System Identification and Loop Shaping

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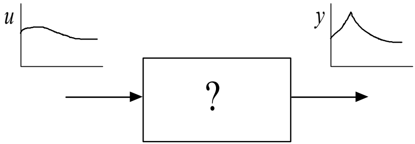
IEE2683 – Laboratorio de Control Automático

Contenido

Problem setup



Elements

1. Unknown process with at least one input and one output
2. Actuator to introduce an user-designed input
3. Sensor to measure corresponding output

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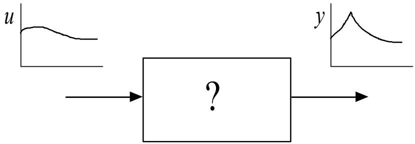

Elements

1. Unknown process with at least one input and one output
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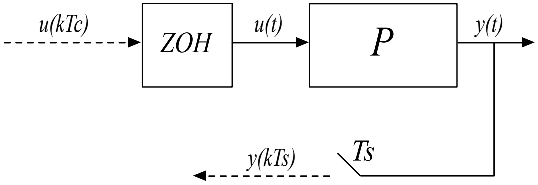
Goals

1. Determine a model using input/output experimental data
2. In the form of a set of equations
3. Or in the form of a characteristic response

Problem setup



Assumptions

1. Continuous-time LTI and (stable) process
2. Arbitrary sampling frequency for sensor and actuator
3. Strictly proper transfer function without non-minimun phase zeros

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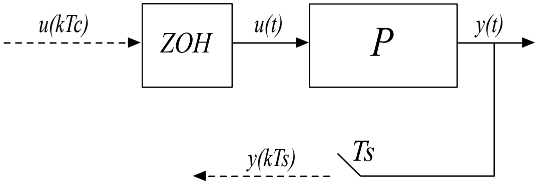

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Challenges

1. Design a set of inputs u to characterize P based on outputs y
2. Design a processing routine to characterize P based on input/output data

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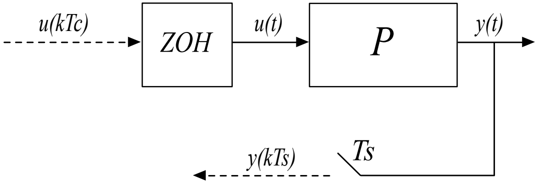

Goals

1. Obtain an estimate of the frequency response $\hat{G}(e^{j\omega})$
2. Obtain the coherence spectrum κ

Input selection

Typical signals

1. Zero function
2. Step function
3. Sinusoidal
4. Periodic signals: sum of sinusoids
5. Pseudo-random binary sequences (PRBS)

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Winners

1. Periodic function: gives estimate of bandwidth
2. PRBS of period M: p.e. of order M with white noise-like spectrum

Filtering

Artifacts

1. Drifts
2. Grid noise
3. Measurement noise

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Filtering

1. Direct detrending on the identification data
2. Time-domain average exploiting periodic nature of input:
   2.1 Separate data into several chunks of length equal to period of input
   2.2 Perform direct averaging to obtain smoothed data
3. Smoothing in the frequency domain (next slide)

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Input design revisited

1. Construct a PRBS of length M
2. Append several periods to obtain input

Spectral leakage

Problem

1. Real signals have finite length
2. Periodicity assumed in DFT depends on sampling
3. Non-integer periods lead to inaccurate estimates

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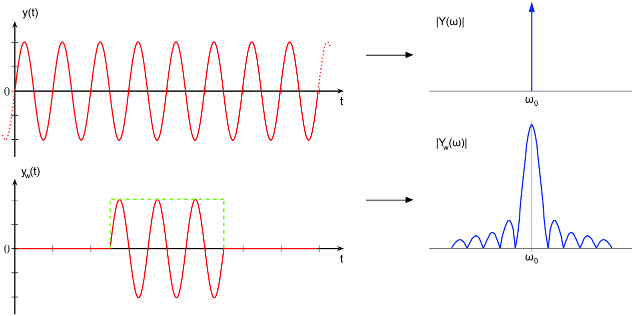

