第二章　时变电磁场

回顾：电磁学的基本量和基本实验定律

电荷及电荷密度

电荷和电流周围空间存在电场和磁场，是激发电磁场的基本源

电荷分布

体电荷密度

$$\rho_V(\mathbf{r}')=\lim_{\Delta V\to 0}\frac{\Delta q}{\Delta V'}=\frac{dq}{dV'}$$

$$q=\int_{V'}\rho_{V'}(\mathbf{r}')dV'$$

面电荷密度

$$\rho_S(\mathbf{r}')=\lim_{\Delta S\to 0}\frac{\Delta q}{\Delta S'}=\frac{dq}{dS'}$$

$$q=\int_{S'}\rho_{S'}(\mathbf{r}')dS'$$

点电荷密度

$$\rho_V(\mathbf{r})=\lim_{\Delta V\to 0}\frac{\Delta q}{\Delta V}=\begin{cases}0 & r\neq 0\\ \infty & r=0\end{cases}$$

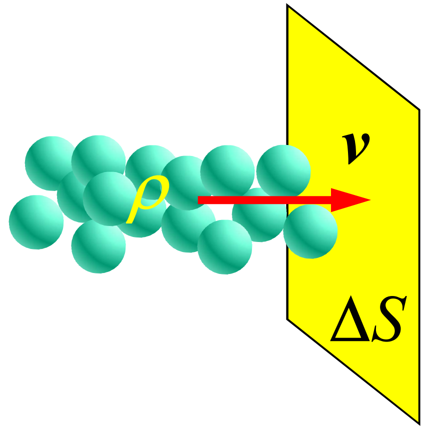
$$=q\delta(0)$$

$$\rho(\mathbf{r})=q\delta(\mathbf{r}-\mathbf{r}')$$

电流及电流密度

电荷在电场的作用下的宏观运动形成电流，定义为单位时间内通过某横截面的电荷：

$$\boldsymbol{i} = \hat{\boldsymbol{i}} \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \hat{\boldsymbol{i}} \frac{dq}{dt}$$

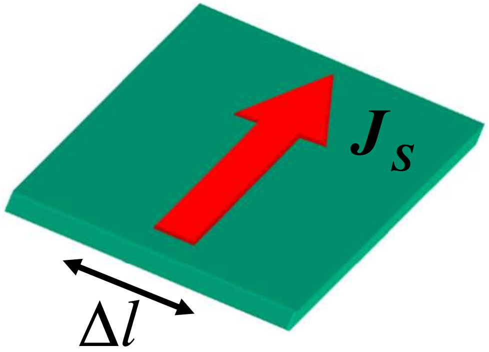


体电流密度

$$\boldsymbol{J} = \boldsymbol{n} \lim_{\Delta S \to 0} \frac{\Delta i}{\Delta S} = \boldsymbol{n} \frac{di}{dS} = \rho \boldsymbol{v}$$

面电流密度

$$\left| \boldsymbol{J}_S \right| = \lim_{\Delta l \to 0} \frac{\Delta i}{\Delta l} = \frac{di}{dl}$$



电流连续性方程

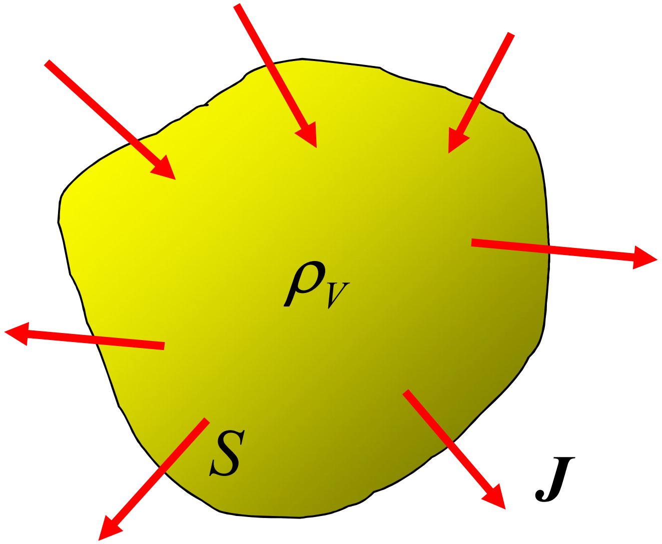
$$i = \oint_S \boldsymbol{J} \bullet d\boldsymbol{S} = -\frac{dq}{dt}$$

$$q = \int_V \rho_V dV$$

散度定理

$$\oint_S \boldsymbol{J} \bullet d\boldsymbol{S} = \int_V (\nabla \bullet \boldsymbol{J}) dV = -\frac{\partial}{\partial t}\int_V \rho_V dV$$

$$\int_V \left(\nabla \bullet \boldsymbol{J} + \frac{\partial \rho_V}{\partial t} \right) dV = 0$$



$$\boxed{\nabla \bullet \boldsymbol{J} + \frac{\partial \rho_V}{\partial t} = 0}$$

微分形式的电流连续方程

稳恒电流：

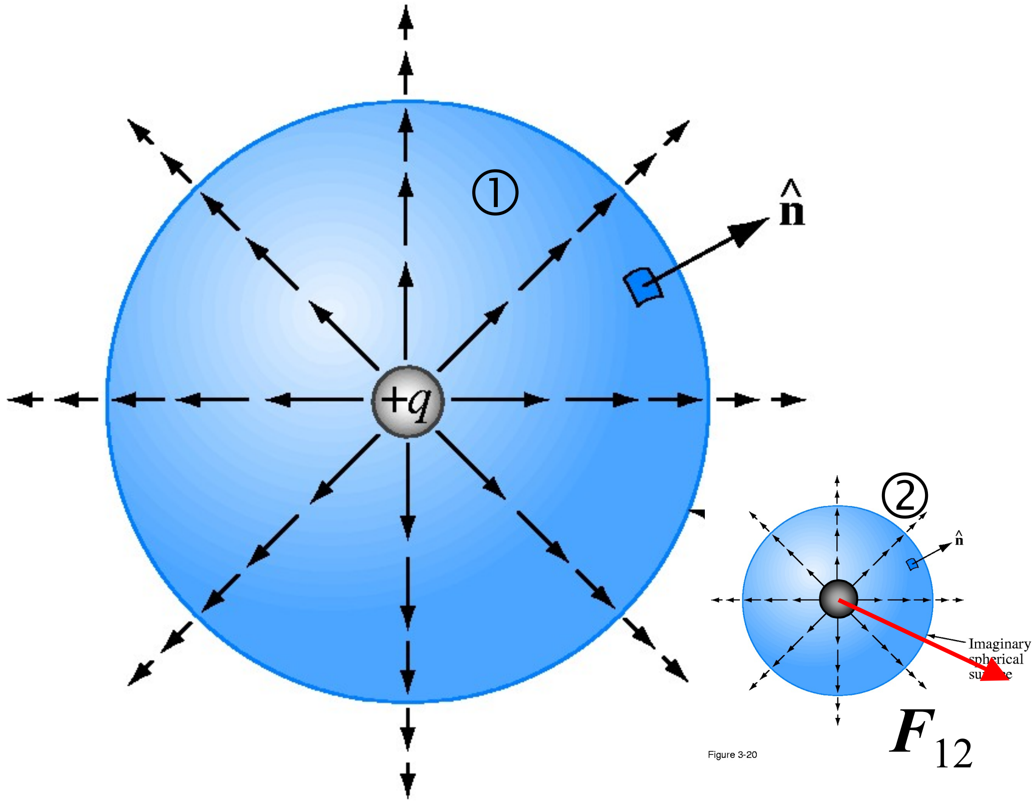
$$\frac{\partial \rho_V}{\partial t} = 0$$

$$\oint_S \boldsymbol{J} \bullet d\boldsymbol{S} = 0 \quad \nabla \bullet \boldsymbol{J} = 0$$

库仑定律和电场强度

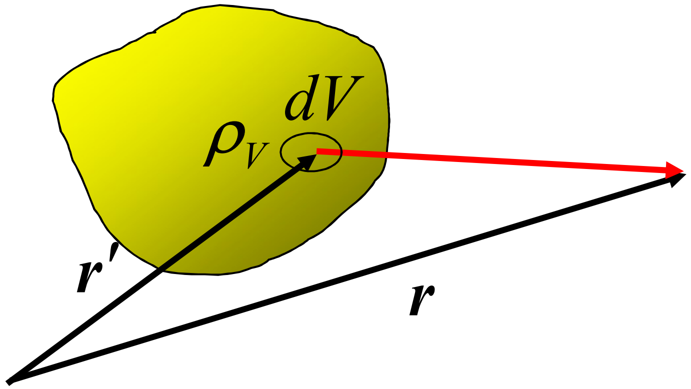
$$\boldsymbol{F}_{12}=\frac{q_1q_2}{4\pi\varepsilon_0R^2}\hat{\boldsymbol{R}}=\frac{q_1q_2}{4\pi\varepsilon_0R^3}\boldsymbol{R}$$

$$\boxed{\boldsymbol{E}=\hat{\boldsymbol{R}}\frac{q}{4\pi\varepsilon_0R^2}\qquad (V/m)}$$



$$\boldsymbol{E}(\boldsymbol{r})=\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\rho_V(\boldsymbol{r}')dV'$$

