### hw1

#### February 25, 2025

## 1 My Github repo:

https://github.com/fantasybarry/MSFT-Prediction/blob/main/hw1.ipynb

```
[124]: | !pip install yfinance # uncomment these to install missing packages if they are...
        ⇔not already installed
       !pip install pandas
      10292.33s - pydevd: Sending message related to process being replaced timed-out
      after 5 seconds
      Requirement already satisfied: yfinance in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (0.2.54)
      Requirement already satisfied: pandas>=1.3.0 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      (2.2.3)
      Requirement already satisfied: numpy>=1.16.5 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      Requirement already satisfied: requests>=2.31 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      Requirement already satisfied: multitasking>=0.0.7 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      Requirement already satisfied: platformdirs>=2.0.0 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      Requirement already satisfied: pytz>=2022.5 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      Requirement already satisfied: frozendict>=2.3.4 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      Requirement already satisfied: peewee>=3.16.2 in
      /home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)
      (3.17.9)
      Requirement already satisfied: beautifulsoup4>=4.11.1 in
```

/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from yfinance)

```
(4.13.3)
Requirement already satisfied: soupsieve>1.2 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
beautifulsoup4>=4.11.1->yfinance) (2.5)
Requirement already satisfied: typing-extensions>=4.0.0 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
beautifulsoup4>=4.11.1->yfinance) (4.12.2)
Requirement already satisfied: python-dateutil>=2.8.2 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
pandas>=1.3.0->yfinance) (2.9.0.post0)
Requirement already satisfied: tzdata>=2022.7 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
pandas>=1.3.0->yfinance) (2025.1)
Requirement already satisfied: charset_normalizer<4,>=2 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
requests>=2.31->yfinance) (3.4.1)
Requirement already satisfied: idna<4,>=2.5 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
requests>=2.31->yfinance) (3.10)
Requirement already satisfied: urllib3<3,>=1.21.1 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
requests>=2.31->yfinance) (2.3.0)
Requirement already satisfied: certifi>=2017.4.17 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from
requests>=2.31->yfinance) (2025.1.31)
Requirement already satisfied: six>=1.5 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from python-
dateutil>=2.8.2->pandas>=1.3.0->yfinance) (1.17.0)
10298.88s - pydevd: Sending message related to process being replaced timed-out
after 5 seconds
Requirement already satisfied: pandas in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (2.2.3)
Requirement already satisfied: numpy>=1.26.0 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from pandas)
(2.2.3)
Requirement already satisfied: python-dateutil>=2.8.2 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from pandas)
(2.9.0.post0)
Requirement already satisfied: pytz>=2020.1 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from pandas)
(2025.1)
Requirement already satisfied: tzdata>=2022.7 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from pandas)
(2025.1)
Requirement already satisfied: six>=1.5 in
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-packages (from python-
dateutil>=2.8.2->pandas) (1.17.0)
```

```
import yfinance as yf
import pandas as pd

def get_price(tick,start='2022-10-01',end=None):
    return yf.Ticker(tick).history(start=start,end=end)['Close']

def get_prices(tickers,start='2022-10-01',end=None):
    df=pd.DataFrame()
    for s in tickers:
        df[s]=get_price(s,start,end)
    return df
```

## 2 Prepare training and testing data sets

```
[43]: feature_stocks=['tsla','meta','goog','amzn','nflx','gbtc','gdx','intc','dal','c']
    predict_stock='msft'

# training set
    start_date_train='2023-1-01'
    end_date_train='2024-6-30'
    X_train = get_prices(feature_stocks, start_date_train, end = end_date_train)
    y_train=get_prices([predict_stock],start=start_date_train,end=end_date_train)

# testing set
    start_date_test='2024-11-01'
    end_date_test='2024-12-31'
    X_test=get_prices(feature_stocks,start=start_date_test,end=end_date_test)
    y_test=get_prices([predict_stock],start=start_date_test,end=end_date_test)
```

```
[44]: X_test
```

```
[44]:
                                     tsla
                                                 meta
                                                                        amzn
                                                             goog
     Date
     2024-11-01 00:00:00-04:00
                               248.979996 566.702881 172.454346 197.929993
     2024-11-04 00:00:00-05:00
                               242.839996
                                           560.228088 170.486572 195.779999
     2024-11-05 00:00:00-05:00
                               251.440002 571.968628 171.215759
                                                                  199.500000
     2024-11-06 00:00:00-05:00
                                288.529999 571.588928 178.127914 207.089996
     2024-11-07 00:00:00-05:00
                                296.910004 591.223145 182.073441
                                                                  210.050003
     2024-11-08 00:00:00-05:00
                                321.220001
                                           588.865051 179.656189
                                                                  208.179993
     2024-11-11 00:00:00-05:00
                                350.000000 582.699951 181.763794 206.839996
     2024-11-12 00:00:00-05:00
                                328.489990 584.348633 183.112274 208.910004
     2024-11-13 00:00:00-05:00
                               330.239990 579.532532 180.285477 214.100006
     2024-11-14 00:00:00-05:00
                               311.179993 576.694824 177.149033 211.479996
     2024-11-15 00:00:00-05:00
                                320.720001 553.633423 173.692947 202.610001
     2024-11-18 00:00:00-05:00
                               338.739990 553.953186 176.599655
                                                                  201.699997
     2024-11-19 00:00:00-05:00
                               346.000000 560.637817 179.376495 204.610001
```

```
2024-11-20 00:00:00-05:00
                           342.029999
                                        565.064209
                                                    177.129044
                                                                202.880005
2024-11-21 00:00:00-05:00
                           339.640015
                                        562.636169
                                                    169.048218
                                                                 198.380005
2024-11-22 00:00:00-05:00
                            352.559998
                                        558.689392
                                                    166.381256
                                                                 197.119995
2024-11-25 00:00:00-05:00
                            338.589996
                                        564.654541
                                                    169.237991
                                                                 201.449997
2024-11-26 00:00:00-05:00
                            338.230011
                                        573.077698
                                                    170.426651
                                                                207.860001
2024-11-27 00:00:00-05:00
                           332.890015
                                        568.741272
                                                    170.626434
                                                                205.740005
2024-11-29 00:00:00-05:00
                           345.160004
                                        573.857117
                                                    170.296799
                                                                207.889999
2024-12-02 00:00:00-05:00
                            357.089996
                                        592.352234
                                                    172.783981
                                                                210.710007
2024-12-03 00:00:00-05:00
                           351.420013
                                        613.155457
                                                    172.823944
                                                                 213.440002
2024-12-04 00:00:00-05:00
                            357.929993
                                        613.285339
                                                    175.890457
                                                                 218.160004
2024-12-05 00:00:00-05:00
                            369.489990
                                        608.439209
                                                    174.112473
                                                                 220.550003
2024-12-06 00:00:00-05:00
                            389.220001
                                        623.267273
                                                    176.290009
                                                                227.029999
2024-12-09 00:00:00-05:00
                           389.790009
                                        613.075500
                                                    177.100006
                                                                226.089996
2024-12-10 00:00:00-05:00
                           400.989990
                                        618.820862
                                                    186.529999
                                                                 225.039993
2024-12-11 00:00:00-05:00
                           424.769989
                                        632.170044
                                                    196.710007
                                                                 230.259995
2024-12-12 00:00:00-05:00
                           418.100006
                                        630.281555
                                                    193.630005
                                                                 228.970001
2024-12-13 00:00:00-05:00
                           436.230011
                                        619.849976
                                                    191.380005
                                                                227.460007
2024-12-16 00:00:00-05:00
                                        624.239990
                           463.019989
                                                    198.160004
                                                                 232.929993
2024-12-17 00:00:00-05:00
                           479.859985
                                        619.440002
                                                    197.119995
                                                                 231.149994
2024-12-18 00:00:00-05:00
                           440.130005
                                        597.190002
                                                    190.149994
                                                                 220.520004
2024-12-19 00:00:00-05:00
                           436.170013
                                        595.570007
                                                    189.699997
                                                                 223.289993
2024-12-20 00:00:00-05:00
                           421.059998
                                        585.250000
                                                    192.960007
                                                                224.919998
2024-12-23 00:00:00-05:00
                           430.600006
                                        599.849976
                                                    195.990005
                                                                225.059998
2024-12-24 00:00:00-05:00
                           462.279999
                                        607.750000
                                                    197.570007
                                                                 229.050003
2024-12-26 00:00:00-05:00
                           454.130005
                                        603.349976
                                                    197.100006
                                                                 227.050003
2024-12-27 00:00:00-05:00
                            431.660004
                                        599.809998
                                                    194.039993
                                                                 223.750000
                           417.410004
2024-12-30 00:00:00-05:00
                                        591.239990
                                                    192.690002
                                                                221.300003
                                 nflx
                                                                    intc \
                                             gbtc
                                                         gdx
Date
2024-11-01 00:00:00-04:00
                           756.099976
                                                               23.200001
                                        55.009998
                                                   39.387589
2024-11-04 00:00:00-05:00
                           755.510010
                                        53.490002
                                                   39.437012
                                                               22.520000
2024-11-05 00:00:00-05:00
                            763.909973
                                        55.169998
                                                   39.644573
                                                               23.320000
2024-11-06 00:00:00-05:00
                           780.210022
                                        60.599998
                                                   38.191631
                                                               25.049999
2024-11-07 00:00:00-05:00
                           796.539978
                                        60.880001
                                                   39.110840
                                                               26.230000
2024-11-08 00:00:00-05:00
                           795.039978
                                        61.049999
                                                   38.567223
                                                               26.200001
                                                   36.303795
2024-11-11 00:00:00-05:00
                           805.440002
                                        69.220001
                                                               25.049999
2024-11-12 00:00:00-05:00
                           819.500000
                                        71.230003
                                                   35.690987
                                                               24.160000
2024-11-13 00:00:00-05:00
                           830.469971
                                        71.309998
                                                   35.147369
                                                               24.920000
2024-11-14 00:00:00-05:00
                           837.260010
                                        69.500000
                                                   35.305515
                                                               25.030001
2024-11-15 00:00:00-05:00
                           823.960022
                                        72.809998
                                                   35.097950
                                                               24.350000
2024-11-18 00:00:00-05:00
                           847.049988
                                        72.769997
                                                   36.590427
                                                               24.840000
2024-11-19 00:00:00-05:00
                           871.320007
                                        73.580002
                                                   37.440449
                                                               24.200001
2024-11-20 00:00:00-05:00
                           883.849976
                                        74.989998
                                                   37.282307
                                                               24.010000
2024-11-21 00:00:00-05:00
                           897.479980
                                        78.050003
                                                   37.697430
                                                               24.440001
2024-11-22 00:00:00-05:00
                           897.789978
                                        78.870003
                                                   37.835808
                                                               24.500000
2024-11-25 00:00:00-05:00
                           865.590027
                                        75.419998
                                                   36.709034
                                                               24.870001
```

2024-11-26 00:00:00-05:00 872.599976 72.169998 36.857296 24.049999 23.650000 2024-11-27 00:00:00-05:00 877.340027 76.820000 37.015438 2024-11-29 00:00:00-05:00 886.809998 77.089996 37.223003 24.049999 2024-12-02 00:00:00-05:00 897.739990 76.029999 36.402634 23.930000 2024-12-03 00:00:00-05:00 75.949997 37.262539 22.469999 902.169983 2024-12-04 00:00:00-05:00 911.059998 78.690002 37.163696 21.959999 917.869995 2024-12-05 00:00:00-05:00 78.690002 37.094509 20.799999 2024-12-06 00:00:00-05:00 934.739990 80.699997 36.392750 20.920000 76.330002 2024-12-09 00:00:00-05:00 913.690002 37.470100 20.809999 2024-12-10 00:00:00-05:00 913.349976 76.559998 37.539288 20.160000 2024-12-11 00:00:00-05:00 936.559998 80.510002 38.636410 20.120001 2024-12-12 00:00:00-05:00 925.549988 79.410004 37.223003 20.780001 2024-12-13 00:00:00-05:00 918.869995 80.769997 36.224724 20.340000 921.080017 84.019997 2024-12-16 00:00:00-05:00 35.997387 20.830000 2024-12-17 00:00:00-05:00 919.130005 84.709999 35.770058 20.440001 2024-12-18 00:00:00-05:00 889.549988 79.809998 34.129322 19.299999 2024-12-19 00:00:00-05:00 902.039978 76.320000 33.990944 19.059999 2024-12-20 00:00:00-05:00 76.470001 34.327000 909.049988 19.520000 2024-12-23 00:00:00-05:00 911.450012 73.709999 34.410000 20.200001 2024-12-24 00:00:00-05:00 78.449997 34.410000 20.400000 932.119995 2024-12-26 00:00:00-05:00 924.140015 75.760002 34.470001 20.440001 2024-12-27 00:00:00-05:00 907.549988 74.879997 34.259998 20.299999 2024-12-30 00:00:00-05:00 900.429993 74.650002 19.820000 33.770000

dal c

#### Date 58.389999 2024-11-01 00:00:00-04:00 62.715710 2024-11-04 00:00:00-05:00 56.889999 61.921215 2024-11-05 00:00:00-05:00 58.290001 63.232136 2024-11-06 00:00:00-05:00 62.320000 68.555275 2024-11-07 00:00:00-05:00 60.430000 67.641602 2024-11-08 00:00:00-05:00 61.049999 68.158028 2024-11-11 00:00:00-05:00 63.560001 69.319984 2024-11-12 00:00:00-05:00 64.050003 68.545341 2024-11-13 00:00:00-05:00 64.459999 68.416237 2024-11-14 00:00:00-05:00 64.849998 67.681328 2024-11-15 00:00:00-05:00 64.070000 68.287132 2024-11-18 00:00:00-05:00 63.240002 68.525482 2024-11-19 00:00:00-05:00 64.750000 68.128235 2024-11-20 00:00:00-05:00 63.639999 67.810432 2024-11-21 00:00:00-05:00 63.340000 68.475822 2024-11-22 00:00:00-05:00 63.340000 69.359703 2024-11-25 00:00:00-05:00 64.489998 70.263451 2024-11-26 00:00:00-05:00 64.139999 69.270325 2024-11-27 00:00:00-05:00 63.619999 69.677505 2024-11-29 00:00:00-05:00 63.820000 70.382622 2024-12-02 00:00:00-05:00 63.410000 70.899048

```
2024-12-03 00:00:00-05:00
                                 62.570000
                                            70.928841
      2024-12-04 00:00:00-05:00
                                 64.260002
                                            71.008293
      2024-12-05 00:00:00-05:00
                                 65.769997
                                            71.733276
      2024-12-06 00:00:00-05:00
                                 64.529999
                                            71.653824
      2024-12-09 00:00:00-05:00
                                 62.259998
                                            71.365814
      2024-12-10 00:00:00-05:00
                                 62.770000
                                            72.001411
      2024-12-11 00:00:00-05:00
                                 63.480000
                                            71.465126
      2024-12-12 00:00:00-05:00
                                 61.630001
                                            70.938774
      2024-12-13 00:00:00-05:00
                                 61.520000
                                            70.521660
      2024-12-16 00:00:00-05:00
                                 61.049999
                                            70.998360
      2024-12-17 00:00:00-05:00
                                 60.810001
                                            70.630905
      2024-12-18 00:00:00-05:00
                                 58.880001
                                            67.651535
      2024-12-19 00:00:00-05:00
                                 60.380001
                                            67.949471
                                 60.930000
      2024-12-20 00:00:00-05:00
                                            68.714180
      2024-12-23 00:00:00-05:00
                                 61.520000
                                            69.290184
      2024-12-24 00:00:00-05:00
                                 62.560001
                                            70.511726
      2024-12-26 00:00:00-05:00
                                 62.400002
                                            70.859322
      2024-12-27 00:00:00-05:00
                                 61.259998
                                            70.511726
      2024-12-30 00:00:00-05:00
                                 60.720001
                                            69.905922
[45]: y_train
[45]:
                                       msft
      Date
      2023-01-03 00:00:00-05:00
                                 235.240005
      2023-01-04 00:00:00-05:00
                                 224.949875
      2023-01-05 00:00:00-05:00
                                 218.282852
      2023-01-06 00:00:00-05:00
                                 220.855377
      2023-01-09 00:00:00-05:00
                                 223.005737
      2024-06-24 00:00:00-04:00
                                 445.079468
      2024-06-25 00:00:00-04:00
                                 448.340454
      2024-06-26 00:00:00-04:00
                                 449.543457
      2024-06-27 00:00:00-04:00
                                 450.229492
      2024-06-28 00:00:00-04:00
                                 444.363647
      [374 rows x 1 columns]
```

## 3 Convert training and testing data into numpy array

```
[46]: import numpy as np

X_train=np.array(X_train)
y_train=np.array(y_train)
X_test=np.array(X_test)
y_test=np.array(y_test)
```

# 4 Use linear regression to predict msft stock price from the other stocks' prices

## 4.1 1. Append a dummy feature to both X\_train and X\_test

	tsla	meta	goog	g amzn	\
Date					
2023-01-03 00:00:00-05:00	108.099998	124.265312	89.378845	85.820000	
2023-01-04 00:00:00-05:00	113.639999	126.885315	88.392403	85.139999	
2023-01-05 00:00:00-05:00	110.339996	126.456947	86.459343	83.120003	
2023-01-06 00:00:00-05:00	113.059998	129.525223	87.844376	86.080002	
2023-01-09 00:00:00-05:00	119.769997	128.977310	88.482079	87.360001	
	•••	•••	•••	•••	
2024-06-24 00:00:00-04:00	182.580002	498.032776	180.347717	185.570007	
2024-06-25 00:00:00-04:00	187.350006	509.702240	185.126007	186.339996	
2024-06-26 00:00:00-04:00	196.369995	512.217773	184.916504	193.610001	
2024-06-27 00:00:00-04:00	197.419998	518.646423	186.402878	3 197.850006	
2024-06-28 00:00:00-04:00	197.880005	503.333435	182.971283	3 193.250000	
	nflx	gbtc	gdx	<pre>intc \</pre>	
Date	nflx	gbtc	gdx	intc \	
Date 2023-01-03 00:00:00-05:00	nflx 294.950012	gbtc 8.200000	gdx 28.842234	intc \ 25.775146	
		-		,	
2023-01-03 00:00:00-05:00	294.950012	8.200000	28.842234	25.775146	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00	294.950012 309.410004	8.200000 8.380000	28.842234 30.067492	25.775146 26.691208	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00 2023-01-05 00:00:00-05:00	294.950012 309.410004 309.700012	8.200000 8.380000 8.450000	28.842234 30.067492 29.804935	25.775146 26.691208 26.575497	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00 2023-01-05 00:00:00-05:00 2023-01-06 00:00:00-05:00	294.950012 309.410004 309.700012 315.549988	8.200000 8.380000 8.450000 8.650000	28.842234 30.067492 29.804935 30.689848	25.775146 26.691208 26.575497 27.703701	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00 2023-01-05 00:00:00-05:00 2023-01-06 00:00:00-05:00	294.950012 309.410004 309.700012 315.549988	8.200000 8.380000 8.450000 8.650000	28.842234 30.067492 29.804935 30.689848	25.775146 26.691208 26.575497 27.703701	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00 2023-01-05 00:00:00-05:00 2023-01-06 00:00:00-05:00 2023-01-09 00:00:00-05:00 	294.950012 309.410004 309.700012 315.549988 315.170013 	8.200000 8.380000 8.450000 8.650000 9.650000	28.842234 30.067492 29.804935 30.689848 30.398119 	25.775146 26.691208 26.575497 27.703701 28.262980	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00 2023-01-05 00:00:00-05:00 2023-01-06 00:00:00-05:00 2023-01-09 00:00:00-05:00  2024-06-24 00:00:00-04:00	294.950012 309.410004 309.700012 315.549988 315.170013  669.020020	8.200000 8.380000 8.450000 8.650000 9.650000  52.610001	28.842234 30.067492 29.804935 30.689848 30.398119  33.852570	25.775146 26.691208 26.575497 27.703701 28.262980 30.377298	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00 2023-01-05 00:00:00-05:00 2023-01-06 00:00:00-05:00 2023-01-09 00:00:00-05:00  2024-06-24 00:00:00-04:00 2024-06-25 00:00:00-04:00	294.950012 309.410004 309.700012 315.549988 315.170013  669.020020 672.409973	8.200000 8.380000 8.450000 8.650000 9.650000  52.610001 55.020000	28.842234 30.067492 29.804935 30.689848 30.398119  33.852570 33.447327	25.775146 26.691208 26.575497 27.703701 28.262980 30.377298 30.546227	
2023-01-03 00:00:00-05:00 2023-01-04 00:00:00-05:00 2023-01-05 00:00:00-05:00 2023-01-06 00:00:00-05:00 2023-01-09 00:00:00-05:00  2024-06-24 00:00:00-04:00 2024-06-25 00:00:00-04:00 2024-06-26 00:00:00-04:00	294.950012 309.410004 309.700012 315.549988 315.170013  669.020020 672.409973 677.690002	8.200000 8.380000 8.450000 9.650000  52.610001 55.020000 54.130001	28.842234 30.067492 29.804935 30.689848 30.398119  33.852570 33.447327 33.427559	25.775146 26.691208 26.575497 27.703701 28.262980 30.377298 30.546227 30.347488	

```
dal
                                               c tsla_dummy meta_dummy \
Date
2023-01-03 00:00:00-05:00
                           32.105251 41.898052
                                                            0
                                                                         0
2023-01-04 00:00:00-05:00
                            33.857697 42.977993
                                                            0
                                                                         0
2023-01-05 00:00:00-05:00
                            34.684692 42.785805
                                                            0
                                                                         0
2023-01-06 00:00:00-05:00
                            35.472309 43.298321
                                                            0
                                                                         0
2023-01-09 00:00:00-05:00
                            36.200859 43.508816
                                                                         0
2024-06-24 00:00:00-04:00
                            49.083549
                                       59.807236
                                                            0
                                                                         1
2024-06-25 00:00:00-04:00
                            48.497330 60.041241
                                                            0
                                                                         1
2024-06-26 00:00:00-04:00
                            47.871361 59.719482
                                                            0
                                                                         1
2024-06-27 00:00:00-04:00
                            48.288670 60.011986
                                                            0
                                                                         1
2024-06-28 00:00:00-04:00
                           47.136101 61.874260
                                                            0
                                                                         1
                            goog_dummy amzn_dummy nflx_dummy gbtc_dummy \
Date
2023-01-03 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                          0
2023-01-04 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                          0
2023-01-05 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                          0
2023-01-06 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                          0
2023-01-09 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                          0
2024-06-24 00:00:00-04:00
                                     1
                                                  1
                                                              1
                                                                          1
2024-06-25 00:00:00-04:00
                                     1
                                                  1
                                                              1
                                                                          1
2024-06-26 00:00:00-04:00
                                     1
                                                  1
                                                              1
                                                                          1
2024-06-27 00:00:00-04:00
                                     1
                                                  1
                                                              1
                                                                           1
2024-06-28 00:00:00-04:00
                                     1
                                                  1
                                                              1
                                                                           1
                            gdx_dummy intc_dummy dal_dummy c_dummy
Date
2023-01-03 00:00:00-05:00
                                    0
                                                 0
                                                            0
                                                                     0
2023-01-04 00:00:00-05:00
                                    1
                                                 0
                                                            0
                                                                     0
2023-01-05 00:00:00-05:00
                                    0
                                                0
                                                            0
                                                                     0
2023-01-06 00:00:00-05:00
                                    1
                                                0
                                                            0
                                                                     0
2023-01-09 00:00:00-05:00
                                    1
                                                 0
                                                            0
                                                                     0
2024-06-24 00:00:00-04:00
                                    1
                                                 0
                                                            1
                                                                     1
2024-06-25 00:00:00-04:00
                                                0
                                    1
                                                            1
                                                                     1
2024-06-26 00:00:00-04:00
                                    1
                                                 0
                                                            1
                                                                     1
2024-06-27 00:00:00-04:00
                                    1
                                                 0
                                                            1
                                                                     1
2024-06-28 00:00:00-04:00
                                    1
                                                 0
                                                            1
                                                                     1
[374 rows x 20 columns]
```

