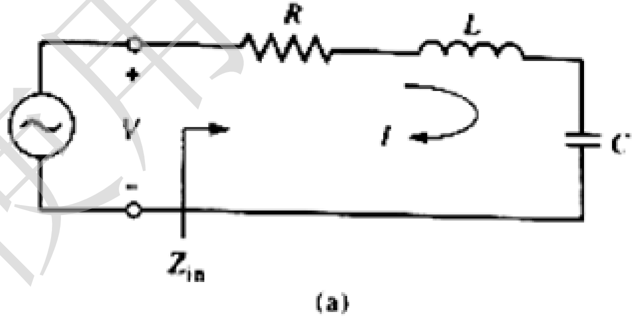
第八章　谐振器（腔）与微波振荡器

谐振电路

- 串联谐振电路



$$Z_{in} = R + jwL - j\frac{1}{wC}$$

$$P_{in} = \frac{1}{2}I^2\left(R + jwL - j\frac{1}{wC}\right) = P_L + 2jw\left(W_m - W_e\right)$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$$Q = w\frac{W_m + W_e}{P_L} = w_0\frac{2W_m}{P_L} = \frac{w_0 L}{R} = \frac{1}{w_0 RC}$$

谐振时ω=ω0，输入功率全部转变为电阻上的损耗

$$P_{in}=\frac{1}{2}I^2\left(R+jwL-j\frac{1}{wC}\right)=P_L$$

当 $|Zin|^2=2R^2$时，输入功率减小一半（半功率点）。

$$\left|Z_{in}\right|^2=R^2+\left(wL-\frac{1}{wC}\right)^2=2R^2\quad\rightarrow\quad\left(wL-\frac{1}{wC}\right)^2=R^2$$

$$P_{in}=\frac{1}{2}I^2\left(R+jwL-j\frac{1}{wC}\right)=P_L\left(1-j\right)$$

当输入功率不变时，有功功率比谐振时减少一半，称为半功率点。

品质因数测量

$$Z_{in}=R+jwL\left(1-\frac{1}{w^2LC}\right)=R+jwL\left(1-\frac{w_0^2}{w^2}\right)$$

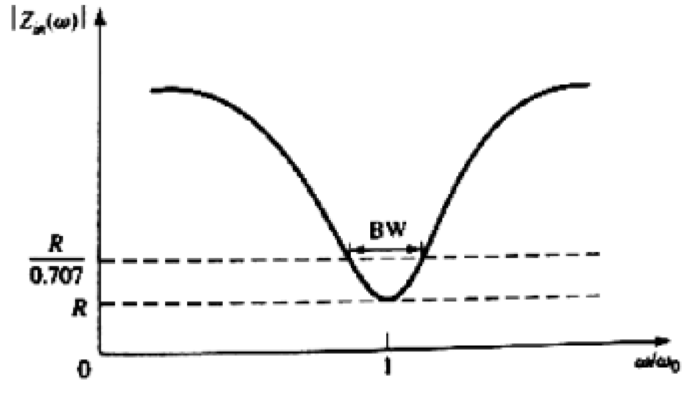
$$\approx R+j2L\Delta w=R+j2\left(\frac{RQ}{w_0}\right)\Delta w$$

当 $|Zin|^2=2R^2$时，有功功率减小一半（半功率点）。

设此时的相对带宽为$B=2\Delta\omega/\omega_0$

$$\left|R+jRQB\right|^2=2R^2$$

$$\mathrm{B}=\frac{1}{Q}$$



(b)

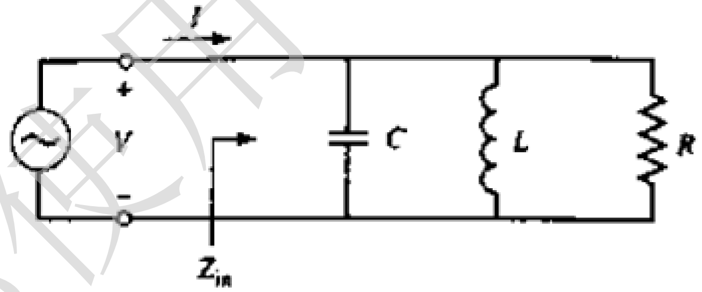
- 并联谐振电路

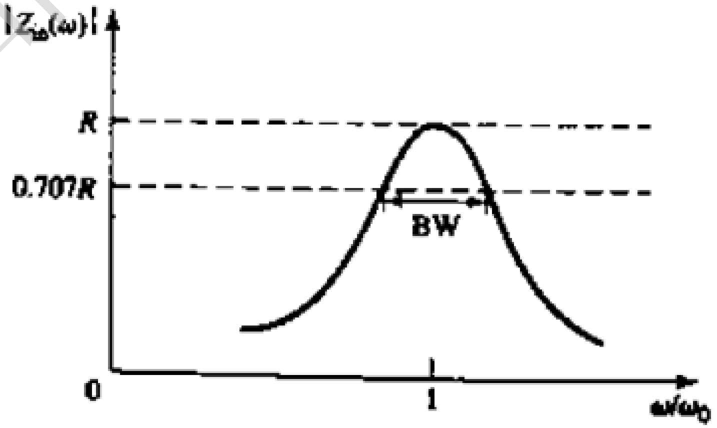
$$Z_{in}=\left(\frac{1}{R}+\frac{1}{jwL}+jwC\right)^{-1}$$

$$P_{in}=\frac{1}{2}V^2\left(\frac{1}{R}+\frac{1}{jwL}+jwC\right)^{-1}$$

$$w_0=\frac{1}{\sqrt{LC}}$$

$$Q=w_0\frac{2W_m}{P_L}=\frac{R}{w_0L}=w_0RC$$



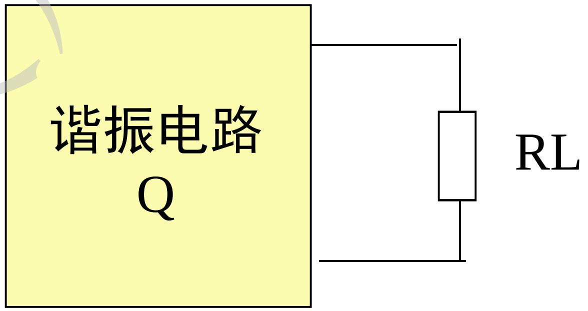


- 有负载和无载Q

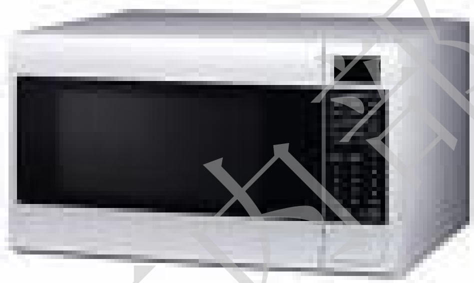
串联谐振电路：Re=R+RL

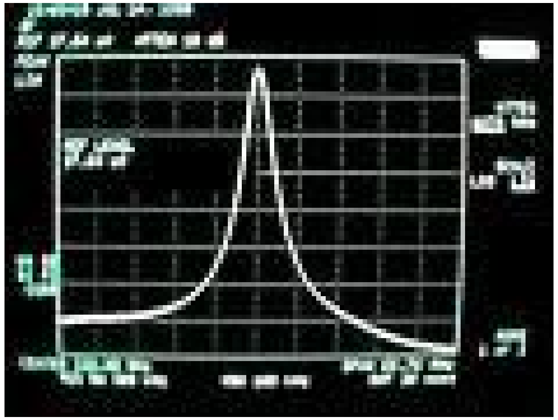
并联谐振电路：Re=R RL/(R+RL)

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q}$$



$$Q_e = \begin{cases} \dfrac{w_0 L}{R_L} & \text{串联谐振电路} \\ \dfrac{R_L}{w_0 L} & \text{并联谐振电路} \end{cases}$$





谐振腔的基本性质

- 谐振腔是微波频率下的谐振元件，相当于低频电路中LC谐振电路。
- 谐振腔可以用各种形式的传输线构造而成。
- 谐振腔在谐振频率附近可以用LRC电路等效。
- 谐振电路的基本性质
  - 谐振频率 f_0
  - 谐振时谐振电路的阻抗为纯电阻
  - 电场储能和磁场储能相互转换
  - 品质因数 Q

