射系数幅度的相位 φ_l，可以测出距负载最近的第一个驻波电压最小点与负载之间的距离 d_{min}。

在该最小点处，有相位关系 $\varphi_l - 2\beta d_{min} = \pi$，因而 $\varphi_l = 2\beta d_{min} + \pi$

由终端反射系数求终端阻抗：

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0} \qquad Z_l = \frac{Z_0(1 + \Gamma_l)}{1 - \Gamma_l}$$

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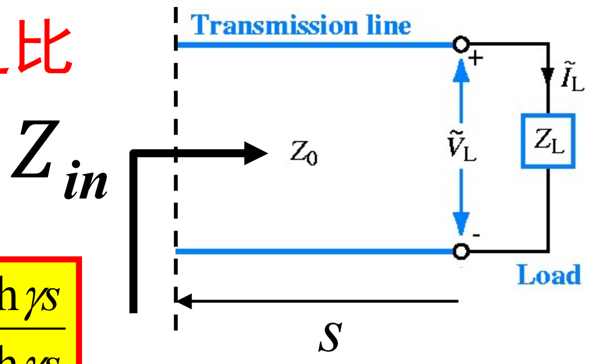
（3）输入阻抗（导纳）

输入阻抗：该点合成波电压与合成波电流之比

$$\boxed{Z_{in}(s)=\frac{U(s)}{I(s)}}$$



$$Z_{in}(s)=Z_0\frac{U_l\cosh\gamma s+Z_0I_l\sinh\gamma s}{U_l\sinh\gamma s+Z_0I_l\cosh\gamma s}$$

$$\boxed{Z_{in}(s)=Z_0\frac{Z_l+Z_0\tanh\gamma s}{Z_0+Z_l\tanh\gamma s}}$$

传输线的输入导纳

$$\boxed{Y_{in}(s)=\frac{1}{Z_{in}(s)}=Y_0\frac{Y_l+Y_0\tanh\gamma s}{Y_0+Y_l\tanh\gamma s}}$$

传输线上任一点的输入阻抗相当于由该点向负载看去的阻抗。

输入阻抗、特性阻抗和反射系数的关系：

$$Z_{in}(s)=\frac{U(s)}{I(s)}=\frac{U^+(s)+U^-(s)}{I^+(s)+I^-(s)}=\frac{U^+(s)[1+\Gamma(s)]}{I^+(s)[1-\Gamma(s)]}=Z_0\frac{1+\Gamma(s)}{1-\Gamma(s)}$$

$$\boxed{\Gamma(s)=\frac{Z_{in}(s)-Z_0}{Z_{in}(s)+Z_0}}$$

对微波波段的无耗线：

$$\boxed{Z_{in}(s)=Z_0\frac{Z_l+\mathrm{j}Z_0\tan\beta s}{Z_0+\mathrm{j}Z_l\tan\beta s}}$$

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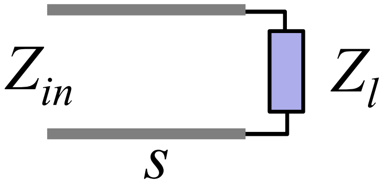
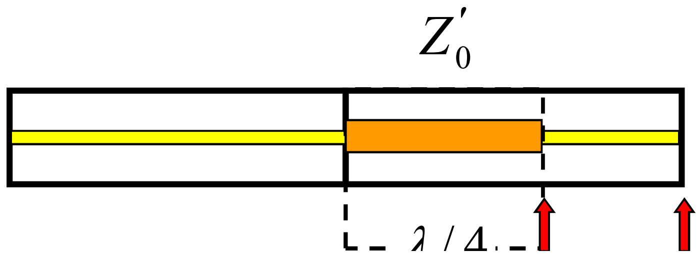
阻抗变换器： $s=\lambda/4$ ➡ $\boxed{Z_{in}=Z_0^2/Z_l}$



$\boldsymbol{Z_l}=0$（短路）$\Leftrightarrow Z_{in}=\infty$ 短路线变为开路线



例1：金属绝缘子：短路变为开路。

例2：微带线接地原理, 开路变为短路。

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线长 s	计算公式	负载 Z_L	输入阻抗Z_{in}	变换作用
$\lambda_g/4$	$Z_{in}=\frac{Z_0^2}{Z_L}$	**0**	∞	短路变开路
		∞	0	开路变短路
		R_L	Z_0^2/R_L	电阻变换器
		jX_L	$-jZ_0^2/X_L$	电容（感）变电感（容）
$\lambda_g/2$	$Z_{in}=Z_L$	Z_L	Z_L	无阻抗变换作用