[48]: # Dummy Feature for X\_test # Load the dataset

		tsla	meta	goog	amzn	١
Date						
2024-11-01	00:00:00-04:00	248.979996	566.702881	172.454346	197.929993	
2024-11-04	00:00:00-05:00	242.839996	560.228088	170.486572	195.779999	
2024-11-05	00:00:00-05:00	251.440002	571.968628	171.215759	199.500000	
2024-11-06	00:00:00-05:00	288.529999	571.588928	178.127914	207.089996	
2024-11-07	00:00:00-05:00	296.910004	591.223145	182.073441	210.050003	
2024-11-08	00:00:00-05:00	321.220001	588.865051	179.656189	208.179993	
2024-11-11	00:00:00-05:00	350.000000	582.699951	181.763794	206.839996	
2024-11-12	00:00:00-05:00	328.489990	584.348633	183.112274	208.910004	
2024-11-13	00:00:00-05:00	330.239990	579.532532	180.285477	214.100006	
2024-11-14	00:00:00-05:00	311.179993	576.694824	177.149033	211.479996	
2024-11-15	00:00:00-05:00	320.720001	553.633423	173.692947	202.610001	
2024-11-18	00:00:00-05:00	338.739990	553.953186	176.599655	201.699997	
2024-11-19	00:00:00-05:00	346.000000	560.637817	179.376495	204.610001	
2024-11-20	00:00:00-05:00	342.029999	565.064209	177.129044	202.880005	
2024-11-21	00:00:00-05:00	339.640015	562.636169	169.048218	198.380005	
2024-11-22	00:00:00-05:00	352.559998	558.689392	166.381256	197.119995	
2024-11-25	00:00:00-05:00	338.589996	564.654541	169.237991	201.449997	
2024-11-26	00:00:00-05:00	338.230011	573.077698	170.426651	207.860001	
2024-11-27	00:00:00-05:00	332.890015	568.741272	170.626434	205.740005	
2024-11-29	00:00:00-05:00	345.160004	573.857117	170.296799	207.889999	
2024-12-02	00:00:00-05:00	357.089996	592.352234	172.783981	210.710007	
2024-12-03	00:00:00-05:00	351.420013	613.155457	172.823944	213.440002	
2024-12-04	00:00:00-05:00	357.929993	613.285339	175.890457	218.160004	
2024-12-05	00:00:00-05:00	369.489990	608.439209	174.112473	220.550003	
2024-12-06	00:00:00-05:00	389.220001	623.267273	176.290009	227.029999	
2024-12-09	00:00:00-05:00	389.790009	613.075500	177.100006	226.089996	
2024-12-10	00:00:00-05:00	400.989990	618.820862	186.529999	225.039993	
2024-12-11	00:00:00-05:00	424.769989	632.170044	196.710007	230.259995	
2024-12-12	00:00:00-05:00	418.100006	630.281555	193.630005	228.970001	
2024-12-13	00:00:00-05:00	436.230011	619.849976	191.380005	227.460007	
2024-12-16	00:00:00-05:00	463.019989	624.239990	198.160004	232.929993	
2024-12-17	00:00:00-05:00	479.859985	619.440002	197.119995	231.149994	
2024-12-18	00:00:00-05:00	440.130005	597.190002	190.149994	220.520004	

```
2024-12-19 00:00:00-05:00
                            436.170013
                                        595.570007
                                                     189.699997
                                                                 223,289993
2024-12-20 00:00:00-05:00
                            421.059998
                                        585.250000
                                                    192.960007
                                                                 224.919998
2024-12-23 00:00:00-05:00
                            430.600006
                                        599.849976
                                                     195.990005
                                                                 225.059998
2024-12-24 00:00:00-05:00
                            462.279999
                                        607.750000
                                                     197.570007
                                                                 229.050003
2024-12-26 00:00:00-05:00
                            454.130005
                                        603.349976
                                                     197.100006
                                                                 227.050003
2024-12-27 00:00:00-05:00
                            431.660004
                                        599.809998
                                                     194.039993
                                                                 223.750000
2024-12-30 00:00:00-05:00
                            417.410004
                                        591.239990
                                                     192.690002
                                                                 221.300003
                                                                    intc \
                                  nflx
                                             gbtc
                                                          gdx
Date
2024-11-01 00:00:00-04:00
                            756.099976
                                                               23.200001
                                        55.009998
                                                   39.387589
2024-11-04 00:00:00-05:00
                            755.510010
                                        53.490002
                                                   39.437012
                                                               22.520000
2024-11-05 00:00:00-05:00
                                        55.169998
                            763.909973
                                                   39.644573
                                                               23.320000
2024-11-06 00:00:00-05:00
                            780.210022
                                        60.599998
                                                   38.191631
                                                               25.049999
2024-11-07 00:00:00-05:00
                            796.539978
                                        60.880001
                                                   39.110840
                                                               26.230000
2024-11-08 00:00:00-05:00
                            795.039978
                                        61.049999
                                                   38.567223
                                                               26.200001
2024-11-11 00:00:00-05:00
                            805.440002
                                        69.220001
                                                   36.303795
                                                               25.049999
2024-11-12 00:00:00-05:00
                                        71.230003
                            819.500000
                                                   35.690987
                                                               24.160000
2024-11-13 00:00:00-05:00
                            830.469971
                                        71.309998
                                                   35.147369
                                                               24.920000
2024-11-14 00:00:00-05:00
                            837.260010
                                        69.500000
                                                   35.305515
                                                               25.030001
2024-11-15 00:00:00-05:00
                            823.960022
                                        72.809998
                                                   35.097950
                                                               24.350000
2024-11-18 00:00:00-05:00
                            847.049988
                                        72.769997
                                                   36.590427
                                                               24.840000
2024-11-19 00:00:00-05:00
                            871.320007
                                        73.580002
                                                   37.440449
                                                               24.200001
2024-11-20 00:00:00-05:00
                            883.849976
                                        74.989998
                                                   37.282307
                                                               24.010000
2024-11-21 00:00:00-05:00
                            897.479980
                                        78.050003
                                                   37.697430
                                                               24.440001
2024-11-22 00:00:00-05:00
                            897.789978
                                        78.870003
                                                   37.835808
                                                               24.500000
2024-11-25 00:00:00-05:00
                            865.590027
                                        75.419998
                                                   36.709034
                                                               24.870001
                                        72.169998
                                                   36.857296
2024-11-26 00:00:00-05:00
                            872.599976
                                                               24.049999
2024-11-27 00:00:00-05:00
                            877.340027
                                        76.820000
                                                   37.015438
                                                               23.650000
2024-11-29 00:00:00-05:00
                            886.809998
                                        77.089996
                                                   37.223003
                                                               24.049999
2024-12-02 00:00:00-05:00
                            897.739990
                                        76.029999
                                                   36.402634
                                                               23.930000
2024-12-03 00:00:00-05:00
                            902.169983
                                        75.949997
                                                   37.262539
                                                               22.469999
                                        78.690002
2024-12-04 00:00:00-05:00
                            911.059998
                                                   37.163696
                                                               21.959999
2024-12-05 00:00:00-05:00
                            917.869995
                                        78.690002
                                                   37.094509
                                                               20.799999
2024-12-06 00:00:00-05:00
                            934.739990
                                        80.699997
                                                   36.392750
                                                               20.920000
2024-12-09 00:00:00-05:00
                            913.690002
                                        76.330002
                                                   37.470100
                                                               20.809999
2024-12-10 00:00:00-05:00
                            913.349976
                                        76.559998
                                                   37.539288
                                                               20.160000
2024-12-11 00:00:00-05:00
                            936.559998
                                        80.510002
                                                   38.636410
                                                               20.120001
2024-12-12 00:00:00-05:00
                            925.549988
                                        79.410004
                                                   37.223003
                                                               20.780001
2024-12-13 00:00:00-05:00
                            918.869995
                                        80.769997
                                                   36.224724
                                                               20.340000
2024-12-16 00:00:00-05:00
                            921.080017
                                        84.019997
                                                   35.997387
                                                               20.830000
2024-12-17 00:00:00-05:00
                                        84.709999
                            919.130005
                                                   35.770058
                                                               20.440001
2024-12-18 00:00:00-05:00
                            889.549988
                                        79.809998
                                                   34.129322
                                                               19.299999
2024-12-19 00:00:00-05:00
                            902.039978
                                        76.320000
                                                   33.990944
                                                               19.059999
2024-12-20 00:00:00-05:00
                            909.049988
                                        76.470001
                                                   34.327000
                                                               19.520000
2024-12-23 00:00:00-05:00
                            911.450012
                                        73.709999
                                                   34.410000
                                                               20.200001
2024-12-24 00:00:00-05:00
                            932.119995
                                        78.449997
                                                   34.410000
                                                               20.400000
2024-12-26 00:00:00-05:00
                            924.140015
                                        75.760002
                                                   34.470001
                                                               20.440001
```

```
2024-12-27 00:00:00-05:00
                            907.549988
                                         74.879997
                                                    34.259998
                                                                20.299999
2024-12-30 00:00:00-05:00
                            900.429993
                                         74.650002
                                                    33.770000
                                                                19.820000
                                  dal
                                                   tsla_dummy
                                                                             \
                                                                meta_dummy
Date
2024-11-01 00:00:00-04:00
                            58.389999
                                                             0
                                        62.715710
                                                                          0
2024-11-04 00:00:00-05:00
                            56.889999
                                        61.921215
                                                             0
                                                                          0
2024-11-05 00:00:00-05:00
                            58.290001
                                        63.232136
                                                             0
                                                                          0
                            62.320000
                                                             0
                                                                          0
2024-11-06 00:00:00-05:00
                                        68.555275
2024-11-07 00:00:00-05:00
                            60.430000
                                        67.641602
                                                             0
                                                                          1
2024-11-08 00:00:00-05:00
                            61.049999
                                        68.158028
                                                             0
                                                                          0
2024-11-11 00:00:00-05:00
                            63.560001
                                                             0
                                                                          0
                                        69.319984
2024-11-12 00:00:00-05:00
                            64.050003
                                                             0
                                                                          0
                                        68.545341
                            64.459999
                                                             0
2024-11-13 00:00:00-05:00
                                        68.416237
                                                                          0
2024-11-14 00:00:00-05:00
                            64.849998
                                        67.681328
                                                             0
                                                                          0
2024-11-15 00:00:00-05:00
                            64.070000
                                                             0
                                                                          0
                                        68.287132
2024-11-18 00:00:00-05:00
                            63.240002
                                        68.525482
                                                             0
                                                                          0
2024-11-19 00:00:00-05:00
                            64.750000
                                        68.128235
                                                             0
                                                                          0
2024-11-20 00:00:00-05:00
                            63.639999
                                        67.810432
                                                             0
                                                                          0
2024-11-21 00:00:00-05:00
                            63.340000
                                        68.475822
                                                             0
                                                                          0
2024-11-22 00:00:00-05:00
                            63.340000
                                        69.359703
                                                             0
                                                                          0
                                                             0
2024-11-25 00:00:00-05:00
                            64.489998
                                        70.263451
                                                                          0
2024-11-26 00:00:00-05:00
                            64.139999
                                        69.270325
                                                             0
                                                                          0
2024-11-27 00:00:00-05:00
                            63.619999
                                        69.677505
                                                             0
                                                                          0
2024-11-29 00:00:00-05:00
                            63.820000
                                        70.382622
                                                             0
                                                                          0
2024-12-02 00:00:00-05:00
                            63.410000
                                        70.899048
                                                             0
                                                                          1
2024-12-03 00:00:00-05:00
                            62.570000
                                                             0
                                        70.928841
                                                                          1
2024-12-04 00:00:00-05:00
                            64.260002
                                       71.008293
                                                             0
                                                                          1
2024-12-05 00:00:00-05:00
                            65.769997
                                        71.733276
                                                             1
                                                                          1
2024-12-06 00:00:00-05:00
                            64.529999
                                        71.653824
                                                             1
                                                                          1
2024-12-09 00:00:00-05:00
                            62.259998
                                        71.365814
                                                             1
                                                                          1
2024-12-10 00:00:00-05:00
                            62.770000
                                       72.001411
                                                             1
                                                                          1
2024-12-11 00:00:00-05:00
                            63.480000
                                        71.465126
                                                             1
                                                                          1
2024-12-12 00:00:00-05:00
                            61.630001
                                       70.938774
                                                             1
                                                                          1
                            61.520000
2024-12-13 00:00:00-05:00
                                        70.521660
                                                             1
                                                                          1
2024-12-16 00:00:00-05:00
                            61.049999
                                        70.998360
                                                             1
                                                                          1
2024-12-17 00:00:00-05:00
                            60.810001
                                        70.630905
                                                             1
                                                                          1
2024-12-18 00:00:00-05:00
                            58.880001
                                        67.651535
                                                             1
                                                                          1
                            60.380001
2024-12-19 00:00:00-05:00
                                        67.949471
                                                             1
                                                                          1
2024-12-20 00:00:00-05:00
                            60.930000
                                        68.714180
                                                             1
                                                                          0
2024-12-23 00:00:00-05:00
                            61.520000
                                                             1
                                                                          1
                                        69.290184
2024-12-24 00:00:00-05:00
                            62.560001
                                                             1
                                                                          1
                                        70.511726
2024-12-26 00:00:00-05:00
                            62.400002
                                        70.859322
                                                             1
                                                                          1
2024-12-27 00:00:00-05:00
                            61.259998
                                        70.511726
                                                             1
                                                                          1
2024-12-30 00:00:00-05:00
                            60.720001
                                        69.905922
                                                             1
                                                                          1
                                         amzn_dummy nflx_dummy
                                                                  gbtc_dummy
```

Date

goog\_dummy

2024-11-01	00:00:00-04:00	0	0		0	0
2024-11-04	00:00:00-05:00	0	0		0	0
2024-11-05	00:00:00-05:00	0	0		0	0
2024-11-06	00:00:00-05:00	0	0		0	0
2024-11-07	00:00:00-05:00	1	0		0	0
2024-11-08	00:00:00-05:00	0	0		0	0
2024-11-11	00:00:00-05:00	1	0		0	0
2024-11-12	00:00:00-05:00	1	0		0	0
2024-11-13	00:00:00-05:00	0	0		0	0
2024-11-14	00:00:00-05:00	0	0		0	0
2024-11-15	00:00:00-05:00	0	0		0	0
2024-11-18	00:00:00-05:00	0	0		0	0
2024-11-19	00:00:00-05:00	0	0		0	1
	00:00:00-05:00	0	0		1	1
	00:00:00-05:00	0	0		1	1
	00:00:00-05:00	0	0		1	1
	00:00:00-05:00	0	0		0	1
	00:00:00-05:00	0	0		0	0
	00:00:00-05:00	0	0		1	1
	00:00:00-05:00	0	0		1	1
	00:00:00-05:00	0	0		1	1
	00:00:00-05:00	0	0		1	1
	00:00:00-05:00	0	1		1	1
	00:00:00-05:00	0	1		1	1
		_	_		_	_
	00:00:00-05:00	0	1		1	1
	00:00:00-05:00	0	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
	00:00:00-05:00	1	1		1	1
2024-12-30	00:00:00-05:00	1	1		1	1
		gdx_dummy	intc_dummy	dal_dummy	$c\_dummy$	
Date						
	00:00:00-04:00	1	1	0	0	
	00:00:00-05:00	1	0	0	0	
	00:00:00-05:00	1	1	0	0	
	00:00:00-05:00	1	1	0	0	
2024-11-07	00:00:00-05:00	1	1	0	0	

```
2024-11-08 00:00:00-05:00
                                                              0
                                                                        0
                                     1
                                                  1
2024-11-11 00:00:00-05:00
                                     0
                                                  1
                                                              1
                                                                        1
2024-11-12 00:00:00-05:00
                                     0
                                                  1
                                                              1
                                                                        0
2024-11-13 00:00:00-05:00
                                     0
                                                              1
                                                                        0
                                                  1
                                     0
2024-11-14 00:00:00-05:00
                                                  1
                                                              1
                                                                        0
2024-11-15 00:00:00-05:00
                                     0
                                                              1
                                                                        0
                                                  1
2024-11-18 00:00:00-05:00
                                     1
                                                  1
                                                              1
                                                                        0
2024-11-19 00:00:00-05:00
                                     1
                                                  1
                                                              1
                                                                        0
2024-11-20 00:00:00-05:00
                                     1
                                                              1
                                                                        0
                                                  1
2024-11-21 00:00:00-05:00
                                     1
                                                  1
                                                              1
                                                                        0
2024-11-22 00:00:00-05:00
                                     1
                                                  1
                                                              1
                                                                        1
2024-11-25 00:00:00-05:00
                                     1
                                                  1
                                                              1
                                                                        1
2024-11-26 00:00:00-05:00
                                      1
                                                              1
                                                                        1
                                                  1
2024-11-27 00:00:00-05:00
                                     1
                                                  1
                                                              1
                                                                        1
2024-11-29 00:00:00-05:00
                                     1
                                                  1
                                                              1
                                                                        1
2024-12-02 00:00:00-05:00
                                     0
                                                  1
                                                              1
                                                                        1
2024-12-03 00:00:00-05:00
                                     1
                                                  0
                                                              1
                                                                        1
2024-12-04 00:00:00-05:00
                                     1
                                                  0
                                                              1
                                                                        1
2024-12-05 00:00:00-05:00
                                     1
                                                  0
                                                              1
                                                                        1
2024-12-06 00:00:00-05:00
                                     0
                                                  0
                                                              1
                                                                        1
2024-12-09 00:00:00-05:00
                                      1
                                                  0
                                                              0
                                                                        1
2024-12-10 00:00:00-05:00
                                     1
                                                  0
                                                              1
                                                                        1
2024-12-11 00:00:00-05:00
                                     1
                                                  0
                                                              1
                                                                        1
2024-12-12 00:00:00-05:00
                                                  0
                                                              0
                                     1
                                                                        1
2024-12-13 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                        1
2024-12-16 00:00:00-05:00
                                     0
                                                              0
                                                  0
                                                                        1
2024-12-17 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                        1
2024-12-18 00:00:00-05:00
                                                              0
                                                                        0
                                     0
                                                  0
2024-12-19 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                        0
2024-12-20 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                        0
2024-12-23 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                        1
2024-12-24 00:00:00-05:00
                                     0
                                                  0
                                                              1
                                                                        1
2024-12-26 00:00:00-05:00
                                     0
                                                  0
                                                              1
                                                                        1
2024-12-27 00:00:00-05:00
                                     0
                                                  0
                                                              0
                                                                        1
2024-12-30 00:00:00-05:00
                                     0
                                                              0
                                                                        1
```

- 4.2 2. Find the best linear regression model based on your training data ( $w = (XX')^{-1}Xy$ )
- 4.2.1 Note that you may need to transpose the matrices to make things work
- **4.2.2** We expect  $\mathbf{w} = (X_{train}X_{train}^T)^{-1}X_{train}y_{train}$  at the optimum

```
[49]: # Your solution here
import numpy as np

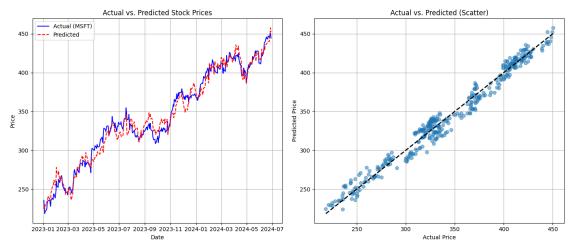
X_train = get_prices(feature_stocks, start_date_train, end_date_train)
```

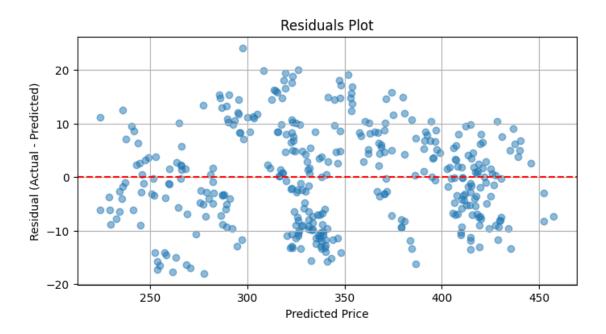
```
predict_stock = 'msft'
y train = get_prices([predict_stock], start_date_train, end_date_train).
 ⇒squeeze()
common_dates = X_train.index.intersection(y_train.index)
X train = X train.loc[common dates]
y_train = y_train.loc[common_dates]
# Add a column of 1s for the intercept term
X = np.column_stack([np.ones(len(X_train)), X_train.values])
# Compute weights using the normal equation
X_{transpose} = X.T
XTX_inv = np.linalg.pinv(X_transpose @ X) # Pseudo-inverse for numerical ∪
 \hookrightarrowstability
w = XTX_inv @ X_transpose @ y_train.values
# Extract coefficients (intercept + feature weights)
intercept = w[0]
coefficients = w[1:]
# Predictions
y_pred = X @ w
# Calculate R-squared (model performance)
ss_res = np.sum((y_train - y_pred) ** 2)
ss tot = np.sum((y train - np.mean(y train)) ** 2)
r_squared = 1 - (ss_res / ss_tot)
print(f"Intercept: {intercept:.4f}")
print(f"Coefficients: {dict(zip(feature_stocks, coefficients.round(4)))}")
print(f"R-squared: {r_squared:.4f}")
Intercept: 16.5294
Coefficients: {'tsla': np.float64(0.1757), 'meta': np.float64(0.2612), 'goog':
np.float64(0.3456), 'amzn': np.float64(0.3001), 'nflx': np.float64(0.1974),
'gbtc': np.float64(-0.4846), 'gdx': np.float64(2.334), 'intc':
np.float64(0.6901), 'dal': np.float64(-1.8977), 'c': np.float64(0.5018)}
R-squared: 0.9771
4.2.3 Linear Regression Plot
```

```
[50]: import matplotlib.pyplot as plt

# Plot actual vs. predicted values over time
plt.figure(figsize=(14, 6))
```

```
# Time series plot
plt.subplot(1, 2, 1)
plt.plot(X_train.index, y_train, label='Actual (MSFT)', color='blue')
plt.plot(X_train.index, y_pred, label='Predicted', color='red', linestyle='--')
plt.title('Actual vs. Predicted Stock Prices')
plt.xlabel('Date')
plt.ylabel('Price')
plt.legend()
plt.grid(True)
# Scatter plot of actual vs. predicted
plt.subplot(1, 2, 2)
plt.scatter(y_train, y_pred, alpha=0.5)
plt.plot([y_train.min(), y_train.max()], [y_train.min(), y_train.max()], 'k--',
 \hookrightarrowlw=2) # 45-degree line
plt.title('Actual vs. Predicted (Scatter)')
plt.xlabel('Actual Price')
plt.ylabel('Predicted Price')
plt.grid(True)
plt.tight_layout()
plt.show()
# Residuals plot (errors)
residuals = y_train - y_pred
plt.figure(figsize=(8, 4))
plt.scatter(y_pred, residuals, alpha=0.5)
plt.axhline(y=0, color='r', linestyle='--')
plt.title('Residuals Plot')
plt.xlabel('Predicted Price')
plt.ylabel('Residual (Actual - Predicted)')
plt.grid(True)
plt.show()
```





- 4.3 3. Report your training and testing error
- 4.3.1 How far your prediction from the actual price. Compute the mean square error for both training and testing

Preparing the testing data(modification)

```
[51]: # Your solution here

start_date_test='2024-11-01'
end_date_test='2024-12-31'

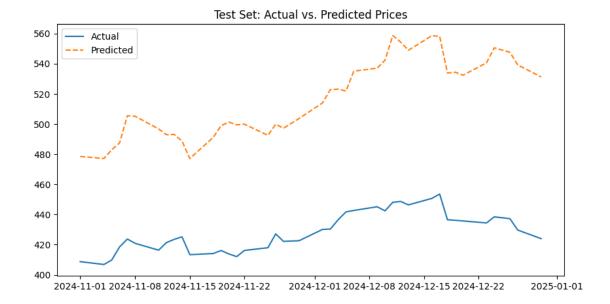
# Fetch testing data
X_test = get_prices(feature_stocks, start_date_test, end_date_test)
y_test = get_prices([predict_stock], start_date_test, end_date_test).squeeze()

# Align dates
common_dates_test = X_test.index.intersection(y_test.index)
X_test = X_test.loc[common_dates_test]
y_test = y_test.loc[common_dates_test]

# Add intercept term to testing data
X_test_design = np.column_stack([np.ones(len(X_test)), X_test.values])
```

#### Compute predictions

```
[52]: y_pred_train = X @ w
      y_pred_test = X_test_design @ w
     Compute Mean squared error
[54]: def mean squared error(y true, y pred):
          return np.mean((y_true - y_pred) ** 2)
      # Training MSE
      mse_train = mean_squared_error(y_train, y_pred_train)
      # Testing MSE
      mse_test = mean_squared_error(y_test, y_pred_test)
      print("=== Training vs. Testing MSE ===")
      print(f"Training MSE(mean square error): {mse_train:.4f}")
      print(f"Testing MSE(mean square error): {mse_test:.4f}")
     === Training vs. Testing MSE ===
     Training MSE(mean square error): 79.9055
     Testing MSE(mean square error): 7940.0156
[55]: # Baseline model (predict mean of y train)
      y_baseline = np.mean(y_train) * np.ones_like(y_test)
      mse_baseline = mean_squared_error(y_test, y_baseline)
      print(f"\nBaseline MSE (Mean Prediction): {mse_baseline:.4f}")
     Baseline MSE (Mean Prediction): 7433.4069
[56]: plt.figure(figsize=(10, 5))
      plt.plot(y_test.index, y_test, label='Actual')
      plt.plot(y_test.index, y_pred_test, label='Predicted', linestyle='--')
      plt.title('Test Set: Actual vs. Predicted Prices')
      plt.legend()
      plt.show()
```