- 微波谐振腔的特点
  - 多个谐振频率

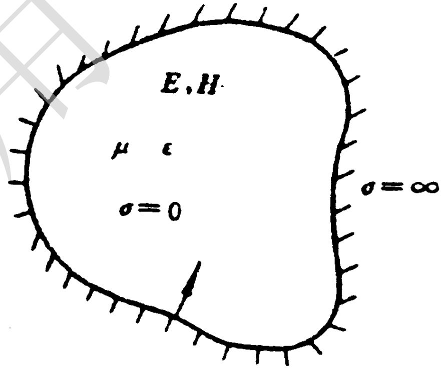
$$\nabla\times\vec{H} = j\omega\varepsilon\vec{E},\quad \nabla\times\vec{E} = -jwm\vec{H}$$

$$\nabla\times\nabla\times\vec{E} = k^2\vec{E}$$

$$\int\vec{E}^*\cdot(\nabla\times\nabla\times\vec{E})\,d\tau = k^2\int\vec{E}^*\cdot\vec{E}\,d\tau$$

$$\int(\nabla\times\vec{E})\cdot(\nabla\times\vec{E}^*)d\tau - \int\nabla\cdot(\vec{E}^*\times\nabla\times\vec{E})d\tau = k^2\int\left|\vec{E}\right|^2 d\tau$$

$$\int\left|(\nabla\times\vec{E})\right|^2 d\tau = k^2\int\left|\vec{E}\right|^2 d\tau$$



Ø 在任意位置|∇×E|＝kE不可能恒成立

Ø 只有对某些特定的 k 值上式才能成立

$$k_i^{\,2}=\frac{\int\left|(\nabla\times\vec{E})\right|^2 d\tau}{\int\left|\vec{E}\right|^2 d\tau}\qquad (i=1,2,3,\ldots)$$

- 谐振时的电磁能量

$$k_i^{\,2}=\frac{\int\left|(\nabla\times\vec{E})\right|^2 d\tau}{\int\left|\vec{E}\right|^2 d\tau}=\frac{2\omega_i^2\mu\left(\frac{1}{2}\int\mu\left|\vec{H}\right|^2 d\tau\right)}{\frac{2}{\varepsilon}\left(\frac{1}{2}\int\varepsilon\left|\vec{E}\right|^2 d\tau\right)}=k_i^{\,2}\frac{W_{m\max}}{W_{e\max}}$$

$$W_{m\max}=W_{e\max}$$

- 品质因数

$$Q_0 = 2\pi \frac{W}{W_T} = \omega_0 \frac{W}{P_L}$$

储能：最大磁能或最大电能

$$W = \frac{1}{2}\mu \int_v \left|\vec{H}\right|^2 dV$$

耗能＝金属损耗＋介质损耗

金属损耗：$$P_c = \frac{1}{2} R_s \oint_s \left|\vec{J}_s\right|^2 dS = \frac{1}{2} R_s \oint_s \left|\vec{H}_t\right|^2 dS$$

介质损耗：$$P_d = \frac{1}{2}\sigma_d \int_v \left|\vec{E}\right|^2 dV$$

$$\boxed{Q_0=\omega_0\frac{W}{P_L}=\omega_0\frac{W}{P_c+P_d}=\left(\frac{1}{Q_c}+\frac{1}{Q_d}\right)^{-1}=\frac{Q_cQ_d}{Q_c+Q_d}}$$

$$Q_c=\frac{\omega_0\mu}{R_s}\frac{\int_v\left|\vec{H}\right|^2dV}{\oint_s\left|\vec{H}_t\right|^2dS}\qquad\qquad Q_d=\omega_0\frac{\varepsilon}{\sigma_d}=\frac{1}{tg\delta}$$

Q的量级

$$Q_c=\frac{\omega_0\mu}{R_s}\frac{\int_v\left|\vec{H}\right|^2dV}{\oint_s\left|\vec{H}_t\right|^2dS}=\frac{2}{\delta}\frac{\left(\left|\vec{H}\right|^2\right)_{av}V}{\left(\left|\vec{H}_t\right|^2\right)_{av}S}\approx\frac{1}{\delta}\frac{V}{S}=\frac{l}{\delta}$$

厘米波段：λ~cm，δ~μm，Q~10^4–10^5

Q值远高于低频谐振电路

- 理想谐振腔有无穷多个分裂的固有频率（谐振频率），只有工作频率为固有频率时，谐振腔中的场才不为零。
- 在固有频率，电磁场有确定的分布状态（模式），可以划分为TE模和TM模。
- 在谐振时，理想谐振腔中没有能量损耗，电场储能和磁场储能相互转换，形成自然振荡。
- 当有耗时，自然振荡随时间衰减；分裂的固有频率扩展为一系列频带（谐振曲线）；谐振曲线的半功率点宽度随随损耗增加而增加。

金属波导矩形谐振腔

- 矩形谐振腔中的电磁场

（1）TE_{mp}模式

TE_{mp}模式波导在z方向的场为

$$E_z = 0; \quad H_z = H_0 \cos\left(k_x x\right)\cos\left(k_y y\right)e^{-jk_z z}$$

矩形波导两端短路，纵向磁场为

$$H_z = H_{zm}^{+} \cos(k_x x)\cos(k_y y)e^{-jk_z z} + H_{zm}^{-} \cos(k_x x)\cos(k_y y)e^{jk_z z}$$
$$= \cos(k_x x)\cos(k_y y)\left(H_{zm}^{+} e^{-jk_z z} + H_{zm}^{-} e^{jk_z z}\right)$$

切向电场：

$$E_x = -\frac{jwmk_y}{k_c^2}\cos(k_x x)\sin(k_y y)\left(H_{zm}^{+}e^{-jk_z z} + H_{zm}^{-}e^{jk_z z}\right)$$

$$E_y = \frac{jwmk_x}{k_c^2}\sin(k_x x)\cos(k_y y)\left(H_{zm}^{+}e^{-jk_z z} + H_{zm}^{-}e^{jk_z z}\right)$$

边界条件：当z=0, d 时 $E_x = E_y = 0$

由边界条件z=0 得 $-H_{zm}^{-} = H_{zm}^{+} = H_{zm}$

由边界条件z=d 得 $k_z = \dfrac{p\pi}{d}$

$H_z\big|_{z=0} = 0;\ H_z\big|_{z=d} = 0$ ？

谐振腔中的场分布

$$E_x = -\frac{jwm\ k_y}{k_c^2} H_{zm} \cos(k_x x) \sin(k_y y) \sin(k_z z) \qquad H_x = -\frac{k_x k_z}{k_c^2} H_{zm} \sin(k_x x) \cos(k_y y) \cos(k_z z)$$

$$E_y = \frac{jwm\ k_x}{k_c^2} H_{zm} \sin(k_x x) \cos(k_y y) \sin(k_z z) \qquad H_y = -\frac{k_y k_z}{k_c^2} H_{zm} \cos(k_x x) \sin(k_y y) \cos(k_z z)$$

$$E_z = 0 \qquad H_z = H_{zm} \cos(k_x x) \cos(k_y y) \sin(k_z z)$$

（2）TM$_{mp}$模式

TE$_{mp}$模式波导在z方向的场为

$$E_z = E_0 \sin\left(k_x x\right)\sin\left(k_y y\right)e^{-jk_z z}$$

$$H_z = 0$$

矩形波导两端短路，纵向磁场为

$$E_z = \sin(k_x x)\sin(k_y y)\left(E_{zm}^{+} e^{-jk_z z} + E_{zm}^{-} e^{jk_z z}\right)$$

切向电场

$$E_x = -\frac{k_x k_z}{k_c^2}\cos(k_x x)\sin(k_y y)\left(E_{zm}^{+} e^{-jk_z z} + E_{zm}^{-} e^{jk_z z}\right)$$

$$E_y = -\frac{k_y k_z}{k_c^2}\sin(k_x x)\cos(k_y y)\left(E_{zm}^{+} e^{-jk_z z} + E_{zm}^{-} e^{jk_z z}\right)$$

