UNIVERSITÀ
DEGLI STUDI
DI PADOVA

Evaluation Measures

Search Engines

Master Degree in Computer Engineering
Master Degree in Data Science
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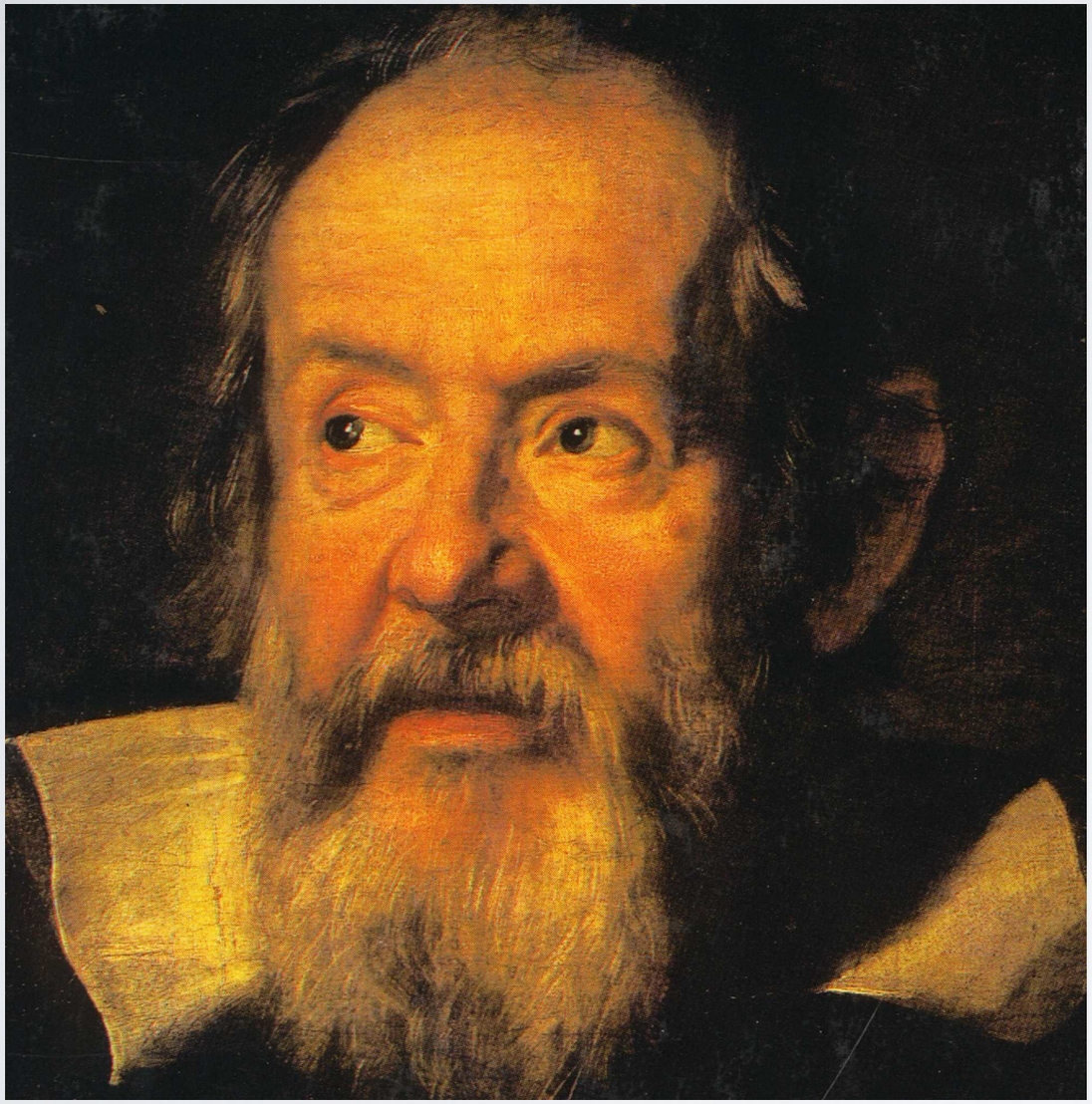
DIPARTIMENTO
DI INGEGNERIA
DELL'INFORMAZIONE

Outline

- Set-based Evaluation Measures
- Rank-based Evaluation Measures

Evaluation Measures



"Measure what is measurable and make measurable what is not"

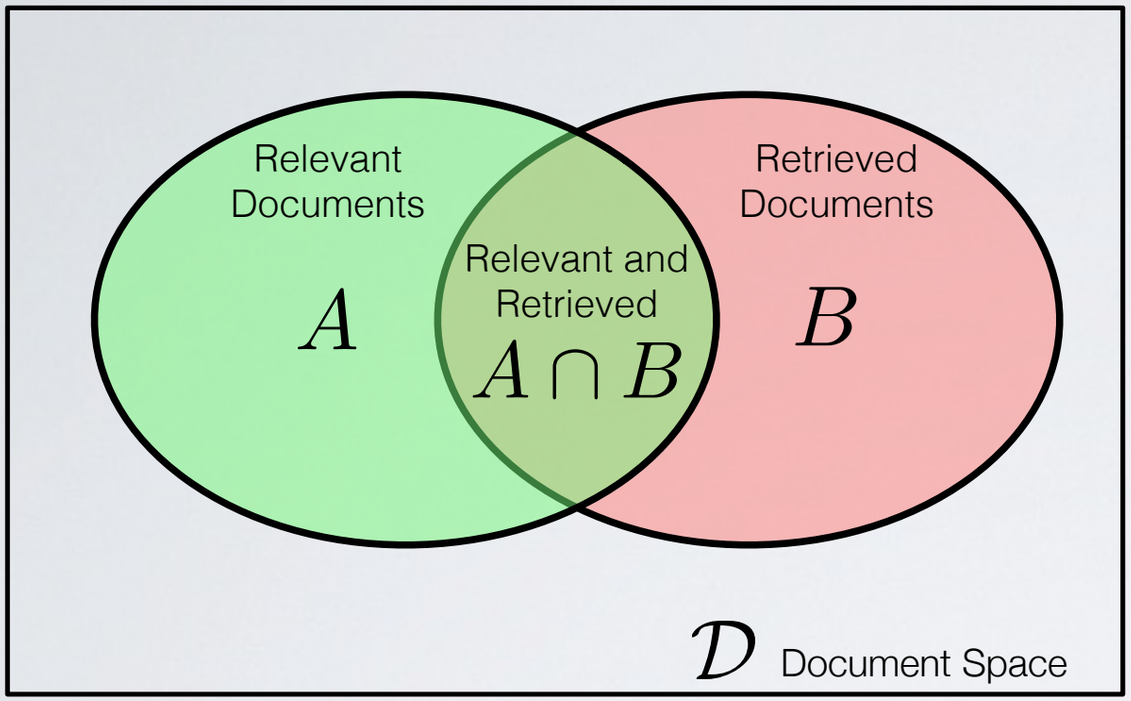
Galileo Galilei (1564-1642)

A Taxonomy of Evaluation Measures

	Set-Based Retrieval	Rank-Based Retrieval
Binary Relevance	Precision (P) Recall (R) F-measure (F)	Precision at Document Cut-off (P@k) Recall at Document Cut-off (R@k) R-Precision (Rprec) Average Precision (AP) Rank-Biased Precision (RBP) …
Multi-graded Relevance	Not widely agreed generalizations of Precision and Recall	Discounted Cumulated Gain (DCG) …

Set-based Evaluation Measures

Set-based Measures: Precision, Recall and F-measure



$$P = \frac{|A \cap B|}{|B|} \qquad R = \frac{|A \cap B|}{|A|}$$

$$F = \frac{2}{\frac{1}{P} + \frac{1}{R}} = 2\frac{P \cdot R}{P + R}$$

- **Precision** is the the proportion of retrieved documents that are actually relevant
- **Recall** is the the proportion of relevant documents actually retrieved
- Together, Precision and Recall measure **retrieval effectiveness**, meant as the ability of a system to retrieve relevant documents while at the same time holding back non-relevant ones
  - maximizing Precision and Recall corresponds to optimal retrieval in the sense of the **Probability Ranking Principle**, i.e. ordering documents by their decreasing probability of being relevant, and creates a tight link between retrieval models and evaluation
- **F-measure** is the harmonic mean of Precision and Recall, summarising them into a single score

van Rijsbergen, C. J. (1974). Foundations of Evaluation. *Journal of Documentation*, 30(4):365–373.
van Rijsbergen, C. J. (1981). Retrieval effectiveness. In Spärck Jones, K., editor, *Information Retrieval Experiment*, pages 32–43. Butterworths, London, United Kingdom.

Set-based Measures: Example

$$P = \frac{4}{10} = 0.40$$

$$R = \frac{4}{8} = 0.50$$

$$F = 2 \cdot \frac{\frac{4}{10} \cdot \frac{4}{8}}{\frac{4}{10} + \frac{4}{8}} = \frac{4}{9} = 0.44$$

Assume

- $|A| = 8$ relevant documents in total
- Lenient mapping to binary relevance degrees

What Type of Mean?

$$s = \frac{d}{t}$$

- Suppose Herbie goes forward a distance d at speed s_1, say 60 km/h, and goes backward the same distance d at speed s_2, say 20 km/h.

What is Herbie's average speed during the travel?

$$\frac{s_1 + s_2}{2} = 40 \text{ km/h} \qquad 2\frac{s_1 \cdot s_2}{s_1 + s_2} = 30 \text{ km/h}$$

Ferger, W. F. (1931). The Nature and Use of the Harmonic Mean. *Journal of the American Statistical Association*, 26(173):36–40.

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$$\bar{s} = \frac{\text{total distance}}{\text{total time}} = \frac{d + d}{\frac{d}{s_1} + \frac{d}{s_2}} = 2\frac{s_1 \cdot s_2}{s_1 + s_2}$$

Ferger, W. F. (1931). The Nature and Use of the Harmonic Mean. *Journal of the American Statistical Association*, 26(173):36–40.

What Type of Mean?

$$s = \frac{d}{t}$$

Mean of two numbers, s_1 and s_2, which are ratios and where the numerator d is fixed

Suppose Herbie goes forward a distance d at speed s_1, say 60 km/h, and goes backward the same distance d at speed s_2, say 20 km/h. What is Herbie's average speed during the travel?