A significant overfitting occurs. Use lasso regression for regularization.

```
Standardize Features
```

#### **Hyperparameter Tuning**

```
[]: scaler = StandardScaler()
    X_train_scaled = scaler.fit_transform(X_train)
    X_test_scaled = scaler.transform(X_test)
```

```
Train Lasso Model with Best Alpha
```

```
[59]: # Define a range of alphas to test
alphas = np.logspace(0, 1, 100)

# Train LassoCV model
lasso_model = LassoCV(alphas=alphas, cv=10, random_state=42)
lasso_model.fit(X_train_scaled, y_train)

# Best alpha from cross-validation
best_alpha = lasso_model.alpha_
print(f"Best alpha (): {best_alpha:.4f}")
```

Best alpha (): 1.0000

#### Evaluate Training and Testing MSE(After regularization)

```
[60]: y_train_pred = lasso_model.predict(X_train_scaled)
    y_test_pred = lasso_model.predict(X_test_scaled)

# Mean Squared Error

mse_train = mean_squared_error(y_train, y_train_pred)
mse_test = mean_squared_error(y_test, y_test_pred)

print(f"\nTraining MSE: {mse_train:.4f}")
print(f"Testing MSE: {mse_test:.4f}")
```

Training MSE: 105.5313 Testing MSE: 2085.9863

```
[61]: # Feature coefficients (standardized scale)
coefficients = lasso_model.coef_

print("\n=== Model Coefficients ===")
for stock, coef in zip(feature_stocks, coefficients):
    print(f"{stock}: {coef:.4f}")

# Number of features retained (non-zero coefficients)
non_zero = sum(coefficients != 0)
print(f"\nNon-zero coefficients: {non_zero}/{len(coefficients)}")
```

```
=== Model Coefficients ===
tsla: 1.4152
meta: 28.6413
goog: 14.6854
amzn: 0.0000
nflx: 10.2608
gbtc: 0.0000
```

gdx: 1.7307 intc: 7.9075 dal: -0.0000 c: 0.0000

Non-zero coefficients: 6/10

## 5 Prediction for MSFT stocks using Neural Networks

5.1 a. Split the stock price data into training, validation and test datasets.

```
[62]: # training set
      start_date_train='2023-1-01'
      end_date_train='2024-6-30'
      X_train=get_prices(feature_stocks,start=start_date_train,end=end_date_train)
      y_train=get_prices([predict_stock],start=start_date_train,end=end_date_train)
      # testing set
      start_date_test='2024-11-01'
      end_date_test='2025-1-01'
      X_test=get_prices(feature_stocks,start=start_date_test,end=end_date_test)
      y_test=get_prices([predict_stock],start=start_date_test,end=end_date_test)
      # validating set
      start_date_validate = '2024-7-01'
      end_date_validate = '2024-11-01'
      X_validate = get_prices(feature_stocks, start = start_date_validate, end = u
       →end_date_validate)
      y_validate = get_prices([predict_stock], start = start_date_validate, end =_u
       ⇔end_date_validate)
      X_{train}
```

```
[62]:
                                                                         amzn \
                                      tsla
                                                  meta
                                                              goog
     Date
     2023-01-03 00:00:00-05:00 108.099998 124.265312
                                                         89.378845
                                                                    85.820000
     2023-01-04 00:00:00-05:00
                                113.639999 126.885315
                                                         88.392403
                                                                    85.139999
                                                                     83.120003
     2023-01-05 00:00:00-05:00
                                110.339996 126.456947
                                                         86.459343
     2023-01-06 00:00:00-05:00
                                113.059998 129.525223
                                                         87.844376
                                                                     86.080002
     2023-01-09 00:00:00-05:00
                                                                     87.360001
                                119.769997
                                            128.977310
                                                         88.482079
     2024-06-24 00:00:00-04:00
                                182.580002 498.032776 180.347717 185.570007
     2024-06-25 00:00:00-04:00
                                187.350006 509.702240 185.126007
                                                                   186.339996
     2024-06-26 00:00:00-04:00
                                196.369995
                                            512.217773 184.916504 193.610001
     2024-06-27 00:00:00-04:00
                                197.419998 518.646423
                                                        186.402878
                                                                    197.850006
     2024-06-28 00:00:00-04:00
                                197.880005
                                            503.333435 182.971283 193.250000
```

```
nflx
                                                   gbtc
                                                                gdx
                                                                          intc \
      Date
      2023-01-03 00:00:00-05:00
                                 294.950012
                                               8.200000
                                                         28.842234
                                                                     25.775146
                                                         30.067492
      2023-01-04 00:00:00-05:00
                                  309.410004
                                               8.380000
                                                                     26.691208
      2023-01-05 00:00:00-05:00
                                  309.700012
                                               8.450000
                                                         29.804935
                                                                     26.575497
      2023-01-06 00:00:00-05:00
                                 315.549988
                                               8.650000
                                                         30.689848
                                                                     27.703701
      2023-01-09 00:00:00-05:00
                                  315.170013
                                               9.650000
                                                         30.398119
                                                                     28.262980
      2024-06-24 00:00:00-04:00
                                  669.020020
                                              52.610001
                                                         33.852570
                                                                    30.377298
      2024-06-25 00:00:00-04:00
                                  672.409973
                                              55.020000
                                                         33.447327
                                                                     30.546227
      2024-06-26 00:00:00-04:00
                                  677.690002
                                              54.130001
                                                         33.427559
                                                                     30.347488
      2024-06-27 00:00:00-04:00
                                  684.340027
                                              54.520000
                                                         33.832802
                                                                     30.397173
      2024-06-28 00:00:00-04:00
                                 674.880005
                                              53.240002
                                                         33.536285
                                                                     30.774776
                                        dal
                                                     С
      Date
      2023-01-03 00:00:00-05:00
                                  32.105251
                                             41.898052
      2023-01-04 00:00:00-05:00
                                  33.857697
                                             42.977993
      2023-01-05 00:00:00-05:00
                                  34.684692
                                             42.785805
      2023-01-06 00:00:00-05:00
                                  35.472309
                                             43.298321
      2023-01-09 00:00:00-05:00
                                  36.200859
                                             43.508816
      2024-06-24 00:00:00-04:00
                                  49.083549
                                             59.807236
      2024-06-25 00:00:00-04:00
                                  48.497330
                                             60.041241
      2024-06-26 00:00:00-04:00
                                  47.871361
                                             59.719482
      2024-06-27 00:00:00-04:00
                                  48.288670
                                             60.011986
      2024-06-28 00:00:00-04:00
                                 47.136101
                                             61.874260
      [374 rows x 10 columns]
[63]:
      y_train
[63]:
                                        msft
      Date
      2023-01-03 00:00:00-05:00
                                  235.240005
      2023-01-04 00:00:00-05:00
                                  224.949875
      2023-01-05 00:00:00-05:00
                                 218.282852
      2023-01-06 00:00:00-05:00
                                  220.855377
      2023-01-09 00:00:00-05:00
                                  223.005737
      2024-06-24 00:00:00-04:00
                                  445.079468
      2024-06-25 00:00:00-04:00
                                  448.340454
      2024-06-26 00:00:00-04:00
                                  449.543457
      2024-06-27 00:00:00-04:00
                                 450.229492
      2024-06-28 00:00:00-04:00
                                 444.363647
```

[64]:	

[64]:	tsla	meta	goog	amzn	\
Date					
2024-11-01 00:00:00-04:0		566.702881	172.454346	197.929993	
2024-11-04 00:00:00-05:0	00 242.839996	560.228088	170.486572	195.779999	
2024-11-05 00:00:00-05:0	00 251.440002	571.968628	171.215759	199.500000	
2024-11-06 00:00:00-05:0	00 288.529999	571.588928	178.127914	207.089996	
2024-11-07 00:00:00-05:0	00 296.910004	591.223145	182.073441	210.050003	
2024-11-08 00:00:00-05:0	00 321.220001	588.865051	179.656189	208.179993	
2024-11-11 00:00:00-05:0	00 350.000000	582.699951	181.763794	206.839996	
2024-11-12 00:00:00-05:0	00 328.489990	584.348633	183.112274	208.910004	
2024-11-13 00:00:00-05:0	00 330.239990	579.532532	180.285477	214.100006	
2024-11-14 00:00:00-05:0	00 311.179993	576.694824	177.149033	211.479996	
2024-11-15 00:00:00-05:0	00 320.720001	553.633423	173.692947	202.610001	
2024-11-18 00:00:00-05:0	00 338.739990	553.953186	176.599655	201.699997	
2024-11-19 00:00:00-05:0	00 346.000000	560.637817	179.376495	204.610001	
2024-11-20 00:00:00-05:0	00 342.029999	565.064209	177.129044	202.880005	
2024-11-21 00:00:00-05:0	00 339.640015	562.636169	169.048218	198.380005	
2024-11-22 00:00:00-05:0	00 352.559998	558.689392	166.381256	197.119995	
2024-11-25 00:00:00-05:0	00 338.589996	564.654541	169.237991	201.449997	
2024-11-26 00:00:00-05:0	00 338.230011	573.077698	170.426651	207.860001	
2024-11-27 00:00:00-05:0	0 332.890015	568.741272	170.626434	205.740005	
2024-11-29 00:00:00-05:0	00 345.160004	573.857117	170.296799	207.889999	
2024-12-02 00:00:00-05:0	00 357.089996	592.352234	172.783981	210.710007	
2024-12-03 00:00:00-05:0	00 351.420013	613.155457	172.823944	213.440002	
2024-12-04 00:00:00-05:0	0 357.929993	613.285339	175.890457	218.160004	
2024-12-05 00:00:00-05:0	00 369.489990	608.439209	174.112473	220.550003	
2024-12-06 00:00:00-05:0	00 389.220001	623.267273	176.290009	227.029999	
2024-12-09 00:00:00-05:0	00 389.790009	613.075500	177.100006	226.089996	
2024-12-10 00:00:00-05:0	00 400.989990	618.820862	186.529999	225.039993	
2024-12-11 00:00:00-05:0	0 424.769989	632.170044	196.710007	230.259995	
2024-12-12 00:00:00-05:0	00 418.100006	630.281555	193.630005	228.970001	
2024-12-13 00:00:00-05:0	0 436.230011	619.849976	191.380005	227.460007	
2024-12-16 00:00:00-05:0	00 463.019989	624.239990	198.160004	232.929993	
2024-12-17 00:00:00-05:0	0 479.859985	619.440002	197.119995	231.149994	
2024-12-18 00:00:00-05:0	0 440.130005	597.190002	190.149994	220.520004	
2024-12-19 00:00:00-05:0	0 436.170013	595.570007	189.699997	223.289993	
2024-12-20 00:00:00-05:0	00 421.059998	585.250000	192.960007	224.919998	
2024-12-23 00:00:00-05:0	00 430.600006	599.849976	195.990005	225.059998	
2024-12-24 00:00:00-05:0		607.750000	197.570007	229.050003	
2024-12-26 00:00:00-05:0		603.349976	197.100006	227.050003	
2024-12-27 00:00:00-05:0	00 431.660004	599.809998	194.039993	223.750000	
2024-12-30 00:00:00-05:0		591.239990	192.690002	221.300003	
2024-12-31 00:00:00-05:0		585.510010	190.440002	219.389999	

```
gdx
                                  nflx
                                              gbtc
                                                                     intc
Date
2024-11-01 00:00:00-04:00
                            756.099976
                                        55.009998
                                                    39.387589
                                                               23.200001
2024-11-04 00:00:00-05:00
                            755.510010
                                        53.490002
                                                    39.437012
                                                               22.520000
2024-11-05 00:00:00-05:00
                                                    39.644573
                                                               23.320000
                            763.909973
                                        55.169998
2024-11-06 00:00:00-05:00
                            780.210022
                                        60.599998
                                                    38.191631
                                                               25.049999
2024-11-07 00:00:00-05:00
                            796.539978
                                        60.880001
                                                    39.110840
                                                               26.230000
2024-11-08 00:00:00-05:00
                            795.039978
                                        61.049999
                                                    38.567223
                                                               26.200001
2024-11-11 00:00:00-05:00
                            805.440002
                                        69.220001
                                                    36.303795
                                                               25.049999
2024-11-12 00:00:00-05:00
                            819.500000
                                        71.230003
                                                    35.690987
                                                               24.160000
2024-11-13 00:00:00-05:00
                                        71.309998
                                                               24.920000
                            830.469971
                                                    35.147369
2024-11-14 00:00:00-05:00
                            837.260010
                                        69.500000
                                                    35.305515
                                                               25.030001
2024-11-15 00:00:00-05:00
                            823.960022
                                        72.809998
                                                    35.097950
                                                               24.350000
2024-11-18 00:00:00-05:00
                            847.049988
                                        72.769997
                                                    36.590427
                                                               24.840000
2024-11-19 00:00:00-05:00
                            871.320007
                                        73.580002
                                                    37.440449
                                                               24.200001
2024-11-20 00:00:00-05:00
                            883.849976
                                        74.989998
                                                    37.282307
                                                               24.010000
2024-11-21 00:00:00-05:00
                            897.479980
                                        78.050003
                                                    37.697430
                                                               24.440001
2024-11-22 00:00:00-05:00
                            897.789978
                                        78.870003
                                                    37.835808
                                                               24.500000
2024-11-25 00:00:00-05:00
                            865.590027
                                        75.419998
                                                    36.709034
                                                               24.870001
2024-11-26 00:00:00-05:00
                            872.599976
                                        72.169998
                                                    36.857296
                                                               24.049999
2024-11-27 00:00:00-05:00
                                        76.820000
                                                    37.015438
                            877.340027
                                                               23.650000
2024-11-29 00:00:00-05:00
                                        77.089996
                                                    37.223003
                                                               24.049999
                            886.809998
2024-12-02 00:00:00-05:00
                            897.739990
                                        76.029999
                                                    36.402634
                                                               23.930000
2024-12-03 00:00:00-05:00
                                        75.949997
                                                    37.262539
                            902.169983
                                                               22.469999
2024-12-04 00:00:00-05:00
                            911.059998
                                        78.690002
                                                    37.163696
                                                               21.959999
2024-12-05 00:00:00-05:00
                            917.869995
                                        78.690002
                                                    37.094509
                                                               20.799999
2024-12-06 00:00:00-05:00
                            934.739990
                                        80.699997
                                                    36.392750
                                                               20.920000
2024-12-09 00:00:00-05:00
                            913.690002
                                        76.330002
                                                    37.470100
                                                               20.809999
2024-12-10 00:00:00-05:00
                            913.349976
                                        76.559998
                                                    37.539288
                                                               20.160000
2024-12-11 00:00:00-05:00
                            936.559998
                                        80.510002
                                                    38.636410
                                                               20.120001
2024-12-12 00:00:00-05:00
                            925.549988
                                        79.410004
                                                    37.223003
                                                               20.780001
2024-12-13 00:00:00-05:00
                            918.869995
                                        80.769997
                                                    36.224724
                                                               20.340000
2024-12-16 00:00:00-05:00
                            921.080017
                                        84.019997
                                                    35.997387
                                                               20.830000
2024-12-17 00:00:00-05:00
                            919.130005
                                        84.709999
                                                    35.770058
                                                               20.440001
2024-12-18 00:00:00-05:00
                            889.549988
                                        79.809998
                                                    34.129322
                                                               19.299999
2024-12-19 00:00:00-05:00
                            902.039978
                                        76.320000
                                                    33.990944
                                                               19.059999
2024-12-20 00:00:00-05:00
                            909.049988
                                        76.470001
                                                    34.327000
                                                               19.520000
2024-12-23 00:00:00-05:00
                            911.450012
                                        73.709999
                                                    34.410000
                                                               20.200001
2024-12-24 00:00:00-05:00
                                        78.449997
                                                    34.410000
                            932.119995
                                                               20.400000
2024-12-26 00:00:00-05:00
                            924.140015
                                        75.760002
                                                    34.470001
                                                               20.440001
2024-12-27 00:00:00-05:00
                            907.549988
                                        74.879997
                                                    34.259998
                                                               20.299999
2024-12-30 00:00:00-05:00
                            900.429993
                                        74.650002
                                                    33.770000
                                                               19.820000
2024-12-31 00:00:00-05:00
                            891.320007
                                        74.019997
                                                    33.910000
                                                               20.049999
```

Date

С

dal

```
2024-11-01 00:00:00-04:00
                            58.389999
                                       62.715710
2024-11-04 00:00:00-05:00
                            56.889999
                                       61.921215
2024-11-05 00:00:00-05:00
                            58.290001
                                       63.232136
2024-11-06 00:00:00-05:00
                            62.320000
                                       68.555275
2024-11-07 00:00:00-05:00
                            60.430000
                                       67.641602
2024-11-08 00:00:00-05:00
                            61.049999
                                       68.158028
2024-11-11 00:00:00-05:00
                            63.560001
                                       69.319984
2024-11-12 00:00:00-05:00
                            64.050003
                                       68.545341
2024-11-13 00:00:00-05:00
                            64.459999
                                       68.416237
2024-11-14 00:00:00-05:00
                            64.849998
                                       67.681328
2024-11-15 00:00:00-05:00
                            64.070000
                                       68.287132
2024-11-18 00:00:00-05:00
                            63.240002
                                       68.525482
2024-11-19 00:00:00-05:00
                            64.750000
                                       68.128235
2024-11-20 00:00:00-05:00
                            63.639999
                                       67.810432
2024-11-21 00:00:00-05:00
                            63.340000
                                       68.475822
2024-11-22 00:00:00-05:00
                            63.340000
                                       69.359703
2024-11-25 00:00:00-05:00
                            64.489998
                                       70.263451
2024-11-26 00:00:00-05:00
                            64.139999
                                       69.270325
2024-11-27 00:00:00-05:00
                            63.619999
                                       69.677505
2024-11-29 00:00:00-05:00
                            63.820000
                                       70.382622
2024-12-02 00:00:00-05:00
                            63.410000
                                       70.899048
                            62.570000
2024-12-03 00:00:00-05:00
                                       70.928841
2024-12-04 00:00:00-05:00
                            64.260002
                                       71.008293
2024-12-05 00:00:00-05:00
                            65.769997
                                       71.733276
2024-12-06 00:00:00-05:00
                            64.529999
                                       71.653824
2024-12-09 00:00:00-05:00
                            62.259998
                                       71.365814
2024-12-10 00:00:00-05:00
                            62.770000
                                       72.001411
2024-12-11 00:00:00-05:00
                            63.480000
                                       71.465126
2024-12-12 00:00:00-05:00
                            61.630001
                                       70.938774
2024-12-13 00:00:00-05:00
                            61.520000
                                       70.521660
2024-12-16 00:00:00-05:00
                            61.049999
                                       70.998360
2024-12-17 00:00:00-05:00
                            60.810001
                                       70.630905
2024-12-18 00:00:00-05:00
                            58.880001
                                       67.651535
                                       67.949471
2024-12-19 00:00:00-05:00
                            60.380001
2024-12-20 00:00:00-05:00
                            60.930000
                                       68.714180
2024-12-23 00:00:00-05:00
                            61.520000
                                       69.290184
2024-12-24 00:00:00-05:00
                            62.560001
                                       70.511726
2024-12-26 00:00:00-05:00
                            62.400002
                                       70.859322
2024-12-27 00:00:00-05:00
                            61.259998
                                       70.511726
2024-12-30 00:00:00-05:00
                            60.720001
                                       69.905922
2024-12-31 00:00:00-05:00
                            60.500000
                                       69.905922
y_test
                                  msft
```

[65]:

[65]:

Date

2024-11-01 00:00:00-04:00 408.730652

```
2024-11-04 00:00:00-05:00
                           406.828278
2024-11-05 00:00:00-05:00
                           409.816315
2024-11-06 00:00:00-05:00
                           418.501465
2024-11-07 00:00:00-05:00
                           423.730530
2024-11-08 00:00:00-05:00
                           420.852051
2024-11-11 00:00:00-05:00
                           416.340149
2024-11-12 00:00:00-05:00
                           421.340088
2024-11-13 00:00:00-05:00
                            423.501465
2024-11-14 00:00:00-05:00
                           425.184692
2024-11-15 00:00:00-05:00
                            413.342194
2024-11-18 00:00:00-05:00
                            414.099152
2024-11-19 00:00:00-05:00
                            416.121063
2024-11-20 00:00:00-05:00
                           413.830200
2024-11-21 00:00:00-05:00
                           412.043793
2024-11-22 00:00:00-05:00
                           416.165527
2024-11-25 00:00:00-05:00
                           417.951965
2024-11-26 00:00:00-05:00
                           427.133545
2024-11-27 00:00:00-05:00
                           422.143555
2024-11-29 00:00:00-05:00
                           422.612610
2024-12-02 00:00:00-05:00
                           430.117584
2024-12-03 00:00:00-05:00
                           430.337128
2024-12-04 00:00:00-05:00
                            436.544678
2024-12-05 00:00:00-05:00
                           441.734253
2024-12-06 00:00:00-05:00
                           442.682373
2024-12-09 00:00:00-05:00
                            445.127441
2024-12-10 00:00:00-05:00
                            442.442841
2024-12-11 00:00:00-05:00
                            448.091522
2024-12-12 00:00:00-05:00
                           448.660370
2024-12-13 00:00:00-05:00
                            446.374939
2024-12-16 00:00:00-05:00
                           450.686310
2024-12-17 00:00:00-05:00
                           453.550568
2024-12-18 00:00:00-05:00
                           436.514740
2024-12-19 00:00:00-05:00
                            436.155457
2024-12-20 00:00:00-05:00
                           435.726318
2024-12-23 00:00:00-05:00
                           434.379028
2024-12-24 00:00:00-05:00
                            438.450836
2024-12-26 00:00:00-05:00
                           437.233276
2024-12-27 00:00:00-05:00
                           429.668457
2024-12-30 00:00:00-05:00
                            423.979858
2024-12-31 00:00:00-05:00
                            420.656525
```

#### [66]: X\_validate

[66]: tsla \ meta goog amzn Date 2024-07-01 00:00:00-04:00 209.860001 503.792603 184.038681 197.199997 2024-07-02 00:00:00-04:00 231.259995 508.604156 186.153488 200.000000

```
2024-07-03 00:00:00-04:00
                           246.389999
                                        509.063354
                                                    186.931580 197.589996
2024-07-05 00:00:00-04:00
                           251.520004
                                        538.960632
                                                    191.490402
                                                                200.000000
2024-07-08 00:00:00-04:00
                            252.940002
                                        528.389282
                                                    190.014008
                                                                199.289993
2024-10-25 00:00:00-04:00
                           269.190002
                                        572.787964
                                                    166.800766
                                                                187.830002
2024-10-28 00:00:00-04:00
                           262.510010
                                        577.693970
                                                    168.149231
                                                                188.389999
2024-10-29 00:00:00-04:00
                           259.519989
                                        592.801880
                                                    170.946060
                                                                190.830002
2024-10-30 00:00:00-04:00
                            257.549988
                                        591.322998
                                                    175.940399
                                                                192.729996
                                        567.122559
                                                    172.494308
2024-10-31 00:00:00-04:00
                           249.850006
                                                                186.399994
                                 nflx
                                             gbtc
                                                         gdx
                                                                    intc
Date
2024-07-01 00:00:00-04:00
                           673.609985
                                        56.090000
                                                   33.496746
                                                              30.645597
2024-07-02 00:00:00-04:00
                           679.580017
                                        54.889999
                                                   33.625240
                                                              30.874147
2024-07-03 00:00:00-04:00
                                        53.660000
                                                   34.870617
                            682.510010
                                                              31.033138
2024-07-05 00:00:00-04:00
                           690.650024
                                        50.150002
                                                   35.849133
                                                              31.818159
2024-07-08 00:00:00-04:00
                           685.739990
                                        50.090000
                                                   35.681103
                                                              33.775742
2024-10-25 00:00:00-04:00
                           754.679993
                                        53.090000
                                                   41.077747
                                                              22.680000
2024-10-28 00:00:00-04:00
                           749.119995
                                        55.380001
                                                   40.880070
                                                              22,920000
2024-10-29 00:00:00-04:00
                           759.440002
                                        57.669998
                                                   41.532410
                                                              22.900000
2024-10-30 00:00:00-04:00
                           753.739990
                                        57.080002
                                                   41.028324
                                                              22.299999
2024-10-31 00:00:00-04:00
                           756.030029
                                        55.610001
                                                   39.871902
                                                              21.520000
                                 dal
                                               C
Date
2024-07-01 00:00:00-04:00
                           46.619431
                                       61.845009
2024-07-02 00:00:00-04:00
                           46.410778
                                       63.063778
2024-07-03 00:00:00-04:00
                            47.086426
                                       62.849270
2024-07-05 00:00:00-04:00
                           45.725201
                                       62.430016
2024-07-08 00:00:00-04:00
                           46.053085
                                       63.122272
2024-10-25 00:00:00-04:00
                            54.119999
                                       60.796143
2024-10-28 00:00:00-04:00
                           55.380001
                                       63.158691
2024-10-29 00:00:00-04:00
                            57.340000
                                       63.680424
2024-10-30 00:00:00-04:00
                           58.459999
                                       63.493385
2024-10-31 00:00:00-04:00
                           57.220001
                                      63.168530
[87 rows x 10 columns]
y_validate
                                 msft
Date
2024-07-01 00:00:00-04:00
                           454.087036
2024-07-02 00:00:00-04:00
                           456.622253
2024-07-03 00:00:00-04:00
                           458.103638
```