由边界条件，切向场连续得：

$$E_{zm}^{-} = E_{zm}^{+} = E_{zm}; \qquad k_z = \frac{p\pi}{d}$$

谐振腔中的场分布

$$E_x = -\frac{k_x k_z}{k_c^2} E_{zm} \cos(k_x x)\sin(k_y y)\sin(k_z z) \qquad H_x = \frac{j\omega\varepsilon k_y}{k_c^2} E_{zm} \sin(k_x x)\cos(k_y y)\cos(k_z z)$$

$$E_y = -\frac{k_y k_z}{k_c^2} E_{zm} \sin(k_x x)\cos(k_y y)\sin(k_z z) \qquad H_y = -\frac{j\omega\varepsilon k_x}{k_c^2} E_{zm} \cos(k_x x)\sin(k_y y)\cos(k_z z)$$

$$E_z = E_{zm} \sin(k_x x)\sin(k_y y)\cos(k_z z) \qquad H_z = 0$$

- 谐振波长和频率

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \left(\frac{mp}{a}\right)^2 + \left(\frac{np}{b}\right)^2 + \left(\frac{pp}{c}\right)^2$$

$$k^2 = w^2 e m$$

$$\boxed{l_0 = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}}$$

$$\boxed{f_0 = \frac{v}{l_0} = \frac{c}{2\sqrt{m_r e_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$$

- 矩形谐振腔的主模：波长最长的谐振模式
  模式数：TE_{101}

Ø 谐振腔波长/频率：$\lambda_0 = \dfrac{2}{\sqrt{(1/a)^2 + (1/c)^2}}$;　　$f_0 = \dfrac{c}{\lambda_0}$

Ø 场分布：

$$\vec{E} = E_y \hat{y} = E_m \sin(k_x x)\sin(k_z z)\hat{y}$$

$$k_z d = \frac{2\pi}{\lambda_g} d = \pi \quad \rightarrow \quad \frac{d}{\lambda_g} = \frac{1}{2}$$

$$\vec{H} = \frac{jE_m k_z}{\omega_0 \mu_0} \sin(k_x x)\cos(k_z z)\hat{x} - \frac{jE_m k_x}{\omega_0 \mu_0}\cos(k_x x)\sin(k_z z)\hat{z}$$

Ø 品质因数：

储能： $W_0 = W_{m\max} = W_{e\max} = \frac{1}{2} e \int E_t \cdot E_t^* dt = \frac{abc}{8} e\ E_m^2$

耗能：

$$P_d = \frac{R_s}{2} \oint_s i \cdot i^* ds = \frac{R_s}{2} \oint_s H_t \cdot H_t^* ds$$

$$= R_s \left\{ \int_0^a \int_0^b \left| H_x \right|_{z=0}^2 dxdy + \int_0^b \int_0^c \left| H_z \right|_{x=0}^2 dydz + \int_0^a \int_0^c \left[\left| H_x \right|^2 + \left| H_z \right|^2 \right]_{y=0} dxdz \right\}$$

$$= \frac{R_s \lambda_0^2}{8\eta^2} E_m^2 \left[\frac{ab}{c^2} + \frac{bc}{a^2} + \frac{1}{2}\left(\frac{a}{c} + \frac{c}{a} \right) \right]$$

$$\boxed{Q_0 = \frac{ph}{4R_s} \left[\frac{2b\left(a^2 + c^2\right)^{3/2}}{ac\left(a^2 + c^2\right) + 2b\left(a^3 + c^3\right)} \right]}$$

例题：矩形波导谐振腔由横截面尺寸为a=4.755cm，b=2.125cm 的波导构成。谐振腔中填满了聚乙烯(εr=2.25，tanδ=0.0004)。如果谐振频率出现在5GHz，试求谐振腔的长度和主模的品质因数。

$$k = \frac{2p\,f}{c}\sqrt{e_r} = 157.08\,\mathrm{m}^{-1} \qquad d = \frac{p}{\sqrt{k^2 - (p/a)^2}} = 2.2\mathrm{cm}$$

假设金属的表面电阻Rs=1.84E-2Ω

$$Q_c = 8403; \quad Q_d = 1/\tan d = 2500$$

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1} = 1927$$

同轴谐振腔

- 适当长度的传输线，两端短路后构成谐振腔
- 谐振时，在传输线中形成驻波

  终端短路：1/2波长谐振腔

  终端开路：1/4波长谐振腔

- 同轴线中的场分布

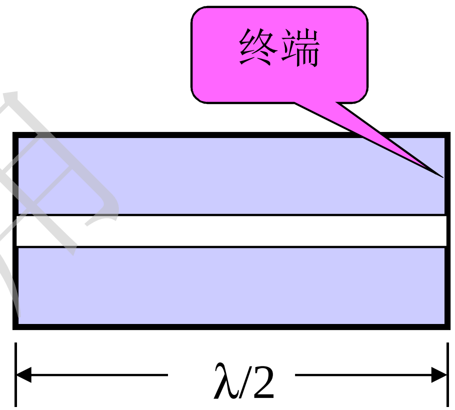
$$\vec{E}(r,\varphi,z)=\hat{r}\,\frac{U_0}{r\ln(b/a)}\left(e^{-j\beta z}+\Gamma e^{j\beta z}\right)$$

$$\vec{H}(r,\varphi,z)=\hat{\varphi}\,\frac{U_0}{\eta r\ln(b/a)}\left(e^{-j\beta z}-\Gamma e^{j\beta z}\right)$$

- λ/2 同轴腔（终端短路）

  z=0，L时 E=0：

  - 谐振器长度：L= λ/2
  - 场分布



$$E_r(r,z,t)=\frac{U_m}{r\ln(b/a)}\sin(\beta z)\sin(\omega t)$$

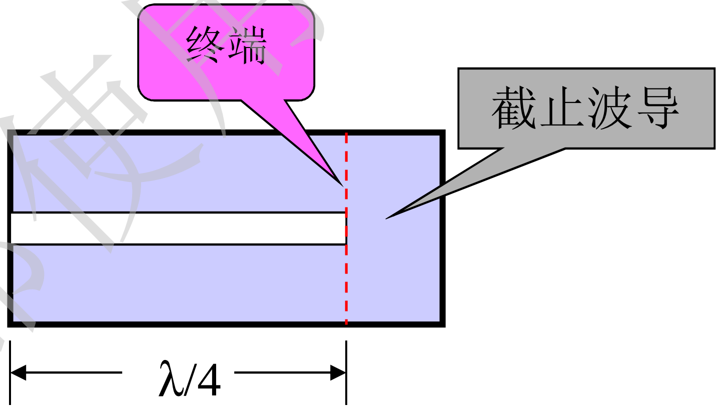
$$H_\varphi(r,z,t)=\frac{I_m}{2\pi r}\cos(\beta z)\cos(\omega t)$$

- λ/4 同轴腔（终端开路）

  z=0 时 E=0,

  z=L 时 H=0 ：

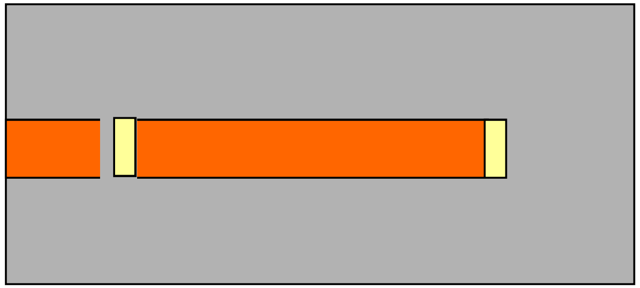
  - 谐振器长度：L= λ/4
  - 场分布



微带谐振器

- 间隙直线谐振器
  - 当线段长度为半波长时，线内形成驻波
  - 终端效应：端点电容效应使得线段的有效长度增加
  - 有效长度可以通过两个长度相差一倍的微带谐振器的频率差确定

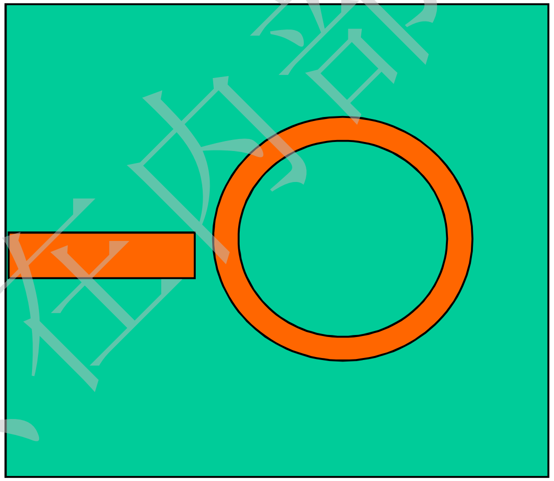
$$l_e = \frac{f_2 l_2 - 2 f_1 l_1}{2 f_1 - f_2}$$