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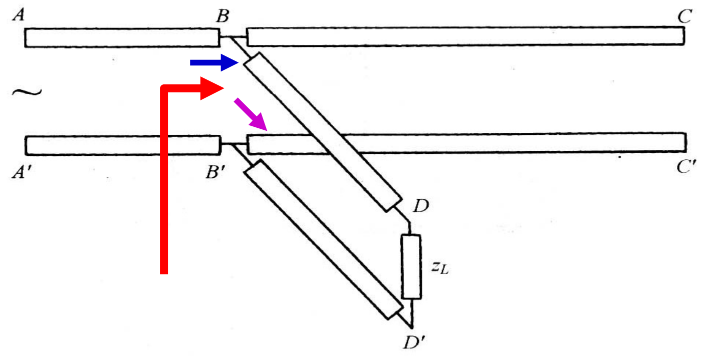
例 2-2 图 2-9 为一传输线网络，其 AB 段、BD 段长为 $\frac{\lambda_p}{4}$，BC 段长 $\frac{\lambda_p}{2}$，各段传输线波阻抗均为 $Z_0=150\Omega$。传输线 CC' 端口开路，DD' 端口接纯阻负载 $Z_L=300\Omega$。求传输线 AA' 端口输入阻抗及各段传输线上的电压驻波比。



解： 直接利用 $\frac{\lambda_p}{4}$ 传输线的阻抗变换性及 $\frac{\lambda_p}{2}$ 传输线的阻抗重复性，则

$$Z_{BB'}=\text{并联}\left\{\begin{matrix} Z_{BB'1}=\infty \\ Z_{BB'2}=\dfrac{150^2}{300}=75 \end{matrix}\right\}=75\Omega$$

$$Z_{AA'}=\frac{150^2}{75}=300\Omega$$

各段传输线的电压驻波比

$$|\Gamma_{BD}|=\frac{300-150}{300+150}=\frac{1}{3}\Rightarrow\rho=2$$

$$|\Gamma_{BC}|=1\Rightarrow\rho=\infty$$

$$|\Gamma_{AB}|=\frac{150-75}{150+75}=\frac{1}{3}\Rightarrow\rho=2$$

$$|\Gamma|=\frac{R_L-Z_0}{R_L+Z_0},\quad R_L>Z_0$$

$$|\Gamma|=\frac{Z_0-R_L}{Z_0+R_L},\quad R_L<Z_0$$

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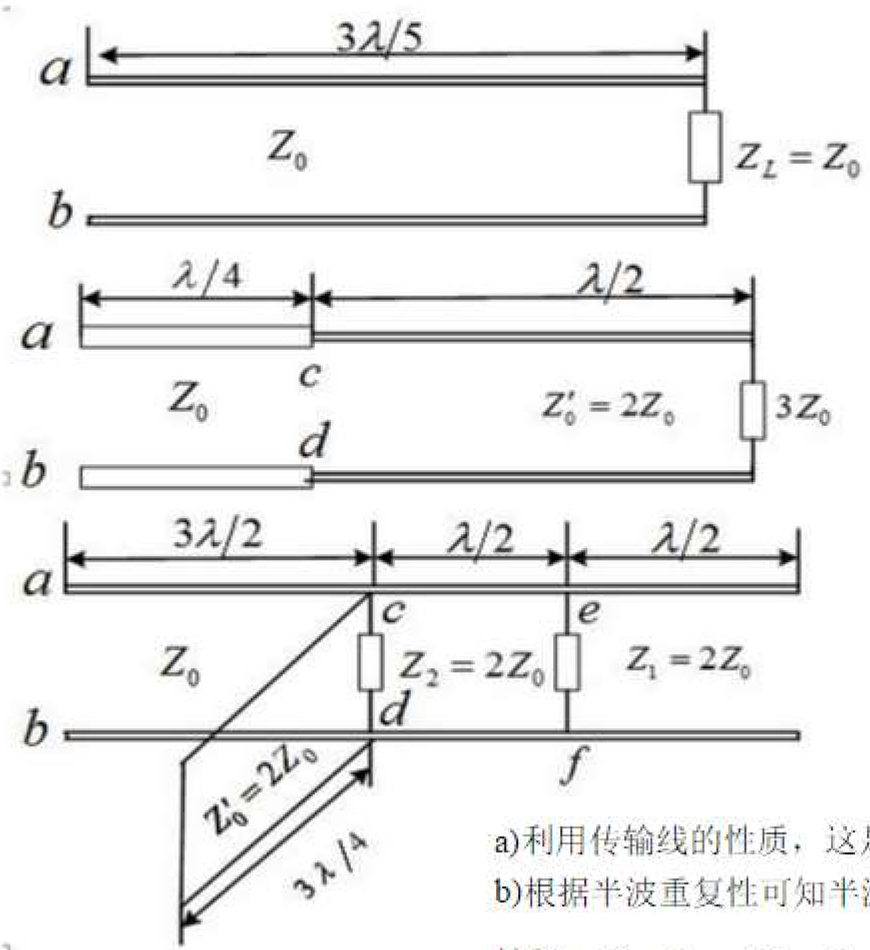
习题 9　试求如题图 2-2 所示各电路的输入阻抗 Z_{ab}。