[67]:

[67]:

```
2024-07-05 00:00:00-04:00 464.854340
2024-07-08 00:00:00-04:00 463.541992
...
2024-10-25 00:00:00-04:00 426.439636
2024-10-28 00:00:00-04:00 424.885895
2024-10-29 00:00:00-04:00 430.224487
2024-10-30 00:00:00-04:00 430.802155
2024-10-31 00:00:00-04:00 404.726746

[87 rows x 1 columns]
```

6 b.Estimation of the MSFT stock price with a fully connected neural network with 5 hidden layers.

```
[68]: import numpy as np
      import pandas as pd
      import yfinance as yf
      from sklearn.preprocessing import MinMaxScaler
      import keras
      import tensorflow as tf
      from keras import Sequential
      from keras import layers
      import matplotlib.pyplot as plt
      # Configuration
      n_lags = 30  # Use past 30 days of data to predict next day
      # Get full dataset
      full_data = get_prices(feature_stocks + [predict_stock], start='2023-01-01',__
       ⇔end='2025-01-01')
      # Create lagged features and targets
      X, y = [], []
      for i in range(n lags, len(full data)):
          # Use past n_lags days of feature stocks to predict current day's MSFT price
          X.append(full data[feature stocks].iloc[i-n lags:i].values.flatten())
          y.append(full_data[predict_stock].iloc[i])
      X = np.array(X)
      y = np.array(y)
      train_end = '2024-06-30'
      val_end = '2024-11-01'
      train_idx = full_data.index <= train_end</pre>
      val_idx = (full_data.index > train_end) & (full_data.index <= val_end)</pre>
      test idx = full data.index > val end
```

```
X_train, y_train = X[train_idx[n_lags:]], y[train_idx[n_lags:]]
X_val, y_val = X[val_idx[n_lags:]], y[val_idx[n_lags:]]
X_test, y_test = X[test_idx[n_lags:]], y[test_idx[n_lags:]]
# Scale data
X scaler = MinMaxScaler()
y_scaler = MinMaxScaler()
X_train_scaled = X_scaler.fit_transform(X_train)
X_val_scaled = X_scaler.transform(X_val)
X_test_scaled = X_scaler.transform(X_test)
y_train_scaled = y_scaler.fit_transform(y_train.reshape(-1, 1))
y_val_scaled = y_scaler.transform(y_val.reshape(-1, 1))
y_test_scaled = y_scaler.transform(y_test.reshape(-1, 1))
# Build model
model = Sequential([
   layers.Dense(20, activation='relu', input_shape=(X_train.shape[1],)),
   layers.Dense(20, activation='relu'),
   layers.Dense(20, activation='relu'),
   layers.Dense(20, activation='relu'),
   layers.Dense(20, activation='relu'),
   layers.Dense(1, activation='linear')
1)
model.compile(loss='mse')
# Train with validation
history = model.fit(
   X_train_scaled,
   y_train_scaled,
   epochs=100,
   batch_size=32,
   validation_data=(X_val_scaled, y_val_scaled),
   verbose=1
)
# Predict on test set
y_pred_scaled = model.predict(X_test_scaled)
y_pred = y_scaler.inverse_transform(y_pred_scaled)
# Plot results
plt.figure(figsize=(12, 6))
plt.plot(full_data.index[-len(y_test):], y_test, label='Actual Price')
plt.plot(full_data.index[-len(y_test):], y_pred, label='Predicted Price')
```

```
plt.title(f'{predict_stock} Stock Price Prediction (Current Timeline)')
plt.xlabel('Date')
plt.ylabel('Price (USD)')
plt.legend()
plt.grid(True)
plt.show()
# Calculate metrics
rmse = np.sqrt(np.mean((y_pred - y_test) ** 2))
print(f"Test RMSE: {rmse:.2f}")
print(f"Last Prediction Date: {full data.index[-1]}")
print(f"Latest Prediction: {y_pred[-1][0]:.2f}")
Epoch 1/100
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-
packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an
`input_shape`/`input_dim` argument to a layer. When using Sequential models,
prefer using an `Input(shape)` object as the first layer in the model instead.
  super().__init__(activity_regularizer=activity_regularizer, **kwargs)
11/11
                  1s 64ms/step -
loss: 0.0578 - val_loss: 0.0142
Epoch 2/100
11/11
                 Os 6ms/step - loss:
0.0062 - val_loss: 0.0168
Epoch 3/100
11/11
                 Os 6ms/step - loss:
0.0124 - val_loss: 0.0242
Epoch 4/100
11/11
                  Os 6ms/step - loss:
0.0089 - val_loss: 0.0285
Epoch 5/100
11/11
                  Os 6ms/step - loss:
0.0096 - val_loss: 0.0139
Epoch 6/100
11/11
                  Os 7ms/step - loss:
0.0073 - val_loss: 0.0068
Epoch 7/100
11/11
                  Os 6ms/step - loss:
0.0046 - val_loss: 0.0186
Epoch 8/100
11/11
                  Os 6ms/step - loss:
0.0058 - val_loss: 0.0174
Epoch 9/100
11/11
                  Os 7ms/step - loss:
0.0082 - val_loss: 0.0205
Epoch 10/100
11/11
                 Os 6ms/step - loss:
```

```
0.0075 - val_loss: 0.0091
Epoch 11/100
11/11
                  Os 6ms/step - loss:
0.0043 - val_loss: 0.0150
Epoch 12/100
11/11
                  Os 6ms/step - loss:
0.0060 - val_loss: 0.0070
Epoch 13/100
11/11
                  Os 6ms/step - loss:
0.0032 - val_loss: 0.0601
Epoch 14/100
11/11
                  Os 6ms/step - loss:
0.0054 - val_loss: 0.0113
Epoch 15/100
11/11
                  Os 6ms/step - loss:
0.0040 - val_loss: 0.0072
Epoch 16/100
11/11
                  Os 6ms/step - loss:
0.0036 - val_loss: 0.0131
Epoch 17/100
11/11
                  Os 6ms/step - loss:
0.0045 - val_loss: 0.0463
Epoch 18/100
11/11
                  Os 7ms/step - loss:
0.0035 - val_loss: 0.0100
Epoch 19/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0151
Epoch 20/100
11/11
                  Os 6ms/step - loss:
0.0034 - val_loss: 0.0117
Epoch 21/100
11/11
                  Os 6ms/step - loss:
0.0018 - val_loss: 0.0091
Epoch 22/100
11/11
                  Os 6ms/step - loss:
0.0070 - val_loss: 0.0429
Epoch 23/100
11/11
                  Os 6ms/step - loss:
0.0033 - val_loss: 0.0102
Epoch 24/100
11/11
                  Os 6ms/step - loss:
0.0042 - val_loss: 0.0326
Epoch 25/100
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0106
Epoch 26/100
11/11
                  Os 6ms/step - loss:
```

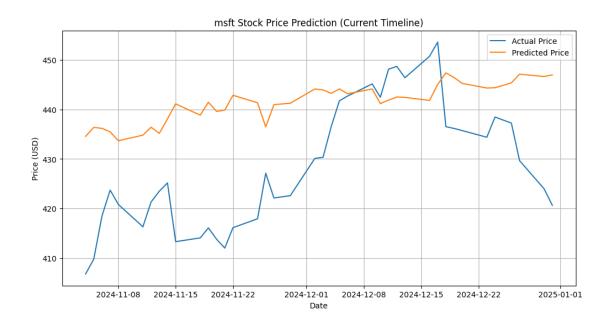
```
0.0022 - val_loss: 0.0156
Epoch 27/100
11/11
                  Os 6ms/step - loss:
0.0045 - val_loss: 0.0163
Epoch 28/100
11/11
                  Os 6ms/step - loss:
0.0029 - val_loss: 0.0459
Epoch 29/100
11/11
                  Os 6ms/step - loss:
0.0028 - val_loss: 0.0520
Epoch 30/100
11/11
                 Os 6ms/step - loss:
0.0023 - val_loss: 0.0544
Epoch 31/100
11/11
                  Os 6ms/step - loss:
0.0033 - val_loss: 0.0721
Epoch 32/100
11/11
                  Os 7ms/step - loss:
0.0045 - val_loss: 0.0687
Epoch 33/100
11/11
                  Os 6ms/step - loss:
0.0053 - val_loss: 0.0324
Epoch 34/100
11/11
                  Os 6ms/step - loss:
0.0038 - val_loss: 0.0101
Epoch 35/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0575
Epoch 36/100
11/11
                  Os 6ms/step - loss:
0.0022 - val_loss: 0.0163
Epoch 37/100
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0363
Epoch 38/100
11/11
                  Os 6ms/step - loss:
0.0035 - val_loss: 0.0158
Epoch 39/100
11/11
                  Os 6ms/step - loss:
0.0021 - val_loss: 0.0161
Epoch 40/100
11/11
                  Os 6ms/step - loss:
0.0032 - val_loss: 0.0294
Epoch 41/100
11/11
                  Os 6ms/step - loss:
0.0013 - val_loss: 0.1177
Epoch 42/100
11/11
                  Os 6ms/step - loss:
```

```
0.0029 - val_loss: 0.0348
Epoch 43/100
11/11
                  Os 6ms/step - loss:
0.0028 - val_loss: 0.0236
Epoch 44/100
11/11
                  Os 6ms/step - loss:
0.0035 - val_loss: 0.0383
Epoch 45/100
11/11
                  Os 6ms/step - loss:
0.0018 - val_loss: 0.0098
Epoch 46/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0598
Epoch 47/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0218
Epoch 48/100
11/11
                  Os 6ms/step - loss:
0.0013 - val_loss: 0.0652
Epoch 49/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0524
Epoch 50/100
11/11
                  Os 5ms/step - loss:
0.0014 - val_loss: 0.0577
Epoch 51/100
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0380
Epoch 52/100
11/11
                  Os 6ms/step - loss:
0.0031 - val_loss: 0.0706
Epoch 53/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0786
Epoch 54/100
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0343
Epoch 55/100
11/11
                  Os 6ms/step - loss:
0.0024 - val_loss: 0.0329
Epoch 56/100
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0819
Epoch 57/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0481
Epoch 58/100
11/11
                  Os 6ms/step - loss:
```

```
0.0024 - val_loss: 0.0434
Epoch 59/100
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0616
Epoch 60/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0401
Epoch 61/100
11/11
                  Os 6ms/step - loss:
0.0021 - val_loss: 0.0765
Epoch 62/100
11/11
                  Os 6ms/step - loss:
0.0036 - val_loss: 0.0321
Epoch 63/100
11/11
                  Os 7ms/step - loss:
0.0024 - val_loss: 0.0341
Epoch 64/100
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0434
Epoch 65/100
11/11
                  Os 6ms/step - loss:
8.5052e-04 - val_loss: 0.0895
Epoch 66/100
11/11
                  Os 6ms/step - loss:
0.0029 - val_loss: 0.0212
Epoch 67/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0204
Epoch 68/100
11/11
                  Os 6ms/step - loss:
0.0013 - val_loss: 0.0134
Epoch 69/100
11/11
                  Os 7ms/step - loss:
0.0029 - val_loss: 0.0252
Epoch 70/100
11/11
                  Os 6ms/step - loss:
0.0020 - val_loss: 0.0235
Epoch 71/100
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0350
Epoch 72/100
11/11
                  Os 6ms/step - loss:
0.0017 - val_loss: 0.0885
Epoch 73/100
11/11
                  Os 6ms/step - loss:
0.0020 - val_loss: 0.0254
Epoch 74/100
11/11
                  Os 6ms/step - loss:
```

```
0.0018 - val_loss: 0.0414
Epoch 75/100
11/11
                  Os 6ms/step - loss:
0.0016 - val_loss: 0.0188
Epoch 76/100
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0259
Epoch 77/100
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0178
Epoch 78/100
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0185
Epoch 79/100
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0279
Epoch 80/100
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0229
Epoch 81/100
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0462
Epoch 82/100
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0252
Epoch 83/100
11/11
                  Os 7ms/step - loss:
9.4611e-04 - val_loss: 0.0220
Epoch 84/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0389
Epoch 85/100
11/11
                  Os 6ms/step - loss:
7.5379e-04 - val_loss: 0.0291
Epoch 86/100
11/11
                  Os 6ms/step - loss:
0.0016 - val_loss: 0.0486
Epoch 87/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0397
Epoch 88/100
11/11
                  Os 6ms/step - loss:
7.1716e-04 - val_loss: 0.0303
Epoch 89/100
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0394
Epoch 90/100
11/11
                  Os 6ms/step - loss:
```

```
9.2117e-04 - val_loss: 0.0104
Epoch 91/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0294
Epoch 92/100
11/11
                  Os 6ms/step - loss:
0.0013 - val_loss: 0.0377
Epoch 93/100
11/11
                 Os 6ms/step - loss:
0.0019 - val_loss: 0.0221
Epoch 94/100
11/11
                 Os 6ms/step - loss:
8.6376e-04 - val_loss: 0.0112
Epoch 95/100
11/11
                 Os 6ms/step - loss:
0.0028 - val_loss: 0.0200
Epoch 96/100
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0141
Epoch 97/100
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0467
Epoch 98/100
11/11
                 Os 6ms/step - loss:
0.0020 - val_loss: 0.0308
Epoch 99/100
11/11
                  Os 6ms/step - loss:
0.0021 - val_loss: 0.0203
Epoch 100/100
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0097
               Os 102ms/step
2/2
```



Test RMSE: 18.19

Last Prediction Date: 2024-12-31 00:00:00-05:00

Latest Prediction: 446.94

## 7 c. using different optimization algorithms

#### 7.1 adam

```
[69]: import numpy as np
      import pandas as pd
      import yfinance as yf
      from sklearn.preprocessing import MinMaxScaler
      import keras
      import tensorflow as tf
      from keras import Sequential
      from keras import layers
      import matplotlib.pyplot as plt
      # Configuration
      n_lags = 30  # Use past 30 days of data to predict next day
      # Get full dataset
      full_data = get_prices(feature_stocks + [predict_stock], start='2023-01-01',_
       \rightarrowend='2025-01-01')
      # Create lagged features and targets
      X, y = [], []
      for i in range(n_lags, len(full_data)):
```

```
# Use past n lags days of feature stocks to predict current day's MSFT price
    X.append(full_data[feature_stocks].iloc[i-n_lags:i].values.flatten())
    y.append(full_data[predict_stock].iloc[i])
X = np.array(X)
y = np.array(y)
train_end = '2024-06-30'
val_end = '2024-11-01'
train idx = full data.index <= train end</pre>
val_idx = (full_data.index > train_end) & (full_data.index <= val_end)</pre>
test_idx = full_data.index > val_end
X_train, y_train = X[train_idx[n_lags:]], y[train_idx[n_lags:]]
X_val, y_val = X[val_idx[n_lags:]], y[val_idx[n_lags:]]
X_test, y_test = X[test_idx[n_lags:]], y[test_idx[n_lags:]]
# Scale data
X_scaler = MinMaxScaler()
y_scaler = MinMaxScaler()
X_train_scaled = X_scaler.fit_transform(X_train)
X_val_scaled = X_scaler.transform(X_val)
X_test_scaled = X_scaler.transform(X_test)
y_train_scaled = y_scaler.fit_transform(y_train.reshape(-1, 1))
y_val_scaled = y_scaler.transform(y_val.reshape(-1, 1))
y_test_scaled = y_scaler.transform(y_test.reshape(-1, 1))
# Build model
model = Sequential([
    layers.Dense(20, activation='relu', input shape=(X_train.shape[1],)),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(1, activation='linear')
])
model.compile(optimizer = 'adam', loss='mse')
# Train with validation
history = model.fit(
    X_train_scaled,
    y_train_scaled,
    epochs=100,
```

```
batch_size=32,
    validation_data=(X_val_scaled, y_val_scaled),
    verbose=1
# Predict on test set
y_pred_scaled = model.predict(X_test_scaled)
y_pred = y_scaler.inverse_transform(y_pred_scaled)
# Plot results
plt.figure(figsize=(12, 6))
plt.plot(full_data.index[-len(y_test):], y_test, label='Actual Price')
plt.plot(full_data.index[-len(y_test):], y_pred, label='Predicted Price')
plt.title(f'{predict_stock} Stock Price Prediction (Current Timeline)')
plt.xlabel('Date')
plt.ylabel('Price (USD)')
plt.legend()
plt.grid(True)
plt.show()
# Calculate metrics
rmse = np.sqrt(np.mean((y_pred - y_test) ** 2))
print(f"Test RMSE: {rmse:.2f}")
print(f"Last Prediction Date: {full data.index[-1]}")
print(f"Latest Prediction: {y_pred[-1][0]:.2f}")
plt.figure(figsize=(12, 6))
plt.plot(history.history['loss'], label='Train Loss')
plt.plot(history.history['val_loss'], label='Validation Loss')
plt.title('Training History with Adam')
plt.xlabel('Epochs')
plt.ylabel('MSE Loss')
plt.legend()
plt.grid(True)
plt.show()
Epoch 1/100
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-
packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an
`input_shape`/`input_dim` argument to a layer. When using Sequential models,
prefer using an `Input(shape)` object as the first layer in the model instead.
  super().__init__(activity_regularizer=activity_regularizer, **kwargs)
```

2s 64ms/step -

Os 5ms/step - loss:

loss: 0.3420 - val\_loss: 0.0209

0.0227 - val\_loss: 0.0656

11/11

11/11

Epoch 2/100

```
Epoch 3/100
11/11
                 Os 6ms/step - loss:
0.0101 - val_loss: 0.0076
Epoch 4/100
11/11
                  Os 6ms/step - loss:
0.0076 - val_loss: 0.0253
Epoch 5/100
11/11
                  Os 5ms/step - loss:
0.0060 - val_loss: 0.0085
Epoch 6/100
11/11
                  Os 6ms/step - loss:
0.0050 - val_loss: 0.0116
Epoch 7/100
11/11
                  Os 6ms/step - loss:
0.0050 - val_loss: 0.0090
Epoch 8/100
11/11
                  Os 6ms/step - loss:
0.0037 - val_loss: 0.0071
Epoch 9/100
11/11
                  Os 5ms/step - loss:
0.0039 - val_loss: 0.0095
Epoch 10/100
11/11
                 Os 6ms/step - loss:
0.0035 - val_loss: 0.0080
Epoch 11/100
11/11
                  Os 5ms/step - loss:
0.0037 - val_loss: 0.0091
Epoch 12/100
11/11
                  Os 5ms/step - loss:
0.0031 - val_loss: 0.0110
Epoch 13/100
11/11
                  Os 6ms/step - loss:
0.0032 - val_loss: 0.0134
Epoch 14/100
11/11
                  Os 6ms/step - loss:
0.0033 - val_loss: 0.0136
Epoch 15/100
11/11
                  Os 6ms/step - loss:
0.0030 - val_loss: 0.0112
Epoch 16/100
11/11
                  Os 6ms/step - loss:
0.0029 - val_loss: 0.0142
Epoch 17/100
11/11
                  Os 5ms/step - loss:
0.0028 - val_loss: 0.0183
Epoch 18/100
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0153
```

```
Epoch 19/100
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0123
Epoch 20/100
11/11
                  Os 6ms/step - loss:
0.0022 - val_loss: 0.0165
Epoch 21/100
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0139
Epoch 22/100
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0178
Epoch 23/100
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0190
Epoch 24/100
11/11
                  Os 6ms/step - loss:
0.0017 - val_loss: 0.0193
Epoch 25/100
11/11
                  Os 5ms/step - loss:
0.0018 - val_loss: 0.0145
Epoch 26/100
11/11
                 Os 5ms/step - loss:
0.0016 - val_loss: 0.0185
Epoch 27/100
11/11
                  Os 5ms/step - loss:
0.0015 - val_loss: 0.0157
Epoch 28/100
11/11
                  Os 6ms/step - loss:
0.0014 - val_loss: 0.0148
Epoch 29/100
11/11
                  Os 5ms/step - loss:
0.0015 - val_loss: 0.0225
Epoch 30/100
11/11
                  Os 6ms/step - loss:
0.0018 - val_loss: 0.0327
Epoch 31/100
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0220
Epoch 32/100
11/11
                  Os 6ms/step - loss:
0.0016 - val_loss: 0.0165
Epoch 33/100
11/11
                  Os 5ms/step - loss:
0.0016 - val_loss: 0.0135
Epoch 34/100
11/11
                  Os 5ms/step - loss:
0.0015 - val_loss: 0.0113
```

```
Epoch 35/100
11/11
                  Os 5ms/step - loss:
0.0014 - val_loss: 0.0142
Epoch 36/100
11/11
                  Os 6ms/step - loss:
0.0013 - val_loss: 0.0179
Epoch 37/100
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0139
Epoch 38/100
11/11
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0193
Epoch 39/100
11/11
                  Os 6ms/step - loss:
0.0010 - val_loss: 0.0220
Epoch 40/100
11/11
                  Os 6ms/step - loss:
0.0012 - val_loss: 0.0170
Epoch 41/100
11/11
                  Os 5ms/step - loss:
0.0010 - val_loss: 0.0234
Epoch 42/100
11/11
                  Os 6ms/step - loss:
0.0010 - val_loss: 0.0177
Epoch 43/100
11/11
                  Os 6ms/step - loss:
9.3928e-04 - val_loss: 0.0169
Epoch 44/100
11/11
                  Os 5ms/step - loss:
9.2184e-04 - val_loss: 0.0147
Epoch 45/100
11/11
                  Os 5ms/step - loss:
9.0446e-04 - val_loss: 0.0165
Epoch 46/100
11/11
                  Os 5ms/step - loss:
8.2539e-04 - val_loss: 0.0229
Epoch 47/100
11/11
                  Os 6ms/step - loss:
9.7495e-04 - val_loss: 0.0204
Epoch 48/100
11/11
                  Os 6ms/step - loss:
8.0372e-04 - val_loss: 0.0168
Epoch 49/100
11/11
                  Os 5ms/step - loss:
8.4484e-04 - val_loss: 0.0209
Epoch 50/100
11/11
                  Os 5ms/step - loss:
8.8616e-04 - val_loss: 0.0166
```

```
Epoch 51/100
11/11
                  Os 5ms/step - loss:
7.4047e-04 - val_loss: 0.0236
Epoch 52/100
11/11
                  Os 5ms/step - loss:
8.5009e-04 - val_loss: 0.0192
Epoch 53/100
11/11
                  Os 5ms/step - loss:
7.2879e-04 - val_loss: 0.0185
Epoch 54/100
11/11
                  Os 5ms/step - loss:
7.9127e-04 - val_loss: 0.0128
Epoch 55/100
11/11
                  Os 5ms/step - loss:
8.8935e-04 - val_loss: 0.0227
Epoch 56/100
11/11
                  Os 6ms/step - loss:
9.2957e-04 - val_loss: 0.0199
Epoch 57/100
11/11
                  Os 5ms/step - loss:
8.7883e-04 - val_loss: 0.0160
Epoch 58/100
11/11
                  Os 5ms/step - loss:
7.5580e-04 - val_loss: 0.0185
Epoch 59/100
11/11
                  Os 5ms/step - loss:
7.1067e-04 - val_loss: 0.0156
Epoch 60/100
                  Os 5ms/step - loss:
11/11
7.2522e-04 - val_loss: 0.0205
Epoch 61/100
11/11
                  Os 5ms/step - loss:
8.2674e-04 - val_loss: 0.0189
Epoch 62/100
11/11
                  Os 5ms/step - loss:
6.5036e-04 - val_loss: 0.0197
Epoch 63/100
11/11
                  Os 5ms/step - loss:
6.9564e-04 - val_loss: 0.0183
Epoch 64/100
11/11
                  Os 5ms/step - loss:
6.2255e-04 - val_loss: 0.0171
Epoch 65/100
11/11
                  Os 5ms/step - loss:
5.9906e-04 - val_loss: 0.0214
Epoch 66/100
11/11
                  Os 5ms/step - loss:
8.2514e-04 - val_loss: 0.0187
```

```
Epoch 67/100
11/11
                  Os 5ms/step - loss:
6.3691e-04 - val_loss: 0.0169
Epoch 68/100
11/11
                  Os 5ms/step - loss:
6.1718e-04 - val_loss: 0.0200
Epoch 69/100
11/11
                  Os 5ms/step - loss:
7.2041e-04 - val_loss: 0.0137
Epoch 70/100
11/11
                  Os 5ms/step - loss:
5.4355e-04 - val_loss: 0.0150
Epoch 71/100
                  Os 5ms/step - loss:
11/11
6.8118e-04 - val_loss: 0.0210
Epoch 72/100
11/11
                  Os 5ms/step - loss:
6.4362e-04 - val_loss: 0.0150
Epoch 73/100
11/11
                  Os 5ms/step - loss:
5.1728e-04 - val_loss: 0.0180
Epoch 74/100
11/11
                  Os 5ms/step - loss:
5.9069e-04 - val_loss: 0.0150
Epoch 75/100
11/11
                  Os 6ms/step - loss:
6.2530e-04 - val_loss: 0.0156
Epoch 76/100
11/11
                  Os 7ms/step - loss:
6.8660e-04 - val_loss: 0.0191
Epoch 77/100
11/11
                  Os 5ms/step - loss:
7.4616e-04 - val_loss: 0.0155
Epoch 78/100
11/11
                  Os 6ms/step - loss:
6.4413e-04 - val_loss: 0.0148
Epoch 79/100
11/11
                  Os 5ms/step - loss:
6.1358e-04 - val_loss: 0.0135
Epoch 80/100
11/11
                  Os 5ms/step - loss:
7.2638e-04 - val_loss: 0.0192
Epoch 81/100
11/11
                  Os 6ms/step - loss:
7.8533e-04 - val_loss: 0.0151
Epoch 82/100
11/11
                  Os 5ms/step - loss:
5.2041e-04 - val_loss: 0.0171
```