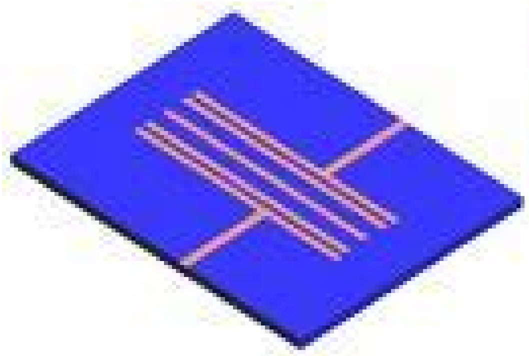


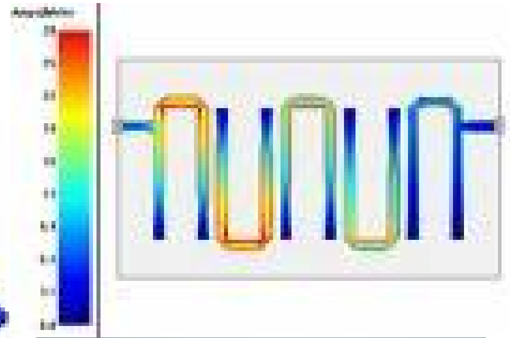
- 环形谐振器

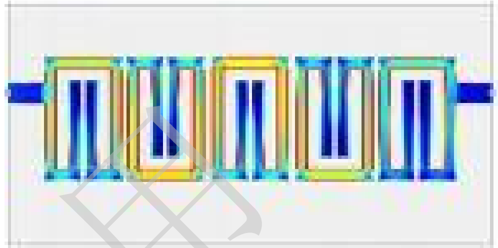
  当w/h<1, l>>w 时，如圆周长为半波长，产生谐振

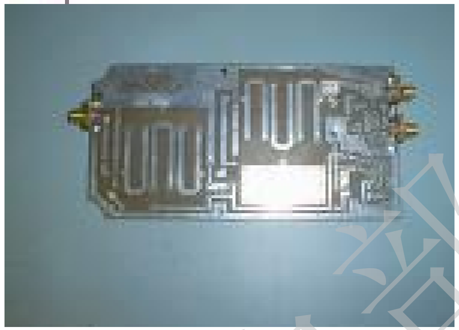
  没有终端效应，辐射损耗较小

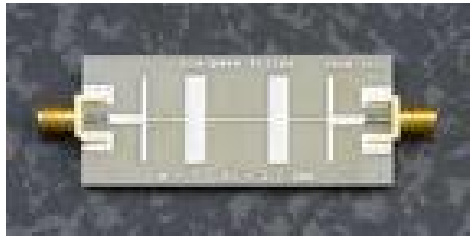


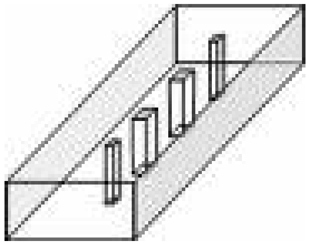


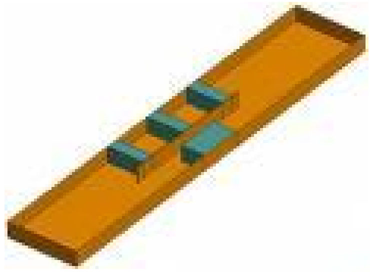


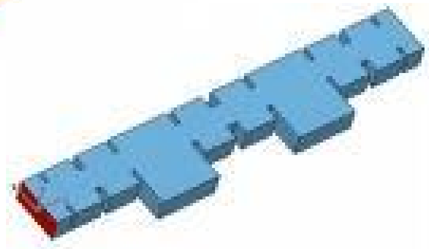


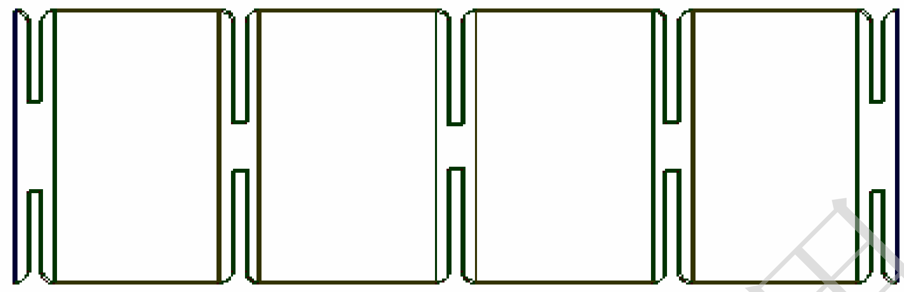


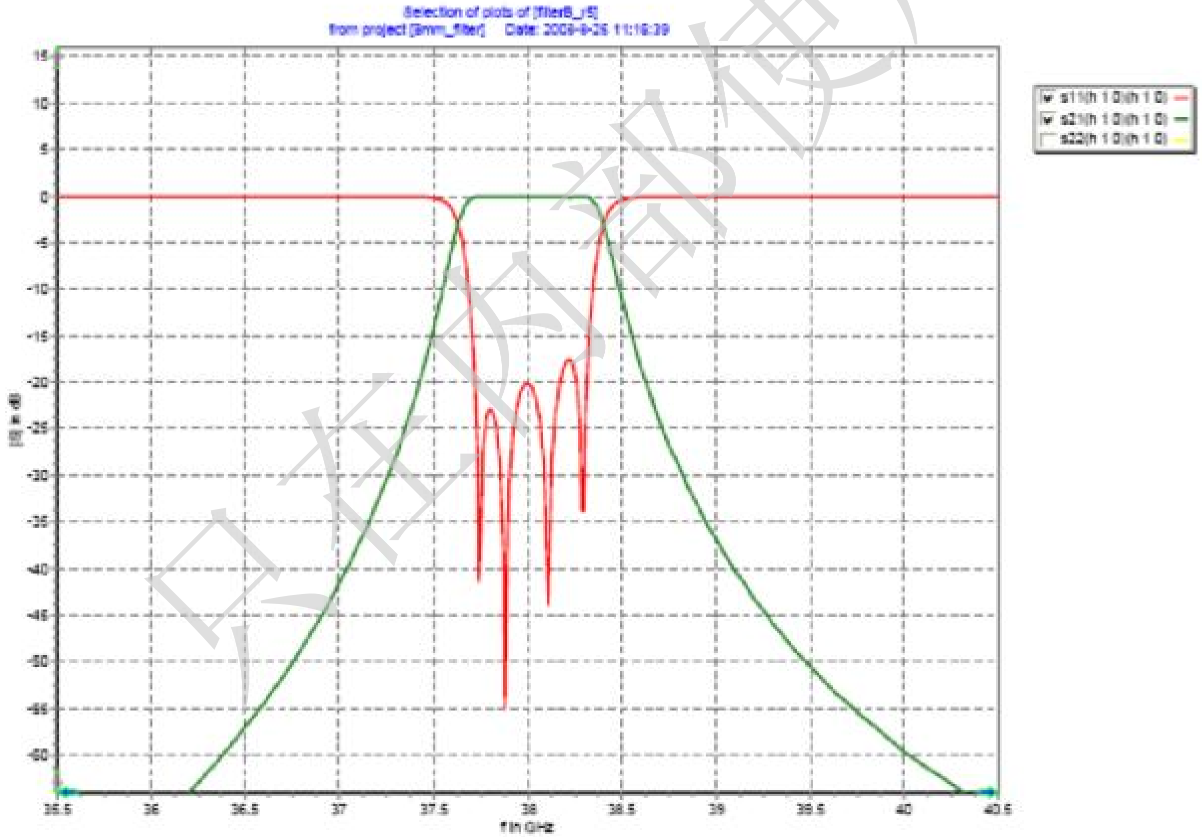












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s22(h 1 0)(h 1 0)