a)利用传输线的性质，这是匹配的情况，$Z_{ab}=Z_{in}=Z_0$；

b)根据半波重复性可知半波长段的输入阻抗等于 $Z_{cd}=Z_L=3Z_0$，再根据四分之一波长的变换性得：$Z_{ab}\times Z_{cd}=Z_0^2 \Rightarrow Z_{ab}=Z_0/3$；

c) 根据半波重复性得：$Z_{ef}=Z_1=2Z_0$，$Z_{cd}=2Z_0 // 2Z_0=Z_0$，$Z_{ab}=Z_0$

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4. 3 无耗工作状态分析

无耗工作状态：

$$\begin{cases} \alpha = 0 \\ \beta = \omega\sqrt{L_0 C_0} \end{cases}$$

终端	Z_l	Γ_l	ρ	Z_{in}	
匹配	Z_0	0	1	Z_0	行波状态
短路	0	-1	∞	$jZ_0 \tan\beta s$	驻波状态
开路	∞	1	∞	$-jZ_0 \operatorname{ctan}\beta s$	驻波状态
纯电抗	jX_l	$\lvert\Gamma_L\rvert = 1$	∞		驻波状态
不匹配电阻	R_L	$\lvert\Gamma_L\rvert < 1$	> 1		行驻波状态
复阻抗	$R_l + jX_l$				

（1）行波状态

$$\begin{cases} U(z) = U^+(z) = \dfrac{U_l + I_l Z_0}{2} e^{-j\beta(z-l)} = A e^{-j\beta z} \\ I(z) = I^+(z) = \dfrac{U_l + I_l Z_0}{2Z_0} e^{-j\beta(z-l)} = \dfrac{A}{Z_0} e^{-j\beta z} \end{cases}$$

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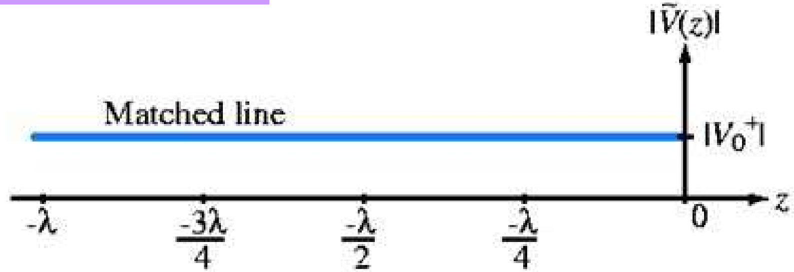
它们的瞬时表示式为：

$$\begin{cases} u(z,t)=|A|\cos(\omega t-\beta z+\varphi_1) \\ i(z,t)=\dfrac{|A|}{Z_0}\cos(\omega t-\beta z+\varphi_1) \end{cases}$$