Spectral leakage

Solution

Windowing of signals: non-square window function to improve spectral properties

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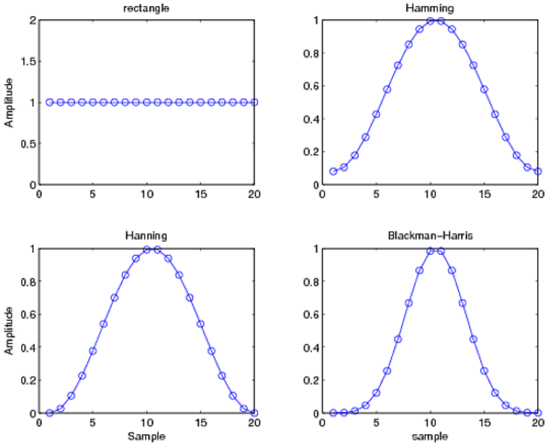

Processing

On a windowed vector of identification data.

Correlation function

$$\hat{R}_u^N(\tau) = \frac{1}{N}\sum_{t=1}^{N} u(t)\,u(t-\tau)\,, \quad \tau \in [-\gamma, \gamma] \tag{1}$$

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Calculate estimations for $\hat{R}_{yu}^N$, $\hat{\Phi}_{yu}^N$, $\hat{R}_y^N$, and $\hat{\Phi}_y^N$

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Smoothed Estimate of Frequency Response

$$\hat{G}_N(\omega) = \frac{\hat{\Phi}_{yu}^N(\omega)}{\hat{\Phi}_u^N(\omega)} \tag{3}$$

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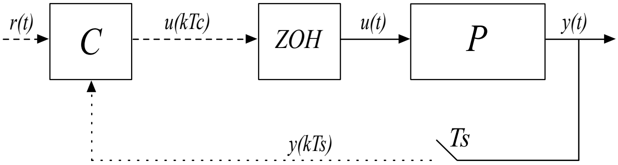
Coherence Spectrum

$$\kappa = \sqrt{\frac{\left|\hat{\Phi}_{yu}^N(\omega)\right|^2}{\hat{\Phi}_y^N(\omega)\hat{\Phi}_u^N(\omega)}} \tag{5}$$

Problem setup



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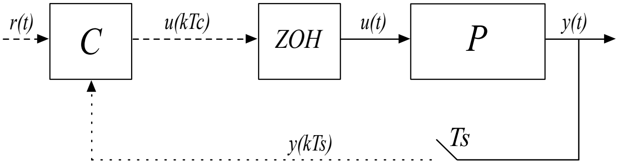

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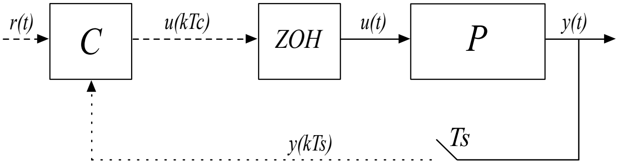

Problem

1. Correlation between inputs and outputs due to feedback !!
2. Controller tracks the set-point: no p.e.

Problem setup

Solution

1. Use a "bad" controller: only stabilization
2. Add an exiting input after the controller
3. Use direct input to the plant to perform identification

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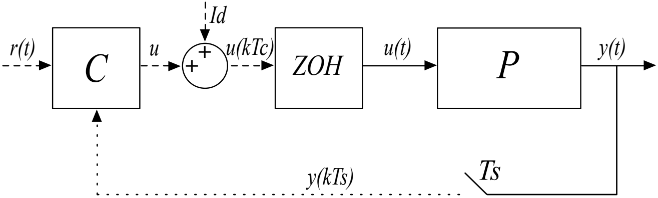

Overview

Methods

1. Least-squares
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Structures

1. ARX
2. ARMAX
3. ARIMAX
4. NARMAX
5. ...

Overview

Subspace methods in the time domain

1. Give a state-space representation of the system
2. Less sensitive to measurement noise
3. Complicated algorithms
4. Often give unstable models
5. See N4SID (exists in Matlab)

