

Deep Generative Models

Lecture 14

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Recap of Previous Lecture

Outline

Discrete or Continuous Diffusion Models?

Reminder: Diffusion models define a forward corruption process and a reverse denoising process. Previously, we studied diffusion models with continuous states $\mathbf{x}(t) \in \mathbb{R}^m$.

Continuous state space

- ▶ **Discrete time** $t \in \{0, 1, \dots, T\} \Rightarrow \text{DDPM / NCSN.}$
- ▶ **Continuous time** $t \in [0, 1] \Rightarrow \text{Score-based SDE models.}$

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Now we turn to diffusion over discrete-value states
 $\mathbf{x}(t) \in \{1, \dots, K\}^m$.

Discrete state space

- ▶ **Discrete time** $t \in \{0, 1, \dots, T\}.$
- ▶ **Continuous time** $t \in [0, 1].$

Let's discuss why we need discrete diffusion models.

Why Discrete Diffusion Models?

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- ▶ **Robustness:** diffusion avoids the "exposure bias" caused by teacher forcing in AR training.
- ▶ **Unified framework:** diffusion generalizes naturally to discrete domains that do not suit continuous Gaussian noise.

Forward Discrete Process

Continuous Diffusion Markov Chain

In continuous diffusion, the forward Markov chain is defined by progressively corrupting data with Gaussian noise:

$$q(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\sqrt{1 - \beta_t} \mathbf{x}_{t-1}, \beta_t \mathbf{I}).$$

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Discrete Diffusion Markov Chain

For discrete data, we instead define a Markov chain over categorical states:

$$q(\mathbf{x}_t | \mathbf{x}_{t-1}) = \text{Categorical}(\mathbf{Q}_t \mathbf{x}_{t-1}),$$

where $\mathbf{Q}_t \in [0, 1]^{K \times K}$ is a **transition matrix** where each column gives transition probabilities from one state to all others, and columns sum to 1:

$$[\mathbf{Q}_t]_{ij} = q(x_t = i | x_{t-1} = j), \quad \sum_{i=1}^K [\mathbf{Q}_t]_{ij} = 1.$$

Forward (noising) process in discrete diffusion

Markov chain definition

$$q(\mathbf{x}_{1:T} | \mathbf{x}_0) = \prod_{t=1}^T q(\mathbf{x}_t | \mathbf{x}_{t-1}), \quad q(\mathbf{x}_t | \mathbf{x}_{t-1}) = \text{Cat}(\mathbf{x}_t; Q_t[\mathbf{x}_{t-1}, :]).$$

- ▶ $Q_t \in \mathbb{R}^{K \times K}$ — transition matrix between symbols.
- ▶ Typical choices:
 - ▶ **Uniform diffusion:** $Q_t = (1 - \beta_t)I + \beta_t U$.
 - ▶ **Absorbing diffusion:** $Q_t(i, m) = \beta_t$ for mask token m .
- ▶ After many steps, all tokens become random or masked.

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- ▶ This process drives the data towards a stationary (often uniform) distribution as $t \rightarrow T$.
- ▶ Analogy:

Continuous diffusion: $\sigma_t^2 \uparrow \Rightarrow x_t \sim \mathcal{N}(0, I)$ \iff Discrete diffusion

Summary