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What Type of Mean?

$$s = \frac{d}{t}$$

- Suppose Herbie goes for a time t, say 10 minutes, at speed s_1, say 60 km/h, and for the same amount of time t at speed s_2, say 20 km/h. What is Herbie's average speed during the travel?

$$\frac{s_1 + s_2}{2} = 40 \text{ km/h} \qquad 2\frac{s_1 \cdot s_2}{s_1 + s_2} = 30 \text{ km/h}$$

What Type of Mean?

$$s = \frac{d}{t}$$

- Suppose Herbie goes for a time t, say 10 minutes, at speed s_1, say 60 km/h, and for the same amount of time t at speed s_2, say 20 km/h.

What is Herbie's average speed during the travel?

$$\frac{s_1 + s_2}{2} = 40 \text{ km/h}$$

$$2\frac{s_1 \cdot s_2}{s_1 + s_2} = 30 \text{ km/h}$$

$$\bar{s} = \frac{\text{total distance}}{\text{total time}} = \frac{s_1 \cdot t + s_2 \cdot t}{t + t} = \frac{s_1 + s_2}{2}$$

What Type of Mean?

$$s = \frac{d}{t}$$

Mean of two numbers, s_1 and s_2, which are ratios and where the denominator t is fixed

Suppose Herbie goes for a time t, say 10 minutes, at speed s_1, say 60 km/h, and for the same amount of time t at speed s_2, say 20 km/h. What is Herbie's average speed during the travel?

$$\frac{s_1 + s_2}{2} = 40 \text{ km/h}$$

$$2\frac{s_1 \cdot s_2}{s_1 + s_2} = 30 \text{ km/h}$$

$$\bar{s} = \frac{\text{total distance}}{\text{total time}} = \frac{s_1 \cdot t + s_2 \cdot t}{t + t} = \frac{s_1 + s_2}{2}$$

Back to Precision, Recall, and F-measure

$$P = \frac{|A \cap B|}{|B|} \qquad R = \frac{|A \cap B|}{|A|}$$

- F-measure is a mean between two ratios whose numerator is fixed, thus the harmonic mean looks more appropriate

$$F = 2\frac{P \cdot R}{P + R} = 2\frac{\frac{|A \cap B|}{|B|} \cdot \frac{|A \cap B|}{|A|}}{\frac{|A \cap B|}{|B|} + \frac{|A \cap B|}{|A|}} = \frac{|A \cap B|}{\frac{|A| + |B|}{2}}$$

- F-measure scores what went good $|A \cap B|$, i.e. the relevant retrieved documents, against the average of what might have been $|A|$, i.e. the relevant documents, and what actually happened $|B|$, i.e. the retrieved documents

Rank-based Evaluation Measures

Rank-based Measures: Precision and Recall

- **Precision at Document Cut-off**:

$$P(k) = \frac{1}{k} \sum_{n=1}^{k} r_n$$

where $r_k \in \{0, 1\}$ is the relevance degree of the n-th document

- **Recall at Document Cut-off**:

$$R(k) = \frac{1}{RB} \sum_{n=1}^{k} r_n$$

where $RB = |A|$ is the **recall base**, i.e. the total number of relevant documents

- **Rprec** is Precision computed at the recall base

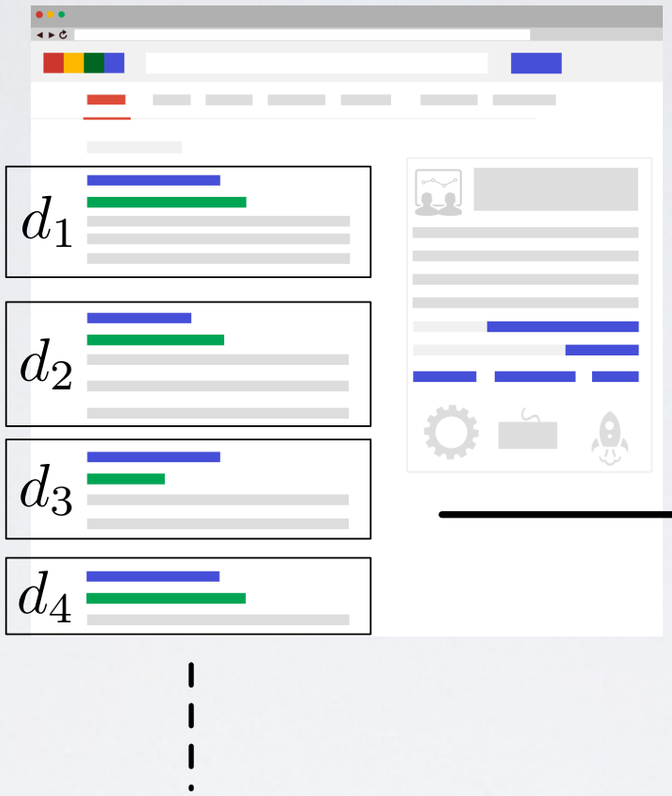
$$Rprec = P(RB)$$

Rank-based Measures: Example of Precision and Recall

Assume

- $RB = 8$ relevant documents in total
- Lenient mapping to binary relevance degrees

Topic



Run

	Assessed Run
1	Highly Relevant
2	Not Relevant
3	Partially Relevant
4	Fairly Relevant
5	Not Relevant
6	Not Relevant
7	Not Relevant
8	Fairly Relevant
9	Not Relevant
10	Not Relevant

Assessed Run

	Binary Weighted Assessed Run
1	1
2	0
3	1
4	1
5	0
6	0
7	0
8	1
9	0
10	0

Binary Weighted Assessed Run

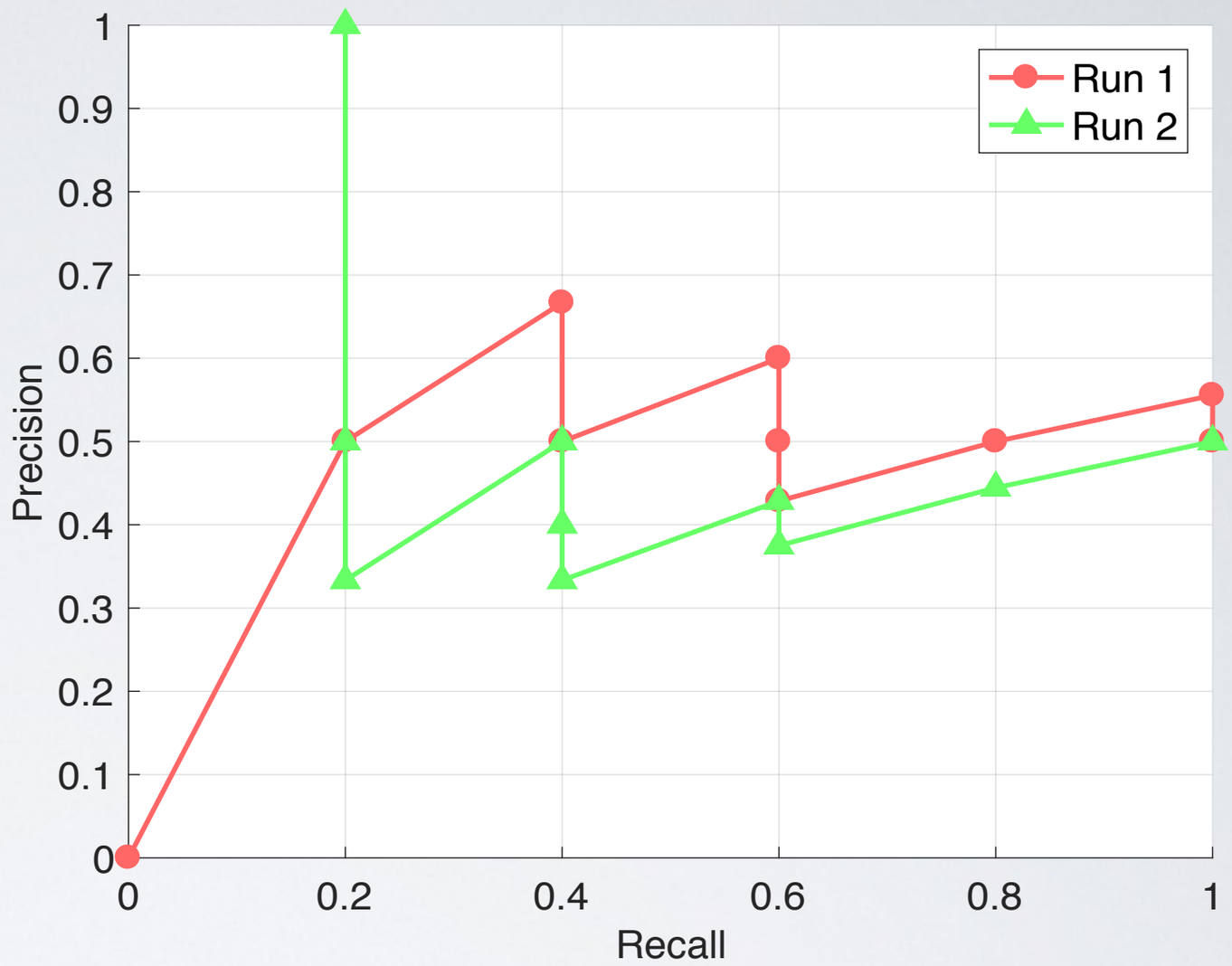
$$P(5) = \frac{3}{5} = 0.600$$

$$R(5) = \frac{3}{8} = 0.375$$

$$Rprec = P(8) = \frac{4}{8} = 0.500$$

Precision-Recall Curve

	Run1		Run2	
1	0	P = 0.00 R = 0.00	1	P = 1.00 R = 0.20
2	1	P = 0.50 R = 0.20	0	P = 0.50 R = 0.20
3	1	P = 0.66 R = 0.40	0	P = 0.33 R = 0.20
4	0	P = 0.50 R = 0.40	1	P = 0.50 R = 0.40
5	1	P = 0.60 R = 0.60	0	P = 0.40 R = 0.40
6	0	P = 0.50 R = 0.60	0	P = 0.33 R = 0.40
7	0	P = 0.42 R = 0.60	1	P = 0.42 R = 0.60
8	1	P = 0.50 R = 0.80	0	P = 0.37 R = 0.60
9	1	P = 0.55 R = 1.00	1	P = 0.44 R = 0.80
10	0	P = 0.50 R = 1.00	1	P = 0.50 R = 1.00