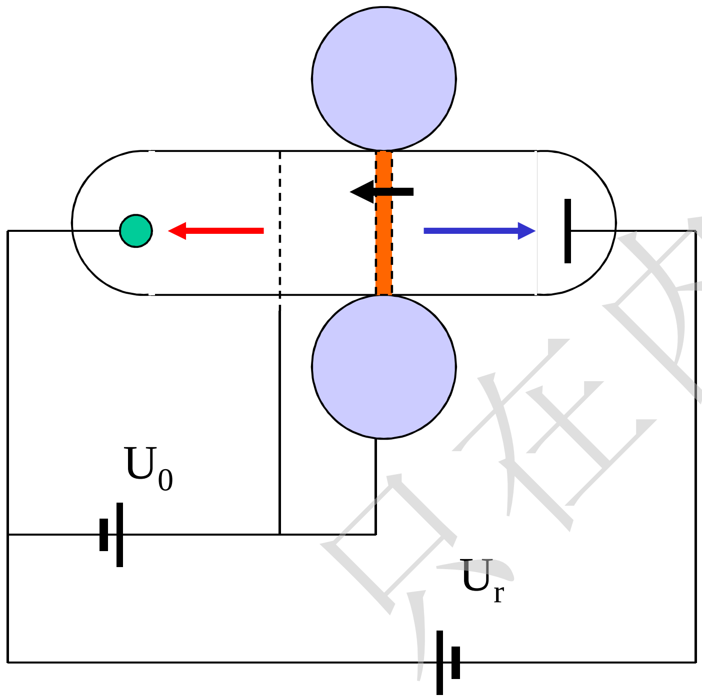
电磁波的产生：振荡器

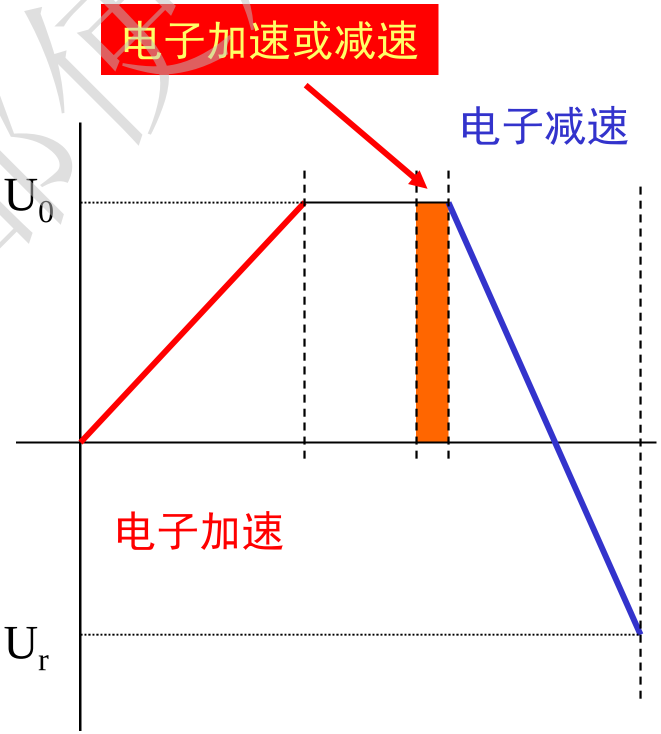
产生电磁波的基本原理

- EM theory can be regarded as the study of **fields produced by electric charges at rest and in motion**.
- Dynamic or time-varying fields are usually due to **accelerated charges or time-varying currents**.

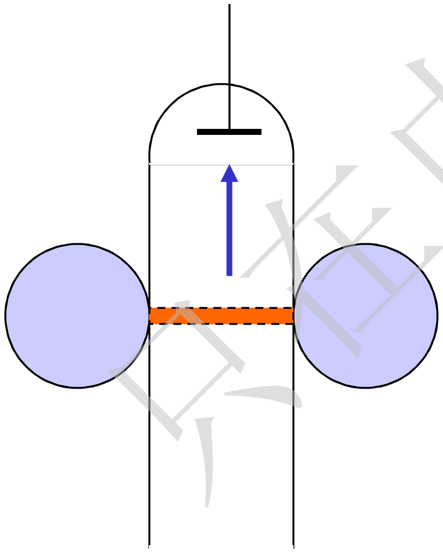
反射速调管振荡器

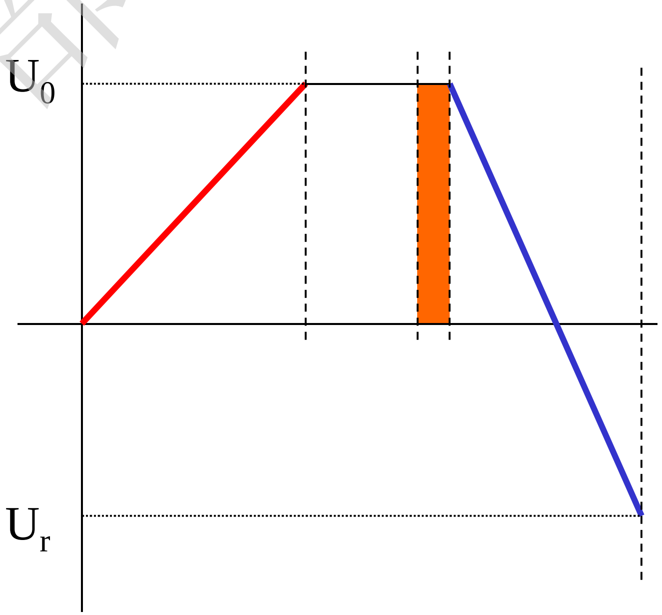
- 反射速调管结构





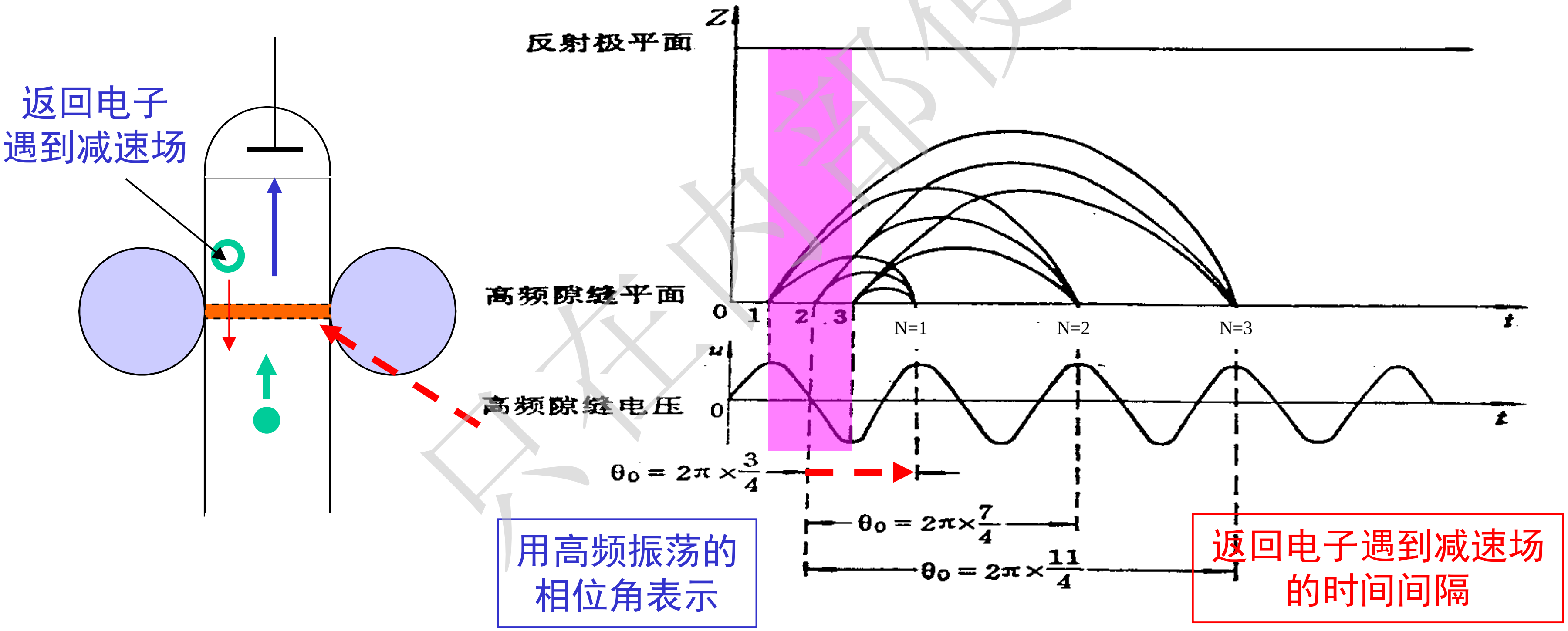
- 电子通过谐振腔时，通过自激建立振荡
- 入射电子通过栅极后的出射速度不同。
- 入射电子在减速区做“上抛运动”。
- 不同出射速度的电子回到栅极所需时间不同。调节反射极电压，使不同出射速度的电子在栅极汇聚。