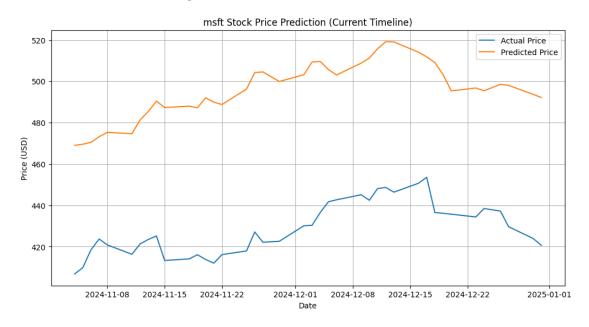
```
Epoch 83/100
11/11
                  Os 5ms/step - loss:
7.2240e-04 - val_loss: 0.0175
Epoch 84/100
11/11
                  Os 6ms/step - loss:
5.8323e-04 - val_loss: 0.0160
Epoch 85/100
11/11
                  Os 5ms/step - loss:
5.0497e-04 - val_loss: 0.0166
Epoch 86/100
11/11
                  Os 5ms/step - loss:
5.2343e-04 - val_loss: 0.0148
Epoch 87/100
                  Os 5ms/step - loss:
11/11
5.0654e-04 - val_loss: 0.0157
Epoch 88/100
11/11
                  Os 5ms/step - loss:
5.2454e-04 - val_loss: 0.0129
Epoch 89/100
11/11
                  Os 6ms/step - loss:
6.9461e-04 - val_loss: 0.0231
Epoch 90/100
11/11
                 Os 5ms/step - loss:
0.0012 - val_loss: 0.0110
Epoch 91/100
11/11
                  Os 5ms/step - loss:
5.7888e-04 - val_loss: 0.0132
Epoch 92/100
                  Os 5ms/step - loss:
11/11
5.1409e-04 - val_loss: 0.0161
Epoch 93/100
11/11
                  Os 5ms/step - loss:
5.7649e-04 - val_loss: 0.0121
Epoch 94/100
11/11
                  Os 6ms/step - loss:
5.1123e-04 - val_loss: 0.0159
Epoch 95/100
11/11
                  Os 5ms/step - loss:
5.0183e-04 - val_loss: 0.0128
Epoch 96/100
11/11
                  Os 5ms/step - loss:
5.9046e-04 - val_loss: 0.0188
Epoch 97/100
11/11
                  Os 5ms/step - loss:
4.9171e-04 - val_loss: 0.0155
Epoch 98/100
11/11
                  Os 5ms/step - loss:
6.5105e-04 - val_loss: 0.0158
```

Epoch 99/100

5.4192e-04 - val\_loss: 0.0127

Epoch 100/100

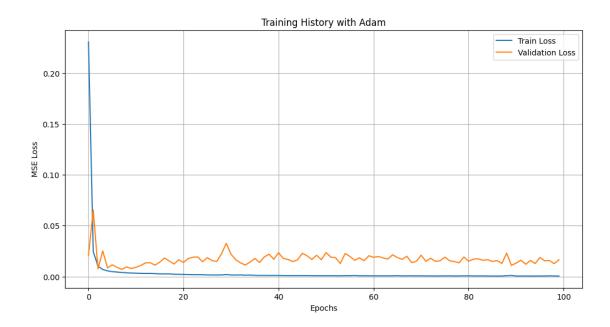
11/11 Os 5ms/step - loss:



Test RMSE: 69.76

Last Prediction Date: 2024-12-31 00:00:00-05:00

Latest Prediction: 492.15



## $7.2 \operatorname{sgd}$

```
[70]: import numpy as np
      import pandas as pd
      import yfinance as yf
      from sklearn.preprocessing import MinMaxScaler
      import keras
      import tensorflow as tf
      from keras import Sequential
      from keras import layers
      import matplotlib.pyplot as plt
      # Configuration
      n_lags = 30  # Use past 30 days of data to predict next day
      # Get full dataset
      full_data = get_prices(feature_stocks + [predict_stock], start='2023-01-01',__
       ⇔end='2025-01-01')
      # Create lagged features and targets
      X, y = [], []
      for i in range(n_lags, len(full_data)):
          # Use past n_lags days of feature stocks to predict current day's MSFT price
          X.append(full_data[feature_stocks].iloc[i-n_lags:i].values.flatten())
          y.append(full_data[predict_stock].iloc[i])
      X = np.array(X)
      y = np.array(y)
```

```
train_end = '2024-06-30'
val_end = '2024-11-01'
train_idx = full_data.index <= train_end</pre>
val_idx = (full_data.index > train_end) & (full_data.index <= val_end)</pre>
test_idx = full_data.index > val_end
X_train, y_train = X[train_idx[n_lags:]], y[train_idx[n_lags:]]
X_val, y_val = X[val_idx[n_lags:]], y[val_idx[n_lags:]]
X_test, y_test = X[test_idx[n_lags:]], y[test_idx[n_lags:]]
# Scale data
X_scaler = MinMaxScaler()
y_scaler = MinMaxScaler()
X_train_scaled = X_scaler.fit_transform(X_train)
X_val_scaled = X_scaler.transform(X_val)
X_test_scaled = X_scaler.transform(X_test)
y_train_scaled = y_scaler.fit_transform(y_train.reshape(-1, 1))
y_val_scaled = y_scaler.transform(y_val.reshape(-1, 1))
y_test_scaled = y_scaler.transform(y_test.reshape(-1, 1))
# Build model
model = Sequential([
    layers.Dense(20, activation='relu', input_shape=(X_train.shape[1],)),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(1, activation='linear')
])
model.compile(optimizer = 'sgd', loss='mse')
# Train with validation
history = model.fit(
    X_train_scaled,
    y_train_scaled,
    epochs=100,
    batch_size=32,
    validation_data=(X_val_scaled, y_val_scaled),
    verbose=1
)
# Predict on test set
```

```
y_pred_scaled = model.predict(X_test_scaled)
y_pred = y_scaler.inverse_transform(y_pred_scaled)
# Plot results
plt.figure(figsize=(12, 6))
plt.plot(full_data.index[-len(y_test):], y_test, label='Actual Price')
plt.plot(full_data index[-len(y_test):], y_pred, label='Predicted Price')
plt.title(f'{predict_stock} Stock Price Prediction (Current Timeline)')
plt.xlabel('Date')
plt.ylabel('Price (USD)')
plt.legend()
plt.grid(True)
plt.show()
# Calculate metrics
rmse = np.sqrt(np.mean((y_pred - y_test) ** 2))
print(f"Test RMSE: {rmse:.2f}")
print(f"Last Prediction Date: {full_data.index[-1]}")
print(f"Latest Prediction: {y_pred[-1][0]:.2f}")
plt.figure(figsize=(12, 6))
plt.plot(history.history['loss'], label='Train Loss')
plt.plot(history.history['val_loss'], label='Validation Loss')
plt.title('Training History with SGD')
plt.xlabel('Epochs')
plt.ylabel('MSE Loss')
plt.legend()
plt.grid(True)
plt.show()
Epoch 1/100
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-
packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an
`input_shape`/`input_dim` argument to a layer. When using Sequential models,
prefer using an `Input(shape)` object as the first layer in the model instead.
  super().__init__(activity_regularizer=activity_regularizer, **kwargs)
11/11
                  1s 55ms/step -
loss: 0.2269 - val_loss: 0.2068
Epoch 2/100
11/11
                 Os 6ms/step - loss:
0.0366 - val_loss: 0.0487
Epoch 3/100
11/11
                  Os 5ms/step - loss:
0.0140 - val_loss: 0.0293
Epoch 4/100
11/11
                  Os 5ms/step - loss:
0.0100 - val_loss: 0.0264
```

```
Epoch 5/100
11/11
                 Os 5ms/step - loss:
0.0097 - val_loss: 0.0250
Epoch 6/100
11/11
                  Os 5ms/step - loss:
0.0106 - val_loss: 0.0229
Epoch 7/100
11/11
                  Os 5ms/step - loss:
0.0084 - val_loss: 0.0248
Epoch 8/100
11/11
                  Os 6ms/step - loss:
0.0078 - val_loss: 0.0216
Epoch 9/100
11/11
                  Os 5ms/step - loss:
0.0072 - val_loss: 0.0230
Epoch 10/100
11/11
                  Os 6ms/step - loss:
0.0082 - val_loss: 0.0234
Epoch 11/100
11/11
                  Os 6ms/step - loss:
0.0081 - val_loss: 0.0234
Epoch 12/100
11/11
                 Os 6ms/step - loss:
0.0075 - val_loss: 0.0225
Epoch 13/100
11/11
                  Os 5ms/step - loss:
0.0069 - val_loss: 0.0227
Epoch 14/100
11/11
                  Os 5ms/step - loss:
0.0068 - val_loss: 0.0252
Epoch 15/100
11/11
                  Os 6ms/step - loss:
0.0066 - val_loss: 0.0229
Epoch 16/100
11/11
                  Os 6ms/step - loss:
0.0061 - val_loss: 0.0229
Epoch 17/100
11/11
                  Os 5ms/step - loss:
0.0063 - val_loss: 0.0216
Epoch 18/100
11/11
                  Os 5ms/step - loss:
0.0063 - val_loss: 0.0214
Epoch 19/100
11/11
                  Os 5ms/step - loss:
0.0057 - val_loss: 0.0212
Epoch 20/100
11/11
                  Os 5ms/step - loss:
0.0057 - val_loss: 0.0206
```

```
Epoch 21/100
11/11
                  Os 6ms/step - loss:
0.0054 - val_loss: 0.0217
Epoch 22/100
11/11
                  Os 5ms/step - loss:
0.0054 - val_loss: 0.0226
Epoch 23/100
11/11
                  Os 6ms/step - loss:
0.0054 - val_loss: 0.0223
Epoch 24/100
11/11
                  Os 6ms/step - loss:
0.0052 - val_loss: 0.0221
Epoch 25/100
11/11
                  Os 5ms/step - loss:
0.0056 - val_loss: 0.0205
Epoch 26/100
11/11
                  Os 5ms/step - loss:
0.0055 - val_loss: 0.0200
Epoch 27/100
11/11
                  Os 6ms/step - loss:
0.0049 - val_loss: 0.0198
Epoch 28/100
11/11
                 Os 5ms/step - loss:
0.0050 - val_loss: 0.0215
Epoch 29/100
11/11
                  Os 6ms/step - loss:
0.0048 - val_loss: 0.0197
Epoch 30/100
11/11
                  Os 5ms/step - loss:
0.0053 - val_loss: 0.0198
Epoch 31/100
11/11
                  Os 6ms/step - loss:
0.0048 - val_loss: 0.0206
Epoch 32/100
11/11
                  Os 6ms/step - loss:
0.0048 - val_loss: 0.0201
Epoch 33/100
11/11
                  Os 6ms/step - loss:
0.0044 - val_loss: 0.0210
Epoch 34/100
11/11
                  Os 6ms/step - loss:
0.0044 - val_loss: 0.0208
Epoch 35/100
11/11
                  Os 5ms/step - loss:
0.0044 - val_loss: 0.0185
Epoch 36/100
11/11
                  Os 6ms/step - loss:
0.0045 - val_loss: 0.0185
```

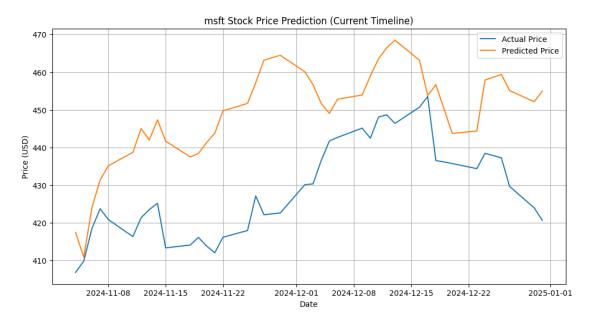
```
Epoch 37/100
11/11
                  Os 5ms/step - loss:
0.0045 - val_loss: 0.0182
Epoch 38/100
11/11
                  Os 6ms/step - loss:
0.0042 - val_loss: 0.0201
Epoch 39/100
11/11
                  Os 5ms/step - loss:
0.0045 - val_loss: 0.0188
Epoch 40/100
11/11
                  Os 6ms/step - loss:
0.0041 - val_loss: 0.0188
Epoch 41/100
11/11
                  Os 5ms/step - loss:
0.0044 - val_loss: 0.0176
Epoch 42/100
11/11
                  Os 6ms/step - loss:
0.0042 - val_loss: 0.0175
Epoch 43/100
11/11
                  Os 6ms/step - loss:
0.0041 - val_loss: 0.0167
Epoch 44/100
11/11
                  Os 6ms/step - loss:
0.0041 - val_loss: 0.0169
Epoch 45/100
11/11
                  Os 5ms/step - loss:
0.0042 - val_loss: 0.0164
Epoch 46/100
11/11
                  Os 6ms/step - loss:
0.0039 - val_loss: 0.0154
Epoch 47/100
11/11
                  Os 5ms/step - loss:
0.0039 - val_loss: 0.0155
Epoch 48/100
11/11
                  Os 6ms/step - loss:
0.0036 - val_loss: 0.0146
Epoch 49/100
11/11
                  Os 6ms/step - loss:
0.0042 - val_loss: 0.0153
Epoch 50/100
11/11
                  Os 5ms/step - loss:
0.0037 - val_loss: 0.0155
Epoch 51/100
11/11
                  Os 5ms/step - loss:
0.0038 - val_loss: 0.0160
Epoch 52/100
11/11
                  Os 5ms/step - loss:
0.0037 - val_loss: 0.0142
```

```
Epoch 53/100
11/11
                  Os 5ms/step - loss:
0.0039 - val_loss: 0.0158
Epoch 54/100
11/11
                  Os 5ms/step - loss:
0.0036 - val_loss: 0.0148
Epoch 55/100
11/11
                  Os 5ms/step - loss:
0.0034 - val_loss: 0.0138
Epoch 56/100
11/11
                  Os 5ms/step - loss:
0.0035 - val_loss: 0.0136
Epoch 57/100
11/11
                  Os 5ms/step - loss:
0.0039 - val_loss: 0.0146
Epoch 58/100
11/11
                  Os 6ms/step - loss:
0.0034 - val_loss: 0.0142
Epoch 59/100
11/11
                  Os 6ms/step - loss:
0.0034 - val_loss: 0.0132
Epoch 60/100
11/11
                  Os 5ms/step - loss:
0.0036 - val_loss: 0.0133
Epoch 61/100
11/11
                  Os 5ms/step - loss:
0.0034 - val_loss: 0.0132
Epoch 62/100
11/11
                  Os 5ms/step - loss:
0.0031 - val_loss: 0.0126
Epoch 63/100
11/11
                  Os 5ms/step - loss:
0.0032 - val_loss: 0.0136
Epoch 64/100
11/11
                  Os 5ms/step - loss:
0.0033 - val_loss: 0.0122
Epoch 65/100
11/11
                  Os 6ms/step - loss:
0.0035 - val_loss: 0.0125
Epoch 66/100
11/11
                  Os 6ms/step - loss:
0.0031 - val_loss: 0.0121
Epoch 67/100
11/11
                  Os 5ms/step - loss:
0.0032 - val_loss: 0.0121
Epoch 68/100
11/11
                  Os 5ms/step - loss:
0.0030 - val_loss: 0.0119
```

```
Epoch 69/100
11/11
                  Os 6ms/step - loss:
0.0032 - val_loss: 0.0123
Epoch 70/100
11/11
                  Os 6ms/step - loss:
0.0033 - val_loss: 0.0120
Epoch 71/100
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0113
Epoch 72/100
11/11
                  Os 6ms/step - loss:
0.0031 - val_loss: 0.0112
Epoch 73/100
11/11
                  Os 5ms/step - loss:
0.0029 - val_loss: 0.0108
Epoch 74/100
11/11
                  Os 6ms/step - loss:
0.0030 - val_loss: 0.0113
Epoch 75/100
11/11
                  Os 5ms/step - loss:
0.0031 - val_loss: 0.0108
Epoch 76/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0106
Epoch 77/100
11/11
                  Os 6ms/step - loss:
0.0028 - val_loss: 0.0110
Epoch 78/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0102
Epoch 79/100
11/11
                  Os 5ms/step - loss:
0.0029 - val_loss: 0.0101
Epoch 80/100
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0106
Epoch 81/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0106
Epoch 82/100
11/11
                  Os 6ms/step - loss:
0.0028 - val_loss: 0.0105
Epoch 83/100
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0101
Epoch 84/100
11/11
                  Os 7ms/step - loss:
0.0028 - val_loss: 0.0101
```

```
Epoch 85/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0103
Epoch 86/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0102
Epoch 87/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0097
Epoch 88/100
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0098
Epoch 89/100
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0094
Epoch 90/100
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0094
Epoch 91/100
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0096
Epoch 92/100
11/11
                  Os 6ms/step - loss:
0.0027 - val_loss: 0.0098
Epoch 93/100
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0092
Epoch 94/100
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0092
Epoch 95/100
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0090
Epoch 96/100
11/11
                  Os 6ms/step - loss:
0.0021 - val_loss: 0.0088
Epoch 97/100
11/11
                  Os 6ms/step - loss:
0.0024 - val_loss: 0.0089
Epoch 98/100
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0089
Epoch 99/100
11/11
                  Os 6ms/step - loss:
0.0024 - val_loss: 0.0090
Epoch 100/100
11/11
                  Os 7ms/step - loss:
0.0023 - val_loss: 0.0093
```

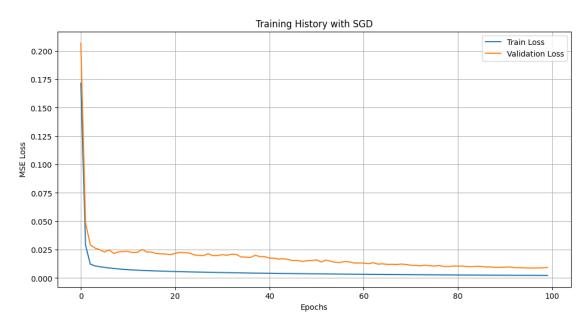
# 2/2 0s 105ms/step



Test RMSE: 26.86

Last Prediction Date: 2024-12-31 00:00:00-05:00

Latest Prediction: 454.95



### 7.3 Momentum

```
[71]: import numpy as np
      import pandas as pd
      import yfinance as yf
      from sklearn.preprocessing import MinMaxScaler
      import keras
      import tensorflow as tf
      from keras import Sequential
      from keras import layers
      import matplotlib.pyplot as plt
      # Configuration
      n_lags = 30  # Use past 30 days of data to predict next day
      # Get full dataset
      full_data = get_prices(feature_stocks + [predict_stock], start='2023-01-01',__
       ⇔end='2025-01-01')
      # Create lagged features and targets
      X, y = [], []
      for i in range(n_lags, len(full_data)):
          # Use past n_lags days of feature stocks to predict current day's MSFT price
          X.append(full_data[feature_stocks].iloc[i-n_lags:i].values.flatten())
          y.append(full_data[predict_stock].iloc[i])
      X = np.array(X)
      y = np.array(y)
      train end = '2024-06-30'
      val_end = '2024-11-01'
      train_idx = full_data.index <= train_end</pre>
      val_idx = (full_data.index > train_end) & (full_data.index <= val_end)</pre>
      test_idx = full_data.index > val_end
      X_train, y_train = X[train_idx[n_lags:]], y[train_idx[n_lags:]]
      X_val, y_val = X[val_idx[n_lags:]], y[val_idx[n_lags:]]
      X_test, y_test = X[test_idx[n_lags:]], y[test_idx[n_lags:]]
      # Scale data
      X_scaler = MinMaxScaler()
      y_scaler = MinMaxScaler()
      X_train_scaled = X_scaler.fit_transform(X_train)
      X_val_scaled = X_scaler.transform(X_val)
      X test scaled = X scaler.transform(X test)
```

```
y_train_scaled = y_scaler.fit_transform(y_train.reshape(-1, 1))
y_val_scaled = y_scaler.transform(y_val.reshape(-1, 1))
y_test_scaled = y_scaler.transform(y_test.reshape(-1, 1))
# Build model
model = Sequential([
   layers.Dense(20, activation='relu', input_shape=(X_train.shape[1],)),
   layers.Dense(20, activation='relu'),
   layers.Dense(20, activation='relu'),
   layers.Dense(20, activation='relu'),
   layers.Dense(20, activation='relu'),
   layers.Dense(1, activation='linear')
])
optimizer = keras.optimizers.SGD(learning_rate=0.001, momentum=0.95, nesterov=
 →False)
model.compile(optimizer = optimizer, loss='mse')
# Train with validation
history = model.fit(
   X train scaled,
   y_train_scaled,
   epochs=150,
   batch_size=32,
   validation_data=(X_val_scaled, y_val_scaled),
   verbose=1
)
# Predict on test set
y_pred_scaled = model.predict(X_test_scaled)
y_pred = y_scaler.inverse_transform(y_pred_scaled)
# Plot results
plt.figure(figsize=(12, 6))
plt.plot(full_data.index[-len(y_test):], y_test, label='Actual Price')
plt.plot(full_data.index[-len(y_test):], y_pred, label='Predicted Price')
plt.title(f'{predict_stock} Stock Price Prediction (Current Timeline)')
plt.xlabel('Date')
plt.ylabel('Price (USD)')
plt.legend()
plt.grid(True)
plt.show()
# Calculate metrics
rmse = np.sqrt(np.mean((y_pred - y_test) ** 2))
print(f"Test RMSE: {rmse:.2f}")
print(f"Last Prediction Date: {full_data.index[-1]}")
```

```
print(f"Latest Prediction: {y_pred[-1][0]:.2f}")
plt.figure(figsize=(12, 6))
plt.plot(history.history['loss'], label='Train Loss')
plt.plot(history.history['val_loss'], label='Validation Loss')
plt.title('Training History with Momentum')
plt.xlabel('Epochs')
plt.ylabel('MSE Loss')
plt.legend()
plt.grid(True)
plt.show()
```

