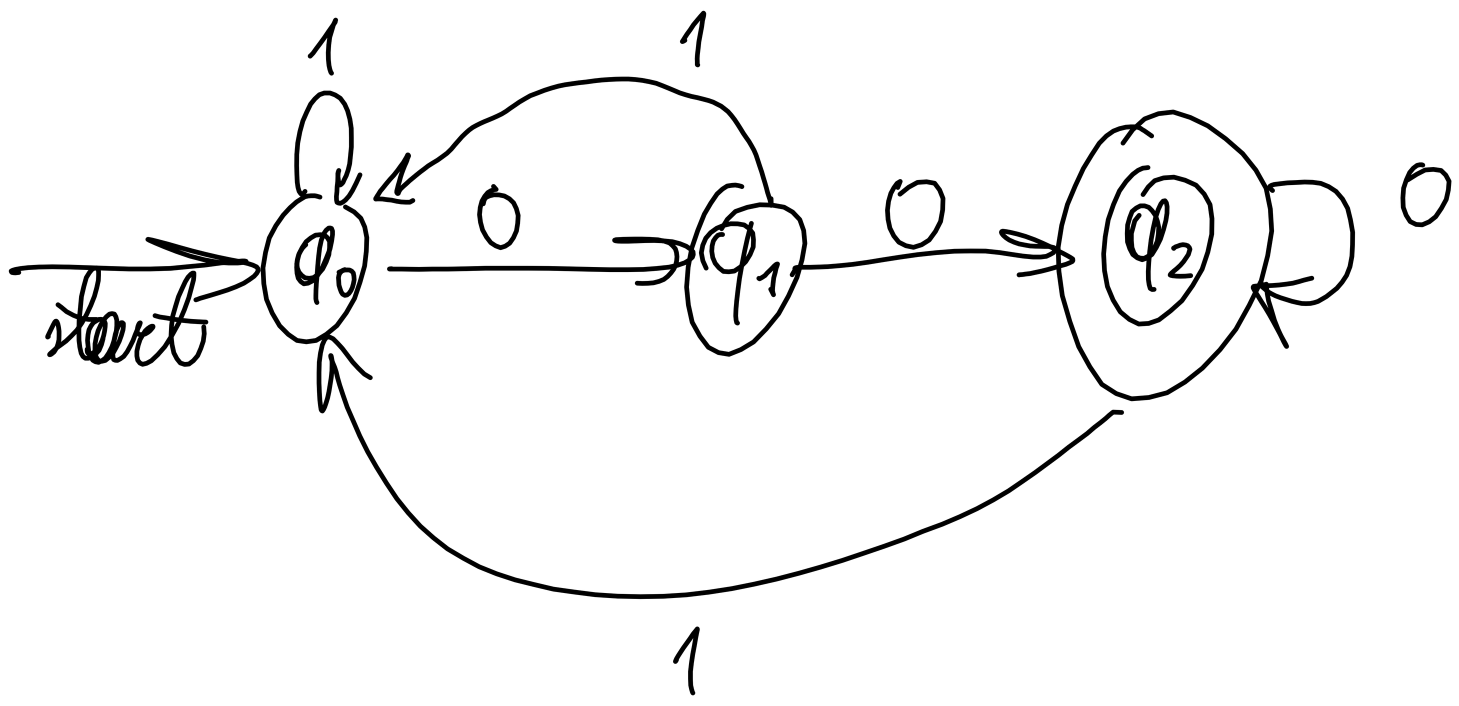
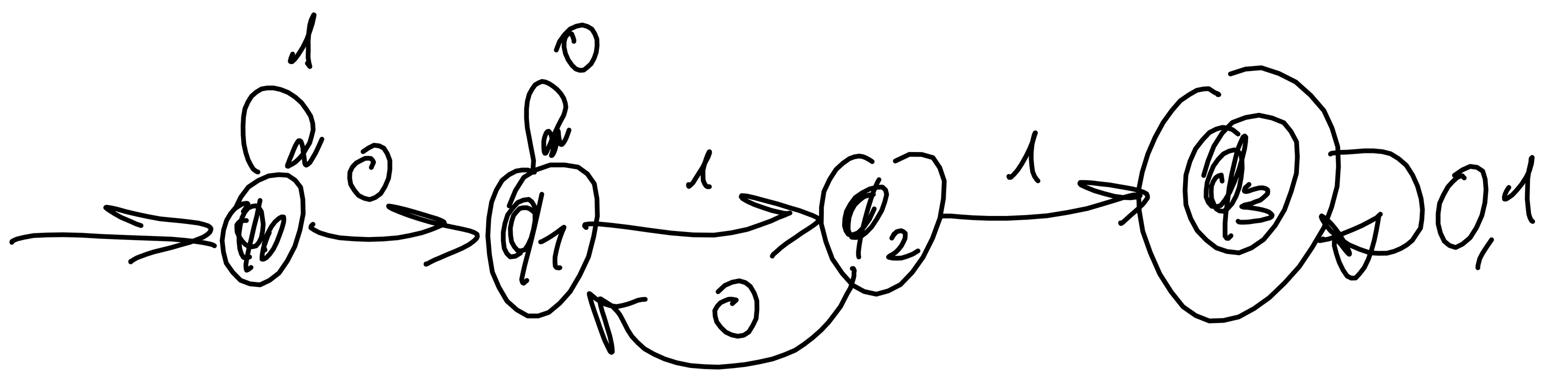
8/10

ES1:



10/10

$L = \{ w \mid w = \{0,1\}^*, 011 \text{ appears within } w \}$

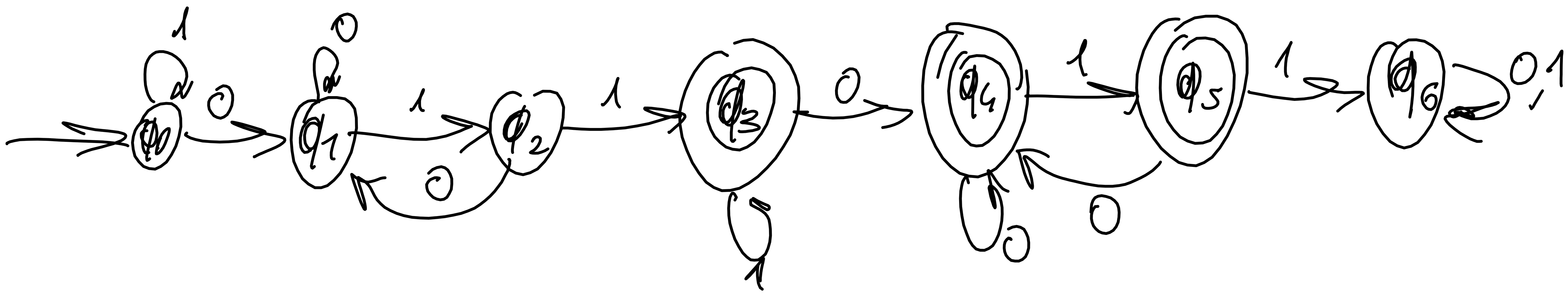


$L_? = \{ w \mid w = \{0,1\}^*, 011 \text{ doesn't appear within } w \} = \{0,1\}^* - L$