$$Z_{in}(z)=Z_0$$

传输线上的传输功率为：

$$P(z)=\frac{1}{2}\mathrm{Re}[U(z)I^*(z)]=\frac{|A|^2}{2Z_0}$$



（2）驻波状态

（a） 终端短路

$$\begin{cases} U(s)=U_l\cos\beta s+jZ_0I_l\sin\beta s \\ I(s)=j\dfrac{U_l}{Z_0}\sin\beta s+I_l\cos\beta s \end{cases}$$

$$\begin{cases} U(s)=\mathrm{j}Z_0I_l\sin\beta s \\ I(s)=I_l\cos\beta s \end{cases}$$

$$U_l^+=\frac{U_l+Z_0I_l}{2}=\frac{Z_0I_l}{2}$$

$$\begin{cases} U(s)=2\mathrm{j}U_l^+\sin\beta s \\ I(s)=\dfrac{2U_l^+}{Z_0}\cos\beta s \end{cases}$$

$$\begin{cases} u(s,t)=2\left|U_l^+\right|\sin\beta s\cos(\omega t+\pi/2+\varphi_l) \\ i(s,t)=\dfrac{2\left|U_l^+\right|}{Z_0}\cos\beta s\cos(\omega t+\varphi_l) \end{cases}$$

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$$P(s)=\mathrm{Re}\left[U(s)I^*(s)\right]/2=0$$

$$Z_{in}(s) = \mathrm{j}Z_0\tan\beta s$$

（b） 终端开路

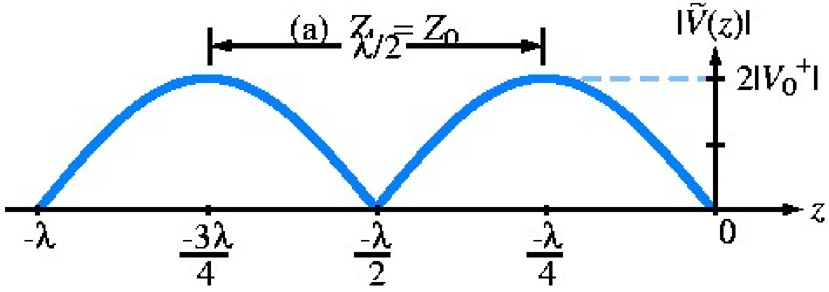
$$\begin{cases} u(s,\ t)=2\left|U_l^+\right|\cos\beta s\cos(\omega t+\varphi_l) \\ i(s,\ t)=\dfrac{2\left|U_l^+\right|}{Z_0}\sin\beta s\cos(\omega t+\dfrac{\pi}{2}+\varphi_l) \end{cases}$$

$$\begin{cases} Z_{in}(s)=-jZ_0 ctan\beta t \\ P(s)=0 \end{cases}$$

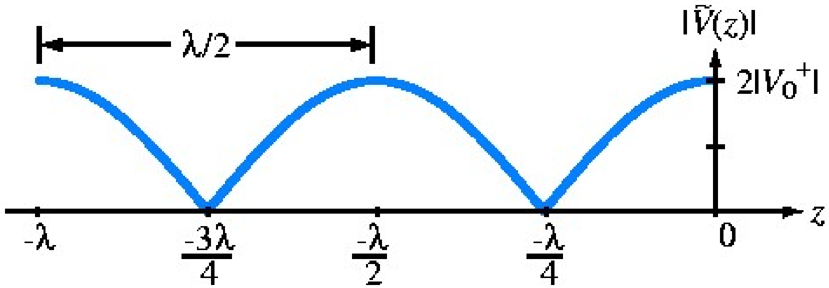
（c）终端接纯电抗负载

$$\Gamma_l=\frac{\mathrm{j}X_l-Z_0}{\mathrm{j}X_l+Z_0}=\left|\Gamma_l\right|\mathrm{e}^{\mathrm{j}\varphi_l}=\mathrm{e}^{\mathrm{j}\varphi_l}$$

$$\varphi_l=\arctan\frac{2X_lZ_0}{X_l^2-Z_0^2}$$



(b) $Z_L = 0$ (short circuit)



(c) $Z_L =$ (open circuit)