0.0060 - val\_loss: 0.0089

```
Epoch 1/150
/home/barrytan/miniconda3/envs/ANN/lib/python3.12/site-
packages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an
`input_shape`/`input_dim` argument to a layer. When using Sequential models,
prefer using an `Input(shape)` object as the first layer in the model instead.
  super().__init__(activity_regularizer=activity_regularizer, **kwargs)
                  1s 57ms/step -
loss: 0.3100 - val_loss: 0.2150
Epoch 2/150
11/11
                  Os 6ms/step - loss:
0.0770 - val_loss: 0.0071
Epoch 3/150
11/11
                  Os 6ms/step - loss:
0.0249 - val_loss: 0.0574
Epoch 4/150
11/11
                  Os 6ms/step - loss:
0.0335 - val_loss: 0.0090
Epoch 5/150
11/11
                  Os 6ms/step - loss:
0.0125 - val_loss: 0.0474
Epoch 6/150
11/11
                  Os 6ms/step - loss:
0.0195 - val_loss: 0.0201
Epoch 7/150
11/11
                  Os 6ms/step - loss:
0.0097 - val_loss: 0.0050
Epoch 8/150
11/11
                  Os 6ms/step - loss:
0.0094 - val_loss: 0.0049
Epoch 9/150
11/11
                  Os 6ms/step - loss:
0.0078 - val_loss: 0.0067
Epoch 10/150
11/11
                  Os 5ms/step - loss:
```

```
Epoch 11/150
11/11
                  Os 6ms/step - loss:
0.0064 - val_loss: 0.0051
Epoch 12/150
11/11
                  Os 6ms/step - loss:
0.0047 - val_loss: 0.0041
Epoch 13/150
11/11
                  Os 10ms/step -
loss: 0.0048 - val_loss: 0.0042
Epoch 14/150
11/11
                  Os 6ms/step - loss:
0.0051 - val_loss: 0.0049
Epoch 15/150
11/11
                  Os 5ms/step - loss:
0.0051 - val_loss: 0.0047
Epoch 16/150
11/11
                  Os 5ms/step - loss:
0.0047 - val_loss: 0.0043
Epoch 17/150
11/11
                  Os 6ms/step - loss:
0.0045 - val_loss: 0.0044
Epoch 18/150
11/11
                  Os 5ms/step - loss:
0.0040 - val_loss: 0.0045
Epoch 19/150
11/11
                  Os 5ms/step - loss:
0.0044 - val_loss: 0.0048
Epoch 20/150
11/11
                  Os 5ms/step - loss:
0.0042 - val_loss: 0.0046
Epoch 21/150
11/11
                  Os 5ms/step - loss:
0.0037 - val_loss: 0.0046
Epoch 22/150
11/11
                  Os 5ms/step - loss:
0.0046 - val_loss: 0.0049
Epoch 23/150
11/11
                  Os 5ms/step - loss:
0.0036 - val_loss: 0.0048
Epoch 24/150
11/11
                  Os 5ms/step - loss:
0.0042 - val_loss: 0.0047
Epoch 25/150
11/11
                  Os 6ms/step - loss:
0.0033 - val_loss: 0.0047
Epoch 26/150
11/11
                  Os 5ms/step - loss:
0.0037 - val_loss: 0.0047
```

```
Epoch 27/150
11/11
                  Os 5ms/step - loss:
0.0035 - val_loss: 0.0048
Epoch 28/150
11/11
                  Os 6ms/step - loss:
0.0034 - val_loss: 0.0047
Epoch 29/150
11/11
                  Os 6ms/step - loss:
0.0031 - val_loss: 0.0047
Epoch 30/150
11/11
                  Os 6ms/step - loss:
0.0032 - val_loss: 0.0048
Epoch 31/150
11/11
                  Os 5ms/step - loss:
0.0034 - val_loss: 0.0048
Epoch 32/150
11/11
                  Os 5ms/step - loss:
0.0037 - val_loss: 0.0048
Epoch 33/150
11/11
                  Os 6ms/step - loss:
0.0037 - val_loss: 0.0048
Epoch 34/150
11/11
                  Os 5ms/step - loss:
0.0033 - val_loss: 0.0048
Epoch 35/150
11/11
                  Os 5ms/step - loss:
0.0035 - val_loss: 0.0048
Epoch 36/150
11/11
                  Os 6ms/step - loss:
0.0034 - val_loss: 0.0048
Epoch 37/150
11/11
                  Os 5ms/step - loss:
0.0034 - val_loss: 0.0048
Epoch 38/150
11/11
                  Os 5ms/step - loss:
0.0035 - val_loss: 0.0048
Epoch 39/150
11/11
                  Os 5ms/step - loss:
0.0029 - val_loss: 0.0048
Epoch 40/150
11/11
                  Os 5ms/step - loss:
0.0032 - val_loss: 0.0048
Epoch 41/150
11/11
                  Os 5ms/step - loss:
0.0031 - val_loss: 0.0048
Epoch 42/150
11/11
                  Os 6ms/step - loss:
0.0033 - val_loss: 0.0048
```

```
Epoch 43/150
11/11
                  Os 5ms/step - loss:
0.0030 - val_loss: 0.0049
Epoch 44/150
11/11
                  Os 5ms/step - loss:
0.0034 - val_loss: 0.0048
Epoch 45/150
11/11
                  Os 5ms/step - loss:
0.0032 - val_loss: 0.0048
Epoch 46/150
11/11
                  Os 6ms/step - loss:
0.0030 - val_loss: 0.0048
Epoch 47/150
11/11
                  Os 6ms/step - loss:
0.0036 - val_loss: 0.0048
Epoch 48/150
11/11
                  Os 6ms/step - loss:
0.0029 - val_loss: 0.0049
Epoch 49/150
11/11
                  Os 5ms/step - loss:
0.0028 - val_loss: 0.0048
Epoch 50/150
11/11
                  Os 5ms/step - loss:
0.0032 - val_loss: 0.0048
Epoch 51/150
11/11
                  Os 5ms/step - loss:
0.0031 - val_loss: 0.0048
Epoch 52/150
11/11
                  Os 5ms/step - loss:
0.0028 - val_loss: 0.0050
Epoch 53/150
11/11
                  Os 5ms/step - loss:
0.0032 - val_loss: 0.0048
Epoch 54/150
11/11
                  Os 5ms/step - loss:
0.0032 - val_loss: 0.0048
Epoch 55/150
11/11
                  Os 5ms/step - loss:
0.0028 - val_loss: 0.0050
Epoch 56/150
11/11
                  Os 5ms/step - loss:
0.0031 - val_loss: 0.0049
Epoch 57/150
11/11
                  Os 5ms/step - loss:
0.0030 - val_loss: 0.0049
Epoch 58/150
11/11
                  Os 5ms/step - loss:
0.0033 - val_loss: 0.0049
```

```
Epoch 59/150
11/11
                  Os 5ms/step - loss:
0.0028 - val_loss: 0.0049
Epoch 60/150
11/11
                  Os 5ms/step - loss:
0.0029 - val_loss: 0.0050
Epoch 61/150
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0049
Epoch 62/150
11/11
                  Os 5ms/step - loss:
0.0029 - val_loss: 0.0049
Epoch 63/150
11/11
                  Os 5ms/step - loss:
0.0028 - val_loss: 0.0050
Epoch 64/150
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0050
Epoch 65/150
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0050
Epoch 66/150
11/11
                  Os 5ms/step - loss:
0.0029 - val_loss: 0.0050
Epoch 67/150
11/11
                  Os 5ms/step - loss:
0.0027 - val_loss: 0.0052
Epoch 68/150
11/11
                  Os 5ms/step - loss:
0.0028 - val_loss: 0.0051
Epoch 69/150
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0051
Epoch 70/150
11/11
                  Os 6ms/step - loss:
0.0028 - val_loss: 0.0053
Epoch 71/150
11/11
                  Os 5ms/step - loss:
0.0027 - val_loss: 0.0052
Epoch 72/150
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0052
Epoch 73/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0051
Epoch 74/150
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0054
```

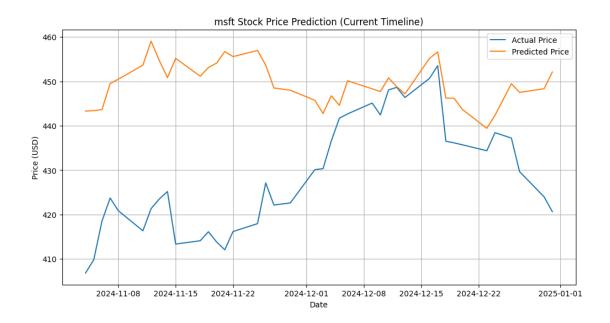
```
Epoch 75/150
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0056
Epoch 76/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0050
Epoch 77/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0052
Epoch 78/150
11/11
                  Os 5ms/step - loss:
0.0027 - val_loss: 0.0057
Epoch 79/150
11/11
                  Os 5ms/step - loss:
0.0029 - val_loss: 0.0054
Epoch 80/150
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0053
Epoch 81/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0055
Epoch 82/150
11/11
                 Os 5ms/step - loss:
0.0026 - val_loss: 0.0056
Epoch 83/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0051
Epoch 84/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0056
Epoch 85/150
11/11
                  Os 6ms/step - loss:
0.0026 - val_loss: 0.0055
Epoch 86/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0055
Epoch 87/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0054
Epoch 88/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0053
Epoch 89/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0059
Epoch 90/150
11/11
                  Os 5ms/step - loss:
0.0027 - val_loss: 0.0054
```

```
Epoch 91/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0054
Epoch 92/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0053
Epoch 93/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0054
Epoch 94/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0056
Epoch 95/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0051
Epoch 96/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0054
Epoch 97/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0051
Epoch 98/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0052
Epoch 99/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0053
Epoch 100/150
11/11
                  Os 5ms/step - loss:
0.0024 - val_loss: 0.0050
Epoch 101/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0048
Epoch 102/150
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0049
Epoch 103/150
11/11
                  Os 5ms/step - loss:
0.0022 - val_loss: 0.0051
Epoch 104/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0049
Epoch 105/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0047
Epoch 106/150
11/11
                  Os 6ms/step - loss:
0.0021 - val_loss: 0.0048
```

```
Epoch 107/150
11/11
                  Os 6ms/step - loss:
0.0024 - val_loss: 0.0051
Epoch 108/150
11/11
                  Os 5ms/step - loss:
0.0022 - val_loss: 0.0048
Epoch 109/150
11/11
                  Os 5ms/step - loss:
0.0022 - val_loss: 0.0047
Epoch 110/150
11/11
                  Os 6ms/step - loss:
0.0023 - val_loss: 0.0049
Epoch 111/150
11/11
                  Os 5ms/step - loss:
0.0022 - val_loss: 0.0049
Epoch 112/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0050
Epoch 113/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0050
Epoch 114/150
11/11
                  Os 6ms/step - loss:
0.0022 - val_loss: 0.0050
Epoch 115/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0049
Epoch 116/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0050
Epoch 117/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0054
Epoch 118/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0051
Epoch 119/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0050
Epoch 120/150
11/11
                  Os 5ms/step - loss:
0.0022 - val_loss: 0.0053
Epoch 121/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0051
Epoch 122/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0054
```

```
Epoch 123/150
11/11
                  Os 5ms/step - loss:
0.0023 - val_loss: 0.0053
Epoch 124/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0052
Epoch 125/150
11/11
                  Os 6ms/step - loss:
0.0022 - val_loss: 0.0054
Epoch 126/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0055
Epoch 127/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0053
Epoch 128/150
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0056
Epoch 129/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0054
Epoch 130/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0053
Epoch 131/150
11/11
                  Os 5ms/step - loss:
0.0018 - val_loss: 0.0057
Epoch 132/150
11/11
                  Os 6ms/step - loss:
0.0020 - val_loss: 0.0055
Epoch 133/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0057
Epoch 134/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0055
Epoch 135/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0057
Epoch 136/150
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0055
Epoch 137/150
                  Os 5ms/step - loss:
11/11
0.0020 - val_loss: 0.0057
Epoch 138/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0060
```

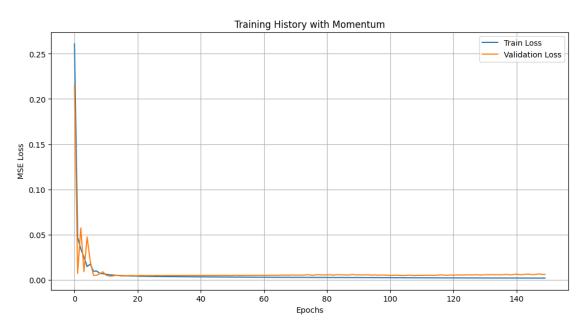
```
Epoch 139/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0054
Epoch 140/150
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0058
Epoch 141/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0063
Epoch 142/150
11/11
                  Os 5ms/step - loss:
0.0017 - val_loss: 0.0056
Epoch 143/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0056
Epoch 144/150
11/11
                  Os 5ms/step - loss:
0.0018 - val_loss: 0.0063
Epoch 145/150
11/11
                  Os 5ms/step - loss:
0.0019 - val_loss: 0.0061
Epoch 146/150
11/11
                  Os 5ms/step - loss:
0.0019 - val_loss: 0.0057
Epoch 147/150
11/11
                  Os 5ms/step - loss:
0.0018 - val_loss: 0.0059
Epoch 148/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0065
Epoch 149/150
11/11
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0060
Epoch 150/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0060
2/2
                0s 102ms/step
```



Test RMSE: 24.72

Last Prediction Date: 2024-12-31 00:00:00-05:00

Latest Prediction: 452.12



## 8 d. learning rate schedulers

```
[]: import numpy as np
     import pandas as pd
     import yfinance as yf
     from sklearn.preprocessing import MinMaxScaler
     import keras
     import tensorflow as tf
     from keras import Sequential
     from keras import layers
     import matplotlib.pyplot as plt
     class CyclicLR(keras.callbacks.Callback):
         def __init__(self, base_lr=0.001, max_lr=0.006, step_size=2000.):
             super().__init__()
             self.base_lr = base_lr
             self.max_lr = max_lr
             self.step_size = step_size
             self.trn_iterations = 0
         def clr(self):
             cycle = np.floor(1 + self.trn_iterations / (2 * self.step_size))
             x = np.abs(self.trn_iterations / self.step_size - 2 * cycle + 1)
             return self.base_lr + (self.max_lr - self.base_lr) * np.maximum(0, (1 - _ _
      →x))
         def on_epoch_end(self, epoch, logs=None):
             logs = logs or {}
             logs['lr'] = self.model.optimizer.learning_rate.numpy()
         def on_batch_end(self, batch, logs={}):
             self.trn iterations += 1
             self._update_lr(self.clr())
         def _update_lr(self, lr):
             self.model.optimizer.learning_rate.assign(lr)
     class OneCycleLR(keras.callbacks.Callback):
         def __init__(self, max_lr=0.006, total_steps=10000, pct_start=0.3):
             super().__init__()
             self.max_lr = max_lr
             self.total_steps = total_steps
             self.pct_start = pct_start
             self.step_num = 0
```

```
def on_epoch_end(self, epoch, logs=None):
        logs = logs or {}
        logs['lr'] = self.model.optimizer.learning_rate.numpy()
    def on_train_begin(self, logs=None):
        self.anneal_steps = int(self.total_steps * self.pct_start)
        self.low lr = self.max lr / 10
        self.high_lr = self.max_lr
    def on_train_batch_begin(self, batch, logs=None):
        if self.step num < self.anneal steps:</pre>
            lr = self.low_lr + (self.high_lr - self.low_lr) * (self.step_num /__
 ⇒self.anneal steps)
        else:
            lr = self.high_lr - (self.high_lr - self.low_lr) * (
                (self.step_num - self.anneal_steps) / (self.total_steps - self.
 →anneal_steps))
        self.model.optimizer.learning_rate.assign(lr)
        self.step_num += 1
class LRSchedulerLogger(keras.callbacks.Callback):
    def on_epoch_end(self, epoch, logs=None):
        logs = logs or {}
        logs['lr'] = self.model.optimizer.learning_rate.numpy()
# Configuration
n_lags = 30  # Use past 30 days of data to predict next day
# Get full dataset
full_data = get_prices(feature_stocks + [predict_stock], start='2023-01-01',__
\Rightarrowend='2025-01-01')
# Create lagged features and targets
X, y = [], []
for i in range(n_lags, len(full_data)):
    # Use past n_lags days of feature stocks to predict current day's MSFT price
    X.append(full_data[feature_stocks].iloc[i-n_lags:i].values.flatten())
    y.append(full_data[predict_stock].iloc[i])
X = np.array(X)
y = np.array(y)
train end = '2024-06-30'
val_end = '2024-11-01'
train_idx = full_data.index <= train_end</pre>
```

```
val_idx = (full_data.index > train_end) & (full_data.index <= val_end)</pre>
test_idx = full_data.index > val_end
X_train, y_train = X[train_idx[n_lags:]], y[train_idx[n_lags:]]
X_val, y_val = X[val_idx[n_lags:]], y[val_idx[n_lags:]]
X_test, y_test = X[test_idx[n_lags:]], y[test_idx[n_lags:]]
# Scale data
X scaler = MinMaxScaler()
y_scaler = MinMaxScaler()
X_train_scaled = X_scaler.fit_transform(X_train)
X_val_scaled = X_scaler.transform(X_val)
X_test_scaled = X_scaler.transform(X_test)
y_train_scaled = y_scaler.fit_transform(y_train.reshape(-1, 1))
y_val_scaled = y_scaler.transform(y_val.reshape(-1, 1))
y_test_scaled = y_scaler.transform(y_test.reshape(-1, 1))
# Build model
model = Sequential([
    layers.Dense(20, activation='relu', input_shape=(X_train.shape[1],)),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(20, activation='relu'),
    layers.Dense(1, activation='linear')
])
# Configure optimizer
optimizer = keras.optimizers.SGD(learning_rate=0.001, momentum=0.95)
model.compile(optimizer=optimizer, loss='mse')
reduce_lr = keras.callbacks.ReduceLROnPlateau(
    monitor='val_loss',
    factor=0.2,
    patience=5,
    min_lr=1e-6,
    verbose=1
)
cyclic_lr = CyclicLR(
    base lr=0.001,
    max_lr=0.006,
    step_size=2000
)
```

```
onecycle_lr = OneCycleLR(
    max_lr=0.006,
    total_steps=150*len(X_train_scaled)//32 # epochs * steps_per_epoch
)
```

/home/barrytan/miniconda3/envs/ANN/lib/python3.12/sitepackages/keras/src/layers/core/dense.py:87: UserWarning: Do not pass an
`input\_shape`/`input\_dim` argument to a layer. When using Sequential models,
prefer using an `Input(shape)` object as the first layer in the model instead.
super().\_\_init\_\_(activity\_regularizer=activity\_regularizer, \*\*kwargs)

### 8.0.1 OneCycleLR

```
[73]: # Train with chosen scheduler
      history = model.fit(
          X_train_scaled,
          y_train_scaled,
          epochs=150,
          batch_size=32,
          validation_data=(X_val_scaled, y_val_scaled),
          callbacks=[onecycle_lr],
          verbose=1
      )
      # After training, plot using epoch-level LR values
      plt.figure(figsize=(12, 6))
      plt.plot(history.history['lr'], label='Learning Rate')
      plt.title('Learning Rate Schedule(OneCycleLR)')
      plt.xlabel('Epochs')
      plt.ylabel('LR Value')
      plt.show()
```

```
Epoch 1/150
11/11
                 4s 355ms/step -
loss: 0.3423 - val_loss: 0.5211 - lr: 7.1180e-04
Epoch 2/150
11/11
                 Os 6ms/step - loss:
0.2191 - val loss: 0.2544 - lr: 8.3478e-04
Epoch 3/150
11/11
                 Os 6ms/step - loss:
0.0794 - val_loss: 0.0444 - lr: 9.5776e-04
Epoch 4/150
11/11
                 Os 6ms/step - loss:
0.0287 - val_loss: 0.0089 - lr: 0.0011
Epoch 5/150
11/11
                  Os 6ms/step - loss:
0.0415 - val_loss: 0.0068 - lr: 0.0012
Epoch 6/150
```

```
11/11
                  Os 6ms/step - loss:
0.0280 - val_loss: 0.0234 - lr: 0.0013
Epoch 7/150
11/11
                  Os 7ms/step - loss:
0.0239 - val_loss: 0.0350 - lr: 0.0014
Epoch 8/150
11/11
                  Os 6ms/step - loss:
0.0201 - val_loss: 0.0148 - lr: 0.0016
Epoch 9/150
11/11
                  Os 6ms/step - loss:
0.0146 - val_loss: 0.0066 - lr: 0.0017
Epoch 10/150
11/11
                  Os 6ms/step - loss:
0.0154 - val_loss: 0.0067 - lr: 0.0018
Epoch 11/150
11/11
                  Os 6ms/step - loss:
0.0120 - val_loss: 0.0087 - lr: 0.0019
Epoch 12/150
11/11
                  Os 6ms/step - loss:
0.0093 - val_loss: 0.0067 - lr: 0.0021
Epoch 13/150
11/11
                  Os 6ms/step - loss:
0.0076 - val_loss: 0.0059 - lr: 0.0022
Epoch 14/150
11/11
                  Os 6ms/step - loss:
0.0078 - val_loss: 0.0066 - lr: 0.0023
Epoch 15/150
11/11
                  Os 6ms/step - loss:
0.0066 - val_loss: 0.0066 - lr: 0.0024
Epoch 16/150
11/11
                  Os 6ms/step - loss:
0.0059 - val_loss: 0.0067 - lr: 0.0026
Epoch 17/150
11/11
                  Os 6ms/step - loss:
0.0047 - val loss: 0.0069 - lr: 0.0027
Epoch 18/150
11/11
                  Os 8ms/step - loss:
0.0051 - val_loss: 0.0067 - lr: 0.0028
Epoch 19/150
11/11
                  Os 8ms/step - loss:
0.0047 - val_loss: 0.0065 - lr: 0.0029
Epoch 20/150
                  Os 6ms/step - loss:
0.0040 - val_loss: 0.0066 - lr: 0.0030
Epoch 21/150
11/11
                  Os 6ms/step - loss:
0.0036 - val_loss: 0.0066 - lr: 0.0032
Epoch 22/150
```

```
Os 6ms/step - loss:
11/11
0.0041 - val_loss: 0.0066 - lr: 0.0033
Epoch 23/150
11/11
                  Os 6ms/step - loss:
0.0034 - val_loss: 0.0066 - lr: 0.0034
Epoch 24/150
11/11
                  Os 6ms/step - loss:
0.0032 - val_loss: 0.0067 - lr: 0.0035
Epoch 25/150
11/11
                  Os 5ms/step - loss:
0.0034 - val_loss: 0.0068 - lr: 0.0037
Epoch 26/150
11/11
                  Os 5ms/step - loss:
0.0030 - val_loss: 0.0068 - lr: 0.0038
Epoch 27/150
11/11
                  Os 6ms/step - loss:
0.0028 - val_loss: 0.0068 - lr: 0.0039
Epoch 28/150
11/11
                  Os 6ms/step - loss:
0.0030 - val_loss: 0.0070 - lr: 0.0040
Epoch 29/150
11/11
                  Os 6ms/step - loss:
0.0029 - val_loss: 0.0070 - lr: 0.0042
Epoch 30/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0071 - lr: 0.0043
Epoch 31/150
11/11
                  Os 5ms/step - loss:
0.0027 - val_loss: 0.0072 - lr: 0.0044
Epoch 32/150
11/11
                  Os 5ms/step - loss:
0.0026 - val_loss: 0.0074 - lr: 0.0045
Epoch 33/150
11/11
                  Os 6ms/step - loss:
0.0027 - val loss: 0.0072 - lr: 0.0046
Epoch 34/150
11/11
                  Os 6ms/step - loss:
0.0024 - val_loss: 0.0076 - lr: 0.0048
Epoch 35/150
11/11
                  Os 5ms/step - loss:
0.0025 - val_loss: 0.0073 - lr: 0.0049
Epoch 36/150
                  Os 5ms/step - loss:
0.0021 - val_loss: 0.0074 - lr: 0.0050
Epoch 37/150
11/11
                  Os 6ms/step - loss:
0.0025 - val_loss: 0.0074 - lr: 0.0051
Epoch 38/150
```

```
11/11
                  Os 5ms/step - loss:
0.0022 - val_loss: 0.0075 - lr: 0.0053
Epoch 39/150
11/11
                  Os 6ms/step - loss:
0.0021 - val_loss: 0.0081 - lr: 0.0054
Epoch 40/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0074 - lr: 0.0055
Epoch 41/150
11/11
                  Os 6ms/step - loss:
0.0021 - val_loss: 0.0077 - lr: 0.0056
Epoch 42/150
11/11
                  Os 6ms/step - loss:
0.0020 - val_loss: 0.0086 - lr: 0.0058
Epoch 43/150
11/11
                  Os 5ms/step - loss:
0.0020 - val_loss: 0.0072 - lr: 0.0059
Epoch 44/150
11/11
                  Os 6ms/step - loss:
0.0019 - val_loss: 0.0096 - lr: 0.0060
Epoch 45/150
11/11
                  Os 5ms/step - loss:
0.0019 - val_loss: 0.0075 - lr: 0.0059
Epoch 46/150
11/11
                  Os 5ms/step - loss:
0.0018 - val_loss: 0.0091 - lr: 0.0059
Epoch 47/150
11/11
                  Os 5ms/step - loss:
0.0018 - val_loss: 0.0079 - lr: 0.0058
Epoch 48/150
                  Os 5ms/step - loss:
11/11
0.0017 - val_loss: 0.0090 - lr: 0.0058
Epoch 49/150
11/11
                  Os 6ms/step - loss:
0.0018 - val loss: 0.0084 - lr: 0.0057
Epoch 50/150
11/11
                  Os 5ms/step - loss:
0.0016 - val_loss: 0.0093 - lr: 0.0057
Epoch 51/150
11/11
                  Os 5ms/step - loss:
0.0017 - val_loss: 0.0085 - lr: 0.0056
Epoch 52/150
                  Os 5ms/step - loss:
0.0016 - val_loss: 0.0093 - lr: 0.0056
Epoch 53/150
                  Os 5ms/step - loss:
0.0016 - val_loss: 0.0087 - lr: 0.0055
Epoch 54/150
```

```
Os 5ms/step - loss:
11/11
0.0017 - val_loss: 0.0095 - lr: 0.0055
Epoch 55/150
11/11
                  Os 6ms/step - loss:
0.0016 - val_loss: 0.0093 - lr: 0.0054
Epoch 56/150
11/11
                  Os 6ms/step - loss:
0.0016 - val_loss: 0.0091 - lr: 0.0054
Epoch 57/150
11/11
                  Os 5ms/step - loss:
0.0016 - val_loss: 0.0099 - lr: 0.0053
Epoch 58/150
11/11
                  Os 6ms/step - loss:
0.0016 - val_loss: 0.0093 - lr: 0.0053
Epoch 59/150
11/11
                  Os 5ms/step - loss:
0.0017 - val_loss: 0.0102 - lr: 0.0052
Epoch 60/150
11/11
                  Os 6ms/step - loss:
0.0014 - val_loss: 0.0093 - lr: 0.0052
Epoch 61/150
11/11
                  Os 5ms/step - loss:
0.0015 - val_loss: 0.0104 - lr: 0.0051
Epoch 62/150
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0098 - lr: 0.0051
Epoch 63/150
11/11
                  Os 6ms/step - loss:
0.0016 - val_loss: 0.0097 - lr: 0.0050
Epoch 64/150
                  Os 7ms/step - loss:
11/11
0.0016 - val_loss: 0.0111 - lr: 0.0049
Epoch 65/150
11/11
                  Os 5ms/step - loss:
0.0015 - val loss: 0.0094 - lr: 0.0049
Epoch 66/150
11/11
                  Os 6ms/step - loss:
0.0017 - val_loss: 0.0122 - lr: 0.0048
Epoch 67/150
11/11
                  Os 6ms/step - loss:
0.0014 - val_loss: 0.0095 - lr: 0.0048
Epoch 68/150
                  Os 6ms/step - loss:
0.0014 - val_loss: 0.0108 - lr: 0.0047
Epoch 69/150
                  Os 8ms/step - loss:
0.0014 - val_loss: 0.0108 - lr: 0.0047
Epoch 70/150
```