- 汇聚电子在通过栅极时遇到减速场，将电子的动能转换成电磁能量，以维持振荡
- 维持振荡需要的渡越时间：(n-1/4)T，

  最佳渡越角：2π(n-1/4)



- 最佳渡越角决定于反射极电压

$$F = -eE = -e(U_0 + U_r)/l$$

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = -\frac{e}{m}\frac{(U_0 + U_r)}{l}$$

- 初始条件 $t = 0; \quad z = 0; \quad v_0 = \sqrt{\dfrac{2eU_0}{m}}$

$$t = \frac{2m}{e}\left(\frac{l}{U_0 + U_r}\right)v_0 = 2\sqrt{\frac{2m}{e}U_0}\,\frac{l}{(U_0 + U_r)}$$

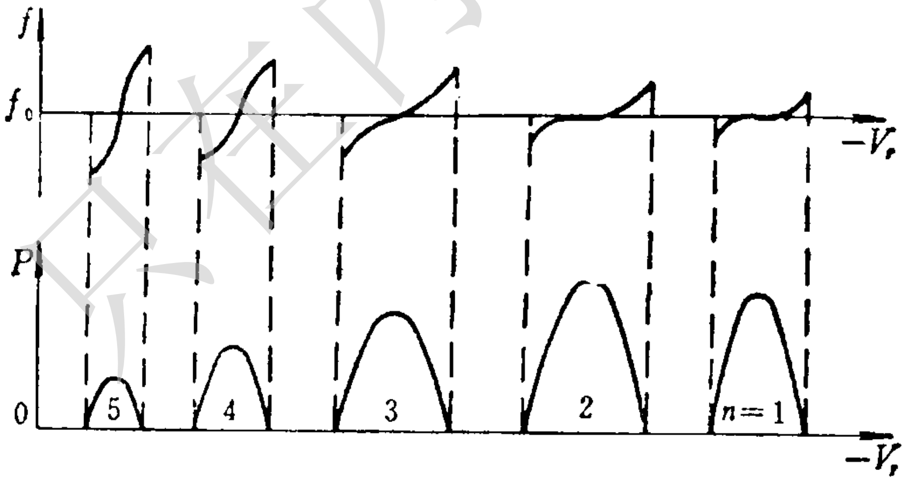
$$q = w\,t = 2w\sqrt{\frac{2m}{e}U_0}\,\frac{l}{(U_0 + U_r)}$$

$$q = 2w\sqrt{\frac{2mU_0}{e}}\frac{l}{(U_0+U_r)} = 2p\left(n-\frac{1}{4}\right) \qquad n = 1,2,3\mathbf{L}$$

$$w = p\sqrt{\frac{e}{2mU_0}}\left(n-\frac{1}{4}\right)\frac{(U_0+U_r)}{l}$$

- 振荡功率与电子调谐

在每一个振荡区，在最佳相位条件上有最大功率输出；偏离最佳相位条件，输出功率减小直至停振



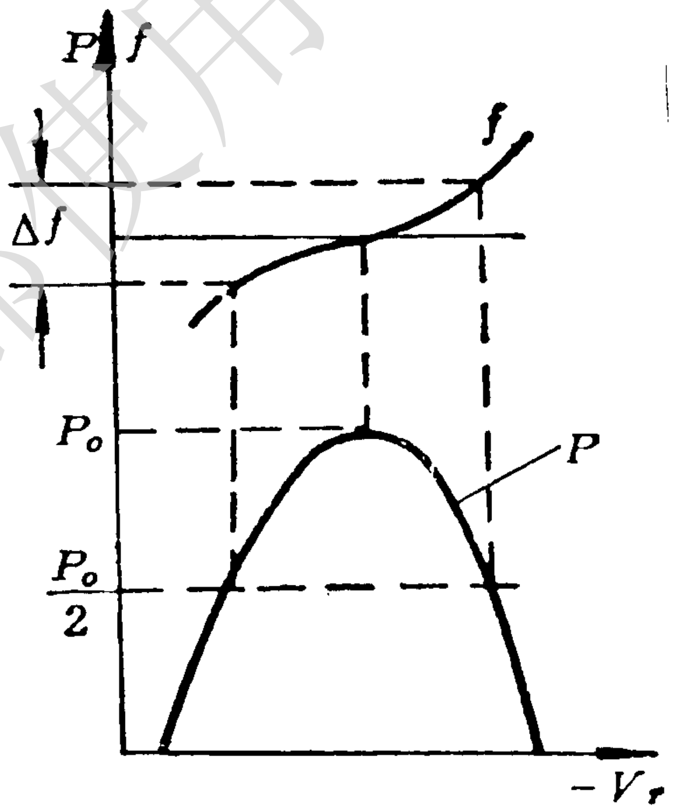
电子调谐：反射极电压的变化引起谐振频率的变化

反映电子调谐的参数

a．电子调谐斜率

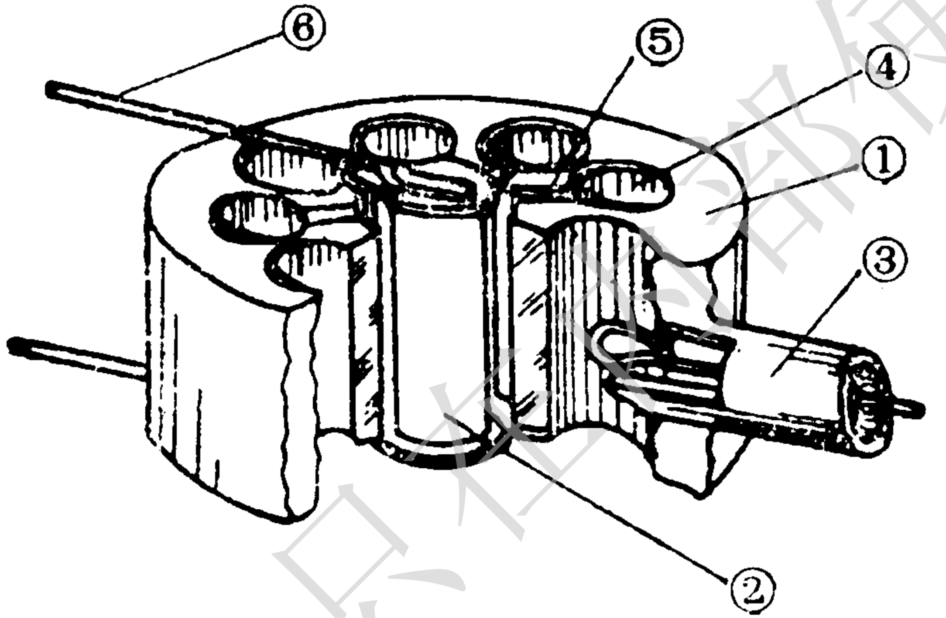
b．电子调谐范围：

- 反射速调管的结构参量及其应用

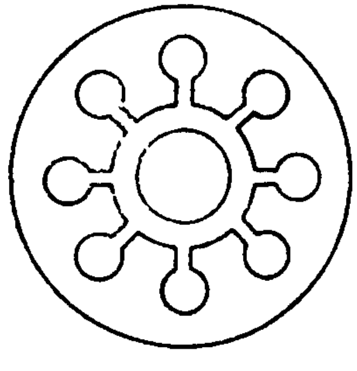


磁控管振荡器

- 磁控管结构



①阳极块②阴极③能量输出器④谐振腔孔⑤谐振腔隙缝⑥热子引线



(a)