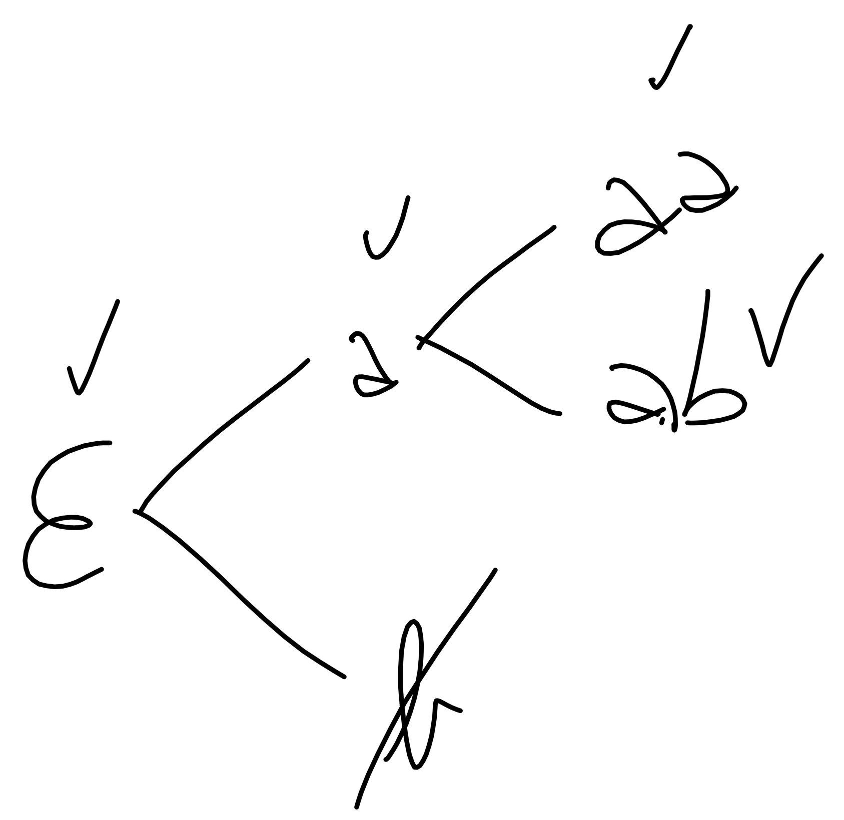
$\Downarrow$

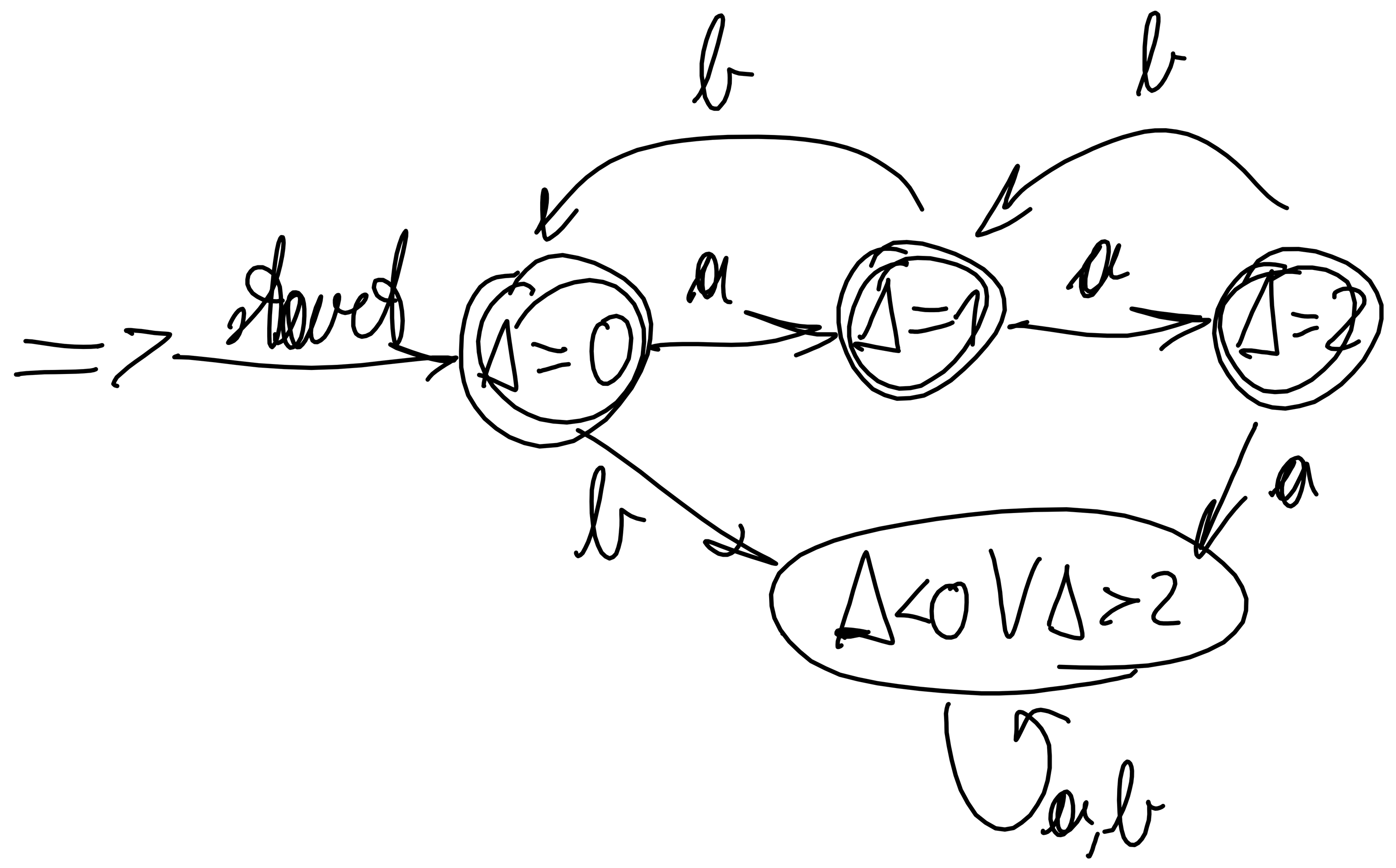
avrei TRAP STATE

$L = \ldots$ exactly once

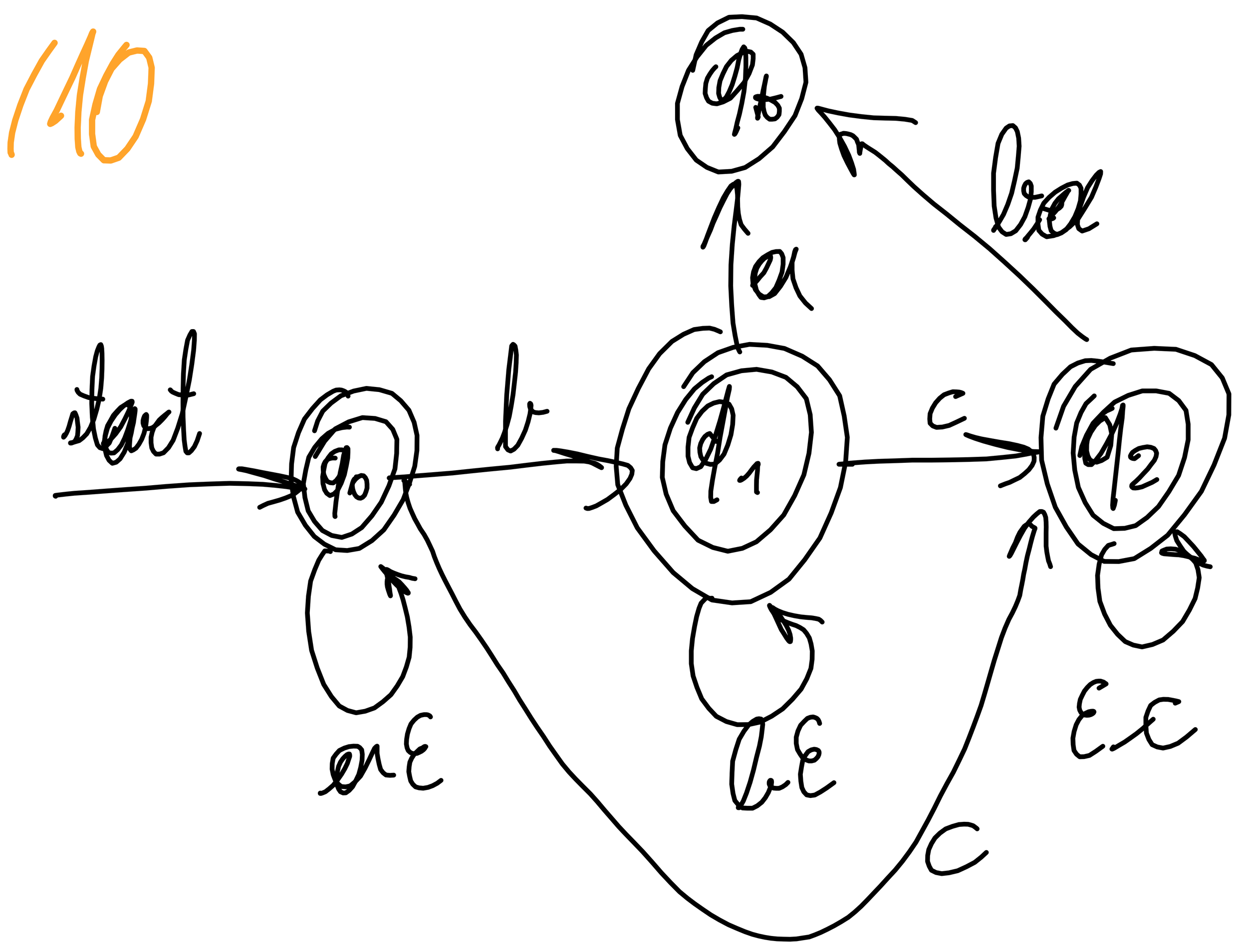


11/10

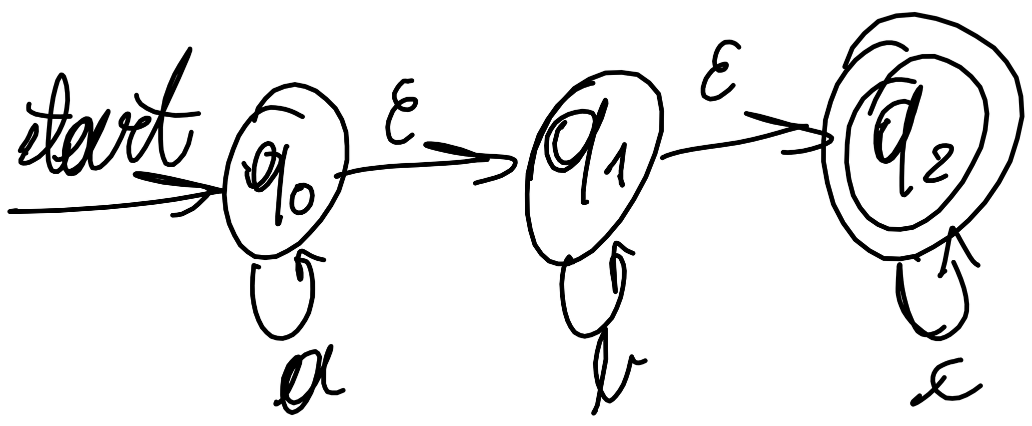




13/10



con 3 stati:



(computazioni morte vengono rifiutate)

29/11

Es 1: $\Sigma = \{a, b\}$, $L_1 = \{a^n b a^n b a^m \mid n, m \geq 1, m \geq n\} \Rightarrow$ CFL?

N cost. $\Rightarrow z = \underset{B1}{a^N} b \underset{B2}{a^N} b \underset{B3}{a^N}$

Casi possibili:

- v o x contengono $b \Rightarrow k=0$: tolgo $b \Rightarrow \notin L_1$
- v, x in stesso blocco $\Rightarrow k=0$: cambio lunghezza di blocchi $\Rightarrow |B1| \neq |B2|$ o $|B3| < |B1| \Rightarrow \notin L_1$
- v in $B1$, x in $B2 \Rightarrow$ almeno uno cresce con $k>1 \Rightarrow$ diventa $|B3| < |B1| \Rightarrow \notin L_1$
- v in $B2$, x in $B3 \Rightarrow k=0$: diventa $|B1| \neq |B2|$ o $|B3| < |B1|$

6/12

$\Sigma = \{a, b, c\}$, $L_1 = \{ w \mid w = XuYvZ,\ X, Y, Z \in \Sigma,\ u, v \in \Sigma^*,\ X = Z,\ |u| = |v| \}$

$L_1 \in REG$?

$$w = X \overset{u}{\overline{\qquad\qquad}} Y \overset{v}{\overline{\qquad\quad}} Z$$

(i) $|w|$ dispari ≥ 3

(ii) primo = ultimo