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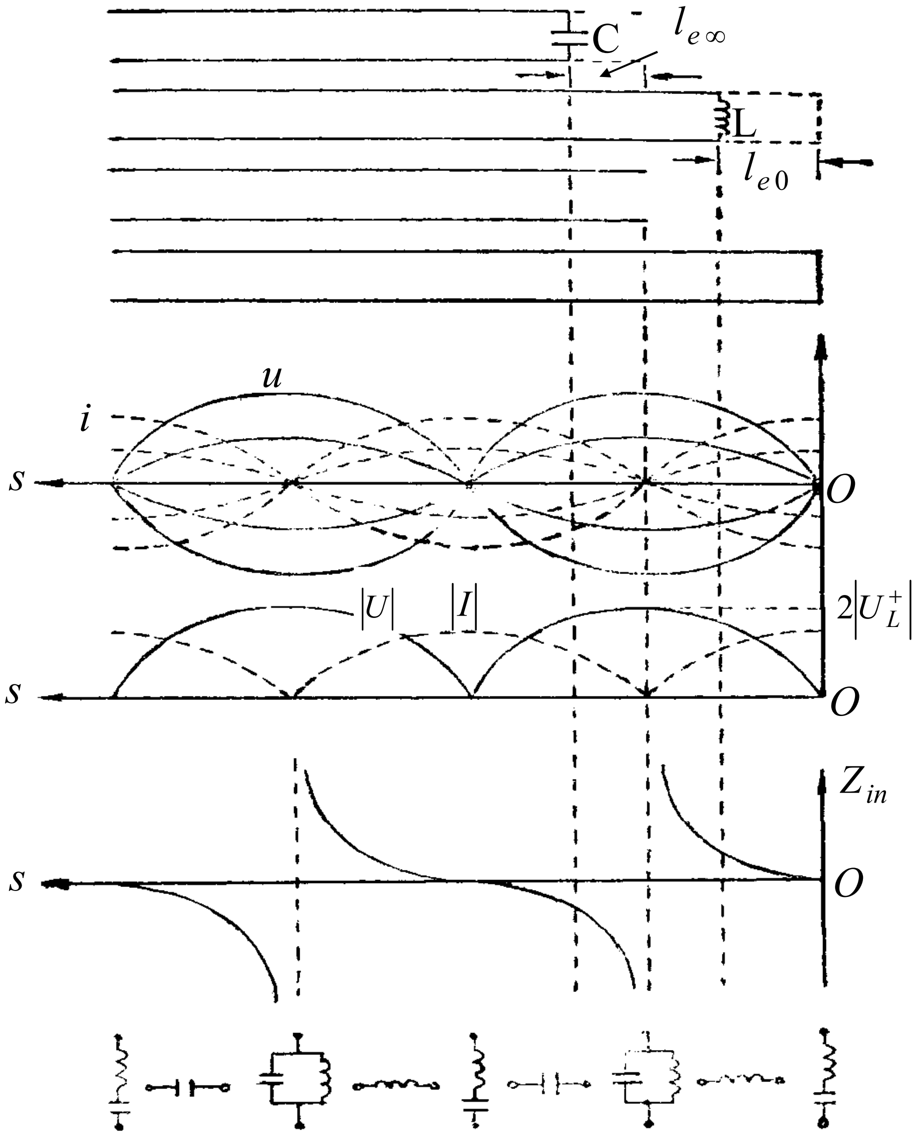

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（3）行驻波状态

当终端接不匹配的电阻负载（$Z_l = R_l \neq Z_0$）或复阻抗（ $Z_l = R_l \pm \mathrm{j}X_l$ ）时，将会产生部分反射而在线上形成行驻波，这时反射系数可写为：

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0} = \frac{R_l \pm \mathrm{j}X_l - Z_0}{R_l \pm \mathrm{j}X_l + Z_0} = |\Gamma_l| \mathrm{e}^{\pm \mathrm{j}\varphi_l}$$

$$\begin{cases} |\Gamma_l| = \sqrt{\dfrac{(R_l - Z_0)^2 + X_l^2}{(R_l + Z_0)^2 + X_l^2}} < 1 \\ \varphi_l = \arctan \dfrac{2X_l Z_0}{R_l^2 + X_l^2 - Z_0^2} \end{cases}$$

$$\begin{cases} U_l^+ = \dfrac{U_l + Z_0 I_l}{2} \\ U_l^- = \dfrac{U_l - Z_0 I_l}{2} \end{cases} \Rightarrow \begin{cases} U(s) = U_l^+ \mathrm{e}^{\mathrm{j}\beta s} + U_l^- \mathrm{e}^{-\mathrm{j}\beta s} \\ \quad = (U_l^+ - U_l^-)\mathrm{e}^{\mathrm{j}\beta s} + 2U_l^- \cos\beta s \\ I(s) = \dfrac{U_l^+ - U_l^-}{Z_0}\mathrm{e}^{\mathrm{j}\beta s} + \dfrac{2\mathrm{j}U_l^-}{Z_0}\sin\beta s \end{cases}$$