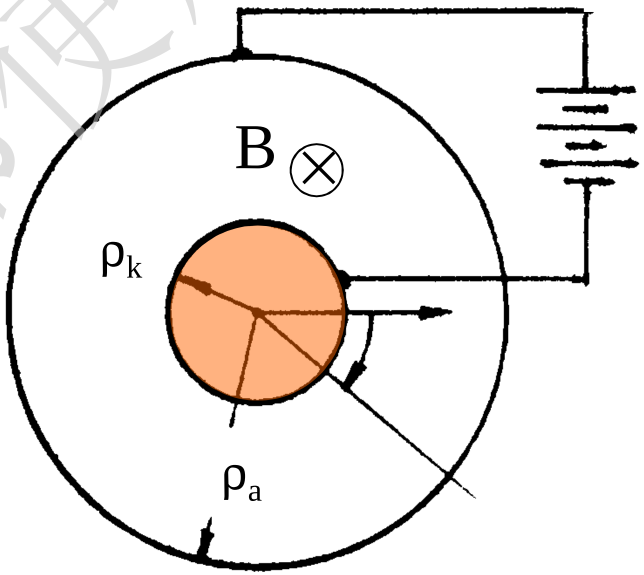
- 电子在磁控管中的运动（静态场）

假定阴极发射电子无z方向速度分量

$$m\frac{d^2\vec{r}}{dt^2}=\vec{F}=-e(\vec{E}+\vec{v}\times\vec{B})$$

$$E_\varphi=E_z=0;\quad E_r=-E(r)$$

$$B_r=B_\varphi=0;\quad B_z=B$$



平面极坐标系中的运动方程

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}-r\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2=-\frac{e}{m}\left(-E+rB\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)$$

$$2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\varphi}{\mathrm{d}t}+r\left(\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}\right)=\frac{eB}{m}\frac{\mathrm{d}r}{\mathrm{d}t}$$

改写（2）式成

$$\frac{d}{dt}\left(\rho^2\frac{d\varphi}{dt}\right)=\frac{eB}{2m}\frac{d}{dt}\left(\rho^2\right)$$

解得

$$\rho^2\left(\frac{d\varphi}{dt}-\frac{eB}{2m}\right)=const.$$

代入初始条件 $t=0,\ r=r_k,\ \mathrm{d}\varphi/\mathrm{d}t=0$

$$\rho^2\left(\frac{d\varphi}{dt}-\frac{eB}{2m}\right)+\rho_k^2\frac{eB}{2m}=0$$

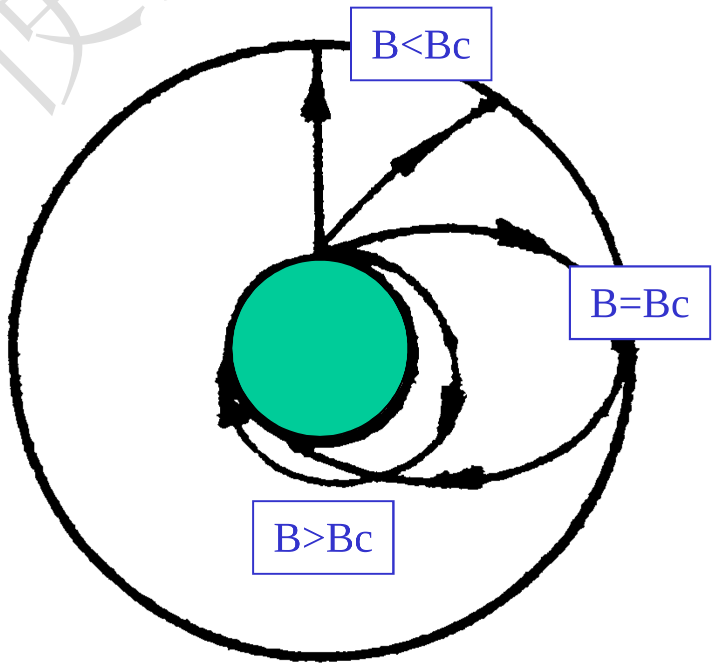
回旋角频率

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t}=\frac{1}{2}\left(\frac{eB}{m}\right)\left(1-\frac{r_k^2}{r^2}\right)=\frac{\omega_c}{2}\left(1-\frac{r_k^2}{r^2}\right)$$

将代入运动方程第一式

$$\frac{d^2\rho}{dt^2}+\rho\frac{\omega_c^2}{4}\left(1-\frac{\rho_k^4}{\rho^4}\right)=\frac{eE}{m}$$

电子的运动轨迹如图



存在一种临界状态，电子速度和阳极相切。临界状态时的磁场和电场？

$$\frac{1}{2}mv_a^2=eU_a$$

$$v_a^2=v_\rho^2+v_t^2=v_t^2$$

其中切向速度为

$$v_t = \rho_a \frac{dj}{dt} = \frac{\omega_c}{2}\left(\frac{\rho_a^2 - \rho_k^2}{\rho_a}\right)$$

从而

$$\frac{1}{2} m \left[\frac{w_c}{2}\left(\frac{r_a^2 - r_k^2}{r_a}\right)\right]^2 = eU_a$$

$$B_c = \rho_a \sqrt{\frac{8mU_a}{e}} \Big/ \left(\rho_a^2 - \rho_k^2\right)$$

当ρ_a与ρ_k相差不大时，令 $\rho_a - \rho_k = d$ 则可得

$$B_c \approx \frac{1}{d}\sqrt{\frac{2mU_a}{e}}$$

- 磁控管多腔谐振系统的性质(交变场部分)

谐振的必要条件：

沿整个阳极圆周上高频振荡的相位变化是2π的整数倍。

设相邻谐振腔中高频振荡讯号的相位差为j

$$
\begin{cases}
u_1 = U_m \sin w t \\
u_2 = U_m \sin(w t - j) \\
u_3 = U_m \sin(w t - 2j) \\
\qquad \mathrm{M} \\
u_n = U_m \sin[w t - (N-1)j] \\
u_{n+1} = U_m \sin(w t - Nj)
\end{cases}
$$

在发生谐振时，$u_{n+1} = u_1$ 　　得在谐振时相位的必要条件：

$$N\varphi = m2\pi \qquad m = 0, 1, 2, \cdots \quad \Rightarrow \quad \varphi = 2\pi\frac{m}{N}$$

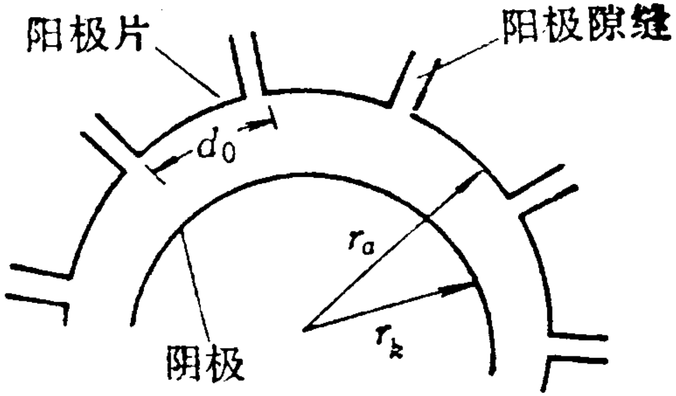
当谐振腔**数目**确定以后，可以在多个不同的相位差φ上满足发生谐振的相位条件

对应的腔体周长：

$$d_0 = \frac{2\pi r_a}{N}$$

$$\varphi = \left(\frac{2\pi}{\lambda_p}\right)d_0$$

$$Nd_0 = m\lambda_p = 2\pi r_a$$



- 给定一个首尾相接的闭合慢波线，若被激励起的高频信号的波长和慢波线总长度之间恰好等于的整数倍，这个闭合的慢波线就出现谐振现象。
- 在谐振时闭合系统吸收或存取高频能量

- 磁控管中自激的产生和阀电压

静态： $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \rightarrow \quad v_e = w_c R_c = E / B$

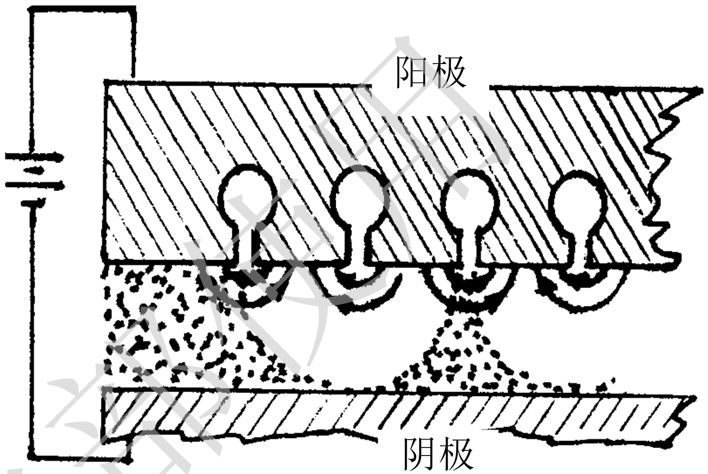
动态： $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B} + \vec{e} + \vec{v} \times \vec{b}) \quad \rightarrow \quad v = v_e + \Delta v$