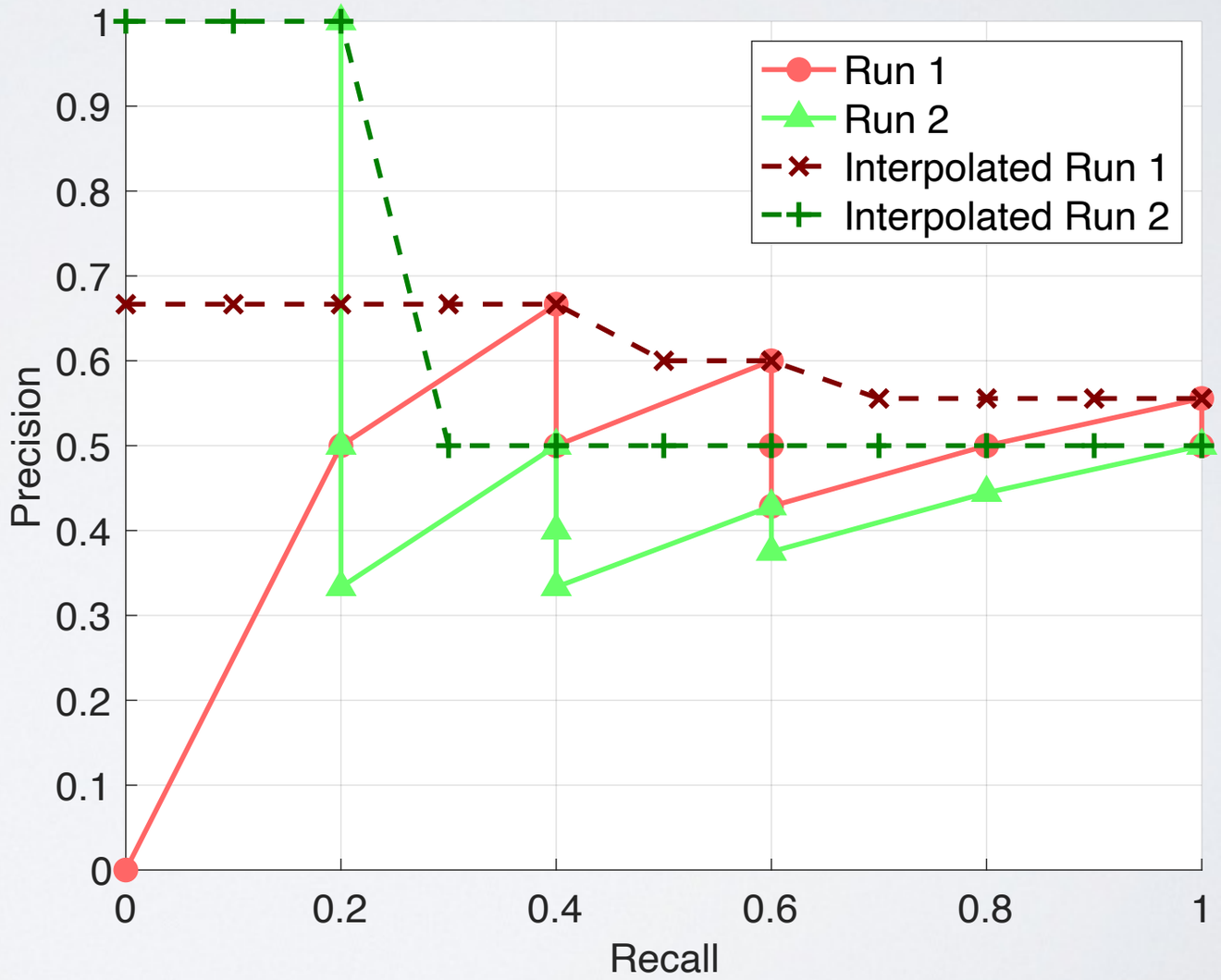
- Assume $RB = 5$ relevant documents in total
- The Precision-Recall curve has a typical saw-tooth shape
  - We may have multiple Precision values for the same Recall value
  - It is difficult to compare runs because they may not have the same Recall values

Interpolated Precision-Recall Curve

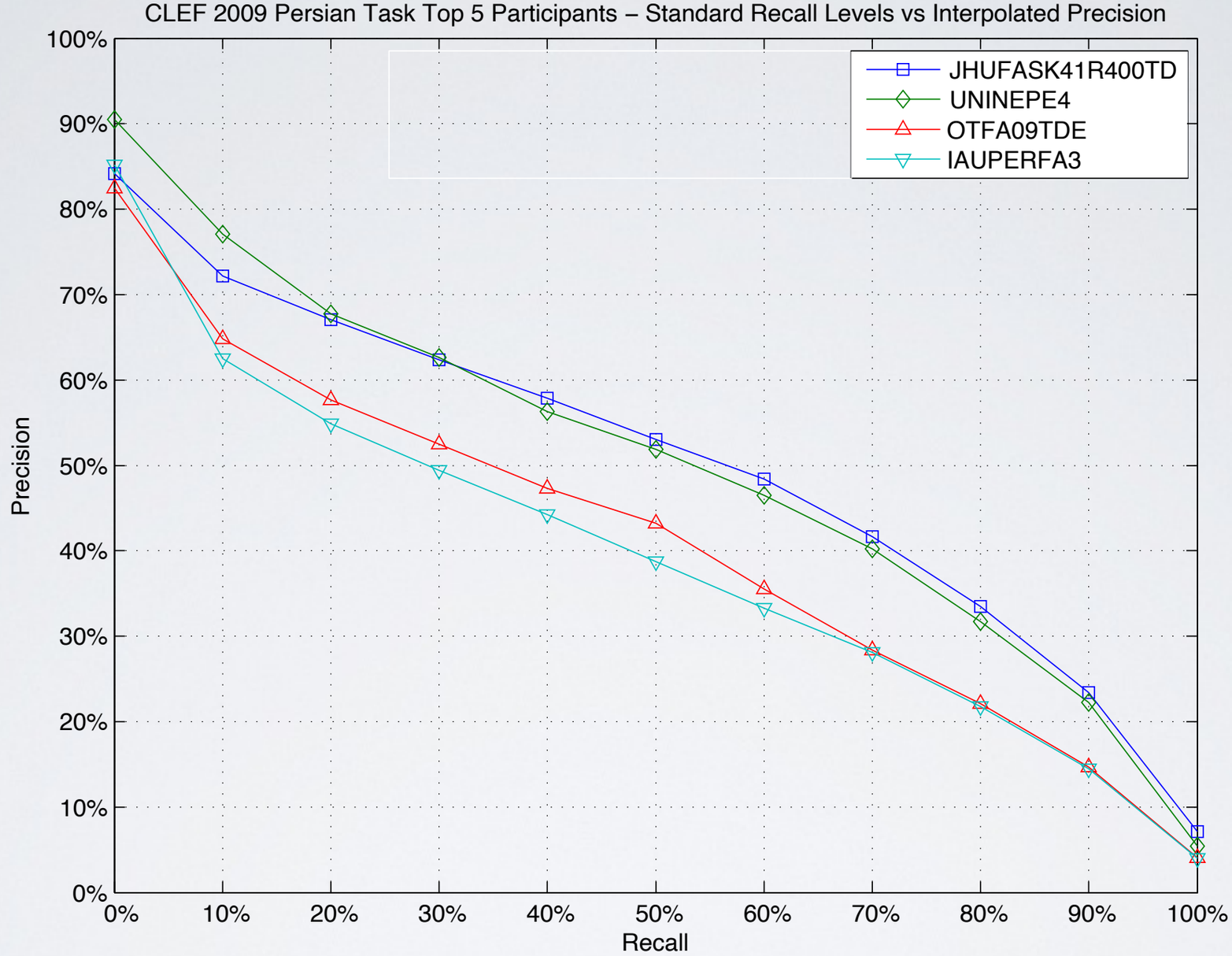
	Run1		Run2	
1	0	**iP = 0.66** P = 0.00 R = 0.00	1	**iP = 1.00** P = 1.00 R = 0.20
2	1	**iP = 0.66** P = 0.50 R = 0.20	0	**iP = 1.00** P = 0.50 R = 0.20
3	1	**iP = 0.66** P = 0.66 R = 0.40	0	**iP = 1.00** P = 0.33 R = 0.20
4	0	**iP = 0.66** P = 0.50 R = 0.40	1	**iP = 0.50** P = 0.50 R = 0.40
5	1	**iP = 0.60** P = 0.60 R = 0.60	0	**iP = 0.50** P = 0.40 R = 0.40
6	0	**iP = 0.60** P = 0.50 R = 0.60	0	**iP = 0.50** P = 0.33 R = 0.40
7	0	**iP = 0.60** P = 0.42 R = 0.60	1	**iP = 0.50** P = 0.42 R = 0.60
8	1	**iP = 0.55** P = 0.50 R = 0.80	0	**iP = 0.50** P = 0.37 R = 0.60
9	1	**iP = 0.55** P = 0.55 R = 1.00	1	**iP = 0.50** P = 0.44 R = 0.80
10	0	**iP = 0.55** P = 0.50 R = 1.00	1	**iP = 0.50** P = 0.50 R = 1.00

- To interpolate Precision at standard Recall value R_j we use the maximum Precision obtained for any actual Recall value R greater than or equal to R_j

$$iP@R_j = \max_{R \geq R_j} P@R$$



11 points Interpolated Precision-Recall Curve



- Standard Interpolated Precision-Recall curves exhibit a typical inverse relationship among Precision and Recall, indicating a trade-off between these two goals of effectiveness

Cleverdon, C. W. (1972). On the inverse relationship of recall and precision. *Journal of Documentation*, 28(3):195–201.

Buckland, M. and Gey, F. (1994). The relationship between Recall and Precision. *Journal of the American Society for Information Science and Technology (JASIST)*, 45(1):12–19.

Egghe, L. (2008). The measures precision, recall, fallout and miss as a function of the number of retrieved documents and their mutual interrelations. *Information Processing & Management*, 44(2):856–876.