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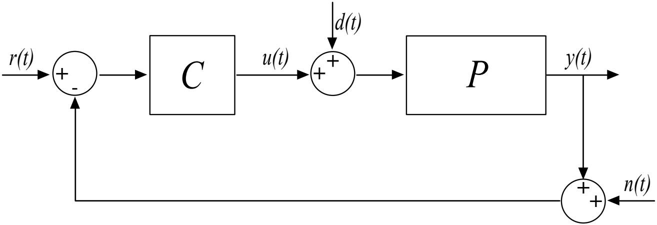
Subspace methods in the frequency domain

1. Give a state-space representation of the system
2. Less sensitive to measurement noise
3. Complicated algorithms using samples of the frequency response
4. Toolboxes for Matlab exist

System Identification
- Preliminaries
- Non-parametric identification in the frequency domain
- Non-parametric closed-loop identification in the frequency domain
- Parametric Identification

Loop-Shaping
- Open-loop vs closed-loop specifications
- Compensators

Overview



Main ideas

1. Look to design a controller C to satisfy closed-loop specifications
2. Convert closed-loop specifications into constrains on the open-loop gain $G_0 = CP$
3. Obtain the plant Bode diagram and shape G_0 starting from $C = 1$
4. "Add" compensators until desired shape is achieved

Specifications

Stability

Given cross-over frequency w_c the phase must be above -180 degrees. The distance between the phase of $G_0(\jmath\omega)$ and -180 degrees is called the phase-margin.

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Reference tracking

A sufficient condition to track a reference with an error at least $k_t \ll 1$ times smaller than the reference over a frequency range $[0, \omega_t]$ is given by:

$$\left|G_0(\jmath\omega)\right| \geq \frac{1}{k_t} + 1, \ \ \forall \omega \in [0, \omega_t] \tag{6}$$

Specifications

Disturbance rejection

A sufficient condition to attenuate a disturbance at the output at least $k_d \ll 1$ times over a frequency range $[0, \omega_d]$ is given by:

$$\left|G_0(\jmath\omega)\right| \geq \frac{\left|P(\jmath\omega)\right|}{k_d} + 1, \ \ \forall \omega \in [0, \omega_d] \tag{7}$$

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Noise rejection

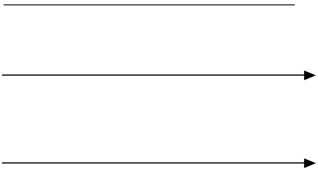
A sufficient condition to attenuate the measurement noise at the output at least $k_n \ll 1$ times over a frequency range $[\omega_n, \infty)$ is given by:

$$\left|G_0(\jmath\omega)\right| \leq \frac{k_n}{k_n + 1}, \;\; \forall\omega \in [\omega_n, \infty) \tag{8}$$

Compensators

P gain

$$C_p = K$$



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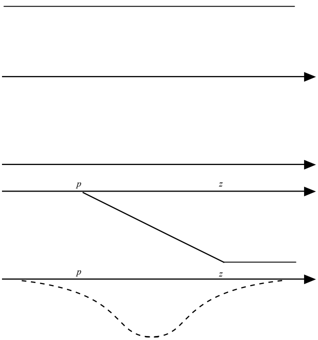
Lag compensator

$$C_{lag} = \frac{s/z+1}{s/p+1}, \; p < z$$



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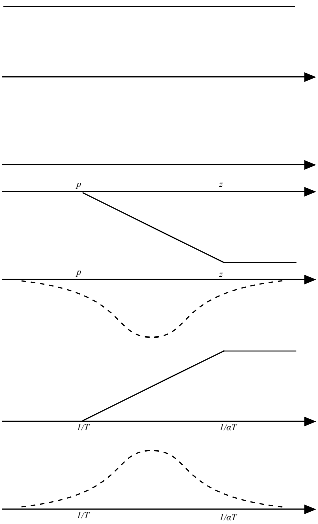
$$C_p = K$$

Lag compensator

$$C_{lag} = \frac{s/z+1}{s/p+1}, \; p < z$$

Lead compensator

$$C_{lead} = \frac{Ts+1}{\alpha Ts+1}, \; \alpha < 1$$



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