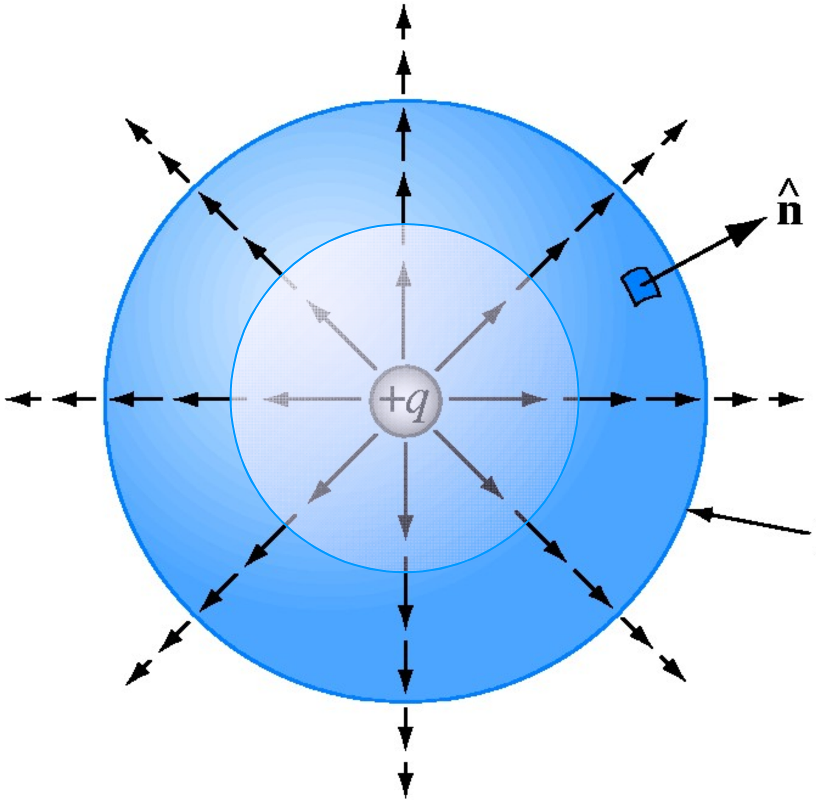
$$\boldsymbol{E}(\boldsymbol{r})=\frac{1}{4\pi\varepsilon_0}\int_{S'}\frac{(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\rho_S(\boldsymbol{r}')dS'$$



高斯定律

$$\boldsymbol{E}=\hat{\boldsymbol{R}}\frac{q}{4\pi\varepsilon_0 R^2}\quad (V/m)$$

$$\oint_S \varepsilon_0 \boldsymbol{E}\bullet d\boldsymbol{S}=\oint_S \varepsilon_0\frac{q}{4\pi\varepsilon_0 R^2}dS=q$$



总电场通量 $=\oint_S \boldsymbol{E}\bullet d\boldsymbol{s}$

$$\oint_S \boldsymbol{E}\bullet d\boldsymbol{s}=\frac{q}{\varepsilon_0}=\frac{1}{\varepsilon_0}\int_V \rho(r)d\tau$$

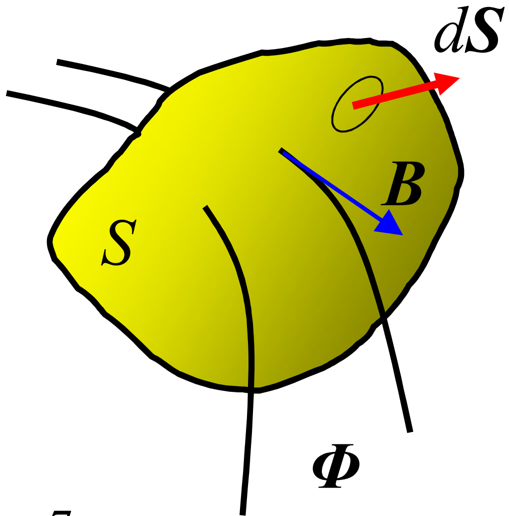
$$\oint_S \boldsymbol{E}\bullet d\boldsymbol{s}=\int_v (\nabla\bullet\boldsymbol{E})d\tau=\frac{1}{\varepsilon_0}\int_v \rho d\tau$$

$$\nabla\bullet\boldsymbol{E}=\frac{\rho}{\varepsilon_0}$$

安培定律和磁感应强度

磁场强度的切向分量沿闭合回路的线积分等于该回路包围的电流的大小。

$$\boxed{\oint_l \boldsymbol{H} \bullet d\boldsymbol{l} = I}$$

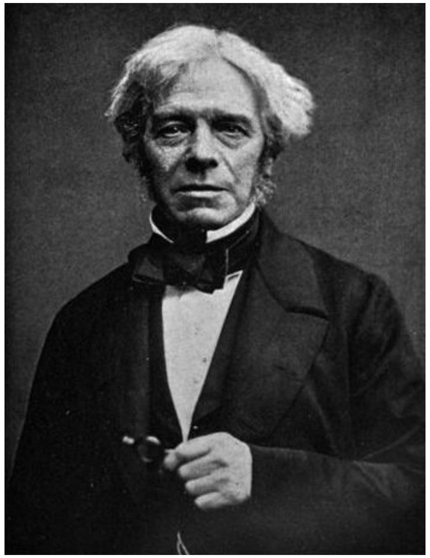


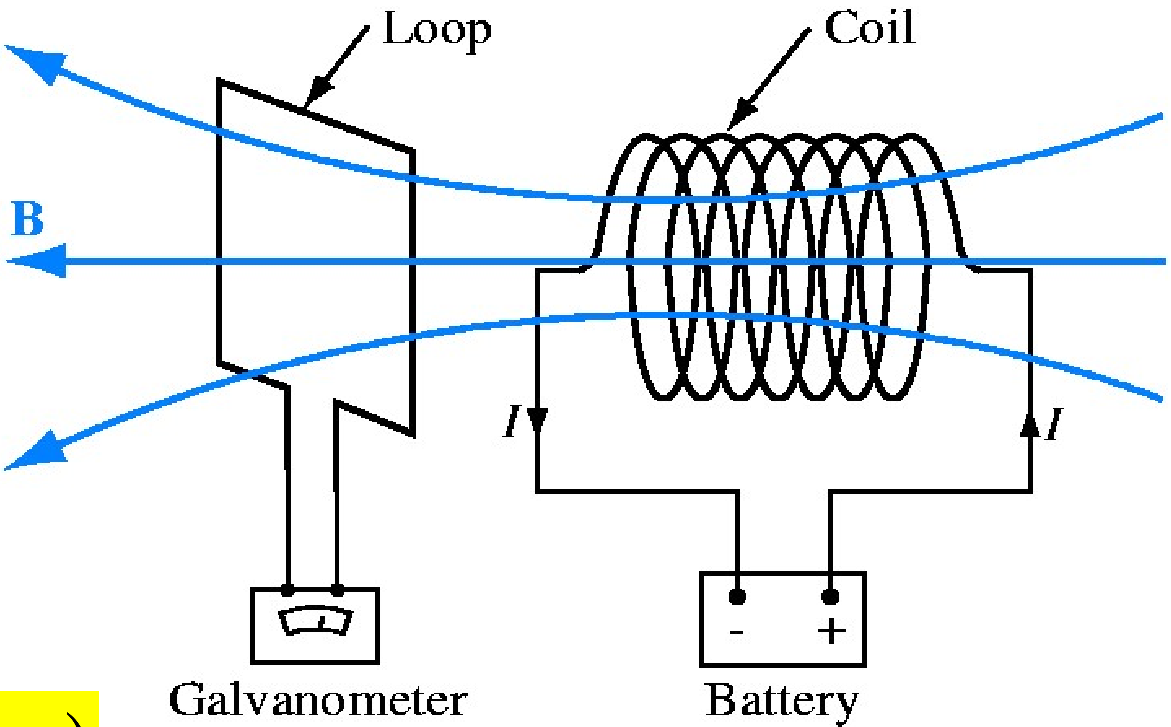
磁感应强度 $$\boxed{\boldsymbol{B} = \mu \boldsymbol{H}}$$

$$\mu = \mu_0 \mu_r \qquad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$$

磁通量 $$\boxed{\Phi = \int_S \boldsymbol{B} \bullet d\boldsymbol{S}}$$

第二章　时变电磁场





Faraday 电磁感应定律

（1831）

磁通量：

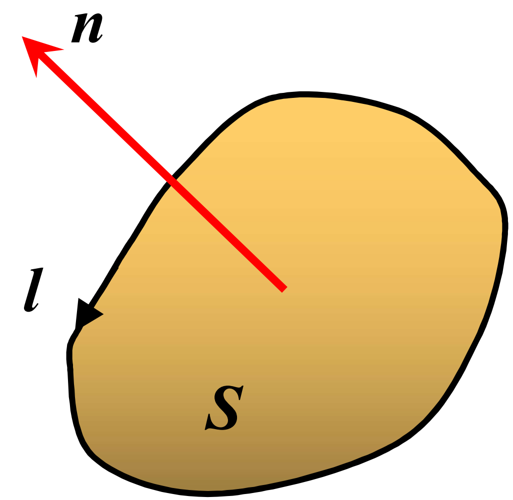
$$\Phi = \int_S \boldsymbol{B} \bullet \mathrm{d}\boldsymbol{s} \quad (\mathrm{Wb})$$

感生电动势 $\varepsilon = -\dfrac{\mathrm{d}}{\mathrm{d}t}\int_S \boldsymbol{B} \bullet \mathrm{d}\boldsymbol{s}$

磁通量

$$\oint_l \boldsymbol{E} \bullet \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \boldsymbol{B} \bullet \mathrm{d}\boldsymbol{s}$$

$$\oint_l \boldsymbol{E} \bullet \mathrm{d}\boldsymbol{l} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \bullet \mathrm{d}\boldsymbol{s}$$



$$\oint_l \boldsymbol{E} \bullet d\boldsymbol{l} = \int_S (\nabla \times \boldsymbol{E}) \bullet d\boldsymbol{s}$$