$R_0 = (a + b + c)$; $\underset{\substack{\Uparrow \\ (i)}}{R_1} = R_0 R_0 R_0 (R_0 R_0)^*$; $\underset{\substack{\Uparrow \\ (ii)}}{R_2} = a R_0^* a + b R_0^* b + c R_0^* c$

$R_1 \cap R_2$ genera L_1

$L_2 = \{ w \mid w = XuYvZ,\ X, Y, Z \in \Sigma,\ u, v \in \Sigma^*,\ X = Z = Y \}$

$R_1 = (a + b + c)^*$, $R_2 = a R_1 a R_1 a + b R_1 b R_1 b + c R_1 c R_1 c$

$L_3 = \{ w \mid w = XuYvZ,\ X, Y, Z \in \Sigma,\ u, v \in \Sigma^*,\ X = Z = Y,\ |u| = |v| \}$

($L_1 \cap L_2$ sbagliato: $L_3 \neq L_1 \cap L_2 \Rightarrow L_1$ ha Y qualunque in mezzo, L_2 ha Y da qualche parte $Y = X = Z$)

15/12

ES1: $\Sigma=\{a,b\}$, $L_1=\{bbxbbx^R \mid x\in\Sigma^*\} \Rightarrow REG?$

$w = bba^Nbba^N$

$\underbrace{bb\,\overbrace{a^N}}_{0 \;\dots\; N}\,bb\,\overbrace{a^N}$

1- y contiene $b \Rightarrow k=0$: tolgo b

2- y contiene solo $a \Rightarrow k=0$: x e x^R non corrispondono

$L_2=\{bbx \mid x\in\Sigma^*\}\cdot\{bbx^R \mid x\in\Sigma^*\} \Rightarrow REG?$

sì: concatenazione di REG

$L_3=\{bbx \mid x\in\Sigma^*\}\cdot L_1\cdot\{bbx^R \mid x\in\Sigma^*\} \Rightarrow REG?$

$w = bb\,bb\,\overbrace{a^N}\,bb\,\overbrace{a^N}\,bb$

($L_4=\{x \mid x\in\Sigma^*\}\cdot\{xx^R \mid x\in\Sigma^*\}\cdot\{x^R \mid x\in\Sigma^*\} \Rightarrow REG?$)

20/12 (settembre 2022)