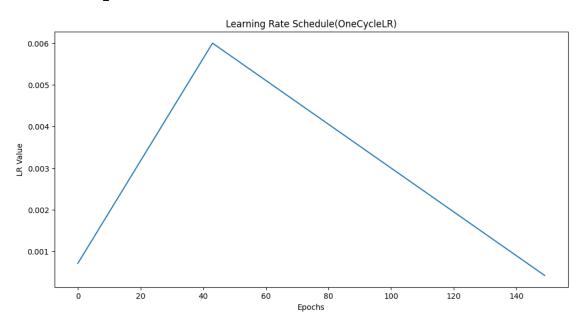
```
11/11
                  Os 7ms/step - loss:
0.0013 - val_loss: 0.0100 - lr: 0.0046
Epoch 71/150
11/11
                  Os 6ms/step - loss:
0.0015 - val_loss: 0.0119 - lr: 0.0046
Epoch 72/150
11/11
                  Os 5ms/step - loss:
0.0014 - val_loss: 0.0106 - lr: 0.0045
Epoch 73/150
11/11
                  Os 6ms/step - loss:
0.0014 - val_loss: 0.0118 - lr: 0.0045
Epoch 74/150
11/11
                  Os 5ms/step - loss:
0.0013 - val_loss: 0.0112 - lr: 0.0044
Epoch 75/150
11/11
                  Os 5ms/step - loss:
0.0013 - val_loss: 0.0117 - lr: 0.0044
Epoch 76/150
11/11
                  Os 5ms/step - loss:
0.0013 - val_loss: 0.0115 - lr: 0.0043
Epoch 77/150
11/11
                  Os 5ms/step - loss:
0.0014 - val_loss: 0.0123 - lr: 0.0043
Epoch 78/150
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0110 - lr: 0.0042
Epoch 79/150
11/11
                  Os 5ms/step - loss:
0.0013 - val_loss: 0.0125 - lr: 0.0042
Epoch 80/150
11/11
                  Os 5ms/step - loss:
0.0013 - val_loss: 0.0109 - lr: 0.0041
Epoch 81/150
11/11
                  Os 5ms/step - loss:
0.0012 - val loss: 0.0121 - lr: 0.0041
Epoch 82/150
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0129 - lr: 0.0040
Epoch 83/150
11/11
                  Os 5ms/step - loss:
0.0015 - val_loss: 0.0116 - lr: 0.0039
Epoch 84/150
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0131 - lr: 0.0039
Epoch 85/150
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0116 - lr: 0.0038
Epoch 86/150
```

```
11/11
                  Os 5ms/step - loss:
0.0013 - val_loss: 0.0133 - lr: 0.0038
Epoch 87/150
11/11
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0122 - lr: 0.0037
Epoch 88/150
11/11
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0127 - lr: 0.0037
Epoch 89/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0127 - lr: 0.0036
Epoch 90/150
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0127 - lr: 0.0036
Epoch 91/150
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0133 - lr: 0.0035
Epoch 92/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0131 - lr: 0.0035
Epoch 93/150
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0128 - lr: 0.0034
Epoch 94/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0134 - lr: 0.0034
Epoch 95/150
11/11
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0132 - lr: 0.0033
Epoch 96/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0128 - lr: 0.0033
Epoch 97/150
11/11
                  Os 5ms/step - loss:
0.0011 - val loss: 0.0143 - lr: 0.0032
Epoch 98/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0126 - lr: 0.0032
Epoch 99/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0141 - lr: 0.0031
Epoch 100/150
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0127 - lr: 0.0031
Epoch 101/150
11/11
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0144 - lr: 0.0030
Epoch 102/150
```

```
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0129 - lr: 0.0029
Epoch 103/150
11/11
                  Os 6ms/step - loss:
0.0011 - val loss: 0.0145 - lr: 0.0029
Epoch 104/150
11/11
                  Os 5ms/step - loss:
0.0012 - val_loss: 0.0135 - lr: 0.0028
Epoch 105/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0141 - lr: 0.0028
Epoch 106/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0138 - lr: 0.0027
Epoch 107/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0137 - lr: 0.0027
Epoch 108/150
11/11
                  Os 5ms/step - loss:
0.0010 - val_loss: 0.0141 - lr: 0.0026
Epoch 109/150
11/11
                  Os 5ms/step - loss:
9.8781e-04 - val_loss: 0.0139 - lr: 0.0026
Epoch 110/150
11/11
                  Os 5ms/step - loss:
0.0010 - val_loss: 0.0139 - lr: 0.0025
Epoch 111/150
11/11
                  Os 6ms/step - loss:
9.5903e-04 - val_loss: 0.0147 - lr: 0.0025
Epoch 112/150
                  Os 5ms/step - loss:
11/11
9.8376e-04 - val_loss: 0.0135 - lr: 0.0024
Epoch 113/150
11/11
                  Os 5ms/step - loss:
0.0011 - val loss: 0.0141 - lr: 0.0024
Epoch 114/150
                  Os 5ms/step - loss:
0.0010 - val_loss: 0.0150 - lr: 0.0023
Epoch 115/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0138 - lr: 0.0023
Epoch 116/150
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0148 - lr: 0.0022
Epoch 117/150
                  Os 6ms/step - loss:
0.0010 - val_loss: 0.0140 - lr: 0.0022
Epoch 118/150
```

```
Os 6ms/step - loss:
11/11
9.2978e-04 - val_loss: 0.0147 - lr: 0.0021
Epoch 119/150
11/11
                  Os 6ms/step - loss:
9.8001e-04 - val loss: 0.0148 - lr: 0.0021
Epoch 120/150
11/11
                  Os 5ms/step - loss:
0.0010 - val_loss: 0.0137 - lr: 0.0020
Epoch 121/150
11/11
                  Os 6ms/step - loss:
9.8899e-04 - val_loss: 0.0153 - lr: 0.0019
Epoch 122/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0138 - lr: 0.0019
Epoch 123/150
                  Os 5ms/step - loss:
11/11
9.5489e-04 - val_loss: 0.0152 - lr: 0.0018
Epoch 124/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0148 - lr: 0.0018
Epoch 125/150
11/11
                  Os 5ms/step - loss:
9.3101e-04 - val_loss: 0.0144 - lr: 0.0017
Epoch 126/150
11/11
                  Os 6ms/step - loss:
0.0011 - val_loss: 0.0149 - lr: 0.0017
Epoch 127/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0149 - lr: 0.0016
Epoch 128/150
                  Os 5ms/step - loss:
11/11
0.0011 - val_loss: 0.0142 - lr: 0.0016
Epoch 129/150
11/11
                  Os 6ms/step - loss:
0.0010 - val loss: 0.0152 - lr: 0.0015
Epoch 130/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0145 - lr: 0.0015
Epoch 131/150
11/11
                  Os 5ms/step - loss:
9.6929e-04 - val_loss: 0.0143 - lr: 0.0014
Epoch 132/150
                  Os 5ms/step - loss:
9.5570e-04 - val_loss: 0.0160 - lr: 0.0014
Epoch 133/150
11/11
                  Os 5ms/step - loss:
0.0011 - val_loss: 0.0144 - lr: 0.0013
Epoch 134/150
```

```
Os 5ms/step - loss:
11/11
9.1600e-04 - val_loss: 0.0145 - lr: 0.0013
Epoch 135/150
11/11
                 Os 5ms/step - loss:
9.9668e-04 - val loss: 0.0150 - lr: 0.0012
Epoch 136/150
11/11
                  Os 5ms/step - loss:
9.7157e-04 - val_loss: 0.0151 - lr: 0.0012
Epoch 137/150
11/11
                  Os 6ms/step - loss:
9.7815e-04 - val_loss: 0.0148 - lr: 0.0011
Epoch 138/150
                  Os 6ms/step - loss:
11/11
9.5463e-04 - val_loss: 0.0150 - lr: 0.0011
Epoch 139/150
                  Os 5ms/step - loss:
11/11
9.6266e-04 - val_loss: 0.0149 - lr: 0.0010
Epoch 140/150
11/11
                  Os 5ms/step - loss:
9.3793e-04 - val_loss: 0.0148 - lr: 9.4916e-04
Epoch 141/150
11/11
                  Os 5ms/step - loss:
9.0075e-04 - val_loss: 0.0147 - lr: 8.9655e-04
Epoch 142/150
11/11
                  Os 6ms/step - loss:
9.0126e-04 - val_loss: 0.0152 - lr: 8.4393e-04
Epoch 143/150
11/11
                  Os 6ms/step - loss:
9.3000e-04 - val_loss: 0.0150 - lr: 7.9132e-04
Epoch 144/150
                  Os 7ms/step - loss:
11/11
8.9571e-04 - val_loss: 0.0151 - lr: 7.3871e-04
Epoch 145/150
11/11
                  Os 6ms/step - loss:
9.3351e-04 - val_loss: 0.0150 - lr: 6.8609e-04
Epoch 146/150
                 Os 6ms/step - loss:
9.1054e-04 - val_loss: 0.0151 - lr: 6.3348e-04
Epoch 147/150
11/11
                  Os 5ms/step - loss:
9.7549e-04 - val_loss: 0.0148 - lr: 5.8087e-04
Epoch 148/150
                  Os 5ms/step - loss:
0.0010 - val_loss: 0.0149 - lr: 5.2826e-04
Epoch 149/150
                 Os 5ms/step - loss:
0.0010 - val_loss: 0.0151 - lr: 4.7564e-04
Epoch 150/150
```



## 8.0.2 CyclicLR

```
[78]: # Train with chosen scheduler
      history = model.fit(
          X_train_scaled,
          y_train_scaled,
          epochs=150,
          batch_size=32,
          validation_data=(X_val_scaled, y_val_scaled),
          callbacks=[cyclic_lr],
          verbose=1
      # After training, plot using epoch-level LR values
      plt.figure(figsize=(12, 6))
      plt.plot(history.history['lr'], label='Learning Rate')
      plt.title('Learning Rate Schedule(CyclicLR)')
      plt.xlabel('Epochs')
      plt.ylabel('LR Value')
      plt.show()
     Epoch 1/150
     11/11
                       Os 10ms/step -
     loss: 6.1718e-04 - val_loss: 0.0162 - lr: 0.0052
     Epoch 2/150
```

```
82
```

```
Os 8ms/step - loss:
11/11
6.2855e-04 - val_loss: 0.0200 - lr: 0.0052
Epoch 3/150
11/11
                  Os 7ms/step - loss:
6.8445e-04 - val loss: 0.0153 - lr: 0.0052
Epoch 4/150
11/11
                  Os 7ms/step - loss:
7.3629e-04 - val_loss: 0.0196 - lr: 0.0052
Epoch 5/150
11/11
                  Os 7ms/step - loss:
6.5860e-04 - val_loss: 0.0159 - lr: 0.0053
Epoch 6/150
11/11
                  Os 10ms/step -
loss: 5.9076e-04 - val_loss: 0.0184 - lr: 0.0053
Epoch 7/150
11/11
                  Os 10ms/step -
loss: 7.2059e-04 - val_loss: 0.0179 - lr: 0.0053
Epoch 8/150
11/11
                  Os 8ms/step - loss:
6.1672e-04 - val_loss: 0.0162 - lr: 0.0053
Epoch 9/150
11/11
                  Os 7ms/step - loss:
7.2431e-04 - val_loss: 0.0204 - lr: 0.0054
Epoch 10/150
11/11
                  Os 6ms/step - loss:
6.8142e-04 - val_loss: 0.0150 - lr: 0.0054
Epoch 11/150
11/11
                  Os 6ms/step - loss:
6.7219e-04 - val_loss: 0.0204 - lr: 0.0054
Epoch 12/150
                  Os 6ms/step - loss:
11/11
7.0424e-04 - val_loss: 0.0158 - lr: 0.0055
Epoch 13/150
11/11
                  Os 8ms/step - loss:
7.2630e-04 - val_loss: 0.0169 - lr: 0.0055
Epoch 14/150
11/11
                  Os 6ms/step - loss:
6.6752e-04 - val_loss: 0.0181 - lr: 0.0055
Epoch 15/150
11/11
                  Os 7ms/step - loss:
6.7271e-04 - val_loss: 0.0166 - lr: 0.0055
Epoch 16/150
                  Os 6ms/step - loss:
11/11
6.8481e-04 - val_loss: 0.0195 - lr: 0.0056
Epoch 17/150
11/11
                  Os 6ms/step - loss:
6.0388e-04 - val_loss: 0.0159 - lr: 0.0056
Epoch 18/150
```

```
Os 6ms/step - loss:
11/11
6.6516e-04 - val_loss: 0.0190 - lr: 0.0056
Epoch 19/150
11/11
                  Os 6ms/step - loss:
6.5215e-04 - val loss: 0.0171 - lr: 0.0056
Epoch 20/150
11/11
                  Os 6ms/step - loss:
6.7954e-04 - val_loss: 0.0196 - lr: 0.0057
Epoch 21/150
11/11
                  Os 6ms/step - loss:
6.3940e-04 - val_loss: 0.0161 - lr: 0.0057
Epoch 22/150
11/11
                  Os 7ms/step - loss:
6.5360e-04 - val_loss: 0.0195 - lr: 0.0057
Epoch 23/150
                  Os 6ms/step - loss:
11/11
6.2483e-04 - val_loss: 0.0158 - lr: 0.0058
Epoch 24/150
11/11
                  Os 6ms/step - loss:
6.0880e-04 - val_loss: 0.0201 - lr: 0.0058
Epoch 25/150
11/11
                  Os 6ms/step - loss:
5.7515e-04 - val_loss: 0.0170 - lr: 0.0058
Epoch 26/150
11/11
                  Os 6ms/step - loss:
6.0512e-04 - val_loss: 0.0188 - lr: 0.0058
Epoch 27/150
11/11
                  Os 6ms/step - loss:
6.4720e-04 - val_loss: 0.0177 - lr: 0.0059
Epoch 28/150
                  Os 6ms/step - loss:
11/11
5.5518e-04 - val_loss: 0.0175 - lr: 0.0059
Epoch 29/150
11/11
                  Os 6ms/step - loss:
5.8552e-04 - val loss: 0.0185 - lr: 0.0059
Epoch 30/150
11/11
                  Os 6ms/step - loss:
6.2182e-04 - val_loss: 0.0190 - lr: 0.0060
Epoch 31/150
11/11
                  Os 6ms/step - loss:
6.9231e-04 - val_loss: 0.0163 - lr: 0.0060
Epoch 32/150
                  Os 6ms/step - loss:
6.1309e-04 - val_loss: 0.0192 - lr: 0.0060
Epoch 33/150
11/11
                  Os 6ms/step - loss:
6.2238e-04 - val_loss: 0.0179 - lr: 0.0060
Epoch 34/150
```

```
Os 5ms/step - loss:
11/11
6.7917e-04 - val_loss: 0.0182 - lr: 0.0059
Epoch 35/150
11/11
                  Os 5ms/step - loss:
5.9586e-04 - val loss: 0.0180 - lr: 0.0059
Epoch 36/150
11/11
                  Os 6ms/step - loss:
6.0039e-04 - val_loss: 0.0188 - lr: 0.0059
Epoch 37/150
11/11
                  Os 6ms/step - loss:
6.0627e-04 - val_loss: 0.0163 - lr: 0.0059
Epoch 38/150
11/11
                  Os 5ms/step - loss:
6.6169e-04 - val_loss: 0.0191 - lr: 0.0058
Epoch 39/150
                  Os 6ms/step - loss:
11/11
6.2402e-04 - val_loss: 0.0192 - lr: 0.0058
Epoch 40/150
11/11
                  Os 6ms/step - loss:
5.9328e-04 - val_loss: 0.0162 - lr: 0.0058
Epoch 41/150
11/11
                  Os 6ms/step - loss:
6.7645e-04 - val_loss: 0.0196 - lr: 0.0057
Epoch 42/150
11/11
                  Os 6ms/step - loss:
5.8776e-04 - val_loss: 0.0173 - lr: 0.0057
Epoch 43/150
11/11
                  Os 6ms/step - loss:
5.5857e-04 - val_loss: 0.0166 - lr: 0.0057
Epoch 44/150
                  Os 6ms/step - loss:
11/11
6.9508e-04 - val_loss: 0.0203 - lr: 0.0057
Epoch 45/150
11/11
                  Os 6ms/step - loss:
6.4428e-04 - val_loss: 0.0170 - lr: 0.0056
Epoch 46/150
11/11
                  Os 7ms/step - loss:
5.9893e-04 - val_loss: 0.0182 - lr: 0.0056
Epoch 47/150
11/11
                  Os 6ms/step - loss:
5.6111e-04 - val_loss: 0.0179 - lr: 0.0056
Epoch 48/150
                  Os 6ms/step - loss:
5.9832e-04 - val_loss: 0.0179 - lr: 0.0056
Epoch 49/150
11/11
                  Os 6ms/step - loss:
5.7887e-04 - val_loss: 0.0189 - lr: 0.0055
Epoch 50/150
```

```
Os 6ms/step - loss:
11/11
5.6461e-04 - val_loss: 0.0167 - lr: 0.0055
Epoch 51/150
11/11
                  Os 6ms/step - loss:
6.0778e-04 - val loss: 0.0207 - lr: 0.0055
Epoch 52/150
11/11
                  Os 6ms/step - loss:
6.2536e-04 - val_loss: 0.0169 - lr: 0.0054
Epoch 53/150
11/11
                  Os 6ms/step - loss:
5.7772e-04 - val_loss: 0.0198 - lr: 0.0054
Epoch 54/150
                  Os 6ms/step - loss:
11/11
6.0937e-04 - val_loss: 0.0189 - lr: 0.0054
Epoch 55/150
                  Os 6ms/step - loss:
11/11
6.8346e-04 - val_loss: 0.0189 - lr: 0.0054
Epoch 56/150
11/11
                  Os 6ms/step - loss:
5.5059e-04 - val_loss: 0.0187 - lr: 0.0053
Epoch 57/150
11/11
                  Os 6ms/step - loss:
6.0885e-04 - val_loss: 0.0178 - lr: 0.0053
Epoch 58/150
11/11
                  Os 6ms/step - loss:
5.6726e-04 - val_loss: 0.0184 - lr: 0.0053
Epoch 59/150
11/11
                  Os 6ms/step - loss:
5.7472e-04 - val_loss: 0.0200 - lr: 0.0053
Epoch 60/150
                  Os 6ms/step - loss:
11/11
5.6026e-04 - val_loss: 0.0173 - lr: 0.0052
Epoch 61/150
11/11
                  Os 6ms/step - loss:
5.7857e-04 - val loss: 0.0202 - lr: 0.0052
Epoch 62/150
11/11
                  Os 6ms/step - loss:
6.1447e-04 - val_loss: 0.0182 - lr: 0.0052
Epoch 63/150
11/11
                  Os 5ms/step - loss:
5.1403e-04 - val_loss: 0.0184 - lr: 0.0051
Epoch 64/150
                  Os 6ms/step - loss:
5.3593e-04 - val_loss: 0.0195 - lr: 0.0051
Epoch 65/150
11/11
                  Os 6ms/step - loss:
6.0869e-04 - val_loss: 0.0172 - lr: 0.0051
Epoch 66/150
```

```
Os 5ms/step - loss:
11/11
5.8765e-04 - val_loss: 0.0207 - lr: 0.0051
Epoch 67/150
11/11
                  Os 6ms/step - loss:
5.0434e-04 - val loss: 0.0168 - lr: 0.0050
Epoch 68/150
11/11
                  Os 6ms/step - loss:
5.8999e-04 - val_loss: 0.0207 - lr: 0.0050
Epoch 69/150
11/11
                  Os 6ms/step - loss:
6.9698e-04 - val_loss: 0.0181 - lr: 0.0050
Epoch 70/150
11/11
                  Os 7ms/step - loss:
5.3760e-04 - val_loss: 0.0177 - lr: 0.0049
Epoch 71/150
                  Os 6ms/step - loss:
11/11
5.3394e-04 - val_loss: 0.0200 - lr: 0.0049
Epoch 72/150
11/11
                  Os 6ms/step - loss:
6.4905e-04 - val_loss: 0.0179 - lr: 0.0049
Epoch 73/150
11/11
                  Os 6ms/step - loss:
4.9324e-04 - val_loss: 0.0187 - lr: 0.0049
Epoch 74/150
11/11
                  Os 7ms/step - loss:
5.1905e-04 - val_loss: 0.0176 - lr: 0.0048
Epoch 75/150
11/11
                  Os 7ms/step - loss:
5.4559e-04 - val_loss: 0.0203 - lr: 0.0048
Epoch 76/150
                  Os 6ms/step - loss:
11/11
5.6797e-04 - val_loss: 0.0180 - lr: 0.0048
Epoch 77/150
11/11
                  Os 6ms/step - loss:
5.2130e-04 - val loss: 0.0179 - lr: 0.0048
Epoch 78/150
11/11
                  Os 6ms/step - loss:
5.5747e-04 - val_loss: 0.0196 - lr: 0.0047
Epoch 79/150
11/11
                  Os 6ms/step - loss:
5.4817e-04 - val_loss: 0.0185 - lr: 0.0047
Epoch 80/150
                  Os 6ms/step - loss:
5.2694e-04 - val_loss: 0.0188 - lr: 0.0047
Epoch 81/150
11/11
                  Os 7ms/step - loss:
5.6345e-04 - val_loss: 0.0191 - lr: 0.0046
Epoch 82/150
```