Stokes 定理

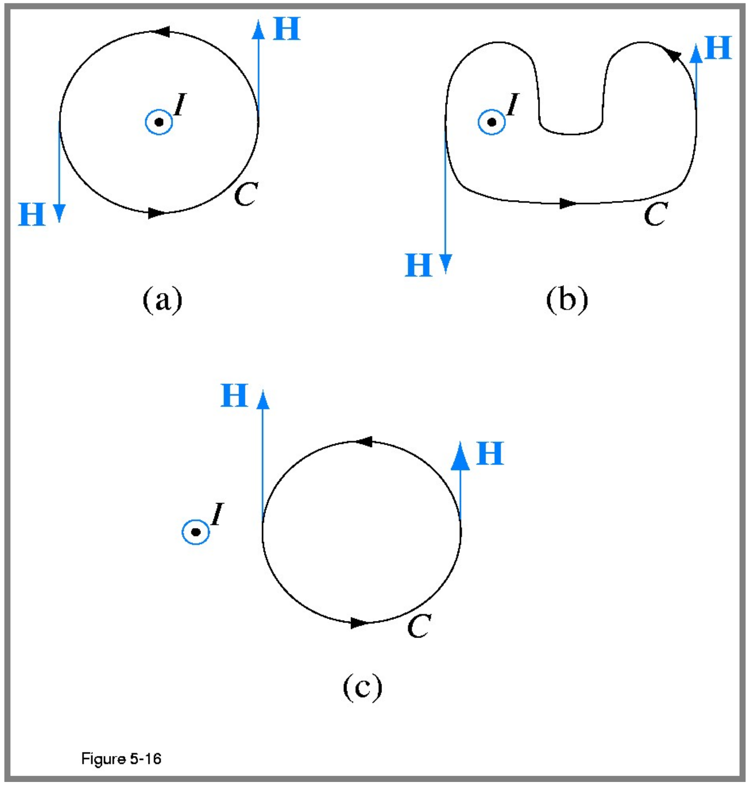
$$\boxed{\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}}$$

与电流电荷无关，适合真空和介质中的宏观电磁场

2.2 位移电流（Displacement Current）

真空稳恒情形下的安培环路定理：

$$\oint_C \boldsymbol{H} \bullet d\boldsymbol{l} = I$$



$$\boxed{\oint_l \boldsymbol{H} \bullet d\boldsymbol{l} = \int_S (\nabla \times \boldsymbol{H}) \bullet d\boldsymbol{s}}$$

$$\int_S (\nabla \times \boldsymbol{H}) \bullet d\boldsymbol{s} = \int_S \boldsymbol{j} \bullet d\boldsymbol{s}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} \Rightarrow \nabla \bullet \boldsymbol{j} = 0$$

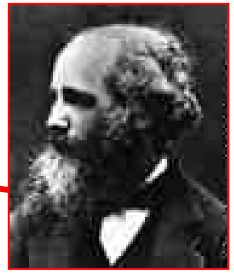
时变情形下： $\nabla \bullet \boldsymbol{j} = -\dfrac{\partial \rho}{\partial t} \neq 0$ 电荷守恒定律（电流连续）

矛盾！

引入位移电流密度物理量： j_D ，满足：

$$\nabla \bullet (j + j_D) = 0$$

$$\nabla \times B = \mu_0 (j + j_D)$$



由电荷守恒定律： $\nabla \bullet j + \frac{\partial \rho}{\partial t} = 0$ 和高斯定律： $\nabla \bullet E = \frac{\rho}{\varepsilon_0}$

$$\nabla \bullet \left(j + \varepsilon_0 \frac{\partial E}{\partial t} \right) = 0$$

$$\boxed{j_D = \varepsilon_0 \frac{\partial E}{\partial t}}$$

（真空情形）

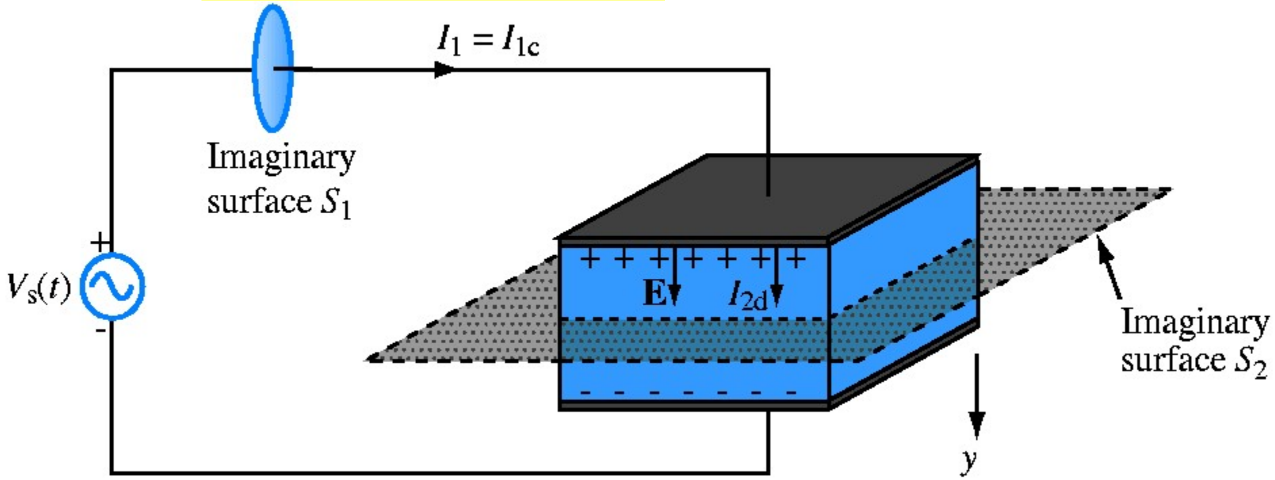
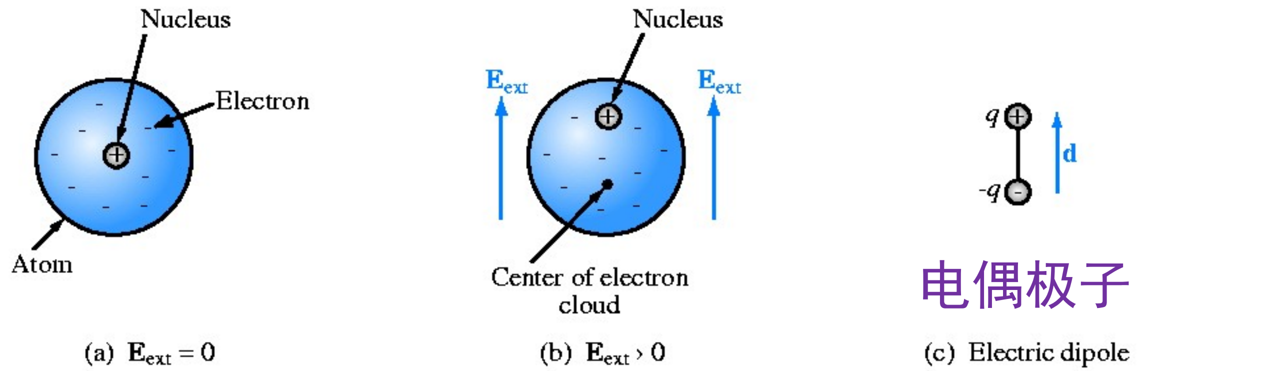


Figure 6-13

介质：真空中的带电粒子系统



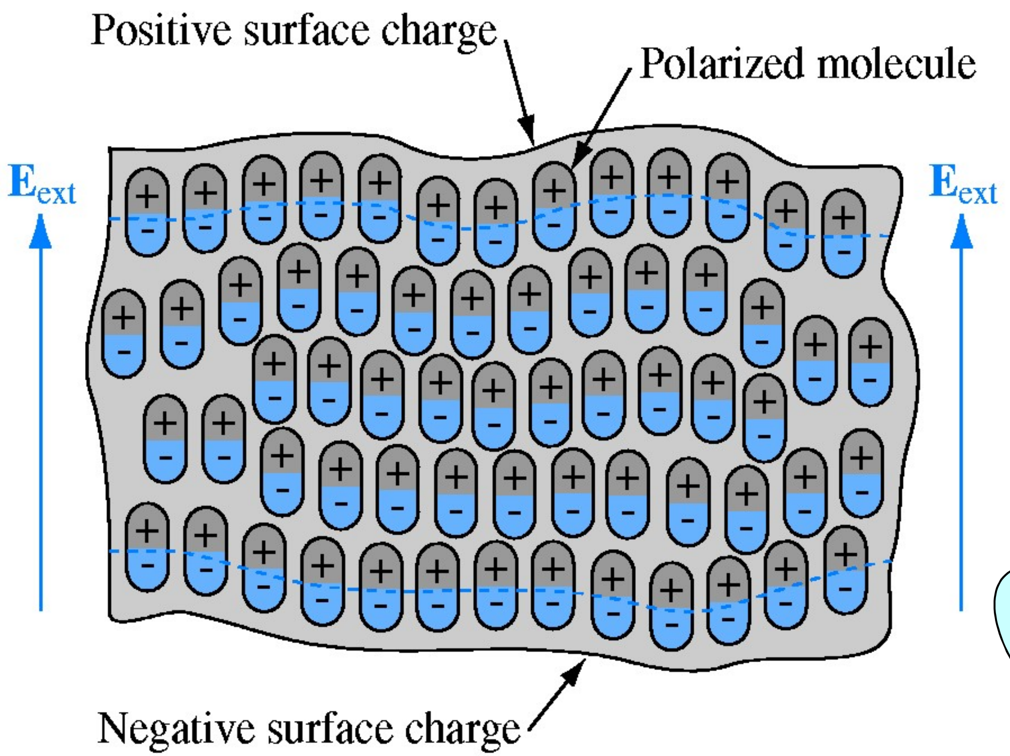


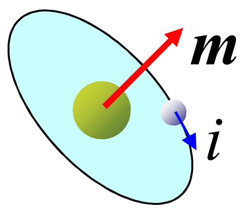
Figure 4-17

微观物理量 **宏观物理量**

电偶极矩$\boldsymbol{p}=q\boldsymbol{d}$ 极化强度 $\boldsymbol{P}$

$$\boldsymbol{P} = \frac{\sum \boldsymbol{p}_i}{\Delta V}$$

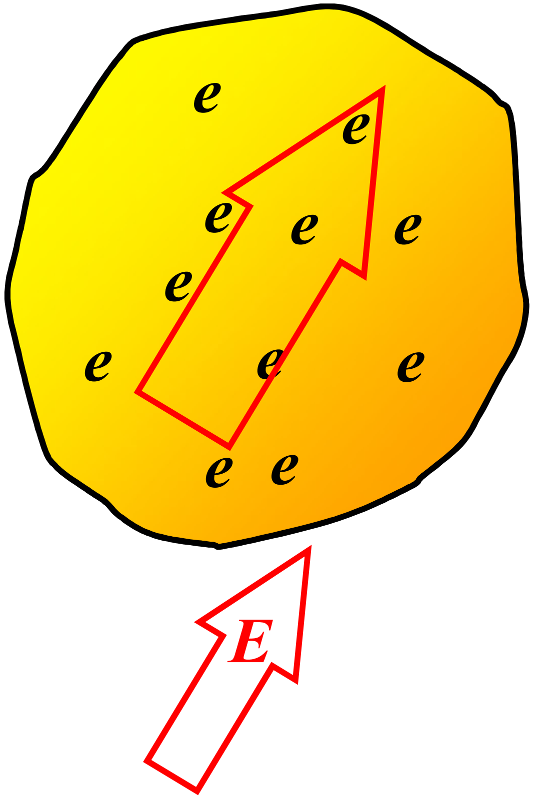
分子磁矩$\boldsymbol{m}=i\boldsymbol{a}$ 磁化强度$\boldsymbol{M}$