$L_1 = \{w \mid w = a^p b a^q,\ p,q \geq 0,\ p+q \geq 1\}$

REG ⇒ non serve confrontare proprio p e q

$R = aa^*ba^* + a^*baa^*$

$L_2 = \{w \mid w = a^p b a^q,\ p,q \geq 0,\ 1 \leq p-q\}$

⇓

$p \geq q+1$ ⇒ confronto: non REG

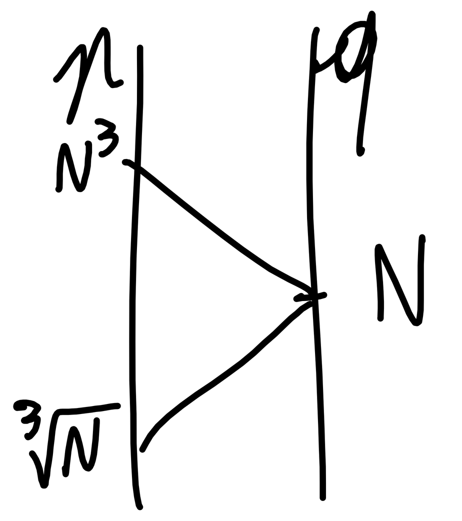
$w = a^{N+1} b a^N$

xy sarà solo in a^{N+1} ⇒ $k=0$: $N+1-|y| \leq N$ ⇒ $xz \notin L_2$

$(|y| \geq 1)$

$L_3 = \{w \mid w = a^p b a^q,\ p,q \geq 0,\ p = q^3 \text{ xor } q = p^3\}$

(non REG: serve conteggio/confronto)



⇒ $w = a^{N^3} b a^N$ ⇒ xy solo in a^{N^3} ⇒ $k>1$: $N+1+(k-1)|y| \neq N^3, \neq \sqrt[3]{N}$ ⇒

⇒ $xy^kz \notin L_3$

22/12

ES1: $\Sigma=\{0,1\}^*$; $L_{ev}=\{w \mid w\in\Sigma^*, |w| \text{ pari}\}$

proprietà DI RE: $P=\{L \mid L\in RE, L_{ev}\subset L\}$

$L_P=\{enc(M) \mid L(M)\in P\}$

(a) $L_P\in REC$? NO $\Rightarrow$ usare Rice $\Rightarrow$ mostrare che P é non-trivial

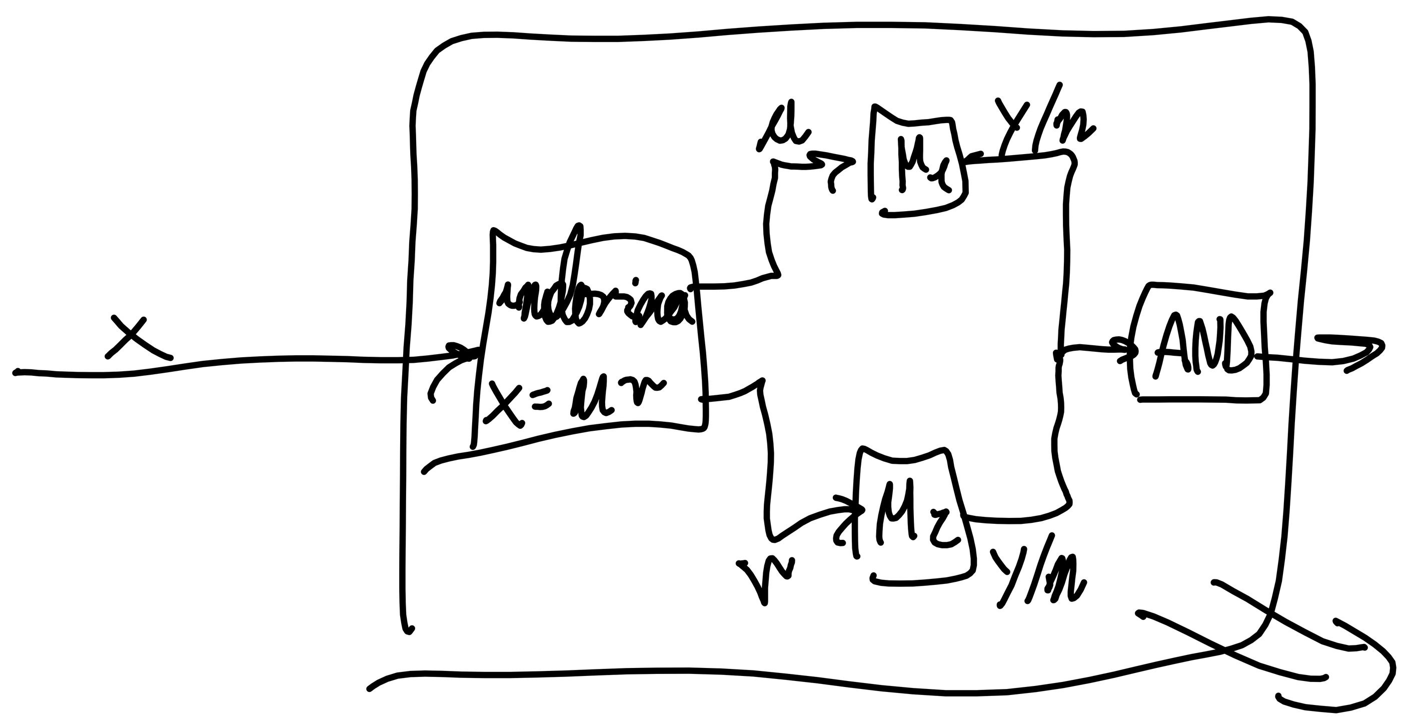
I) $P\neq\emptyset \Rightarrow L=\Sigma^*\in P$

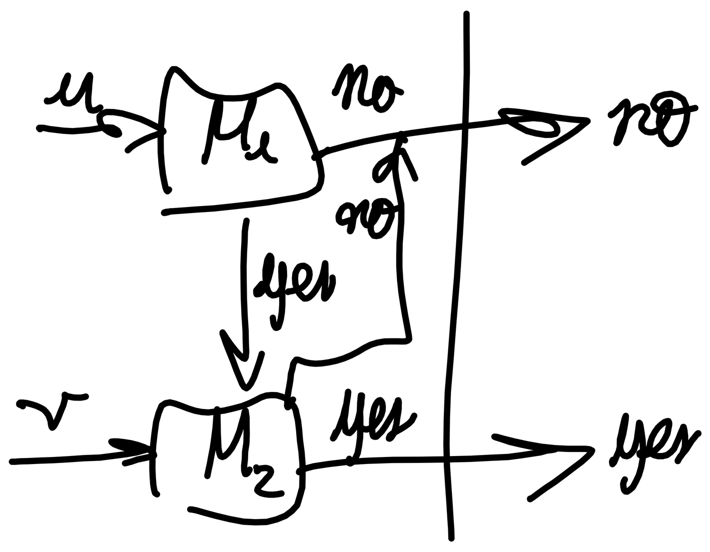
II) $P\neq RE \Rightarrow L=L_{ev}\notin P$, $L_{ev}\in RE$
$L=\emptyset$

ES2. $L_1, L_2\in REC$; $L_1L_2\in REC$? SÍ

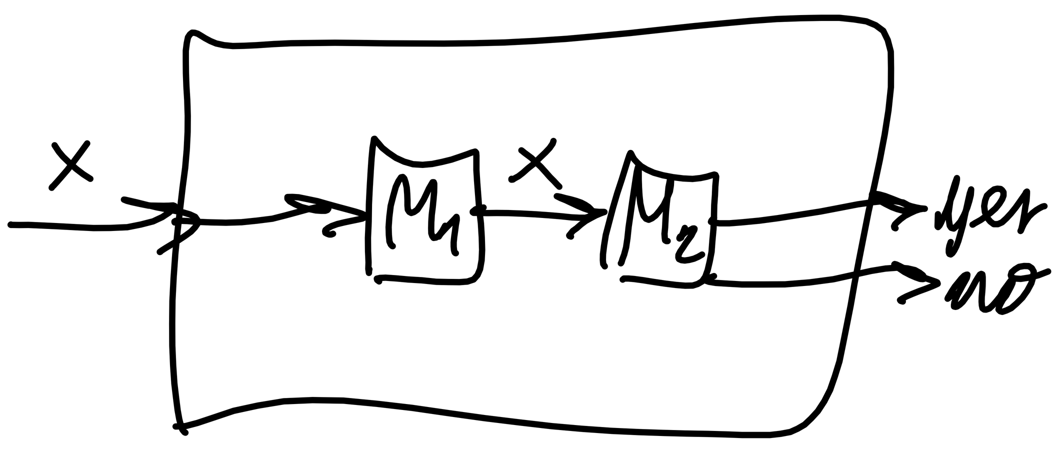
$\exists M_1, M_2 \mid L(M_1)=L_1,\ L(M_2)=L_2$, M_1, M_2 si fermano sempre

costruiamo $M_\cdot \mid L(M_\cdot)=L_1L_2 \Rightarrow$ NTM





sol. SBAGLIATA:



- M_1 non può dare stringhe in output

$L=\{enc(M_1,M_2) \mid L(M_1)\subseteq L(M_2)\} \Rightarrow L\in RE$?