行波　　驻波

电压、电流的复振幅也可以用反射系数来表示：

$$\begin{cases} U(s) = U_l^+ \mathrm{e}^{\mathrm{j}\beta s}\left[1 + |\Gamma_l| \mathrm{e}^{\mathrm{j}(\varphi_l - 2\beta s)}\right] \\ I(s) = \dfrac{U_l^+}{Z_0} \mathrm{e}^{\mathrm{j}\beta s}\left[1 - |\Gamma_l| \mathrm{e}^{\mathrm{j}(\varphi_l - 2\beta s)}\right] \end{cases}$$

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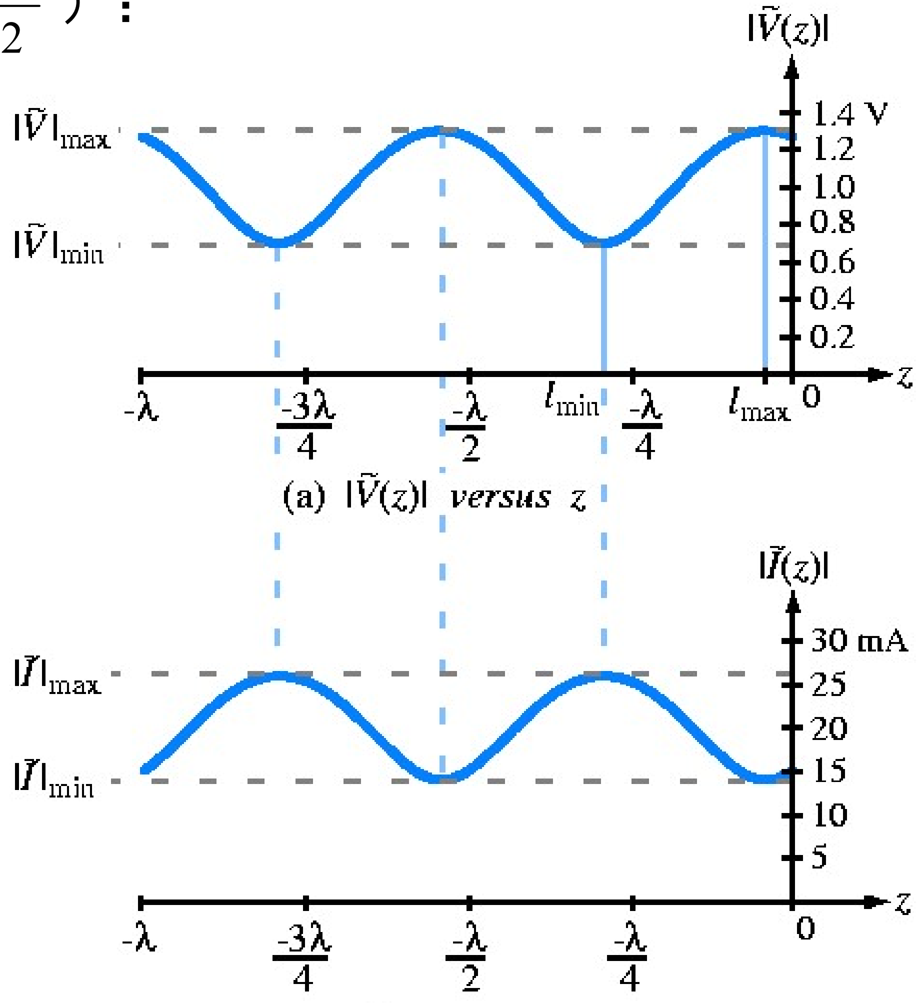
在 $\varphi_l - 2\beta s = \pm 2n\pi$ 处（即 $s = \frac{\varphi_l \lambda}{4\pi} + \frac{n\lambda}{2}$ ）：

$$\begin{cases} |U|_{max} = |U_l^+|[1+|\Gamma_l|] \\ |I|_{min} = \frac{|U_l^+|}{Z_0}[1-|\Gamma_l|] \end{cases}$$