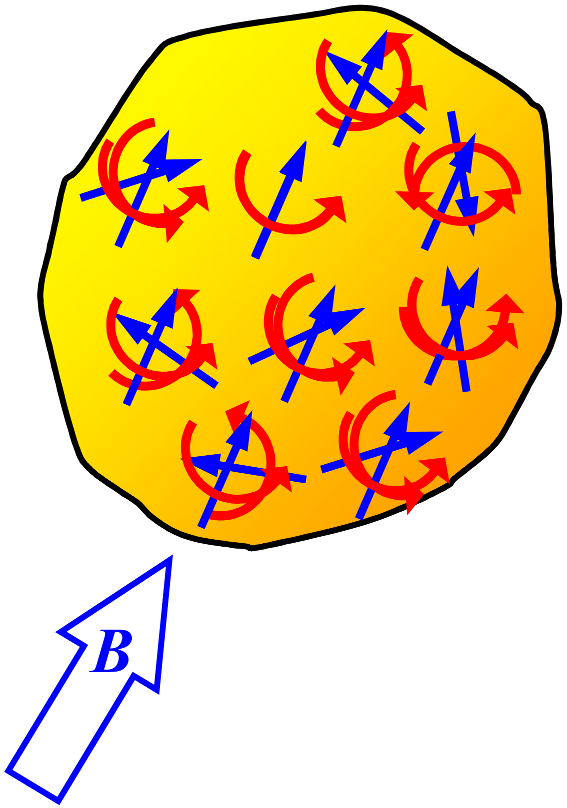


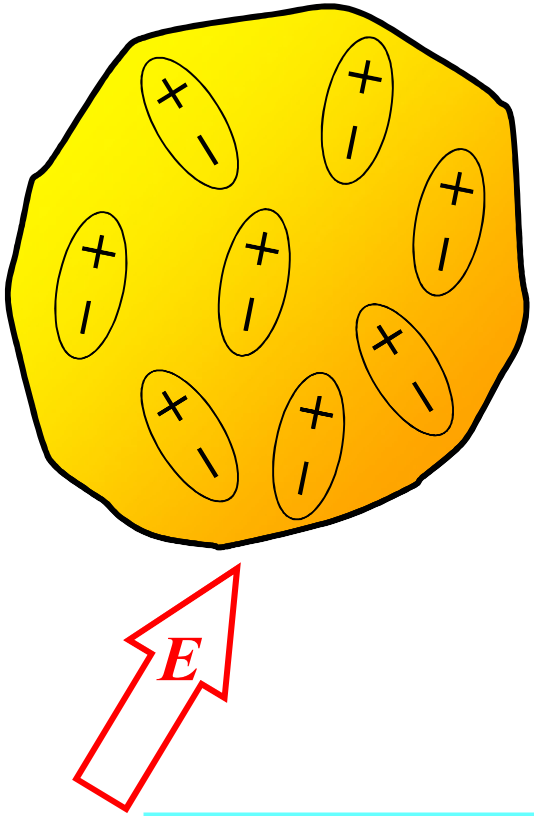
$$\boldsymbol{M} = \frac{\sum \boldsymbol{m}_i}{\Delta V}$$

有介质的情形：介质是真空中的带电粒子系统

电流：自由电子的传导电流、分子电流、极化电流







$$\boldsymbol{j} = \sigma \boldsymbol{E}$$

$$\boldsymbol{m} = i\boldsymbol{a} \quad \boldsymbol{M} = \frac{\sum \boldsymbol{m}_i}{\Delta V}$$

$$\boldsymbol{p}_i = q_i \boldsymbol{l} \quad \boldsymbol{P} = \frac{\sum \boldsymbol{p}_i}{\Delta V}$$

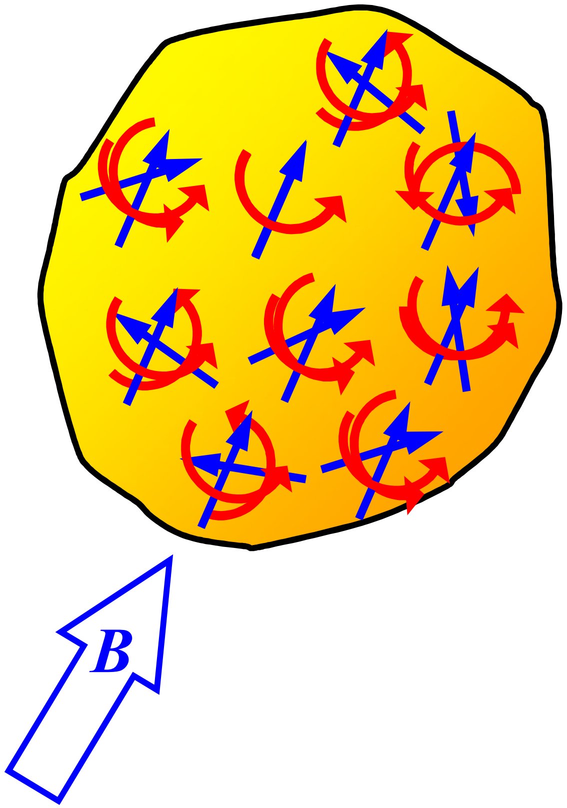
微观安培环路定理：　　自由电子传导电流密度　$\boldsymbol{j}_f$

分子电流密度　$\boldsymbol{j}_m$

$$\nabla\times\boldsymbol{b}=\mu_0(\boldsymbol{j}_f+\boldsymbol{j}_m)$$

时空平均：

$$\nabla\times\boldsymbol{B}=\mu_0(\boldsymbol{j}_f+\bar{\boldsymbol{j}}_m)$$



磁化电流密度　$\bar{\boldsymbol{j}}_m$　$\boldsymbol{j}_M$

无外场时，电流取向无规则：$\bar{\boldsymbol{j}}_m=0$

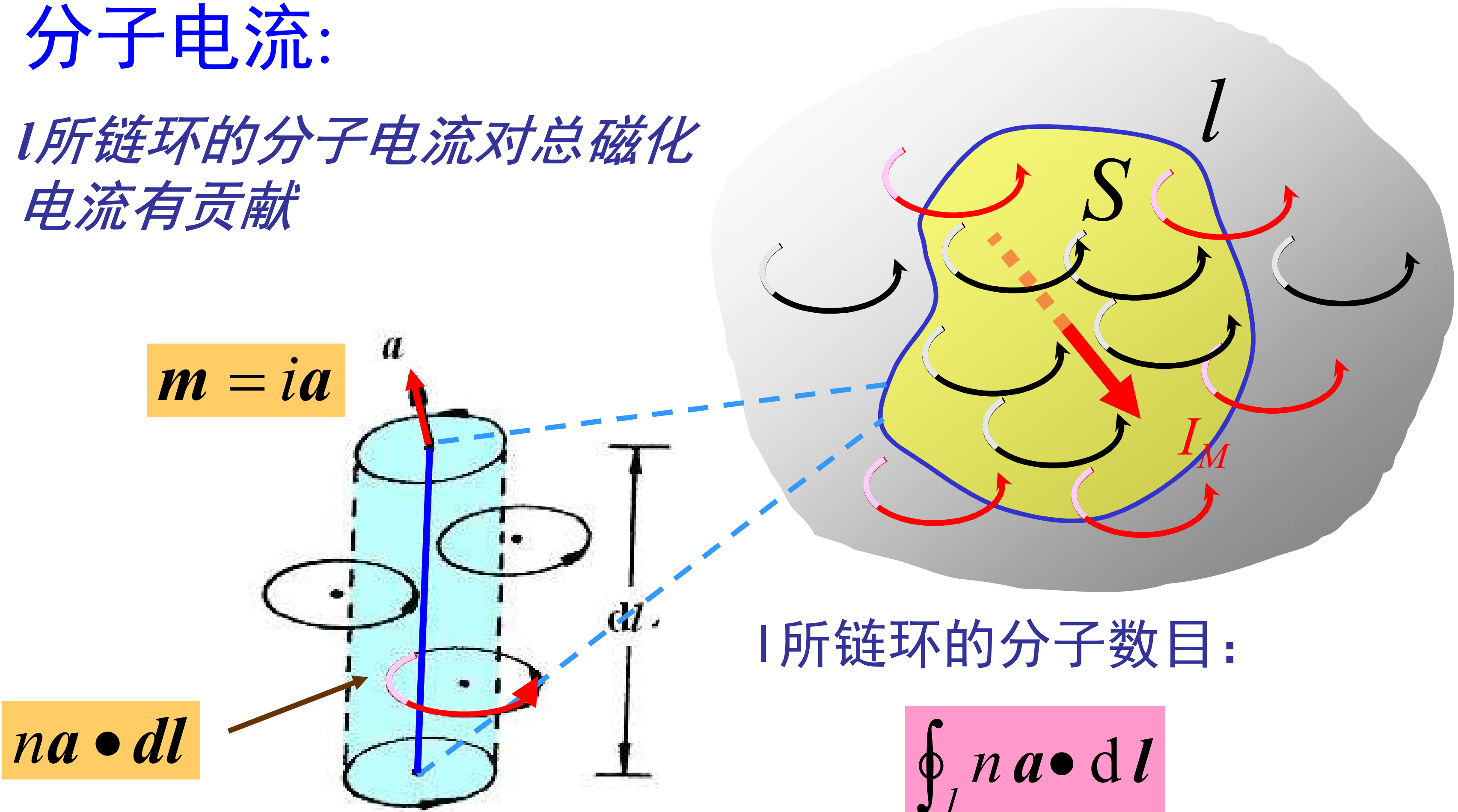
有外磁场时，介质被磁化，

定义宏观磁化强度：

$$\boldsymbol{M}=\frac{\sum \boldsymbol{m}_i}{\Delta\tau}$$

分子电流:

l所链环的分子电流对总磁化电流有贡献



l所链环的分子数目:

$$\oint_l n\,\boldsymbol{a} \bullet \mathrm{d}\,\boldsymbol{l}$$

$$I_M = \oint_l ni\,\boldsymbol{a} \bullet \mathrm{d}\,\boldsymbol{l} = \oint_l n\,\boldsymbol{m} \bullet \mathrm{d}\,\boldsymbol{l} = \oint_l \boldsymbol{M} \bullet \mathrm{d}\,\boldsymbol{l}$$

$$I_M = \int_S \bar{j}_m \bullet \mathrm{d}\boldsymbol{s}$$

$$\int_S \bar{j}_m \bullet \mathrm{d}\boldsymbol{s} = \oint_l \boldsymbol{M} \bullet \mathrm{d}\boldsymbol{l}$$