$L_e\leq_m L \Rightarrow$ $\underset{enc(M)}{L_e} \xrightarrow{red} \underset{enc(M_1,M_2)}{L}$

poniamo $M_1=M$, $M_2=M \Rightarrow$ YES: $L(M)=\emptyset$

Y2Y: $enc(M)\in L_e \Rightarrow L(M)=\emptyset$ (def. L_e) $\Rightarrow L(M_1)=L(M_2)=\emptyset$ (def red) $\Rightarrow$
$\Rightarrow L(M_1)\subseteq L(M_2) \Rightarrow enc(M_1,M_2)\in L$

N2N: $enc(M) \notin L_e \Rightarrow L(M) \neq \emptyset \Rightarrow L(M_1) = L(M_2) \neq \emptyset \Rightarrow \dots \Rightarrow$ non funziona

SOL: $L_e \leq_m L \Rightarrow$ porre $M_1 = M \Rightarrow$ porre $M_\emptyset \mid L(M_\emptyset) = \emptyset \Rightarrow$ porre $M_2 = M_\emptyset$

YZY uguale

N2N: $enc(M) \notin L_e \Rightarrow L(M) \neq \emptyset \Rightarrow L(M_1) \neq \emptyset \Rightarrow L(M_1) \neq L(M_2) = \emptyset \Rightarrow enc(M_1, M_2) \notin L$

riduzione corretta

9/1

01/2020 [6 pt]

a) $L_1, L_2 \notin REG \Rightarrow L_1 \cap L_2 \notin REG$? false

$L_1 = \{a^n b^n \mid n \geq 0\}$, $L_2 = \{b^n a^n \mid n \geq 0\} \Rightarrow L_1 \cap L_2 = \{\varepsilon\} \in REG$

b) $L_1 \in REG$, $L_2 \in CFL \Rightarrow L_1 L_2 \in REG$? false

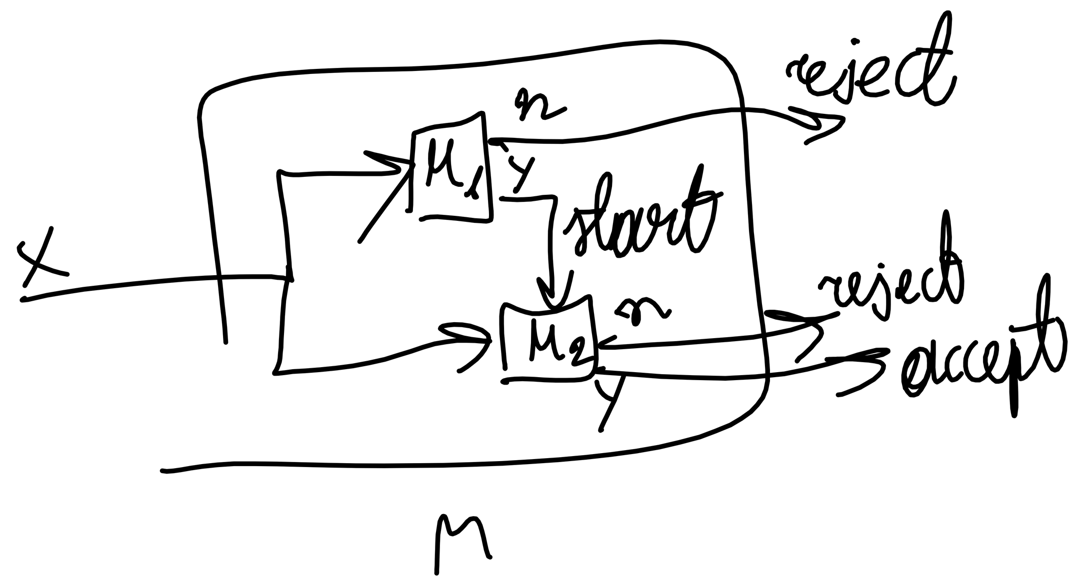
$L_1 = \{\varepsilon\}$, $L_2 = \{a^n b^n \mid \dots\} \Rightarrow L_1 L_2 = L_2 \notin REG$

c) $L \in CFL \Rightarrow \bar{L} \notin CFL$? false

$L_1 = \Sigma^* \Rightarrow \bar{L}_1 = \emptyset \in CFL$

d) $L_1, L_2 \in CFL \Rightarrow L_1 \cap L_2 \in REC$? true

posso costruire TM che accetti $L_1 \cap L_2$



oppure:
- REC chiuso sotto intersezione
- $CFL \subseteq REC$

10/01

ES1: $P=\{L \mid L\in RE,\ w\in L\} \Rightarrow L_P=\{enc(M)\mid L(M)\in P\}$

a) $L_P\in REC$? false

o riduzione o (in certi casi) posso usare Rice

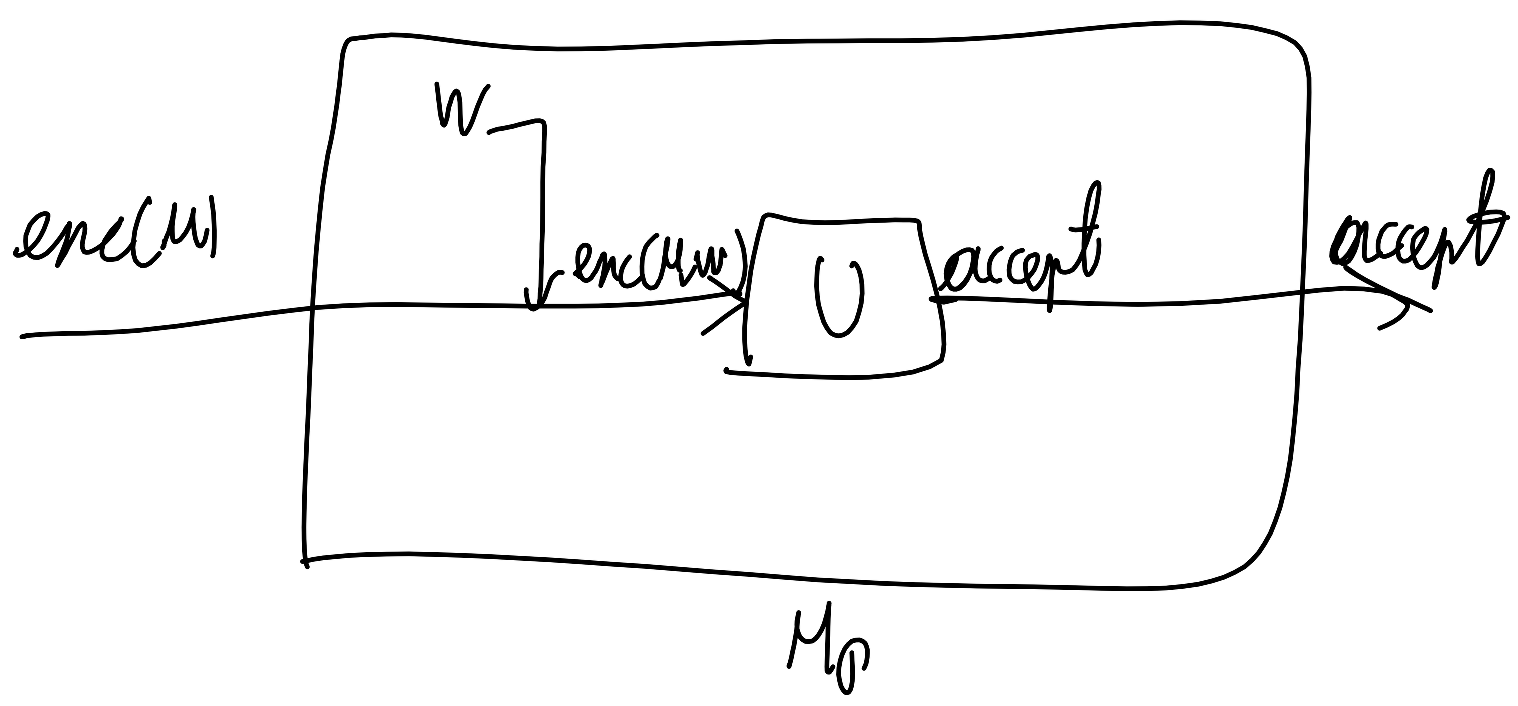
controlliamo se P è triviale:

- $P=\emptyset \Rightarrow RE\neq\emptyset$ ($\{w\}, \Sigma^*\in P$)
- $P=RE \Rightarrow \emptyset\notin P$ ($w\notin\emptyset$), $w\in\Sigma^*$, prendo $u\neq w$: $\{u\}\notin P$

non triviale $\Rightarrow L_P\notin REC$

b) $L_P\in RE$? true

M_P tale che $L(M_P)=L_P$



ES2: (09/23)

x INFIX di w se $w=uxv$, $u,v\in\Sigma^*$

$P=\{L\mid L\in RE, \forall w\in L$ ha infix $01110\}$

($L=\{01110\}\in P$, $\emptyset\in P$)

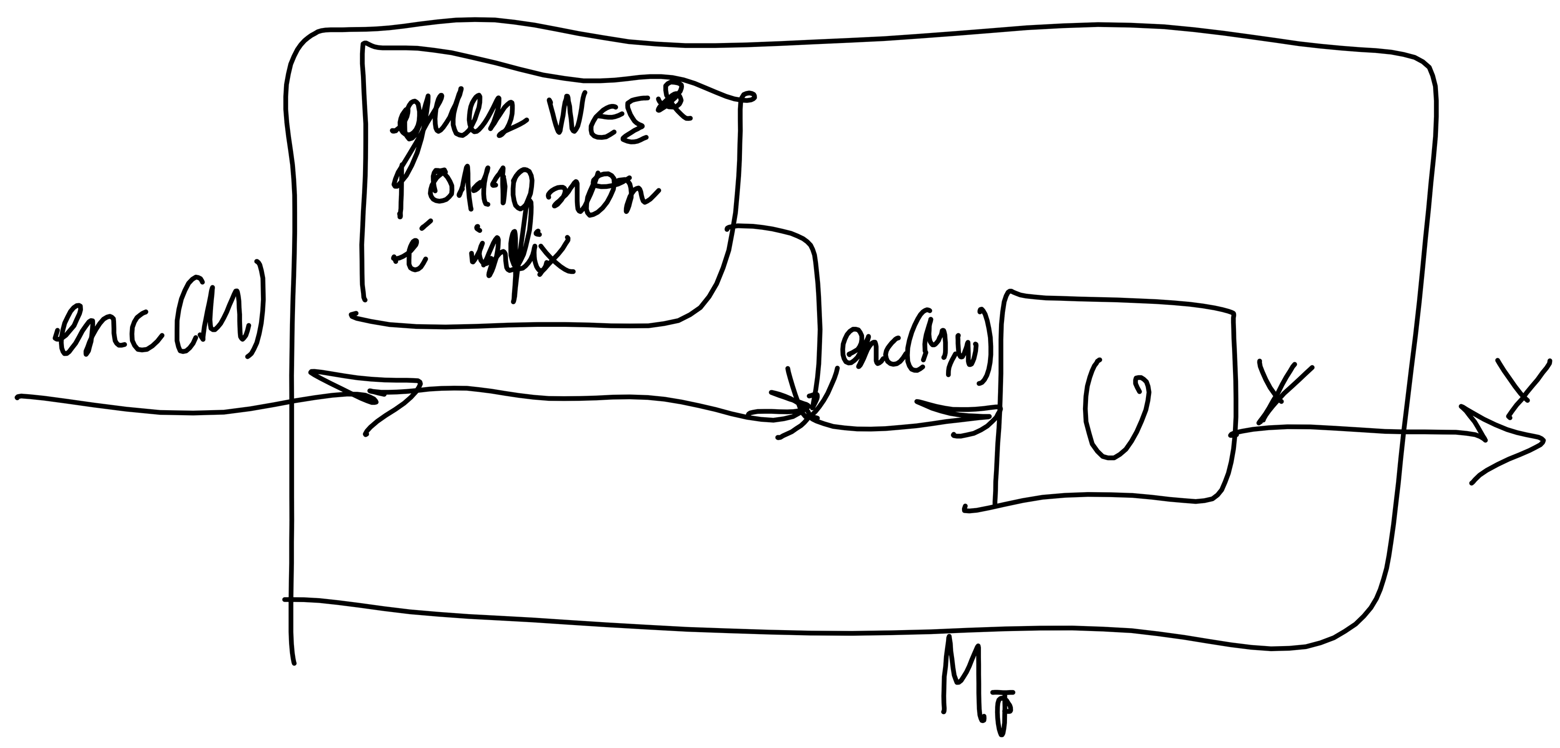
a) $L_P\in REC$? false

- $L_P=\emptyset$? $\{01110\}\in P \Rightarrow L_P\neq\emptyset$
- $L_P=RE$? $\{00\}\notin P \Rightarrow L_P\neq RE$

b) $L_P\in RE$? false

dovrei controllare ogni possibile stringa -> per qualche potrebbe non fermarsi

prendiamo $\overline{L_P}=\{L\mid L\in RE, \exists w\in L \mid$ non ha infix $01110\}$



$L(M_{\overline{P}})=\overline{L_P} \Rightarrow \overline{L_P}\in RE/REC \Rightarrow$