在 $\varphi_l - 2\beta s = \pm(2n+1)\pi$ 处

（即 $s = \frac{\varphi_l \lambda}{4\pi} + (2n+1)\frac{\lambda}{4}$ ）：

$$\begin{cases} |U|_{min} = |U_l^+|[1-|\Gamma_l|] \\ |I|_{max} = \frac{|U_l^+|}{Z_0}[1+|\Gamma_l|] \end{cases}$$



(a) $|\tilde{V}(z)|$ *versus* z

(b) $|\tilde{I}(z)|$ *versus* z

行驻波状态

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行驻波状态沿线各点的输入阻抗一般为复数，但在电压驻波最大点处和电压驻波最小点处的输入阻抗是纯电阻：

$$Z_{in(max)} = \frac{1+|\Gamma_l|}{1-|\Gamma_l|}Z_0 = \rho Z_0 \qquad Z_{in(min)} = \frac{1-|\Gamma_l|}{1+|\Gamma_l|}Z_0 = \frac{Z_0}{\rho}$$

相邻 $Z_{in(max)}$、 $Z_{in(min)}$ 的距离是 $\frac{\lambda}{4}$，且满足关系：$Z_{in(max)}Z_{in(min)} = Z_0^2$

行驻波下传输线的传输功率：

$$\begin{aligned}
\boldsymbol{P}(\boldsymbol{s}) &= \frac{1}{2}\mathrm{Re}[\boldsymbol{U}(\boldsymbol{s})\boldsymbol{I}^*(\boldsymbol{s})] \\
&= \frac{|\boldsymbol{U}_l^+|^2}{2\boldsymbol{Z}_0}\mathrm{Re}\left\{\left[1+|\Gamma_l|\mathrm{e}^{\mathrm{j}(\varphi_l-2\beta s)}\right]\left[1-|\Gamma_l|\mathrm{e}^{-\mathrm{j}(\varphi_l-2\beta s)}\right]\right\} \\
&= \frac{|\boldsymbol{U}_l^+|^2}{2\boldsymbol{Z}_0}\left[1-|\Gamma_l|^2\right] = \boldsymbol{P}^+ - \boldsymbol{P}^-
\end{aligned}$$

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式中P^+和P^-分别代表入射功率和反射功率：

$$\begin{cases} P^+ = \dfrac{\left|U_l^+\right|^2}{2Z_0} \\ P^- = \dfrac{\left|U_l^+\right|^2}{2Z_0}\left|\Gamma_l\right|^2 \end{cases}$$

负载所吸收的功率：

$$P(s) = P^+ - P^- = \frac{1}{2}|U|_{\max}|I|_{\min} = \frac{1}{2}|U|_{\min}|I|_{\max}$$

$$Z_0 = \frac{|U|_{max}}{|I|_{max}} = \frac{|U|_{min}}{|I|_{min}}$$

$$\boxed{P = \frac{|U|_{max}^2}{2\rho Z_0} = \frac{\rho|U|_{min}^2}{2Z_0}}$$

传输线的功率容量（极限功率）可由上式得到：

$$\boxed{P_{br} = \frac{\left|U_{br}\right|^2}{2\rho Z_0}}$$

U_{br} 为线间的击穿电压。

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Table 2-3: Properties of standing waves on a lossless transmission line.

Voltage maximum Voltage minimum	$\lvert\widetilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1+\lvert\Gamma\rvert]$ $\lvert\widetilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1-\lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima) Position of first maximum (also position of first current minimum)	$l_{\max} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$ $l_{\max} = \begin{cases} \dfrac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \dfrac{\theta_r\lambda}{4\pi} + \dfrac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of first current maxima) Position of first minimum (also position of first current maximum)	$l_{\min} = \dfrac{\theta_r\lambda}{4\pi} + \dfrac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$ $l_{\min} = \dfrac{\lambda}{4}\left(1 + \dfrac{\theta_r}{\pi}\right)$
Input impedance	$Z_{in} = Z_0\left(\dfrac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\dfrac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\dfrac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$ Z_{in} of matched line	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$ $Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave, $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians.

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