磁化电流定义

$$\oint_l \boldsymbol{M} \bullet \mathrm{d}\boldsymbol{l} = \int (\nabla \times \boldsymbol{M}) \bullet \mathrm{d}\boldsymbol{s}$$

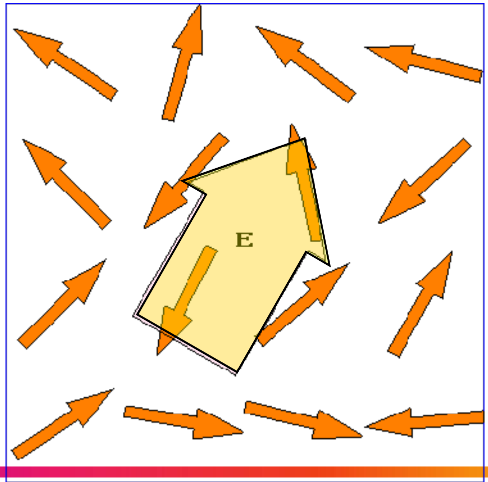
$$\bar{j}_m = \nabla \times \boldsymbol{M} = \boldsymbol{j}_M$$

极化电流:

介质极化具有平均分子电偶极矩，$\boldsymbol{p} = q\boldsymbol{l}$ 产生极化电流密度：$\bar{\boldsymbol{j}}_P$

单位时间穿过截面的电量：

$$I_P = \int \bar{j}_P \bullet \mathrm{d}\boldsymbol{s} = \int \rho \boldsymbol{v} \bullet \mathrm{d}\boldsymbol{s} = \int nq \frac{\mathrm{d}\boldsymbol{l}}{\mathrm{d}t} \bullet \mathrm{d}\boldsymbol{s}$$



$$\int \bar{j}_P \bullet \mathrm{d}\boldsymbol{s} = \int n \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \bullet \mathrm{d}\boldsymbol{s} = \int \frac{\partial \boldsymbol{P}}{\partial t} \bullet \mathrm{d}\boldsymbol{s}$$

$$\int (\bar{j}_P - \frac{\partial \boldsymbol{P}}{\partial t}) \bullet \mathrm{d}\boldsymbol{s} = 0 \Rightarrow \boxed{\bar{j}_P = \frac{\partial \boldsymbol{P}}{\partial t}}$$

$$\nabla\times\boldsymbol{B}=\mu_0\{\boldsymbol{j}_{\mathrm{f}}+\overline{\boldsymbol{j}}_m+\overline{\boldsymbol{j}}_P+\varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}\}$$

$$\nabla\times\frac{\boldsymbol{B}}{\mu_0}=\boldsymbol{j}_{\mathrm{f}}+\nabla\times\boldsymbol{M}+\frac{\partial\boldsymbol{P}}{\partial t}+\varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}$$

$$\boldsymbol{D}=\varepsilon_0\boldsymbol{E}+\boldsymbol{P}\longrightarrow\frac{\partial\boldsymbol{D}}{\partial t}=\varepsilon_0\frac{\partial\boldsymbol{E}}{\partial t}+\frac{\partial\boldsymbol{P}}{\partial t}$$

$$\nabla\times(\frac{\boldsymbol{B}}{\mu_0}-\boldsymbol{M})=\boldsymbol{j}_{\mathrm{f}}+\frac{\partial\boldsymbol{D}}{\partial t}$$

$$\boldsymbol{H}=\frac{\boldsymbol{B}}{\mu_0}-\boldsymbol{M}\longrightarrow\boxed{\nabla\times\boldsymbol{H}=\boldsymbol{j}_{\mathrm{f}}+\frac{\partial\boldsymbol{D}}{\partial t}}$$

全电流定律：

$$\boxed{\oint_l\boldsymbol{H}\bullet\mathrm{d}\boldsymbol{l}=\int_S(\boldsymbol{j}_f+\frac{\partial\boldsymbol{D}}{\partial t})\bullet\mathrm{d}\boldsymbol{s}}$$

全电流连续性方程：

$$\boxed{\nabla\bullet(\boldsymbol{j}+\frac{\partial\boldsymbol{D}}{\partial t})=0}$$

2.3 麦克斯韦(Maxwell)方程组

$$\begin{cases}\oint_l \boldsymbol{E}\bullet \mathrm{d}\boldsymbol{l} = -\int_S \dfrac{\partial \boldsymbol{B}}{\partial t}\bullet \mathrm{d}\boldsymbol{s} \\ \oint_l \boldsymbol{H}\bullet \mathrm{d}\boldsymbol{l} = \int_S (\boldsymbol{j}_f + \dfrac{\partial \boldsymbol{D}}{\partial t})\bullet \mathrm{d}\boldsymbol{s} \\ \oint_S \boldsymbol{D}\bullet \mathrm{d}\boldsymbol{s} = Q_f \\ \oint_S \boldsymbol{B}\bullet \mathrm{d}\boldsymbol{s} = 0\end{cases}$$

积分形式

$$\begin{cases}\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \times \boldsymbol{H} = \boldsymbol{j}_f + \dfrac{\partial \boldsymbol{D}}{\partial t} \\ \nabla \bullet \boldsymbol{D} = \rho \\ \nabla \bullet \boldsymbol{B} = 0\end{cases}$$

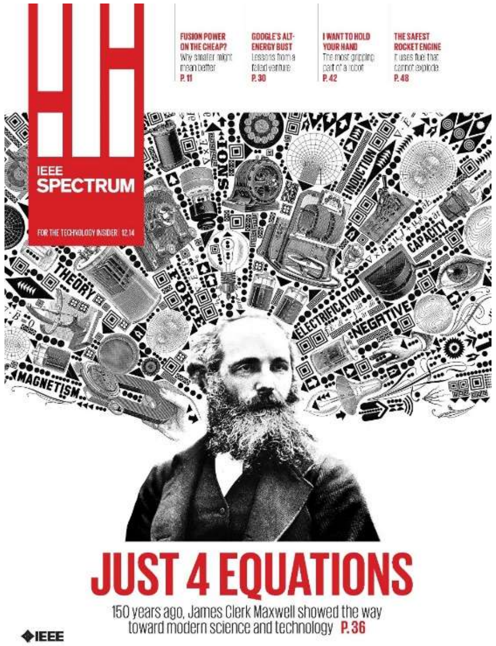
微分形式

- ❑ 积分形式→可适用于不连续边界☑，微分形式→ ☒。
- ❑ 线性方程，可以用叠加原理；电磁场是互易的。
- ❑ 满足Mexwell方程的解具有唯一性。

2004年，英国《物理世界》期刊举办活动：让读者选出科学史上最伟大的公式；

麦克斯韦方程组力压质能方程、欧拉公式、牛顿第二定律、勾股定理、薛定谔方程等”方程界“的巨擘，高居榜首

2.3 麦克斯韦(Maxwell)方程组



James Clerk Maxwell

J. C. Maxwell opened his great paper "A Dynamical Theory of the Electromagnetic Field" published in the fall of 1864.

156 周年！

The aim of science is, on the one hand, a comprehension as complete as possible, of the connection between sense experiences in their totality, and, on the other hand, the accomplishment of this aim by the use of a minimum of primary concepts and relations.

-- Albert Einstein

完整、简洁明了！

- ***The first ties a circulating electric field which would produce a current in a loop of wire, with a magnetic field changing with time (a moving magnet). It is Faraday's Law.*** *法拉第电磁感应定律*
- ***The second links a circulating magnetic field, curling through space, with an electric field changing with time (an electric current). It expands Ampere's Law.*** *安培环路定律*
- ***The third equation ---- Coulomb's Law.*** *库伦定律（电荷是电磁的源）*
- ***None has ever been found of the magnetic "charge."*** *磁荷不存在（磁力线总是闭合的）*

Maxwell方程组揭示了电磁场的内部作用和运动。

不仅电荷和电流可以激发电磁场，而且变化的电场和磁场也可以互相激发。只要某处有电磁扰动，由于电磁场的互相激发，它就在空间中运动传播，形成电磁波。电磁场可以独立于电荷之外而存在。

电磁场本身是一种特殊的物质。

介质(材料)的电磁性质方程： $\boldsymbol{D}=\varepsilon\,\boldsymbol{E}=\varepsilon_0\,\boldsymbol{E}+\boldsymbol{P}$

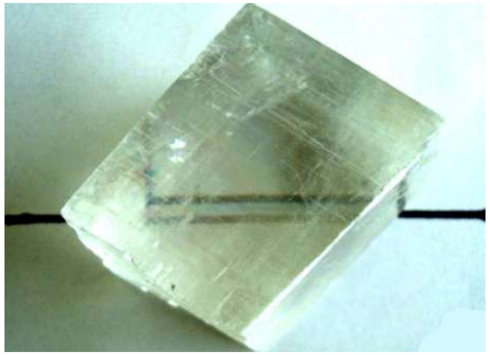
材料的本构参数： $\boldsymbol{B}=\mu\,\boldsymbol{H}=\mu_0(\boldsymbol{H}+\boldsymbol{M})$