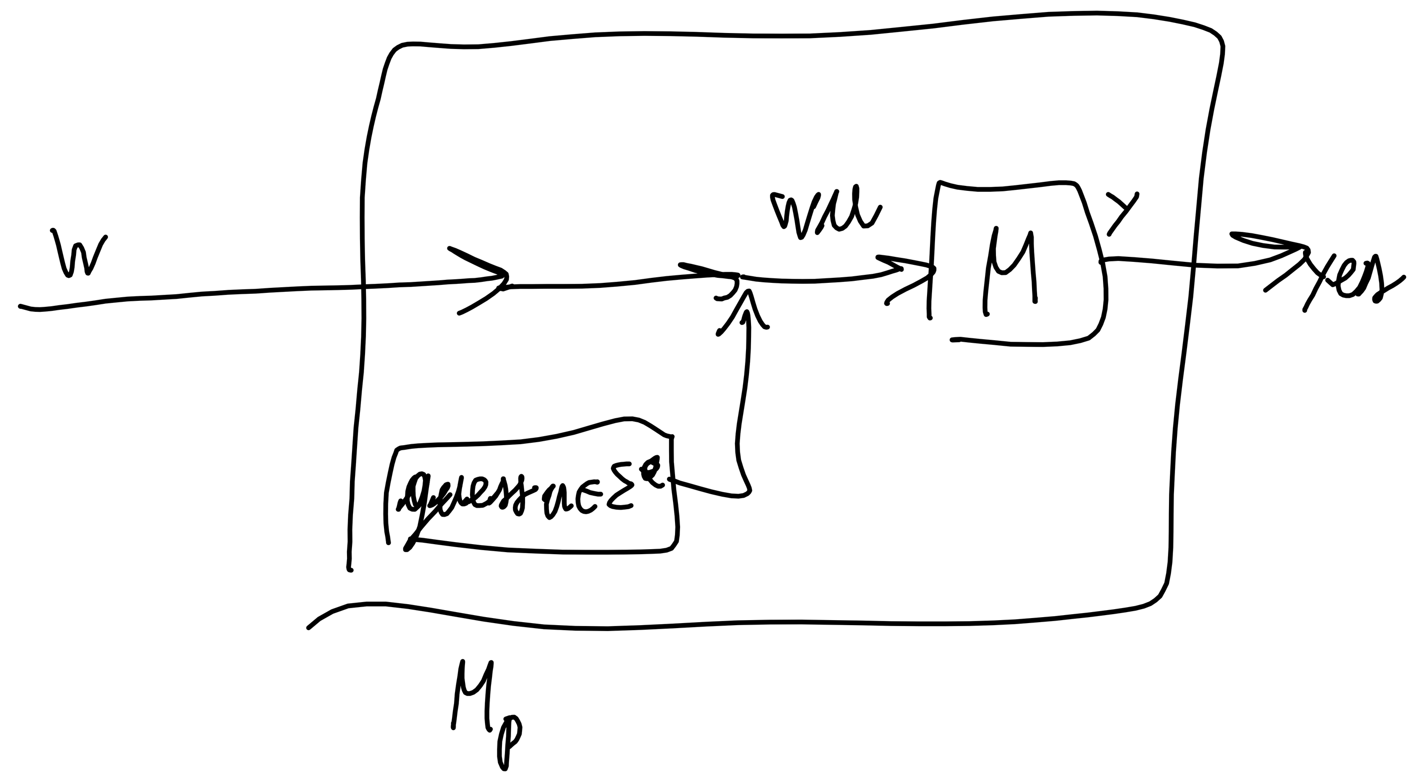
$\Rightarrow L_P\notin RE$

ES2: RE è chiuso sotto operatore prefisso? true

($L \in RE \Rightarrow prefix(L) \in RE$?)

(Ex: $L = \{01110\} \Rightarrow prefix(L) = \{\varepsilon, 0, 01, 011, 0111, 01110\}$)

sia M TM | $L(M) = L \Rightarrow$ facciamo M_p | $L(M_p) = prefix(L) \Rightarrow$ accetto.

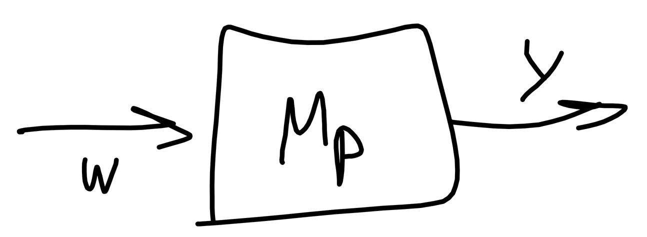


se $w \in prefix(L)$, allora $\exists u \in \Sigma^* \mid wu \in L$, allora M_p indovinerà u, allora wu viene dato a M, allora M_p accetta $prefix(L) \subseteq L(M_p)$

$L(M_p) \subseteq prefix(L)$:

se $w \in L(M_p) \Rightarrow \exists u \in \Sigma^* \mid wu \in L(M) \Rightarrow wu \in L \Rightarrow w \in prefix(L)$

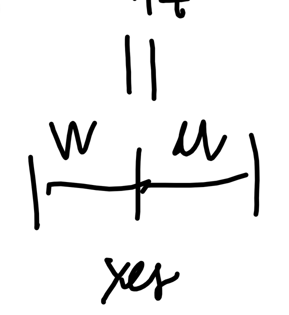
allora $prefix(L) = L(M_p)$



if $w \in prefix(L)$

input $w \Rightarrow \forall u \in \Sigma^*$ se $u \in L(M)$, $\forall$ cut $u = v_1 v_2$, se $v_1 = w$, allora accetta

supponiamo $w_1 \dots w_{17}$



PAIR GENERATOR: $(i,j) \mid i+j =$ costante

$2 \Rightarrow (1,1)$

$3 \Rightarrow (1,2), (2,1)$

$\vdots$

prendo $w \Rightarrow$ genero prossima coppia $(i,j) \Rightarrow$ scelgo $w_i \in \Sigma^* \Rightarrow$

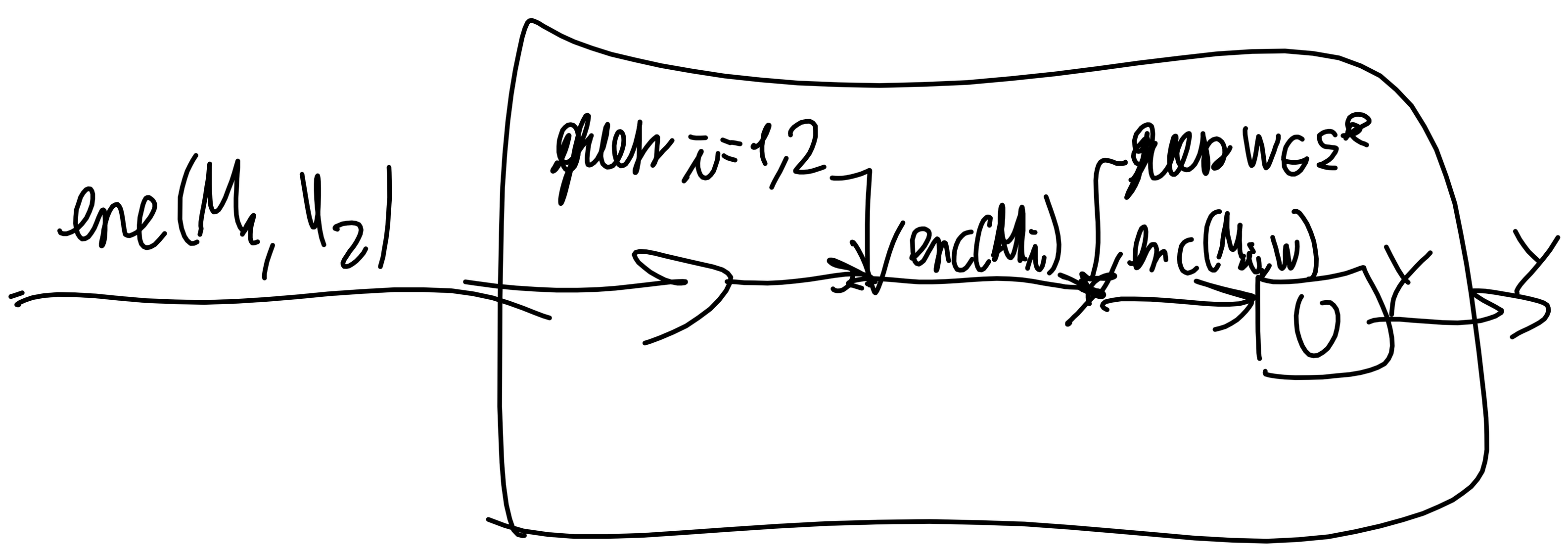
$\Rightarrow$ simulo ww_i su M per esattamente j passi $\Rightarrow$ ripeto

w_g: simulo ww_g per

ES3: $L_1 = \{enc(M_1, M_2) \mid L(M_1) \cup L(M_2) = \emptyset\}$, $L_2 = \{enc(M_1, M_2) \mid L(M_1) \cup L(M_2) \neq \emptyset\}$