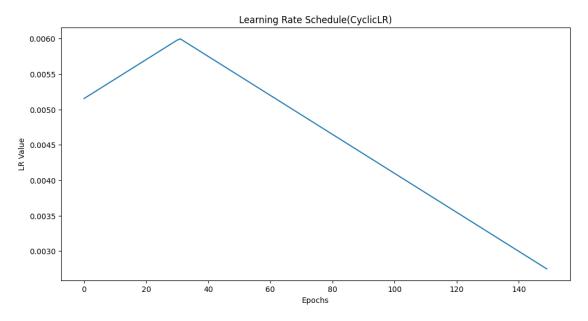
```
Os 6ms/step - loss:
11/11
5.1638e-04 - val_loss: 0.0171 - lr: 0.0046
Epoch 83/150
11/11
                  Os 6ms/step - loss:
5.9451e-04 - val loss: 0.0204 - lr: 0.0046
Epoch 84/150
11/11
                  Os 6ms/step - loss:
5.6380e-04 - val_loss: 0.0170 - lr: 0.0046
Epoch 85/150
11/11
                  Os 6ms/step - loss:
5.7882e-04 - val_loss: 0.0200 - lr: 0.0045
Epoch 86/150
11/11
                  Os 5ms/step - loss:
4.9270e-04 - val_loss: 0.0188 - lr: 0.0045
Epoch 87/150
                  Os 5ms/step - loss:
11/11
5.5273e-04 - val_loss: 0.0191 - lr: 0.0045
Epoch 88/150
11/11
                  Os 6ms/step - loss:
5.2433e-04 - val_loss: 0.0181 - lr: 0.0045
Epoch 89/150
11/11
                  Os 5ms/step - loss:
5.0706e-04 - val_loss: 0.0191 - lr: 0.0044
Epoch 90/150
11/11
                  Os 6ms/step - loss:
5.6304e-04 - val_loss: 0.0183 - lr: 0.0044
Epoch 91/150
11/11
                  Os 5ms/step - loss:
4.9369e-04 - val_loss: 0.0187 - lr: 0.0044
Epoch 92/150
                  Os 5ms/step - loss:
11/11
5.0918e-04 - val_loss: 0.0187 - lr: 0.0043
Epoch 93/150
11/11
                  Os 6ms/step - loss:
5.2590e-04 - val loss: 0.0188 - lr: 0.0043
Epoch 94/150
11/11
                  Os 5ms/step - loss:
4.8096e-04 - val_loss: 0.0189 - lr: 0.0043
Epoch 95/150
11/11
                  Os 5ms/step - loss:
4.9355e-04 - val_loss: 0.0187 - lr: 0.0043
Epoch 96/150
                  Os 6ms/step - loss:
5.6666e-04 - val_loss: 0.0193 - lr: 0.0042
Epoch 97/150
11/11
                  Os 6ms/step - loss:
5.3988e-04 - val_loss: 0.0182 - lr: 0.0042
Epoch 98/150
```

```
Os 6ms/step - loss:
11/11
5.0401e-04 - val_loss: 0.0185 - lr: 0.0042
Epoch 99/150
11/11
                  Os 6ms/step - loss:
5.7996e-04 - val loss: 0.0195 - lr: 0.0042
Epoch 100/150
11/11
                  Os 5ms/step - loss:
5.4447e-04 - val_loss: 0.0174 - lr: 0.0041
Epoch 101/150
11/11
                  Os 5ms/step - loss:
5.8881e-04 - val_loss: 0.0200 - lr: 0.0041
Epoch 102/150
                  Os 6ms/step - loss:
11/11
5.5538e-04 - val_loss: 0.0174 - lr: 0.0041
Epoch 103/150
                  Os 6ms/step - loss:
11/11
5.2569e-04 - val_loss: 0.0204 - lr: 0.0040
Epoch 104/150
11/11
                  Os 6ms/step - loss:
5.2711e-04 - val_loss: 0.0178 - lr: 0.0040
Epoch 105/150
11/11
                  Os 6ms/step - loss:
4.9847e-04 - val_loss: 0.0197 - lr: 0.0040
Epoch 106/150
11/11
                  Os 6ms/step - loss:
5.8778e-04 - val_loss: 0.0187 - lr: 0.0040
Epoch 107/150
11/11
                  Os 6ms/step - loss:
5.4936e-04 - val_loss: 0.0191 - lr: 0.0039
Epoch 108/150
                  Os 6ms/step - loss:
11/11
5.1661e-04 - val_loss: 0.0188 - lr: 0.0039
Epoch 109/150
11/11
                  Os 6ms/step - loss:
4.9099e-04 - val loss: 0.0189 - lr: 0.0039
Epoch 110/150
                  Os 6ms/step - loss:
4.5497e-04 - val_loss: 0.0186 - lr: 0.0038
Epoch 111/150
11/11
                  Os 6ms/step - loss:
5.6171e-04 - val_loss: 0.0189 - lr: 0.0038
Epoch 112/150
                  Os 6ms/step - loss:
5.1675e-04 - val_loss: 0.0190 - lr: 0.0038
Epoch 113/150
11/11
                  Os 6ms/step - loss:
4.6014e-04 - val_loss: 0.0186 - lr: 0.0038
Epoch 114/150
```

```
Os 8ms/step - loss:
11/11
5.3525e-04 - val_loss: 0.0199 - lr: 0.0037
Epoch 115/150
11/11
                  Os 6ms/step - loss:
5.4228e-04 - val loss: 0.0180 - lr: 0.0037
Epoch 116/150
11/11
                  Os 6ms/step - loss:
5.3561e-04 - val_loss: 0.0196 - lr: 0.0037
Epoch 117/150
11/11
                  Os 6ms/step - loss:
5.1131e-04 - val_loss: 0.0187 - lr: 0.0037
Epoch 118/150
                  Os 6ms/step - loss:
11/11
5.2103e-04 - val_loss: 0.0194 - lr: 0.0036
Epoch 119/150
                  Os 6ms/step - loss:
11/11
4.9730e-04 - val_loss: 0.0183 - lr: 0.0036
Epoch 120/150
11/11
                  Os 6ms/step - loss:
5.0407e-04 - val_loss: 0.0187 - lr: 0.0036
Epoch 121/150
11/11
                  Os 6ms/step - loss:
5.3506e-04 - val_loss: 0.0193 - lr: 0.0035
Epoch 122/150
11/11
                  Os 6ms/step - loss:
5.2479e-04 - val_loss: 0.0188 - lr: 0.0035
Epoch 123/150
11/11
                  Os 5ms/step - loss:
5.4407e-04 - val_loss: 0.0197 - lr: 0.0035
Epoch 124/150
                  Os 6ms/step - loss:
11/11
5.2843e-04 - val_loss: 0.0193 - lr: 0.0035
Epoch 125/150
11/11
                  Os 6ms/step - loss:
4.9627e-04 - val loss: 0.0184 - lr: 0.0034
Epoch 126/150
11/11
                  Os 5ms/step - loss:
4.8852e-04 - val_loss: 0.0192 - lr: 0.0034
Epoch 127/150
11/11
                  Os 5ms/step - loss:
5.5869e-04 - val_loss: 0.0195 - lr: 0.0034
Epoch 128/150
                  Os 5ms/step - loss:
4.3769e-04 - val_loss: 0.0187 - lr: 0.0034
Epoch 129/150
11/11
                  Os 6ms/step - loss:
4.6064e-04 - val_loss: 0.0191 - lr: 0.0033
Epoch 130/150
```

```
Os 6ms/step - loss:
11/11
5.0697e-04 - val_loss: 0.0188 - lr: 0.0033
Epoch 131/150
11/11
                  Os 6ms/step - loss:
5.2767e-04 - val loss: 0.0196 - lr: 0.0033
Epoch 132/150
11/11
                  Os 6ms/step - loss:
4.9334e-04 - val_loss: 0.0191 - lr: 0.0032
Epoch 133/150
11/11
                  Os 6ms/step - loss:
5.2255e-04 - val_loss: 0.0189 - lr: 0.0032
Epoch 134/150
11/11
                  Os 5ms/step - loss:
5.3765e-04 - val_loss: 0.0196 - lr: 0.0032
Epoch 135/150
                  Os 6ms/step - loss:
11/11
4.1954e-04 - val_loss: 0.0189 - lr: 0.0032
Epoch 136/150
11/11
                  Os 5ms/step - loss:
5.4906e-04 - val_loss: 0.0191 - lr: 0.0031
Epoch 137/150
11/11
                  Os 5ms/step - loss:
5.3429e-04 - val_loss: 0.0195 - lr: 0.0031
Epoch 138/150
11/11
                  Os 7ms/step - loss:
5.4580e-04 - val_loss: 0.0189 - lr: 0.0031
Epoch 139/150
11/11
                  Os 7ms/step - loss:
4.8868e-04 - val_loss: 0.0191 - lr: 0.0031
Epoch 140/150
                  Os 6ms/step - loss:
11/11
4.8992e-04 - val_loss: 0.0199 - lr: 0.0030
Epoch 141/150
11/11
                  Os 6ms/step - loss:
4.7001e-04 - val loss: 0.0182 - lr: 0.0030
Epoch 142/150
                  Os 6ms/step - loss:
4.9499e-04 - val_loss: 0.0199 - lr: 0.0030
Epoch 143/150
11/11
                  Os 6ms/step - loss:
4.7000e-04 - val_loss: 0.0190 - lr: 0.0029
Epoch 144/150
                  Os 6ms/step - loss:
4.6570e-04 - val_loss: 0.0194 - lr: 0.0029
Epoch 145/150
                  Os 6ms/step - loss:
4.4883e-04 - val_loss: 0.0193 - lr: 0.0029
Epoch 146/150
```

```
11/11
                  Os 6ms/step - loss:
4.7082e-04 - val_loss: 0.0192 - lr: 0.0029
Epoch 147/150
11/11
                  Os 5ms/step - loss:
5.1966e-04 - val_loss: 0.0188 - lr: 0.0028
Epoch 148/150
11/11
                  Os 6ms/step - loss:
4.7205e-04 - val_loss: 0.0197 - lr: 0.0028
Epoch 149/150
11/11
                  Os 6ms/step - loss:
5.0823e-04 - val_loss: 0.0190 - lr: 0.0028
Epoch 150/150
                  Os 6ms/step - loss:
11/11
4.6952e-04 - val_loss: 0.0196 - lr: 0.0027
```



## 8.0.3 ReduceLROnPlateau

```
[79]: # Train with BOTH callbacks
history = model.fit(
    X_train_scaled,
    y_train_scaled,
    epochs=150,
    batch_size=32,
    validation_data=(X_val_scaled, y_val_scaled),
    callbacks=[reduce_lr, LRSchedulerLogger()], # Add both
    verbose=1
)
```

```
plt.figure(figsize=(12, 6))
plt.plot(history.history['lr'], label='Learning Rate')
plt.title('Learning Rate Schedule(ReduceLROnPlateau Schedule)')
plt.xlabel('Epochs')
plt.ylabel('LR Value')
plt.show()
Epoch 1/150
11/11
                  Os 9ms/step - loss:
4.8866e-04 - val_loss: 0.0191 - learning_rate: 0.0027 - lr: 0.0027
Epoch 2/150
                  Os 9ms/step - loss:
11/11
4.7676e-04 - val_loss: 0.0197 - learning_rate: 0.0027 - lr: 0.0027
Epoch 3/150
11/11
                  Os 6ms/step - loss:
4.8786e-04 - val_loss: 0.0192 - learning_rate: 0.0027 - lr: 0.0027
Epoch 4/150
11/11
                  Os 6ms/step - loss:
4.8266e-04 - val_loss: 0.0189 - learning_rate: 0.0027 - lr: 0.0027
Epoch 5/150
11/11
                  Os 6ms/step - loss:
4.6455e-04 - val_loss: 0.0200 - learning_rate: 0.0027 - lr: 0.0027
Epoch 6/150
11/11
                  Os 9ms/step - loss:
4.8680e-04 - val_loss: 0.0180 - learning_rate: 0.0027 - lr: 0.0027
Epoch 7/150
11/11
                 Os 8ms/step - loss:
4.4341e-04 - val_loss: 0.0210 - learning_rate: 0.0027 - lr: 0.0027
Epoch 8/150
11/11
                  Os 8ms/step - loss:
5.0124e-04 - val_loss: 0.0179 - learning_rate: 0.0027 - lr: 0.0027
Epoch 9/150
11/11
                  Os 6ms/step - loss:
5.0275e-04 - val_loss: 0.0198 - learning_rate: 0.0027 - lr: 0.0027
Epoch 10/150
11/11
                  Os 6ms/step - loss:
5.1213e-04 - val_loss: 0.0196 - learning_rate: 0.0027 - lr: 0.0027
Epoch 11/150
1/11
                  Os 13ms/step -
loss: 3.3946e-04
Epoch 11: ReduceLROnPlateau reducing learning rate to 0.0005499999970197678.
                  Os 6ms/step - loss:
11/11
4.8444e-04 - val_loss: 0.0184 - learning_rate: 0.0027 - lr: 5.5000e-04
Epoch 12/150
11/11
                  Os 6ms/step - loss:
5.1277e-04 - val_loss: 0.0190 - learning_rate: 5.5000e-04 - lr: 5.5000e-04
Epoch 13/150
11/11
                 Os 5ms/step - loss:
```

```
4.9949e-04 - val_loss: 0.0194 - learning rate: 5.5000e-04 - lr: 5.5000e-04
Epoch 14/150
11/11
                 Os 5ms/step - loss:
4.7676e-04 - val_loss: 0.0198 - learning_rate: 5.5000e-04 - lr: 5.5000e-04
Epoch 15/150
11/11
                 Os 6ms/step - loss:
4.3418e-04 - val loss: 0.0195 - learning rate: 5.5000e-04 - lr: 5.5000e-04
Epoch 16/150
1/11
                 0s 12ms/step -
loss: 5.4671e-04
Epoch 16: ReduceLROnPlateau reducing learning rate to 0.00010999999940395356.
                 Os 6ms/step - loss:
4.9817e-04 - val_loss: 0.0195 - learning_rate: 5.5000e-04 - lr: 1.1000e-04
Epoch 17/150
11/11
                 Os 7ms/step - loss:
4.8087e-04 - val_loss: 0.0194 - learning_rate: 1.1000e-04 - lr: 1.1000e-04
Epoch 18/150
11/11
                 Os 6ms/step - loss:
4.6405e-04 - val_loss: 0.0192 - learning_rate: 1.1000e-04 - lr: 1.1000e-04
Epoch 19/150
11/11
                 Os 6ms/step - loss:
5.0814e-04 - val_loss: 0.0192 - learning_rate: 1.1000e-04 - lr: 1.1000e-04
Epoch 20/150
                 Os 6ms/step - loss:
11/11
5.1979e-04 - val_loss: 0.0192 - learning_rate: 1.1000e-04 - lr: 1.1000e-04
Epoch 21/150
1/11
                 Os 17ms/step -
loss: 5.8100e-04
Epoch 21: ReduceLROnPlateau reducing learning rate to 2.2000000171829015e-05.
11/11
                 Os 6ms/step - loss:
5.0792e-04 - val_loss: 0.0192 - learning_rate: 1.1000e-04 - lr: 2.2000e-05
Epoch 22/150
11/11
                 Os 6ms/step - loss:
4.6439e-04 - val_loss: 0.0193 - learning_rate: 2.2000e-05 - lr: 2.2000e-05
Epoch 23/150
11/11
                 Os 6ms/step - loss:
4.9816e-04 - val_loss: 0.0193 - learning_rate: 2.2000e-05 - lr: 2.2000e-05
Epoch 24/150
11/11
                 Os 6ms/step - loss:
4.9209e-04 - val_loss: 0.0194 - learning_rate: 2.2000e-05 - lr: 2.2000e-05
Epoch 25/150
11/11
                 Os 5ms/step - loss:
4.7625e-04 - val_loss: 0.0194 - learning_rate: 2.2000e-05 - lr: 2.2000e-05
Epoch 26/150
1/11
                 Os 14ms/step -
loss: 2.4195e-04
Epoch 26: ReduceLROnPlateau reducing learning rate to 4.400000034365803e-06.
11/11
                 Os 6ms/step - loss:
```

```
4.0239e-04 - val_loss: 0.0194 - learning_rate: 2.2000e-05 - lr: 4.4000e-06
Epoch 27/150
11/11
                 Os 5ms/step - loss:
4.4104e-04 - val_loss: 0.0194 - learning_rate: 4.4000e-06 - lr: 4.4000e-06
Epoch 28/150
11/11
                 Os 6ms/step - loss:
4.7390e-04 - val_loss: 0.0194 - learning_rate: 4.4000e-06 - lr: 4.4000e-06
Epoch 29/150
11/11
                 Os 6ms/step - loss:
4.8957e-04 - val_loss: 0.0194 - learning_rate: 4.4000e-06 - lr: 4.4000e-06
Epoch 30/150
11/11
                 Os 5ms/step - loss:
4.8249e-04 - val_loss: 0.0194 - learning_rate: 4.4000e-06 - lr: 4.4000e-06
Epoch 31/150
 1/11
                 Os 13ms/step -
loss: 6.6360e-04
Epoch 31: ReduceLROnPlateau reducing learning rate to 1e-06.
                 Os 5ms/step - loss:
5.0499e-04 - val_loss: 0.0194 - learning_rate: 4.4000e-06 - lr: 1.0000e-06
Epoch 32/150
11/11
                 Os 6ms/step - loss:
4.5791e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 33/150
11/11
                 Os 6ms/step - loss:
4.4779e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 34/150
11/11
                 Os 6ms/step - loss:
5.1368e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 35/150
11/11
                 Os 6ms/step - loss:
5.2132e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 36/150
11/11
                 Os 6ms/step - loss:
4.8416e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 37/150
11/11
                 Os 5ms/step - loss:
4.9338e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 38/150
11/11
                 Os 6ms/step - loss:
5.2458e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 39/150
11/11
                 Os 6ms/step - loss:
4.7329e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 40/150
11/11
                 Os 6ms/step - loss:
5.0682e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 41/150
11/11
                 Os 6ms/step - loss:
```

```
5.1612e-04 - val_loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 42/150
11/11
                 Os 5ms/step - loss:
4.8081e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 43/150
11/11
                 Os 6ms/step - loss:
4.5610e-04 - val loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 44/150
11/11
                 Os 5ms/step - loss:
5.2556e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 45/150
11/11
                 Os 6ms/step - loss:
4.5928e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 46/150
11/11
                 Os 6ms/step - loss:
4.7602e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 47/150
11/11
                 Os 6ms/step - loss:
4.5851e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 48/150
11/11
                 Os 6ms/step - loss:
4.6926e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 49/150
                 Os 6ms/step - loss:
11/11
4.3359e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 50/150
11/11
                 Os 5ms/step - loss:
4.7717e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 51/150
11/11
                 Os 6ms/step - loss:
4.4058e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 52/150
11/11
                 Os 5ms/step - loss:
4.7562e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 53/150
11/11
                 Os 6ms/step - loss:
4.8746e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 54/150
11/11
                 Os 6ms/step - loss:
5.0532e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 55/150
11/11
                 Os 6ms/step - loss:
4.5040e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 56/150
11/11
                 Os 6ms/step - loss:
5.1111e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 57/150
11/11
                 Os 6ms/step - loss:
```

```
4.8577e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 58/150
11/11
                 Os 5ms/step - loss:
4.6074e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 59/150
11/11
                 Os 5ms/step - loss:
5.2573e-04 - val loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 60/150
11/11
                 Os 5ms/step - loss:
4.9553e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 61/150
11/11
                 Os 6ms/step - loss:
5.0387e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 62/150
11/11
                 Os 5ms/step - loss:
4.9314e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 63/150
11/11
                 Os 7ms/step - loss:
4.2725e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 64/150
11/11
                 Os 6ms/step - loss:
4.7565e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 65/150
                 Os 6ms/step - loss:
11/11
4.3027e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 66/150
11/11
                 Os 5ms/step - loss:
4.4966e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 67/150
11/11
                 Os 5ms/step - loss:
4.4454e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 68/150
11/11
                 Os 5ms/step - loss:
4.6110e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 69/150
11/11
                 Os 6ms/step - loss:
4.6559e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 70/150
                 Os 6ms/step - loss:
11/11
4.9664e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 71/150
11/11
                 Os 5ms/step - loss:
4.2145e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 72/150
11/11
                 Os 6ms/step - loss:
4.1552e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 73/150
11/11
                 Os 5ms/step - loss:
```

```
4.9665e-04 - val_loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 74/150
11/11
                 Os 6ms/step - loss:
4.4421e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 75/150
11/11
                 Os 6ms/step - loss:
4.4426e-04 - val loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 76/150
11/11
                 Os 6ms/step - loss:
4.2835e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 77/150
11/11
                 Os 5ms/step - loss:
4.1533e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 78/150
11/11
                 3s 251ms/step -
loss: 4.1614e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 79/150
11/11
                 Os 6ms/step - loss:
5.0043e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 80/150
11/11
                 Os 6ms/step - loss:
4.7789e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 81/150
                 Os 6ms/step - loss:
11/11
4.8280e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 82/150
11/11
                 Os 6ms/step - loss:
4.9565e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 83/150
11/11
                 Os 6ms/step - loss:
5.1318e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 84/150
11/11
                 Os 6ms/step - loss:
4.2898e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 85/150
11/11
                 Os 5ms/step - loss:
4.4539e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 86/150
                 Os 6ms/step - loss:
11/11
5.5921e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 87/150
11/11
                 Os 5ms/step - loss:
5.3876e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 88/150
11/11
                 Os 6ms/step - loss:
4.7837e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 89/150
11/11
                 Os 6ms/step - loss:
```

```
4.6534e-04 - val_loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 90/150
11/11
                 Os 6ms/step - loss:
5.0214e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 91/150
11/11
                 Os 6ms/step - loss:
5.0995e-04 - val loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 92/150
11/11
                 Os 6ms/step - loss:
5.1759e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 93/150
11/11
                 Os 6ms/step - loss:
4.6933e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 94/150
11/11
                 Os 6ms/step - loss:
4.7179e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 95/150
11/11
                 Os 6ms/step - loss:
5.0329e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 96/150
11/11
                 Os 6ms/step - loss:
5.1024e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 97/150
11/11
                 Os 5ms/step - loss:
4.3900e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 98/150
11/11
                 Os 6ms/step - loss:
5.0145e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 99/150
11/11
                 Os 6ms/step - loss:
4.9392e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 100/150
11/11
                 Os 6ms/step - loss:
5.1583e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 101/150
                 Os 6ms/step - loss:
11/11
5.0772e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 102/150
11/11
                 Os 6ms/step - loss:
5.4968e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 103/150
11/11
                 Os 6ms/step - loss:
4.6325e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 104/150
11/11
                 Os 6ms/step - loss:
4.8792e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 105/150
11/11
                 Os 6ms/step - loss:
```

```
4.9879e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 106/150
11/11
                  Os 6ms/step - loss:
4.4892e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 107/150
11/11
                  Os 6ms/step - loss:
4.4985e-04 - val loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 108/150
11/11
                  Os 6ms/step - loss:
5.1936e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 109/150
11/11
                  Os 6ms/step - loss:
4.5423e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 110/150
11/11
                  Os 6ms/step - loss:
4.4978e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 111/150
11/11
                  Os 6ms/step - loss:
4.7026e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 112/150
11/11
                  Os 6ms/step - loss:
4.8581e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 113/150
11/11
                  Os 5ms/step - loss:
4.9457e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 114/150
11/11
                  Os 6ms/step - loss:
4.7740e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 115/150
11/11
                  Os 6ms/step - loss:
5.2520e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 116/150
11/11
                  Os 6ms/step - loss:
4.5405e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 117/150
11/11
                  Os 6ms/step - loss:
4.4754e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 118/150
11/11
                  Os 6ms/step - loss:
4.7820e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 119/150
11/11
                  Os 6ms/step - loss:
4.8471e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 120/150
11/11
                  Os 6ms/step - loss:
4.8642e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 121/150
11/11
                 Os 6ms/step - loss:
```

```
4.2724e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 122/150
11/11
                 Os 6ms/step - loss:
5.2129e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 123/150
11/11
                 Os 6ms/step - loss:
5.2920e-04 - val loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 124/150
11/11
                 Os 6ms/step - loss:
4.7188e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 125/150
11/11
                 Os 6ms/step - loss:
4.8082e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 126/150
11/11
                 Os 5ms/step - loss:
4.9996e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 127/150
11/11
                 Os 6ms/step - loss:
5.0889e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 128/150
11/11
                 Os 5ms/step - loss:
4.8411e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 129/150
11/11
                 Os 6ms/step - loss:
4.8678e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 130/150
11/11
                 Os 6ms/step - loss:
4.9214e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 131/150
11/11
                 Os 6ms/step - loss:
4.7915e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 132/150
11/11
                 Os 5ms/step - loss:
4.8184e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 133/150
11/11
                 Os 5ms/step - loss:
5.4375e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 134/150
11/11
                 Os 6ms/step - loss:
4.7928e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 135/150
11/11
                 Os 6ms/step - loss:
5.1148e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 136/150
11/11
                 Os 6ms/step - loss:
4.8026e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 137/150
11/11
                 Os 6ms/step - loss:
```

```
5.0427e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 138/150
11/11
                 Os 6ms/step - loss:
4.9384e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 139/150
11/11
                 Os 7ms/step - loss:
4.7458e-04 - val loss: 0.0194 - learning rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 140/150
11/11
                 Os 7ms/step - loss:
4.9261e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 141/150
11/11
                 Os 7ms/step - loss:
5.1742e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 142/150
11/11
                 Os 7ms/step - loss:
4.3871e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 143/150
11/11
                 Os 5ms/step - loss:
4.7018e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 144/150
11/11
                 Os 6ms/step - loss:
4.8180e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 145/150
                 Os 6ms/step - loss:
11/11
4.7828e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 146/150
11/11
                 Os 6ms/step - loss:
4.9427e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 147/150
11/11
                 Os 6ms/step - loss:
4.6387e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 148/150
11/11
                 Os 6ms/step - loss:
4.7722e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 149/150
11/11
                 Os 6ms/step - loss:
4.7293e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
Epoch 150/150
                 Os 6ms/step - loss:
11/11
5.1676e-04 - val_loss: 0.0194 - learning_rate: 1.0000e-06 - lr: 1.0000e-06
```