Using trec_eval to Compute Set-based Measures

- To compute set-based evaluation measures (Precision, Recall, F-measure) run
  `trec_eval -q -m set pool.txt run.txt`
  - `-q` prints topic-by-topic results
  - `-m` selects which measures to compute,
    use `set` for set-based evaluation measures

```
[ferro~trec_eval.9.0$ ./trec_eval -q -m set ../data/CLEF2009-Persian/pool/
AH-PERSIAN-CLEF2009.txt ../data/CLEF2009-Persian/runs/UNINEPE1.txt
num_ret                 601-AH  1000
num_rel                 601-AH  89
num_rel_ret             601-AH  67
utility                 601-AH  -866.0000
set_P                   601-AH  0.0670
set_relative_P          601-AH  0.7528
set_recall              601-AH  0.7528
set_map                 601-AH  0.0504
set_F                   601-AH  0.1230
num_ret                 602-AH  1000
num_rel                 602-AH  93
num_rel_ret             602-AH  92
```

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num_ret                 601-AH  1000
num_rel                 601-AH  89
num_rel_ret             601-AH  67
utility                 601-AH  -866.0000
set_P                   601-AH  0.0670
set_relative_P          601-AH  0.7528
set_recall              601-AH  0.7528
set_map                 601-AH  0.0504
set_F                   601-AH  0.1230
num_ret                 602-AH  1000
num_rel                 602-AH  93
num_rel_ret             602-AH  92
```

`num_ret` is the total number of retrieved documents, i.e. $|B|$ - typically 1,000 documents

`num_rel` is the total number of relevant documents, i.e. $|A|$

`num_rel_ret` is the total number of relevant retrieved documents, i.e. $|A \cap B|$

Using trec_eval to Compute Set-based Measures

- To compute set-based evaluation measures (Precision, Recall, F-measure) run
  `trec_eval -q -m set pool.txt run.txt`
  - `-q` prints topic-by-topic results
  - `-m` selects which measures to compute,
    use `set` for set-based evaluation measures

`set_P` is set-based Precision

`set_recall` is set-based Recall

`set_F` is set-based F-measure

```
[ferro~trec_eval.9.0$ ./trec_eval -q -m set ../data/CLEF2009-Persian/pool/
AH-PERSIAN-CLEF2009.txt ../data/CLEF2009-Persian/run/
num_ret                 601-AH  1000
num_rel                 601-AH  89
num_rel_ret             601-AH  67
utility                 601-AH  -866.0000
set_P                   601-AH  0.0670
set_relative_P          601-AH  0.7528
set_recall              601-AH  0.7528
set_map                 601-AH  0.0504
set_F                   601-AH  0.1230
num_ret                 602-AH  1000
num_rel                 602-AH  93
num_rel_ret             602-AH  92
```

Using trec_eval to Compute Rank-based Measures

```
ferro~trec_eval.9.0$ ./trec_eval -q -m all_trec ../data/CLEF2009-Persian/
pool/AH-PERSIAN-CLEF2009.txt ../data/CLEF2009-Persian/runs/UNINEPE1.txt
num_ret                 601-AH  1000
num_rel                 601-AH  89
num_rel_ret             601-AH  67
map                     601-AH  0.1856
Rprec                   601-AH  0.2360
bpref                   601-AH  0.1788
recip_rank              601-AH  1.0000
iprec_at_recall_0.00    601-AH  1.0000
iprec_at_recall_0.10    601-AH  0.4737
iprec_at_recall_0.20    601-AH  0.2500
iprec_at_recall_0.30    601-AH  0.2248
iprec_at_recall_0.40    601-AH  0.2139
iprec_at_recall_0.50    601-AH  0.1852
iprec_at_recall_0.60    601-AH  0.1333
iprec_at_recall_0.70    601-AH  0.0947
iprec_at_recall_0.80    601-AH  0.0000
iprec_at_recall_0.90    601-AH  0.0000
iprec_at_recall_1.00    601-AH  0.0000
P_5                     601-AH  0.4000
P_10                    601-AH  0.3000
P_15                    601-AH  0.4000
P_20                    601-AH  0.4500
P_30                    601-AH  0.3667
P_100                   601-AH  0.2300
P_200                   601-AH  0.1950
P_500                   601-AH  0.1200
P_1000                  601-AH  0.0670
relstring               601-AH  '1100001000'
recall_5                601-AH  0.0225
recall_10               601-AH  0.0337
recall_15               601-AH  0.0674
recall_20               601-AH  0.1011
recall_30               601-AH  0.1236
recall_100              601-AH  0.2584
recall_200              601-AH  0.4382
recall_500              601-AH  0.6742
recall_1000             601-AH  0.7528
infAP                   601-AH  0.1856
Rprec_mult_0.20         601-AH  0.4444
Rprec_mult_0.40         601-AH  0.3056
```

- To compute all the evaluation measures run **`trec_eval -q -m all_trec pool.txt run.txt`**
- **`num_ret`**, **`num_rel`**, and **`num_rel_ret`** have the usual meaning
- **`Rprec`** is Precision at the recall base
- **`iprec_*`** is interpolated Precision
- **`P_*`** is Precision at Document Cut-off
- **`R_*`** is Recall at Document Cut-off

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

% Recall Base
RB = 5;

% Recall at each rank position
recall = cumsum(run) ./ RB;

% Precision at each rank position
precision = cumsum(run)./ (1:length(run));

% Rprec
rprec = NaN;
if (RB <= length(run))
    rprec = precision(RB);
end

% Standard Recall levels
recallLevel = 0:0.1:1;

% 11-points iterpolated precision
iPrecision = zeros(1, length(recallLevel));

for i = 1:length(recallLevel)

    % find the recall values R that are greater than or equal to the
    % current standard recall level R_i
    idx = recall >= recallLevel(i);

    % if there is any such recall value, take the maximum precision
    % otherwise interpolated precision is zero at that standard recall
    % level, i.e. skip any assignment
    if (any(idx))
        iPrecision(i) = max(precision(idx));
    end
end
```

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

% Recall Base
RB = 5;

% Recall at each rank position
recall = cumsum(run) ./ RB;

% Precision at each rank position
precision = cumsum(run)./ (1:length(run));

% Rprec
rprec = NaN;
if (RB <= length(run))
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end

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for i = 1:length(recallLevel)

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    % current standard recall level R_i
    idx = recall >= recallLevel(i);

    % if there is any such recall value, take the maximum precision
    % otherwise interpolated precision is zero at that standard recall
    % level, i.e. skip any assignment
    if (any(idx))
        iPrecision(i) = max(precision(idx));
    end
end
```

What happens in corner cases?

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

% Recall Base
RB = 5;

% Recall at each rank position
recall = cumsum(run) ./ RB;

% Precision at each rank position
precision = cumsum(run)./ (1:length(run));

% Rprec
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    idx = recall >= recallLevel(i);

    % if there is any such recall value, take the maximum precision
    % otherwise interpolated precision is zero at that standard recall
    % level, i.e. skip any assignment
    if (any(idx))
        iPrecision(i) = max(precision(idx));
    end
end
```

What if I wish set-based Precision and Recall?

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

% Recall Base
RB = 5;

% Recall at each rank position
recall = cumsum(run) ./ RB;

% Precision at each rank position
precision = cumsum(run)./ (1:length(run));

% Rprec
rprec = NaN;
if (RB <= length(run))
    rprec = precision(RB);
end

% Standard Recall levels
recallLevel = 0:0.1:1;

% 11-points iterpolated precision
iPrecision = zeros(1, length(recallLevel));

for i = 1:length(recallLevel)

    % find the recall values R that are greater than or equal to the
    % current standard recall level R_i
    idx = recall >= recallLevel(i);

    % if there is any such recall value, take the maximum precision
    % otherwise interpolated precision is zero at that standard recall
    % level, i.e. skip any assignment
    if (any(idx))
        iPrecision(i) = max(precision(idx));
    end
end
```

What happens if there are no relevant documents?

Rank-based Measures: Average Precision

$$AP = \frac{1}{RB} \sum_{k \in \mathcal{R}} P(k) = \frac{1}{RB} \sum_{n=1}^{N} \underbrace{\left(\frac{1}{n} \sum_{m=1}^{n} r_m \right)}_{P(n)} r_n =$$

$$= \underbrace{\frac{rr}{RB}}_{\text{Recall}} \cdot \underbrace{\frac{1}{rr} \sum_{k \in \mathcal{R}} P(k)}_{\text{arithmetic mean of } P(k)}$$

where

- $\mathcal{R}$ is the set of the rank positions of the relevant retrieved documents
- $rr = |\mathcal{R}|$ is the total number of relevant retrieved documents
- N is the total number of retrieved documents, i.e. the length of the run
- The **Mean Average Precision (MAP)** is the mean of AP over a set of topics
  - Differently from the other measures, this mean has its own name since it is the most widely used single number to summarise the whole performance of a system

Chris Buckley

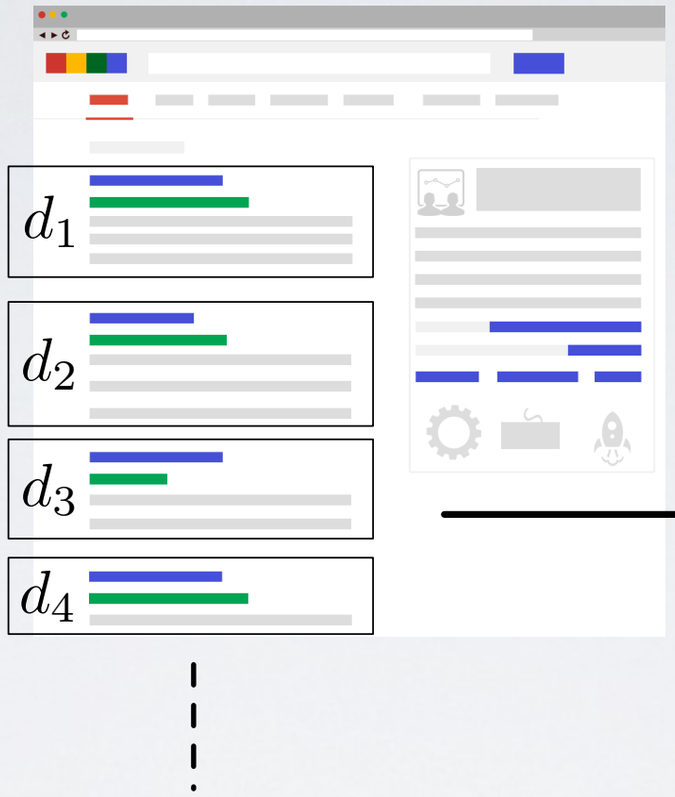
Buckley, C. and Voorhees, E. M. (2005). Retrieval System Evaluation. In Harman, D. K. and Voorhees, E. M., editors, *TREC. Experiment and Evaluation in Information Retrieval*, pages 53–78. MIT Press, Cambridge (MA), USA.

Rank-based Measures: Example of Average Precision

Assume

- $RB = 8$ relevant documents in total
- Lenient mapping to binary relevance degrees

Topic



Run

Assessed Run

1	Highly Relevant
2	Not Relevant
3	Partially Relevant
4	Fairly Relevant
5	Not Relevant
6	Not Relevant
7	Not Relevant
8	Fairly Relevant
9	Not Relevant
10	Not Relevant

Binary Weighted Assessed Run

1	1
2	0
3	1
4	1
5	0
6	0
7	0
8	1
9	0
10	0

$$AP = \frac{1}{RB}\Big(P(1) + P(3) + P(4) + P(8)\Big)$$

$$= \frac{1}{8}\left(1 + \frac{2}{3} + \frac{3}{4} + \frac{4}{8}\right) = \frac{35}{96} = 0.36$$

Computing AP Using trec_eval