- 振荡的稳定

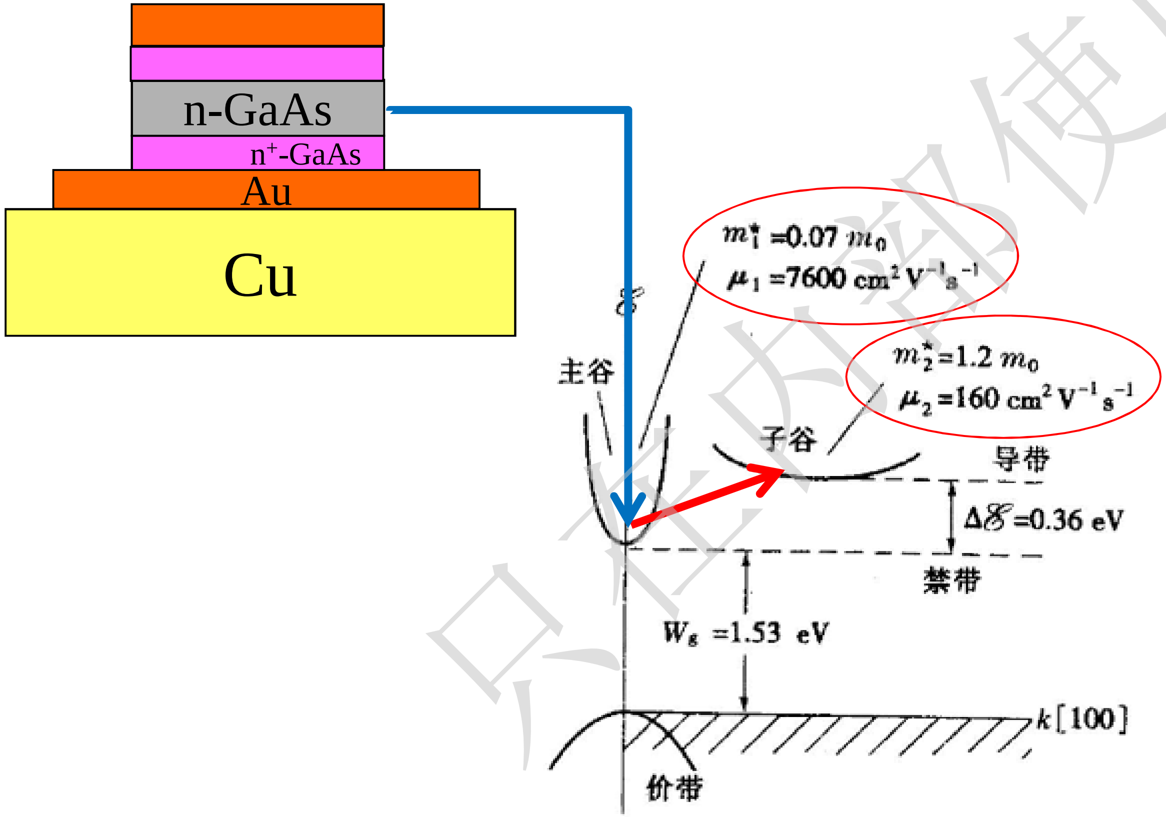
  阳极电压：

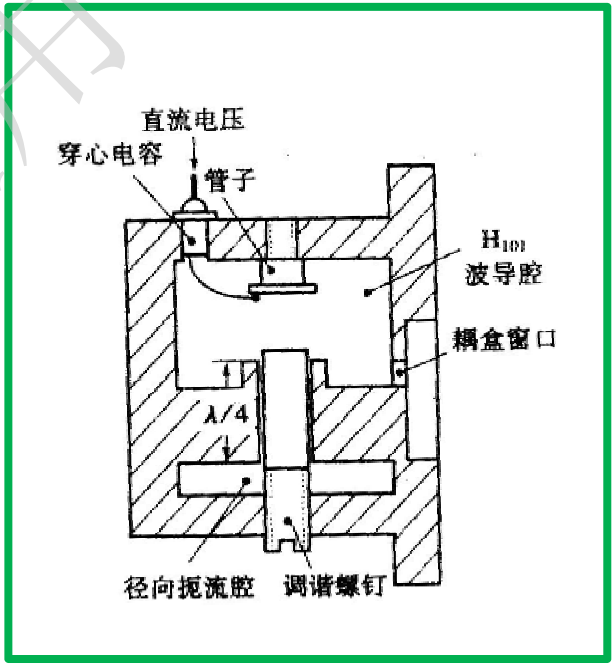
- 模式的稳定

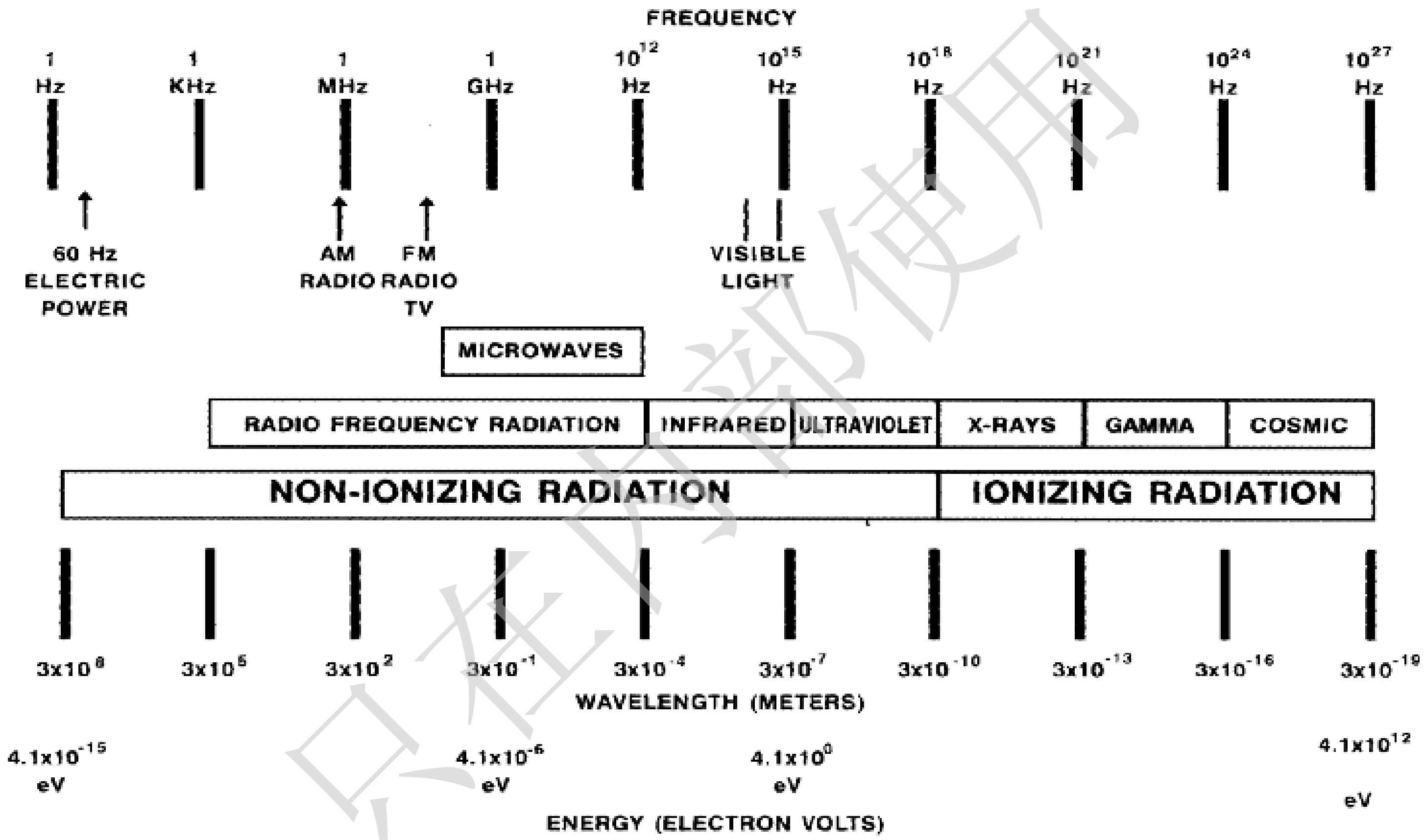
  腔体结构：



微波固态信号源：半导体振荡器







Electromagnetic Spectrum