Constitutional parameters $\boldsymbol{j}=\sigma\,\boldsymbol{E}$

各向异性介质：介电常数和磁导率为张量 $\overleftrightarrow{\varepsilon},\overleftrightarrow{\mu}$ 如铁氧体。

$$\overleftrightarrow{\varepsilon}\qquad \begin{bmatrix} D_1\\ D_2\\ D_3\end{bmatrix}=\begin{bmatrix}\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13}\\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23}\\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}\end{bmatrix}\begin{bmatrix} E_1\\ E_2\\ E_3\end{bmatrix}$$

$$\overleftrightarrow{\mu}\qquad \begin{bmatrix} B_1\\ B_2\\ B_3\end{bmatrix}=\begin{bmatrix}\mu_{11} & \mu_{12} & \mu_{13}\\ \mu_{21} & \mu_{22} & \mu_{23}\\ \mu_{31} & \mu_{32} & \mu_{33}\end{bmatrix}\begin{bmatrix} H_1\\ H_2\\ H_3\end{bmatrix}$$



Calcite 方解石

非线性介质：ε 和 μ 与 $\boldsymbol{E}$ 和 $\boldsymbol{H}$ 有关，如铁磁物质。

正弦电磁场（时谐电磁场）

场源以单一频率随时间正弦变化，其场量也随时间正弦变化，任意时变周期场可由傅立叶级数分解为正弦谐波分量的叠加.

$$\boldsymbol{E}(\boldsymbol{r},t)=\boldsymbol{e_x}E_{xm}(\boldsymbol{r})\cos(\omega t+\phi_x(\boldsymbol{r}))+\boldsymbol{e_y}E_{ym}(\boldsymbol{r})\cos(\omega t+\phi_y(\boldsymbol{r}))$$
$$+\boldsymbol{e_z}E_{zm}(\boldsymbol{r})\cos(\omega t+\phi_z(\boldsymbol{r}))$$
$$=\boldsymbol{e_x}\,\mathrm{Re}\left(E_{xm}(\boldsymbol{r})e^{j\phi_x(\boldsymbol{r})}e^{j\omega t}\right)+\boldsymbol{e_y}\,\mathrm{Re}\left(E_{ym}(\boldsymbol{r})e^{j\phi_y(\boldsymbol{r})}e^{j\omega t}\right)$$
$$+\boldsymbol{e_z}\,\mathrm{Re}\left(E_{zm}(\boldsymbol{r})e^{j\phi_z(\boldsymbol{r})}e^{j\omega t}\right)$$
$$=\boldsymbol{e_x}\,\mathrm{Re}\left(\dot{E}_x e^{j\omega t}\right)+\boldsymbol{e_y}\,\mathrm{Re}\left(\dot{E}_y e^{j\omega t}\right)+\boldsymbol{e_z}\,\mathrm{Re}\left(\dot{E}_z e^{j\omega t}\right)$$

正弦电磁场的复数形式（复矢量）

$$\boxed{\boldsymbol{E}(\boldsymbol{r},t)=\mathrm{Re}\left(\boldsymbol{e_x}\dot{E}_x e^{j\omega t}+\boldsymbol{e_y}\dot{E}_y e^{j\omega t}+\boldsymbol{e_z}\dot{E}_z e^{j\omega t}\right)=\mathrm{Re}\left(\dot{\boldsymbol{E}}e^{j\omega t}\right)}$$

麦氏方程的复数形式

$$\frac{\partial}{\partial t}\boldsymbol{E}(\boldsymbol{r},t)=\frac{\partial}{\partial t}\mathrm{Re}\left(\dot{\boldsymbol{E}}e^{j\omega t}\right)=\mathrm{Re}\left(j\omega\dot{\boldsymbol{E}}e^{j\omega t}\right)$$

$$\frac{\partial}{\partial t^2}\boldsymbol{E}^2(\boldsymbol{r},t)=\frac{\partial^2}{\partial t^2}\mathrm{Re}\left(\dot{\boldsymbol{E}}e^{j\omega t}\right)=\mathrm{Re}\left(-\omega^2\dot{\boldsymbol{E}}e^{j\omega t}\right)$$

麦氏第二方程：

$$\nabla\times\mathrm{Re}\left(\dot{\boldsymbol{H}}e^{j\omega t}\right)=\mathrm{Re}\left(\dot{\boldsymbol{J}}e^{j\omega t}\right)+\mathrm{Re}\left(j\omega\dot{\boldsymbol{D}}e^{j\omega t}\right)$$

$$\mathrm{Re}\left(\nabla\times\dot{\boldsymbol{H}}e^{j\omega t}\right)=\mathrm{Re}\left(\dot{\boldsymbol{J}}e^{j\omega t}\right)+\mathrm{Re}\left(j\omega\dot{\boldsymbol{D}}e^{j\omega t}\right)$$

对任意时间t都成立：

$$\nabla\times\dot{\boldsymbol{H}}e^{j\omega t}=\dot{\boldsymbol{J}}e^{j\omega t}+j\omega\dot{\boldsymbol{D}}e^{j\omega t}$$

$$\nabla\times\dot{\boldsymbol{H}}=\dot{\boldsymbol{J}}+j\omega\dot{\boldsymbol{D}}$$

麦氏方程的复数形式

$$\boxed{\begin{array}{ll}\nabla\times\dot{\boldsymbol{E}}=-j\omega\dot{\boldsymbol{B}} & \nabla\bullet\dot{\boldsymbol{D}}=\rho\\ \nabla\times\dot{\boldsymbol{H}}=\dot{\boldsymbol{J}}+j\omega\dot{\boldsymbol{D}} & \nabla\bullet\dot{\boldsymbol{B}}=0\end{array}}$$

例：已知某无源区域的电场强度瞬时矢量为：

$$\boldsymbol{E}(x,z,t)=\boldsymbol{e}_y E_{ym}\sin k_x x\cos(\omega t-k_z z)$$

求与之相伴的磁场强度复矢量和瞬时矢量。

电场强度复矢量 $\boldsymbol{E}(x,z,t)=\mathrm{Re}\left[\boldsymbol{e}_y E_{ym}\sin k_x x e^{j(\omega t-k_z z)}\right]$

$$\dot{\boldsymbol{E}}(x,z)=\boldsymbol{e}_y E_{ym}\sin k_x x e^{-jk_z z}$$

磁场强度由麦氏方程： $\nabla\times\dot{\boldsymbol{E}}=-j\omega\mu\dot{\boldsymbol{H}}$

$$\dot{\boldsymbol{H}}(x,z)=-\frac{1}{j\omega\mu}\nabla\times\dot{\boldsymbol{E}}(x,z)$$

$$=-\frac{1}{j\omega\mu}\begin{vmatrix}\boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \dot{E}_x & \dot{E}_y & \dot{E}_z\end{vmatrix}=-\frac{1}{j\omega\mu}\left(-\boldsymbol{e}_x\frac{\partial\dot{E}_y}{\partial z}+\boldsymbol{e}_z\frac{\partial\dot{E}_y}{\partial x}\right)$$

$$=-\boldsymbol{e}_x\frac{k_z E_m}{\omega\mu}\sin k_x x e^{-jk_z z}+\boldsymbol{e}_z\frac{jk_x E_m}{\omega\mu}\cos k_x x e^{-jk_z z}$$

$$\boldsymbol{H}(x,z,t)=\mathrm{Re}\left[\dot{\boldsymbol{H}}(x,z)e^{j\omega t}\right]$$

$$=\mathrm{Re}\left[-\boldsymbol{e_x}\frac{k_z E_m}{\omega\mu}\sin k_x x e^{-jk_z z}e^{j\omega t}+\boldsymbol{e_z}\frac{jk_x E_m}{\omega\mu}\cos k_x x e^{-jk_z z}e^{j\omega t}\right]$$

$$=-\boldsymbol{e_x}\frac{k_z E_m}{\omega\mu}\sin k_x x\cos(\omega t-k_z z)-\boldsymbol{e_z}\frac{k_x E_m}{\omega\mu}\cos k_x x\sin(\omega t-k_z z)$$

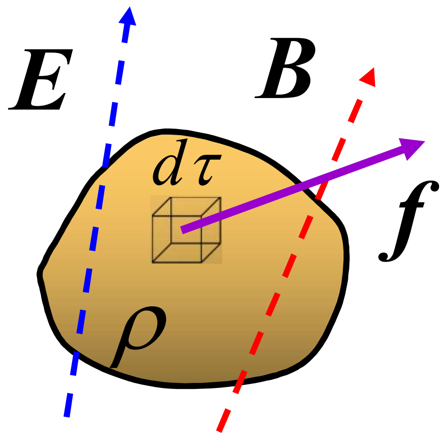
2.4 洛仑兹力

库仑定律：$d\boldsymbol{F} = Q\boldsymbol{E} = \rho\boldsymbol{E}d\tau$

安培定律：$\mathrm{d}\boldsymbol{F} = \boldsymbol{j}\times\boldsymbol{B}\,\mathrm{d}\tau$

洛仑兹力密度：$\boxed{\boldsymbol{f} = \rho\boldsymbol{E} + \boldsymbol{j}\times\boldsymbol{B}}$

洛仑兹力公式：$\boxed{\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v}\times\boldsymbol{B}}$