```
[ferro~trec_eval.9.0$ ./trec_eval -q -m all_trec ../data/CLEF2009-Persian/
pool/AH-PERSIAN-CLEF2009.txt ../data/CLEF2009-Persian/runs/UNINEPE1.txt
num_ret                 601-AH  1000
num_rel                 601-AH  89
num_rel_ret             601-AH  67
map                     601-AH  0.1856
Rprec                   601-AH  0.2360
bpref                   601-AH  0.1788
recip_rank              601-AH  1.0000
iprec_at_recall_0.00    601-AH  1.0000
iprec_at_recall_0.10    601-AH  0.4737
iprec_at_recall_0.20    601-AH  0.2500
iprec_at_recall_0.30    601-AH  0.2240
iprec_at_recall_0.40    601-AH  0.2139
iprec_at_recall_0.50    601-AH  0.1852
iprec_at_recall_0.60    601-AH  0.1333
```

Beware of `trec_eval` calling **`map`** what actually is AP on a topic. However, when **`all`** is reported, i.e. when `trec_eval` is averaging measures over topics, what it still calls `map` is the true MAP, i.e. the mean of AP over topics

- To compute all the evaluation measures run

  `trec_eval -q -m all_trec pool.txt run.txt`

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

% Recall Base
RB = 5;

% Find the rank positions of the relevant retrieved documents
ranks = find(run);
```

```
% compute average precision, if any relevant document has been retrieved
if ~isempty(ranks)
    ap = sum(cumsum(run(ranks)) ./ ranks) ./ RB;
else
    ap = 0;
end
```

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

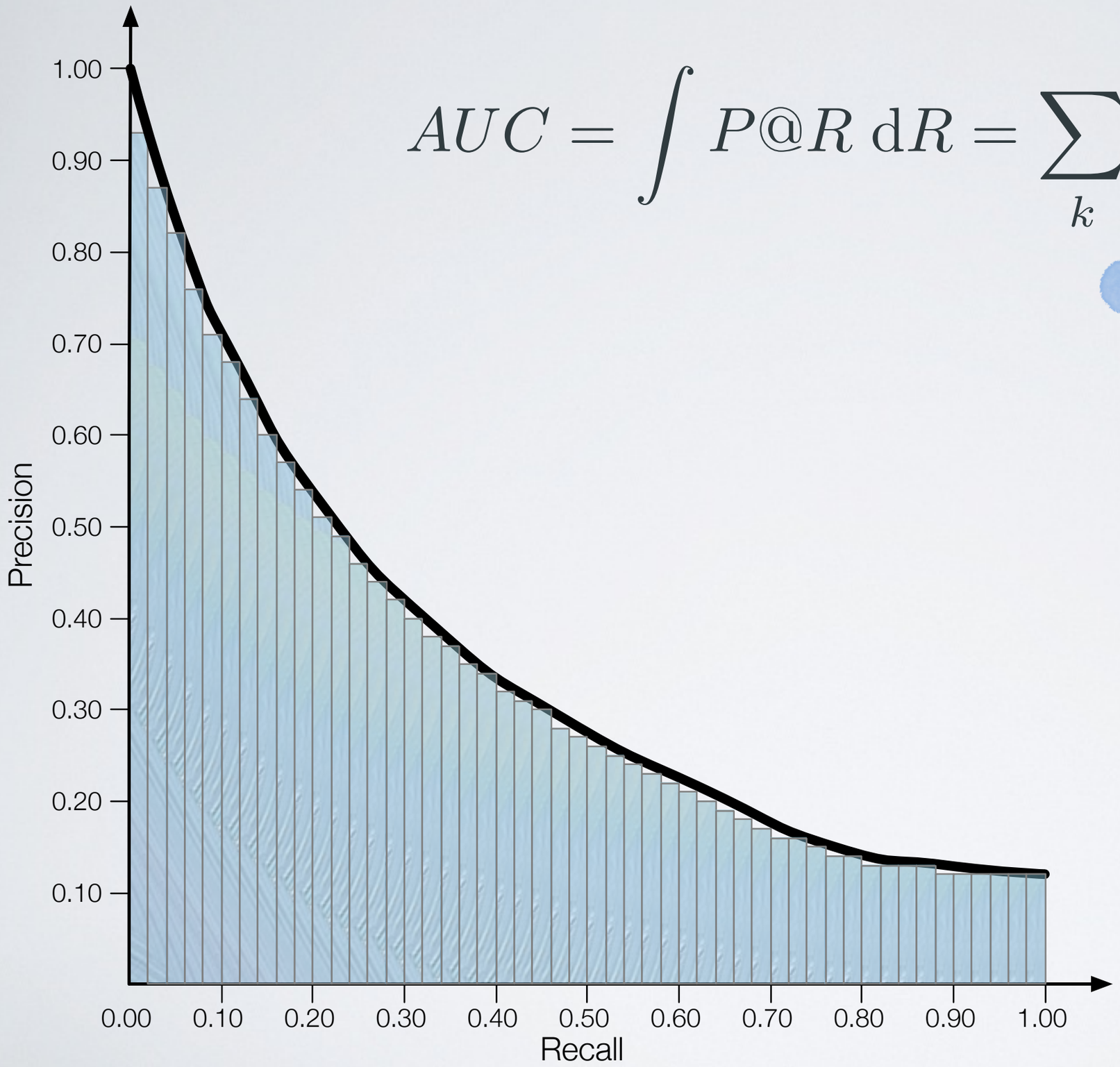
% Recall Base
RB = 5;

% Find the rank positions of the relevant retrieved documents
ranks = find(run);

