

Game theory exercises – Static games of complete information
L. Badia – Game theory course 2020/21

Exercise 1 Consider the following static games of complete information played by A and B where the normal-form representation of the game is given. For all of them, find the entire set of Nash equilibria.

*g dominata da f
↓
dono el.,
G dominata da F*

(a)

		B	
		f	g
A	F	2, 4	0, 1
	G	1, 6	3, 5

(b)

		B	
		f	g
A	F	0, 4	3, 0
	G	6, 0	0, 5

(c)

		B	
		f	g
A	F	9, 3	2, 2
	G	0, 0	3, 9

(d)

		B	
		f	g
A	F	2, 2	0, 6
	G	6, 0	1, 1

*⇒ f dominata da G
↓
dono el.,
f dominata da G*

simile a pari/dispari ⇒ no NE in strategie pure

Exercise 2 Consider the following static game of complete information played by A and B where the normal-form representation is given below.

		B			
		J	K	L	M
A	X	6, 7	5, 5	3, 8	8, 1
	Y	4, 9	9, 2	0, 4	2, 3
	Z	8, 4	2, 8	4, 2	3, 6

1. Prove that there is no Nash equilibrium in pure strategies
2. Prove that these (m_A, m_B) are Nash equilibria in mixed strategies:

- $m_A = (2/3, 0, 1/3)$, $m_B = (5/11, 4/11, 2/11, 0)$
- $m_A = (0, 4/11, 7/11)$, $m_B = (7/11, 4/11, 0, 0)$

3. List all of the joint pure strategies that are Pareto optimal.

Exercise 3 Two students, Charlotte (C) and Daniel (D), need to write their MS Thesis. They need to choose (independently and unbeknownst to each other) a supervising professor. Three professors are available for this role: Xavier, Yuan, and Zingberry. The utility of a student is given by the amount of help he/she receives from the supervisor, which is quantified as 40 for Xavier, 60 for Yuan, 50 for Zingberry. However, if the two students select the *same* professor as their supervisor, they only get 70% of the utility that they would get if the professor had only one of them to supervise.

1. Write the game in normal form.
2. Find all the Nash equilibria of the game in pure strategies
3. Find all the Nash equilibria of the game

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Exercise 4 A strategic interaction takes place between a taxpayer **T** and the tax inspector **I**. **T** is supposed to pay a share S of its income so as to have a net income after paying taxes equal to R . However, **T** is considering two alternatives: hide part of the taxes (**H**) to get an additional dishonest income of L (so the tax paid is $S - L$ and the net income is $R + L$) or pay all due taxes in full (**P**). Player **I** also has two choices: check **T** for tax evasion (**C**) or do not check **T** (**D**). Performing a check has a cost equal to E . If the tax inspector discovers that **T** did not pay the tax, then the taxpayer will be fined and will have to pay an additional amount equal to F that goes into the inspector. The probability of being discovered by a tax inspector after a check is p . The purpose of the taxpayer is to get the maximum possible amount of money. The goal of the tax inspector is to maximize the collected amount (minus the cost). Formalize this conflict in the form of a static game of complete information and find its Nash equilibria.

1b) prob. che B scelga $f: \alpha$ } entrambi vogliono che altro
 // // A // $F: \beta$ } non indifferente

		B	
		f	g
A	F	0,4	3,0
	G	6,0	0,5

$$u_A(d, f) = 3(1-\alpha), \quad u_A(d, g) = 6\alpha \Rightarrow$$

$$\Rightarrow 3(1-\alpha) > 6\alpha \Rightarrow 9d < 3 \Rightarrow \alpha < 1/3$$

strategie per trovare NE:

IEDS, evidenziazione best responses