- 磁力只改变电子运动方向，不改变其动能。
- 电力远大于磁力。

2.5 电磁场的边值关系

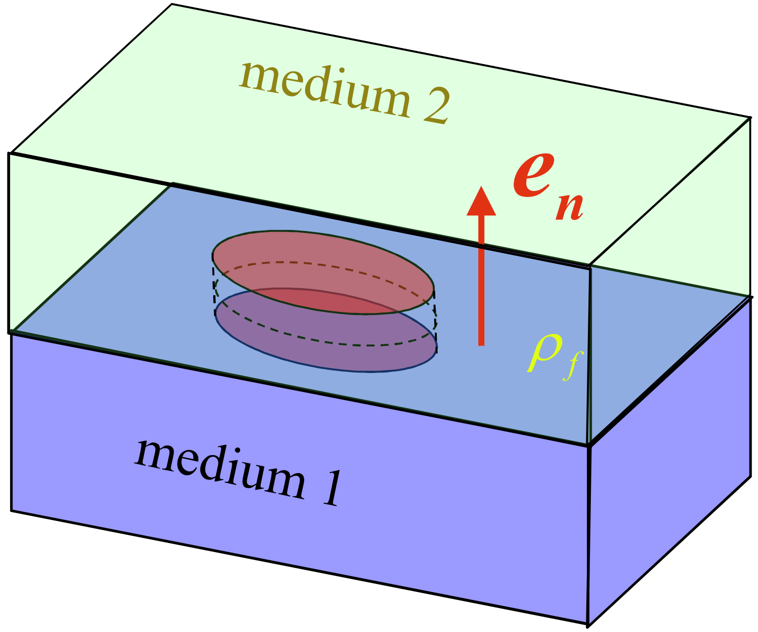
$\boldsymbol{B}$与$\boldsymbol{D}$的法向分量在介质界面上的行为与电磁学中静态场边值关系一样。

$$\oint \boldsymbol{D} \bullet d\boldsymbol{s} = Q$$

$$\oint \boldsymbol{B} \bullet d\boldsymbol{s} = 0$$

$$\boldsymbol{e}_n \cdot \left(\boldsymbol{D}_2 - \boldsymbol{D}_1 \right) = \rho_f$$

$$\boldsymbol{B}_{1n} = \boldsymbol{B}_{2n}$$



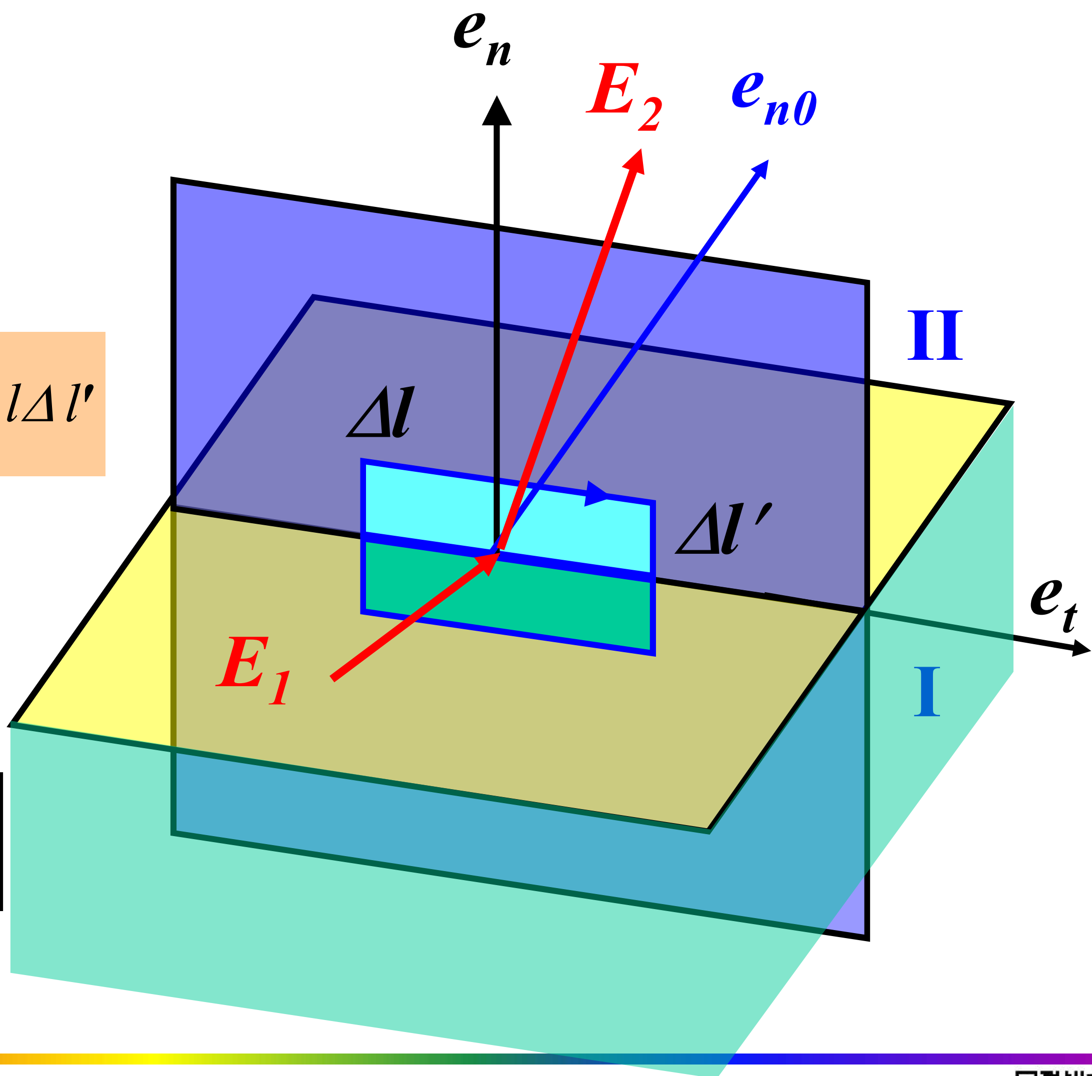
2.5 电磁场的边值关系

$$\oint_l \boldsymbol{E}\bullet \mathrm{d}\boldsymbol{l} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t}\bullet \mathrm{d}\boldsymbol{s}$$

$$(\boldsymbol{E}_2 - \boldsymbol{E}_1)\bullet \boldsymbol{e}_t \,\Delta l = -\frac{\partial \boldsymbol{B}}{\partial t}\bullet \boldsymbol{e}_{n_0}\,\Delta l \Delta l'$$

$$\boxed{(\boldsymbol{E}_2 - \boldsymbol{E}_1)\bullet \boldsymbol{e}_t = 0}$$

$$\boxed{\boldsymbol{e}_n \times (\boldsymbol{E}_2 - \boldsymbol{E}_1) = 0}$$



$$\oint_l \boldsymbol{H}\bullet \mathrm{d}\boldsymbol{l} = \int_S (\boldsymbol{j}_f + \frac{\partial \boldsymbol{D}}{\partial t})\bullet \mathrm{d}\boldsymbol{s}$$

$$(\boldsymbol{H}_2 - \boldsymbol{H}_1)\bullet \boldsymbol{e}_t \,\Delta l = (\boldsymbol{j}_f + \frac{\partial \boldsymbol{D}}{\partial t})\bullet \boldsymbol{e}_{n_0}\,\Delta l \Delta l'$$

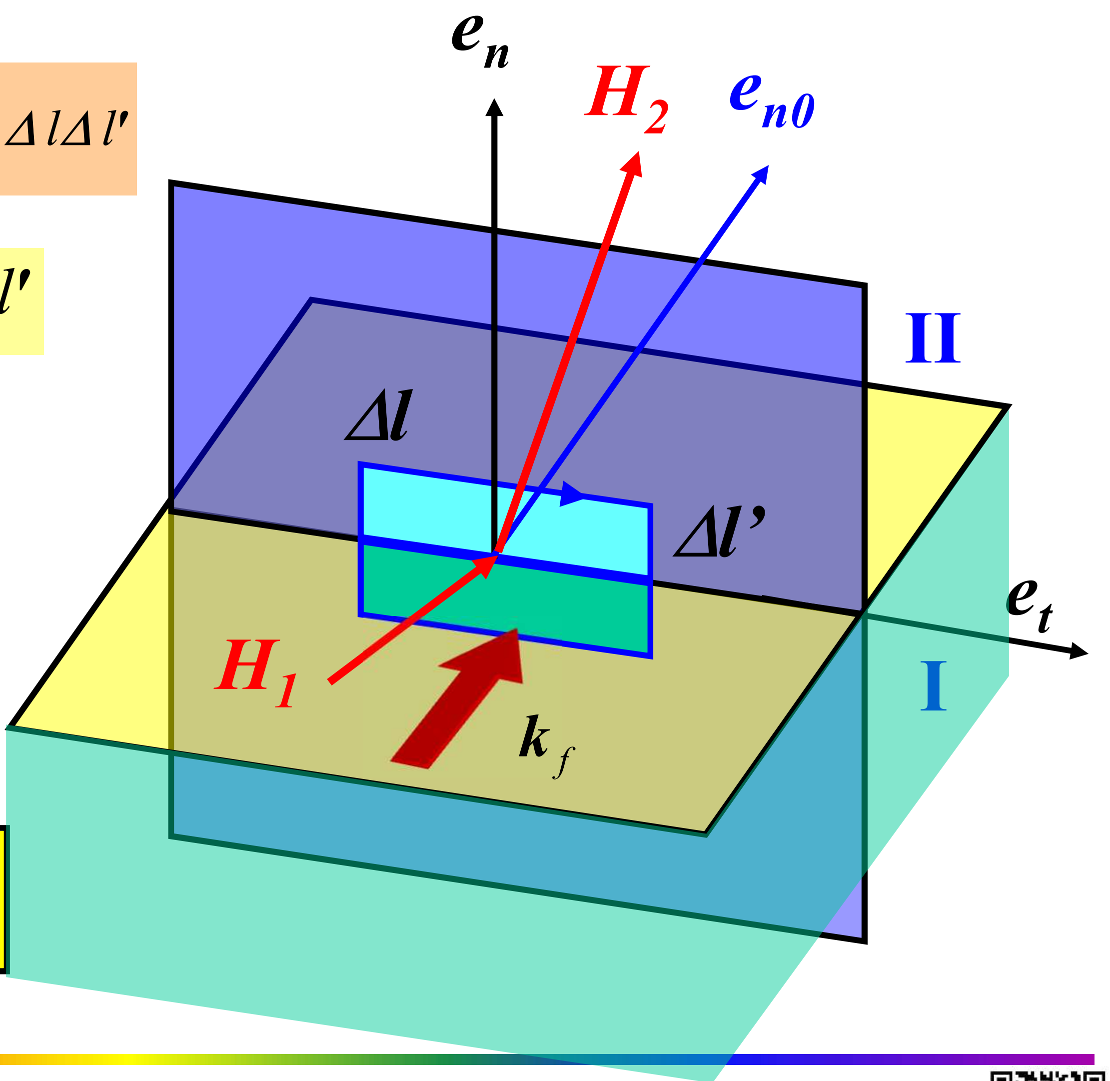
$$(\boldsymbol{H}_2 - \boldsymbol{H}_1)\bullet \boldsymbol{e}_t = \boldsymbol{j}_f \bullet \boldsymbol{e}_{n_0}\,\Delta l'$$