% compute average precision, if any relevant document has been retrieved
if ~isempty(ranks)
    ap = sum(cumsum(run(ranks)) ./ ranks) ./ RB;
else
    ap = 0;
end
```

What happens if there are no relevant documents?

Area under the Precision-Recall Curve



$$AUC = \int P@R \, \mathrm{d}R = \sum_k P(k)\Delta R(k)$$

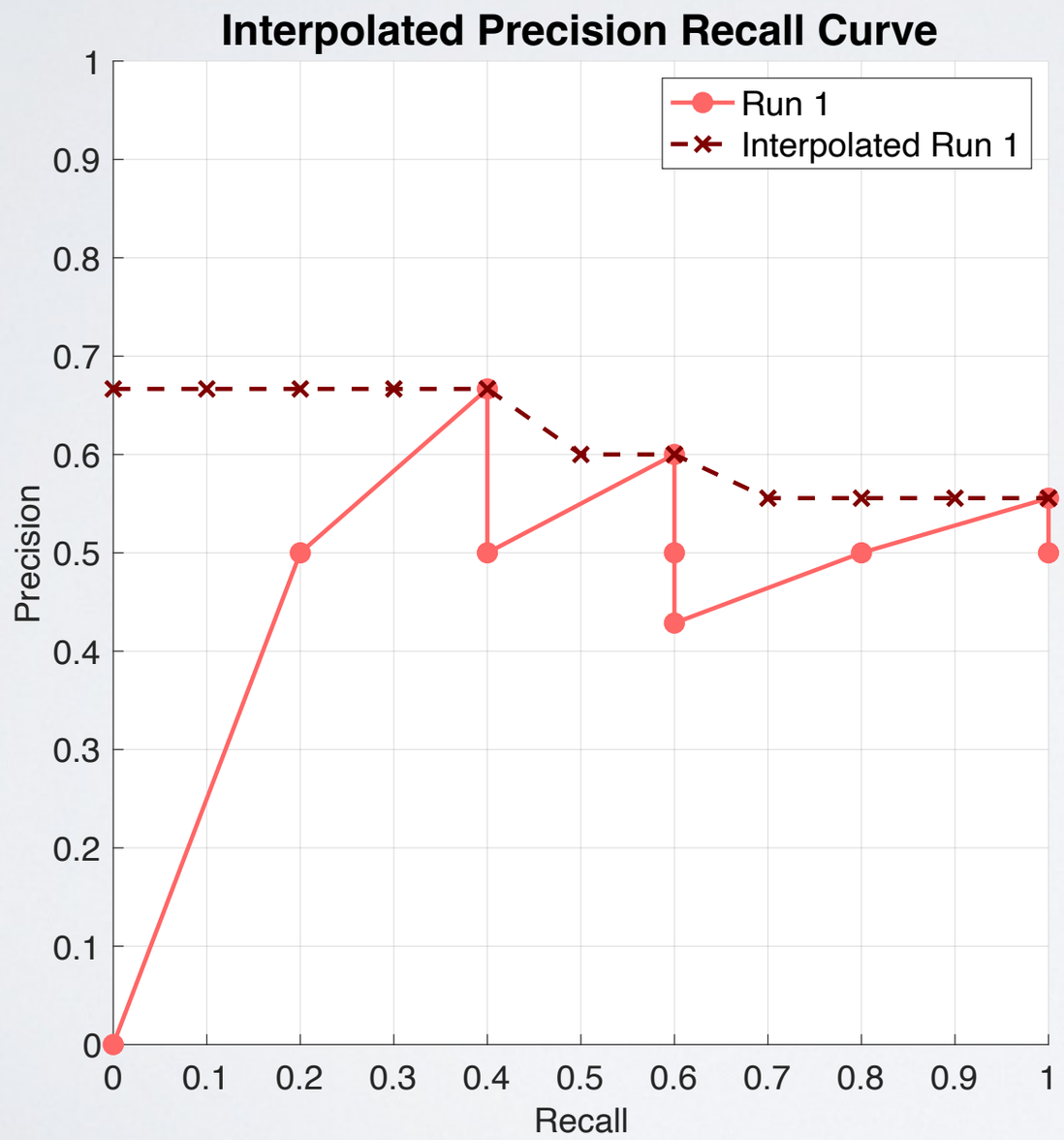
- The **Area Under the** Precision-Recall **Curve (AUC)** is an important indicator of the overall system effectiveness, summarising the trade-off between Precision and Recall

Computing the Area under the Precision-Recall Curve

$$AUC = \sum_{n=1}^{N} P(n)\Big(R(n) - R(n-1)\Big) \quad \text{assuming } R(0) = 0$$

Run1

1	0	P = 0.00 R = 0.00
2	1	P = 0.50 R = 0.20
3	1	P = 0.66 R = 0.40
4	0	P = 0.50 R = 0.40
5	1	P = 0.60 R = 0.60
6	0	P = 0.50 R = 0.60
7	0	P = 0.42 R = 0.60
8	1	P = 0.50 R = 0.80
9	1	P = 0.55 R = 1.00
10	0	P = 0.50 R = 1.00



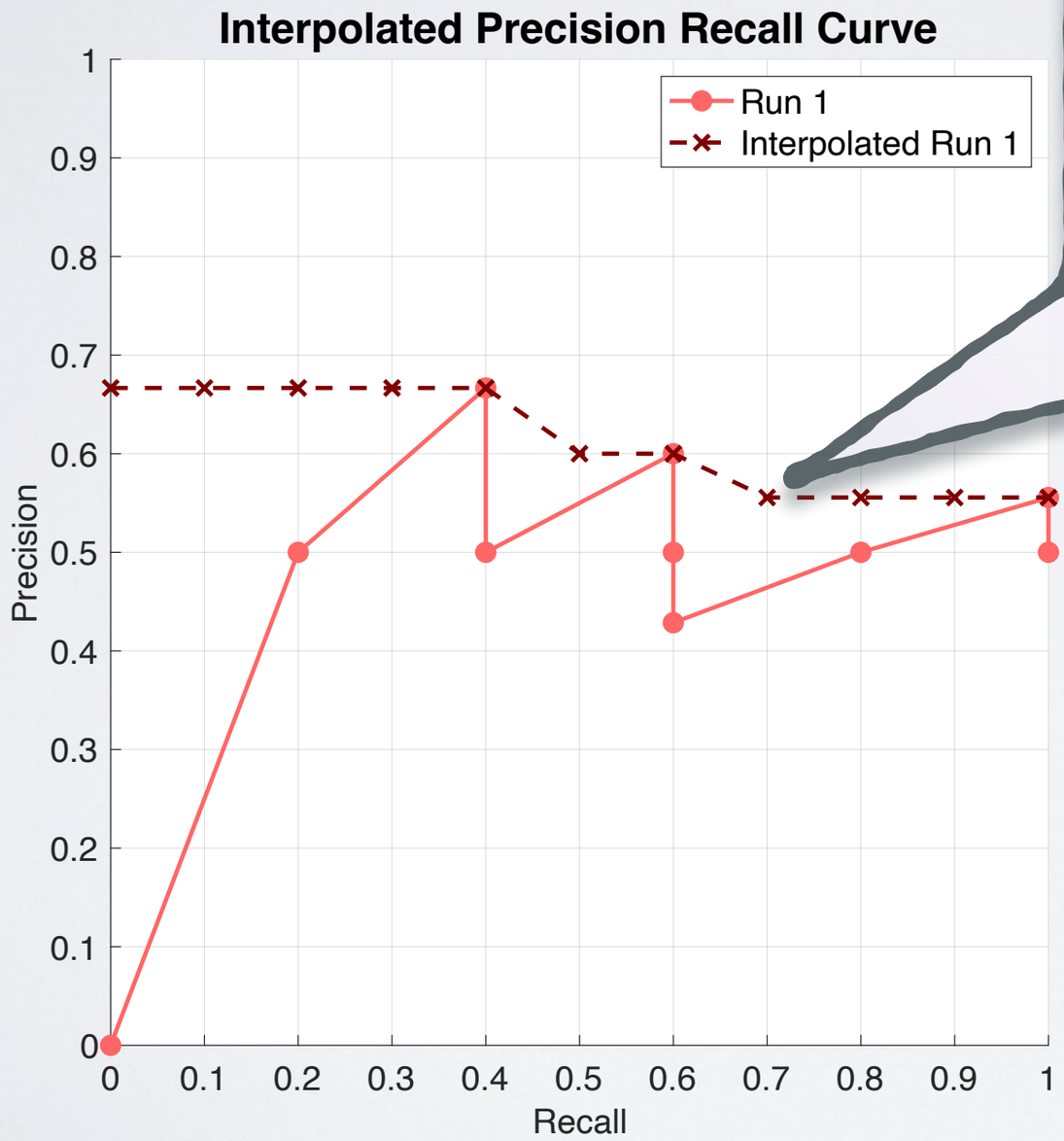
$$\begin{aligned} AUC = \quad & 0.00(0.00 - 0.00) + 0.50(0.20 - 0.00) + \\ & 0.66(0.40 - 0.20) + 0.50(0.40 - 0.40) + \\ & 0.60(0.60 - 0.40) + 0.50(0.60 - 0.60) + \\ & 0.42(0.60 - 0.60) + 0.50(0.80 - 0.60) + \\ & 0.55(1.00 - 0.80) + 0.50(1.00 - 1.00) = \\ = \quad & 0.5620 \end{aligned}$$

Computing the Area under the Precision-Recall Curve

$$AUC = \sum_{n=1}^{N} P(n)\Big(R(n) - R(n-1)\Big) \quad \text{assuming } R(0) = 0$$

Run1

1	0	P = 0.00 R = 0.00
2	1	P = 0.50 R = 0.20
3	1	P = 0.66 R = 0.40
4	0	P = 0.50 R = 0.40
5	1	P = 0.60 R = 0.60
6	0	P = 0.50 R = 0.60
7	0	P = 0.42 R = 0.60
8	1	P = 0.50 R = 0.80
9	1	P = 0.55 R = 1.00
10	0	P = 0.50 R = 1.00



What happens if the interpolated Precision at Standard Recall Levels curve is used instead?

$$\begin{aligned} AUC = \; & 0.00(0.00 - 0.00) + 0.50(0.20 - 0.00) + \\ & 0.66(0.40 - 0.20) + 0.50(0.40 - 0.40) + \\ & 0.60(0.60 - 0.40) + 0.50(0.60 - 0.60) + \\ & 0.42(0.60 - 0.60) + 0.50(0.80 - 0.60) + \\ & 0.55(1.00 - 0.80) + 0.50(1.00 - 1.00) = \\ = \; & 0.5620 \end{aligned}$$

Area under the Precision-Recall Curve and Average Precision

$$AUC = \sum_{n=1}^{N} P(n)\Big(R(n) - R(n-1)\Big)$$

- When the n-th document is not relevant, $R(n)$ is equal to $R(n-1)$ and their difference goes to zero
- Therefore, we can sum only on $\mathcal{R}$, i.e. the set of the rank positions of the relevant retrieved documents

$$AUC = \sum_{k \in \mathcal{R}} P(k)\Big(R(k) - R(k-1)\Big)$$

Area under the Precision-Recall Curve and Average Precision

$$AUC = \sum_{k \in \mathcal{R}} P(k) \Big(R(k) - R(k-1) \Big)$$

- Two adjacent rank positions differ just for one relevant document and thus

$$R(k) - R(k-1) = \underbrace{\frac{1}{RB} \sum_{n=1}^{k} r_n}_{R(k)} - \underbrace{\frac{1}{RB} \sum_{n=1}^{k-1} r_n}_{R(k-1)} = \frac{r_k}{RB} = \frac{1}{RB}$$

- Therefore AUC is equal to AP
  - this is one motivation of the importance of AP

$$AUC = \frac{1}{RB} \sum_{k \in \mathcal{R}} P(k) = AP$$

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

% Recall Base
RB = 5;

% Recall at each rank position
recall = cumsum(run) ./ RB;

% Precision at each rank position
precision = cumsum(run) ./ (1:length(run));

% Find the rank positions of the relevant retrieved documents
ranks = find(run);

% compute average precision, if any relevant document has been retrieved
if ~isempty(ranks)
    ap = sum(cumsum(run(ranks)) ./ ranks) ./ RB;
else
    ap = 0;
end

% compute aread under the precision-recall curve
% Note that [0 recall] is the same as assuming R(0) = 0
auc = sum(precision .* diff([0 recall]));
```

Rank-based Measures: Discounted Cumulated Gain

$$DCG(k) = \begin{cases} \sum_{n=1}^{k} r_n & \text{if } k < b \\ DCG(k-1) + \frac{r_k}{\log_b(k)} & \text{if } k >= b \end{cases} =$$

$$= \sum_{n=1}^{k} \frac{r_n}{\max(1, \log_b(n))}$$

- where the base of the logarithm b indicates the patience of the user in scanning the result list
  - $b = 2$ is an impatient user
  - $b = 10$ is a patient user
- DCG naturally handles multi-graded relevance
- DCG does not depend on the recall base
- DCG is not bounded in [0, 1]

Kalervo Järvelin

Jaana Kekäläinen

Järvelin, K. and Kekäläinen, J. (2002). Cumulated Gain-Based Evaluation of IR Techniques. *ACM Transactions on Information Systems (TOIS)*, 20(4):422–446

Rank-based Measures: Discounted Cumulated Gain

$$DCG(k) = \begin{cases} \sum_{n=1}^{k} r_n & \text{if } k < b \\ DCG(k-1) + \frac{r_k}{\log_b(k)} & \text{if } k >= b \end{cases} =$$

$$= \sum_{n=1}^{k} \frac{r_n}{\max(1, \log_b(n))}$$

Cumulated Gain (CG)