$L_1 \in RE$? false

$\overline{L_1} = L_2$



$L_e \leq_m L_1 \Rightarrow enc(M) \rightarrow \dots \rightarrow enc(M_1, M_2)$

(1) $M_1 = M$, $M_2 = M$

(2) se $enc(M) \in L_e \Rightarrow L(M) = \emptyset \Rightarrow L(M) \cup L(M) = \emptyset \Rightarrow L(M_1) \cup L(M_2) = \emptyset \Rightarrow$

$\Rightarrow enc(M_1, M_2) \in L_1$

se $enc(M) \notin L_e \Rightarrow L(M) \neq \emptyset \Rightarrow L(M) \cup L(M) \neq \emptyset \Rightarrow L(M_1) \cup L(M_2) \neq \emptyset \Rightarrow$

$\Rightarrow enc(M_1, M_2) \notin L_1$

12/1

ES1: (giugno 2021)

$\Sigma = \{a, b, c\}$, $L_1 = \{a^n a^m b^n c^m \mid n, m \geq 1\}$; $L_1 \in CFL$? true

G: $S \to aSc \mid aBc$

$B \to aBb \mid ab$

$L_2 = \{a^n a^n b^n c^n \mid n \geq 1\}$; $L_2 \in CFL$? false

pumping lemma

ES2: (es 3.8)

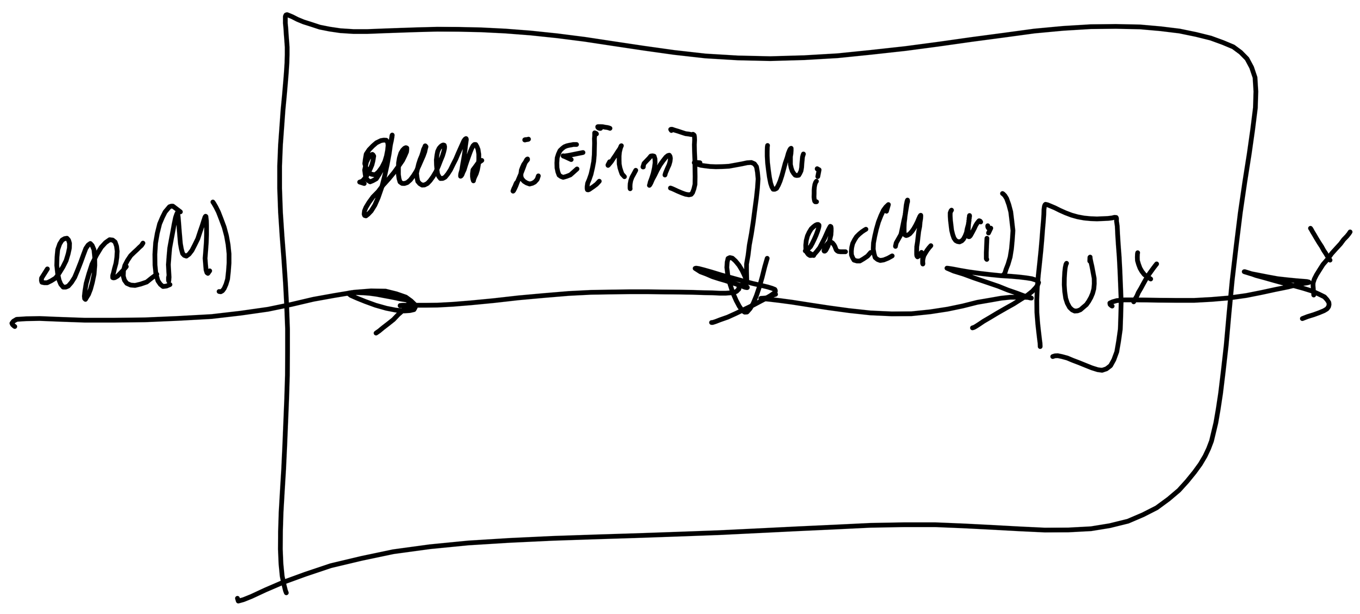
L_1 finito; $P = \{L \mid L \in RE, L_1 \cap L \neq \emptyset\}$

$L_P \in REC$? false

- $L_1 = \emptyset \Rightarrow \forall L \in RE \; L \cap L_1 = \emptyset \Rightarrow P = \emptyset$
- $L_1 \neq \emptyset$ ✓
  - $P \neq \emptyset \Rightarrow L_1 \in P$
  - $P \neq RE \Rightarrow \exists w \in \Sigma^* \mid w \notin L_1 \Rightarrow L_w = \{w\} \in RE \Rightarrow L_w \cap L_1 = \emptyset$

$L_P \in RE$? true

$L_1 \neq \emptyset \Rightarrow L_1 = \{w_1, \dots, w_n\}$



ES3: (giugno 2021)

$L_R \in REG$; $P = \{L \mid L \in RE, L \cup L_R = \Sigma^*\}$; $L_1 = L_P = \{enc(M) \mid L(M) \in L_P\}$; $L_1 \in REC$?

- $L_R = \Sigma^* \Rightarrow$ sempre vero $\Rightarrow$ trivial $\Rightarrow L_P \in REC$
- $L_R \neq \Sigma^* \Rightarrow P$ non triviale $\Rightarrow L_P \notin REC$
  - $P \neq \emptyset \Rightarrow \Sigma^* \in P$
  - $P \neq RE \Rightarrow \emptyset \notin P$

$L_2 = \{enc(M_1, M_2) \mid L(M_1) \cup L(M_2) \cup L_R = \Sigma^*\}$; $L_2 \in REC$? false

$L_1 \leq_m L_2$

① input: enc(M); output: enc(M_1, M_2)

metto $M_1 = M$, $M_2 = M_\emptyset$

(2) YZY NZN

$enc(M) \in L_1 \Rightarrow L(M) \cup L_R = \Sigma^* \Rightarrow L(M) \cup L(M_\emptyset) \cup L_R = \Sigma^* \Rightarrow L(M_1) \cup L(M_2) \cup L_R = \Sigma^* \Rightarrow$

$\Rightarrow enc(M_1, M_2) \in L_2$

$enc(M) \notin L_1 \Rightarrow L(M) \cup L_R \neq \Sigma^* \Rightarrow L(M) \cup L(M_\emptyset) \cup L_R \neq \Sigma^* \Rightarrow L(M_1) \cup L(M_2) \cup L_R \neq \Sigma^* \Rightarrow$

$\Rightarrow enc(M_1, M_2) \notin L_2$

ES: (gennaio 2020)

$\Sigma = \{a, b\}$; $L_1 = \{xyx^Ry^R \mid x, y \in \Sigma^+\}$; $L_1 \in CFL$? false

$R = ba^+b$; $R_4 = RRRR \Rightarrow L(R_4) = \{ba^{n_1}bba^{n_2}bba^{n_3}bba^{n_4} \mid n_i \geq 1 \ \forall i = 1,2,3,4\}$

$L_2 = L(R_4) \cap L_1 \notin CFL \Rightarrow$ contrad.

$L_2 = \{xyy^Rx^R \mid x, y \in \Sigma^+\} \left(\Rightarrow \{\ldots\ldots \overset{aa}{bb} \ldots\ldots\} \Rightarrow\right) L_2 \in CFL$? true

$L_3 = \{xyy^Rx^R \mid x, y \in \Sigma^+, |x| = |y|\} \left(\Rightarrow \{ww^R \mid w \in \Sigma^+, |w| \text{ even}, |w| \geq 2\} \Rightarrow\right) \Rightarrow L_3 \in CFL$? true