在稳恒或时变场下，都可能出现自由电流面密度：

$$\boldsymbol{k}_f = \boldsymbol{j}_f \,\Delta l'$$

$\Delta l' \to 0$, $\boldsymbol{k}_f$ 为有限值

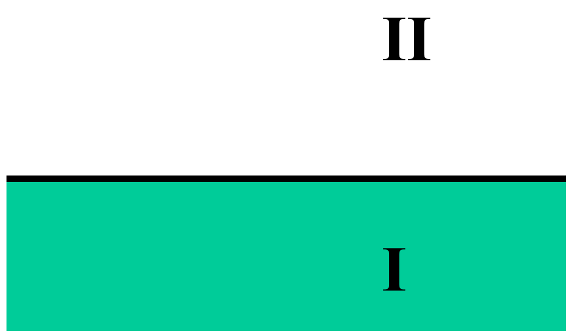
$$\boxed{\boldsymbol{e}_n \times (\boldsymbol{H}_2 - \boldsymbol{H}_1) = \boldsymbol{k}_f}$$



时变电磁场的边界条件：

$$
\begin{cases}
\boldsymbol{e}_n \times (\boldsymbol{E}_2 - \boldsymbol{E}_1) = 0 \\
\boldsymbol{e}_n \times (\boldsymbol{H}_2 - \boldsymbol{H}_1) = \boldsymbol{k}_f \\
\boldsymbol{e}_n \bullet (\boldsymbol{D}_2 - \boldsymbol{D}_1) = \sigma_f \\
\boldsymbol{e}_n \bullet (\boldsymbol{B}_2 - \boldsymbol{B}_1) = 0
\end{cases}
$$

理想导体（I）和介质（II）的边界条件：



$$
\begin{cases}
\boldsymbol{e}_n \times \boldsymbol{E}_2 = 0 \\
\boldsymbol{e}_n \times \boldsymbol{H}_2 = \boldsymbol{k}_f \\
\boldsymbol{e}_n \bullet \boldsymbol{D}_2 = \sigma_f \\
\boldsymbol{e}_n \bullet \boldsymbol{B}_2 = 0
\end{cases}
$$

电磁场的唯一性定理：

当给定区域内的源，并且整个边界面上的切向电场和切向磁场均已给定，此区域内的电磁场解是唯一确定的。

满足麦克斯韦方程和边界条件的电磁场是唯一的！

电磁场理论中的镜像法

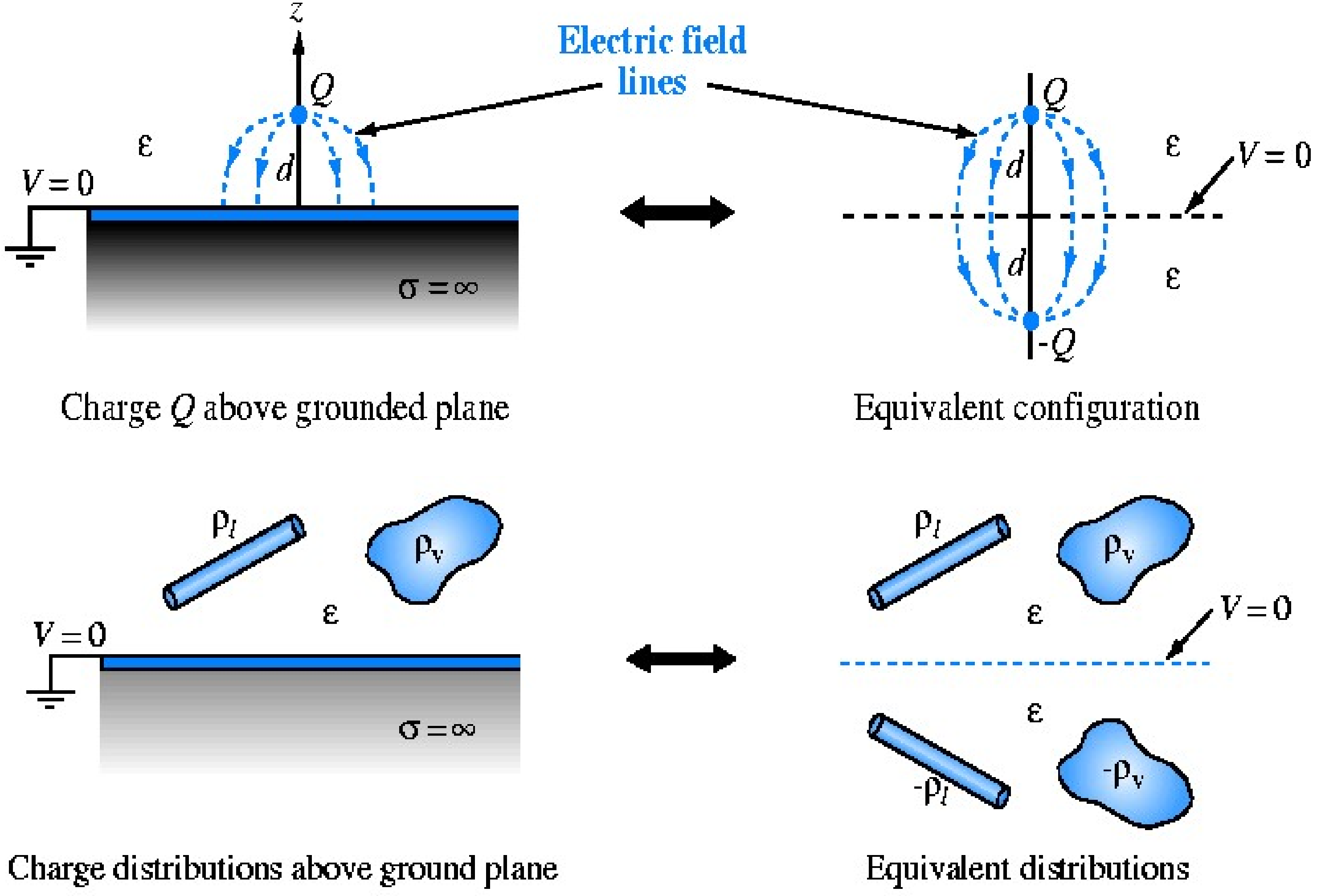


Figure 4-27

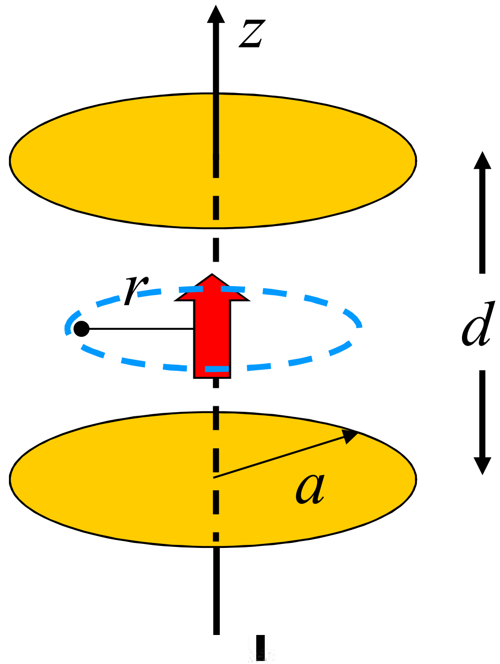
例：漏电的圆盘电容器，漏电导率为σ，介电常数为ε，磁导率为μ_0。圆盘面积足够大可以忽略边缘效应。电容电压为 $U = U_0 cos\omega t$，求电容器中任意点的磁场强度$\boldsymbol{H}$。

传导电流 $\boldsymbol{J} = \sigma \boldsymbol{E}$

位移电流 $\boldsymbol{J}_d = \dfrac{\partial \boldsymbol{D}}{\partial t} = \varepsilon \dfrac{\partial \boldsymbol{E}}{\partial t}$

$$\boldsymbol{E} = \boldsymbol{e}_z \frac{U}{d} = \boldsymbol{e}_z \frac{U_0}{d} \cos \omega t$$

$$\boldsymbol{J} + \boldsymbol{J}_d = \boldsymbol{e}_z \left(\frac{\sigma U_0}{d} \cos \omega t - \frac{\varepsilon \omega U_0}{d} \sin \omega t \right)$$



安培环路定律：$\boldsymbol{H} = \boldsymbol{e}_\phi \dfrac{\pi r^2}{2\pi r}(J + J_d) = \boldsymbol{e}_\phi \dfrac{rU_0}{2d}(\sigma \cos \omega t - \varepsilon \omega \sin \omega t)$

$$\nabla\times\boldsymbol{H}=\left(\boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t}\right)=\boldsymbol{e}_z\left(J+J_d\right)$$

$$\boldsymbol{e}_z\left(\frac{1}{r}\frac{\partial\left(rH_\phi\right)}{\partial r}-\frac{1}{r}\frac{\partial H_r}{\partial\phi}\right)$$

$$\frac{1}{r}\frac{\partial\left(rH_\phi\right)}{\partial r}=J+J_d$$

$$rH_\phi=\int_0^r r\left(J+J_d\right)dr=\frac{1}{2}r^2\left(J+J_d\right)$$

$$H_\phi=\frac{1}{2}r\frac{U_0}{d}\left(\sigma\cos\omega t-\omega\varepsilon\sin\omega t\right)$$