- where the base of the logarithm b indicates the patience of the user in scanning the result list
  - $b = 2$ is an impatient user
  - $b = 10$ is a patient user
- DCG naturally handles multi-graded relevance
- DCG does not depend on the recall base
- DCG is not bounded in [0, 1]

Kalervo Järvelin

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Järvelin, K. and Kekäläinen, J. (2002). Cumulated Gain-Based Evaluation of IR Techniques. *ACM Transactions on Information Systems (TOIS)*, 20(4):422–446

Rank-based Measures: Discounted Cumulated Gain

$$DCG(k) = \begin{cases} \sum_{n=1}^{k} r_n & \text{if } k < b \\ DCG(k-1) + \frac{r_k}{\log_b(k)} & \text{if } k >= b \end{cases} =$$

$$= \sum_{n=1}^{k} \frac{r_n}{\max(1, \log_b(n))}$$

- where the base of the logarithm b indicates the patience of the user in scanning the result list
  - $b = 2$ is an impatient user
  - $b = 10$ is a patient user
- DCG naturally handles multi-graded relevance
- DCG does not depend on the recall base
- DCG is not bounded in [0, 1]

Kalervo Järvelin

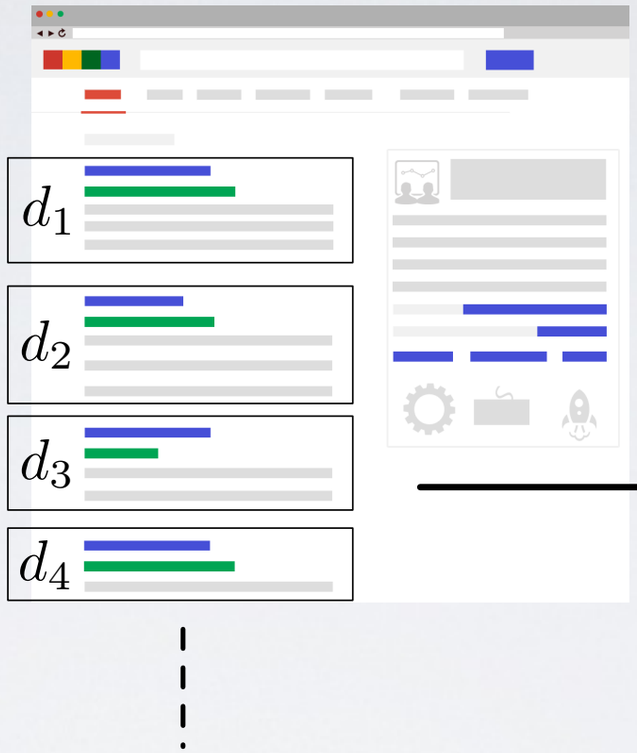
Jaana Kekäläinen

Järvelin, K. and Kekäläinen, J. (2002). Cumulated Gain-Based Evaluation of IR Techniques. *ACM Transactions on Information Systems (TOIS)*, 20(4):422–446

Rank-based Measures: Example of Discounted Cumulated Gain

Assume

- $RB = 8$ relevant documents in total
- An impatient user



	Topic	Run	Assessed Run	Weighted Assessed Run
1			Highly Relevant	3
2			Not Relevant	0
3			Partially Relevant	1
4			Fairly Relevant	2
5			Not Relevant	0
6			Not Relevant	0
7			Not Relevant	0
8			Fairly Relevant	2
9			Not Relevant	0
10			Not Relevant	0

$$DCG = 3 + \frac{1}{\log_2(3)} + \frac{2}{\log_2(4)} + \frac{2}{\log_2(8)} = 5.2976$$

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume graded relevance: 0 for not relevant; 1 for partially relevant;
% 2 for fairly relevant; 3 for highly relevant
run =   [3 0 1 2 0 0 0 2 0 0];

% the base of the logarithm
b = 2;

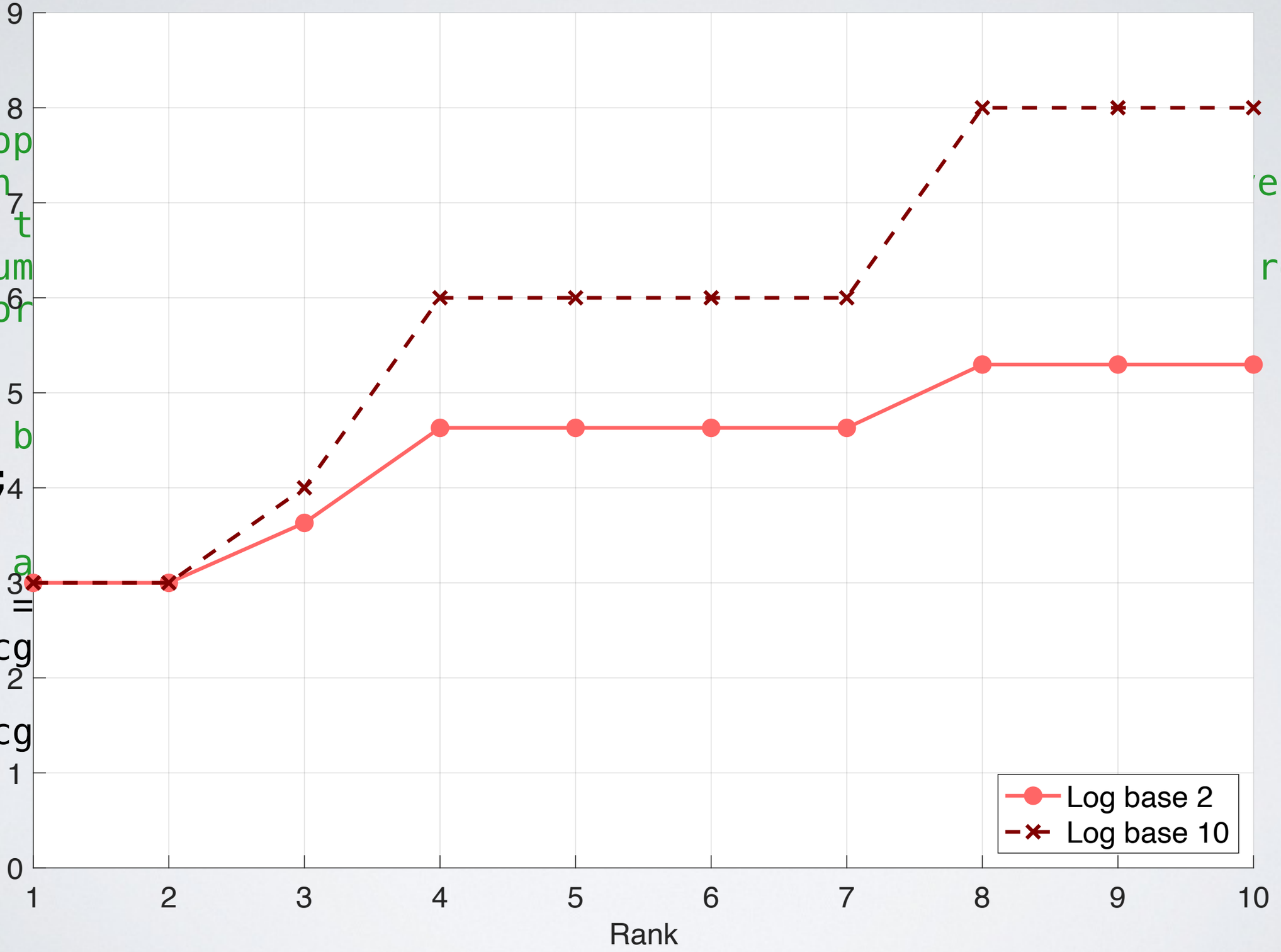
% DCG at each rank position
if (b == 2)      % impatient user
    dcg = cumsum(run ./ max(1, log2(1:length(run))));
else             % patient user
    dcg = cumsum(run ./ max(1, log10(1:length(run))));
end
```

Compute it Yourself!

```
% A top
% Each                                    ector is
% the t
% Assum                                   relevant;
% 2 for
run =

% the b
b = 2;

% DCG a
if (b =
    dcg
else
    dcg
end
```



Rank-based Measures: Normalized Discounted Cumulated Gain

- To normalize DCG in [0, 1], you need to compute the ideal run, i.e. the pool sorted in descending order of relevance, which represents the best retrieval possible and the maximum value of DCG

$$nDCG(k) = \frac{DCG(k)}{iDCG(k)}$$

- nDCG is given by the DCG of the run divided by the DCG of the ideal run

Rank-based Measures: Example of Normalized Discounted Cumulated Gain

Assume

- $RB = 8$ relevant documents in total
- An impatient user

$$DCG = 5.2976$$

$$iDCG = 10.1996$$

$$nDCG = 0.5194$$

Topic	Run	Assessed Run	Weighted Assessed Run	Weighted Assessed Ideal Run
	d_1	1 Highly Relevant	1 3	1 3
	d_2	2 Not Relevant	2 0	2 3
	d_3	3 Partially Relevant	3 1	3 2
	d_4	4 Fairly Relevant	4 2	4 2
	⋮	5 Not Relevant	5 0	5 2
		6 Not Relevant	6 0	6 1
		7 Not Relevant	7 0	7 1
		8 Fairly Relevant	8 2	8 1
		9 Not Relevant	9 0	9 0
		10 Not Relevant	10 0	10 0

Computing nDCG Using trec_eval

```
ferro~trec_eval.9.0$ ./trec_eval -q -m all_trec ../data/CLEF2009-Persian/pool/AH-PERSIAN-CLEF2009.txt ../data/CLEF2009-Persian/runs/UNINEPE1.txt
num_ret                 601-AH  1000
num_rel                 601-AH  89
num_rel_ret             601-AH  67
map                     601-AH  0.1856
Rprec                   601-AH  0.2360
bpref                   601-AH  0.1788
recip_rank              601-AH  1.0000
iprec_at_recall_0.00    601-AH  1.0000
iprec_at_recall_0.10    601-AH  0.4737
iprec_at_recall_0.20    601-AH  0.2500
...
G                       601-AH  0.1405
ndcg                    601-AH  0.5920
ndcg_rel                601-AH  0.4902
Rndcg                   601-AH  0.4411
ndcg_cut_5              601-AH  0.5531
ndcg_cut_10             601-AH  0.4323
ndcg_cut_15             601-AH  0.4697
ndcg_cut_20             601-AH  0.4927
ndcg_cut_30             601-AH  0.4246
ndcg_cut_100            601-AH  0.3061
ndcg_cut_200            601-AH  0.4219
ndcg_cut_500            601-AH  0.5536
ndcg_cut_1000           601-AH  0.5920
map_cut_5               601-AH  0.0225
```

`trec_eval` computes only nDCG (and not also DCG) and it uses b = 2 as base of the logarithm

`trec_eval` uses the values specified in the pool file as nDCG weights

- To compute all the evaluation measures run

  `trec_eval -q -m all_trec pool.txt run.txt`

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume graded relevance: 0 for not relevant; 1 for partially relevant;
% 2 for fairly relevant; 3 for highly relevant
run =   [3 0 1 2 0 0 0 2 0 0];

% the ideal run
ideal = [3 3 2 2 2 1 1 1 0 0];

% the base of the logarithm
b = 2;

% DCG at each rank position
if (b == 2)      % impatient user
    dcg = cumsum(run ./ max(1, log2(1:length(run))));
    dcg_ideal = cumsum(ideal ./ max(1, log2(1:length(ideal))));
else             % patient user
    dcg = cumsum(run ./ max(1, log10(1:length(run))));
    dcg_ideal = cumsum(ideal ./ max(1, log10(1:length(ideal))));
end

ndcg = dcg ./ dcg_ideal;
```

Compute it Yourself!

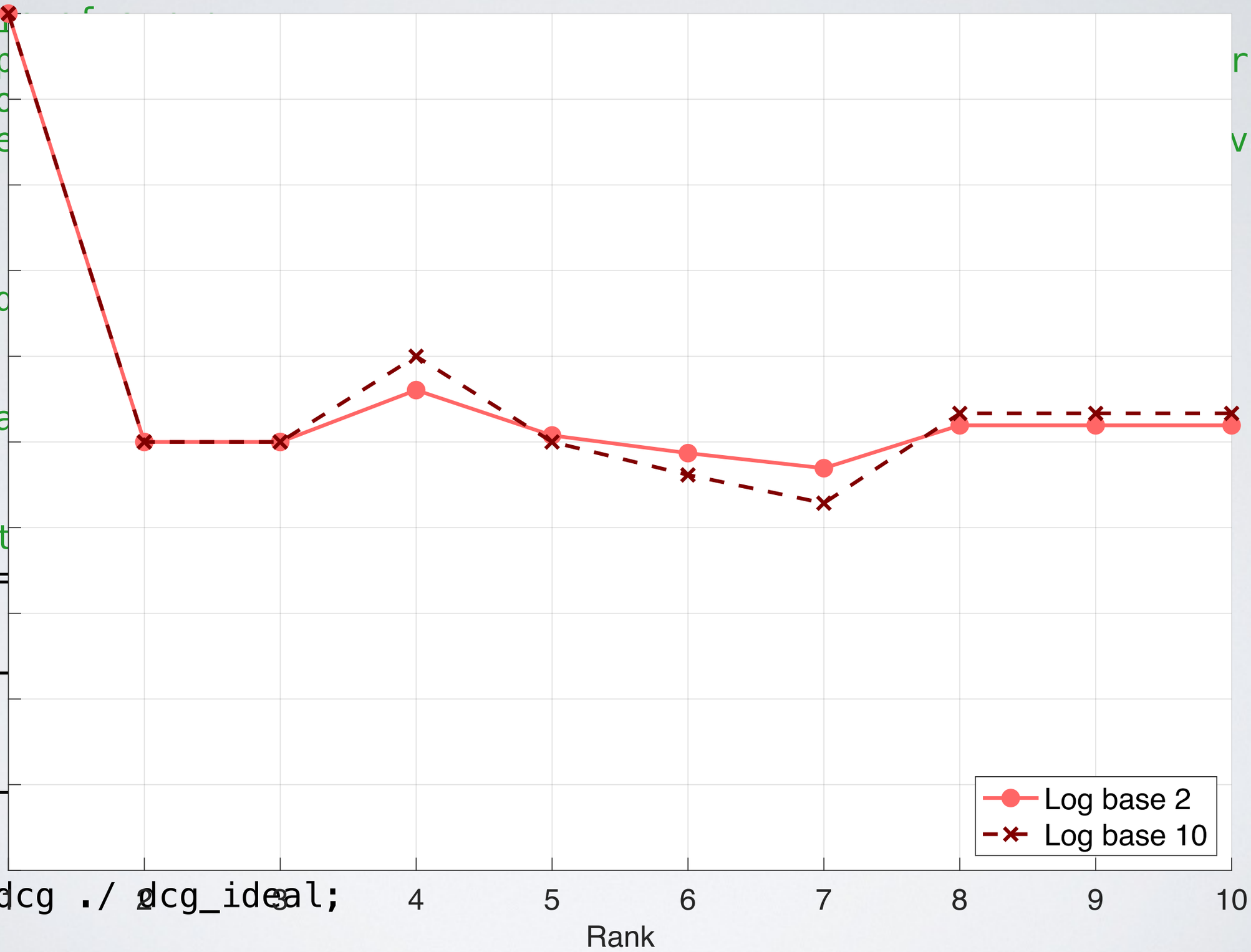
```
% A topi                                                    f
% Each p                                                    r is
% the DC
% Assume                                                    vant;
% 2 for
run =

% the id
ideal =

% the ba
b = 2;

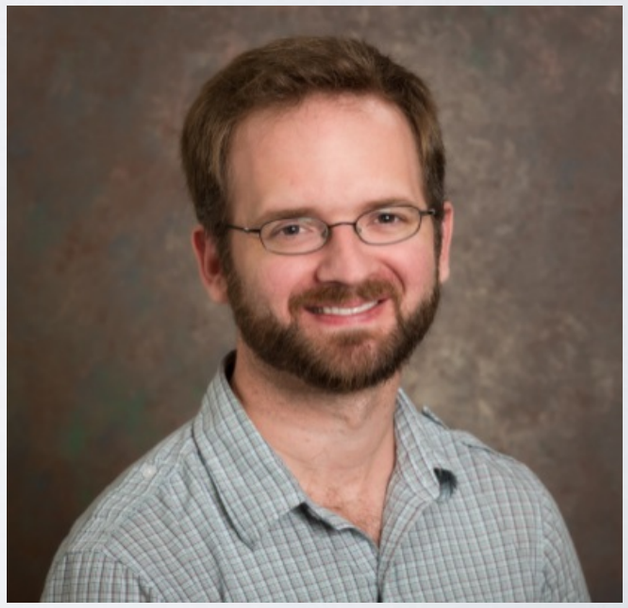
% DCG at
if (b ==
    dcg
    dcg_
else
    dcg
    dcg_
end

ndcg = dcg ./ dcg_ideal;
```



Rank-based Measures: User Models

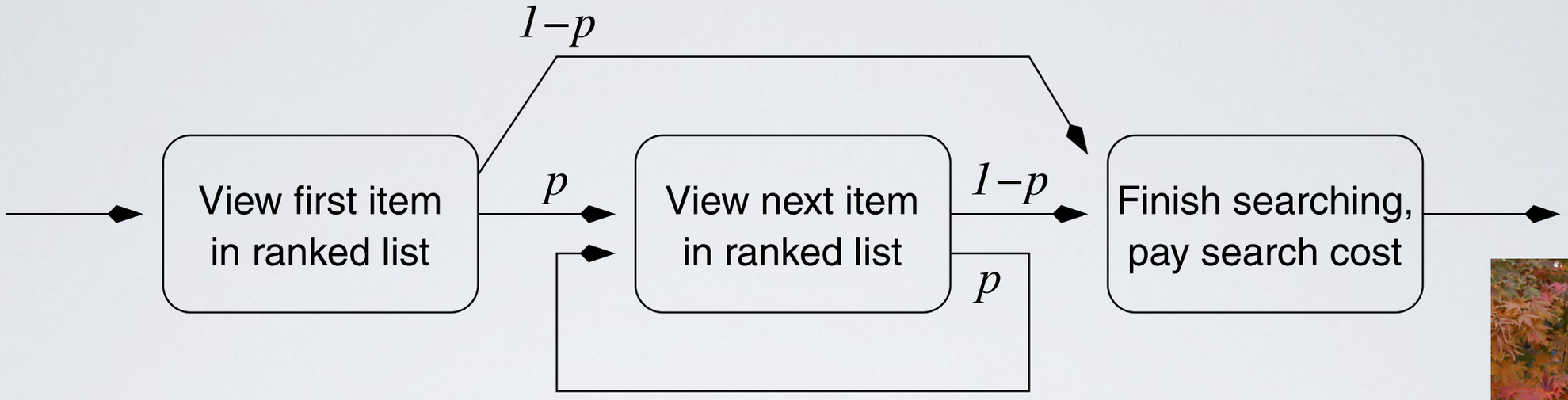
- Rank-based evaluation measures, implicitly or explicitly, embed a **user model** comprising
  - a **browsing model** that describes how a user interacts with results;
  - a **model of document utility**, describing how a user derives utility from individual relevant documents;
  - a **utility accumulation model** that describes how a user accumulates utility in the course of browsing.
- User models may be more or less **artificial** and may be more or less **correlated** with **actual user behaviour** and preferences
- In the case of DCG
  - a **browsing model**: user steps down the ranked results one-by-one, until s/he reaches the stopping rank k which is picked with a probability proportional to the log of the rank
  - a **model of document utility**: user gains something from each relevant document, proportional to its relevance degree
  - a **utility accumulation model**: user gains from all of the relevant documents from ranks 1 through k



Ben Carterette

Carterette, B. A. (2011). System Effectiveness, User Models, and User Utility: A Conceptual Framework for Investigation. In Ma, W.-Y., Nie, J.-Y., Baeza-Yaetes, R., Chua, T.-S., and Croft, W. B., editors, *Proc. 34th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR 2011)*, pages 903–912. ACM Press, New York, USA.

Rank-based Measures: Rank-Biased Precision



- The user starts from the top ranked document and with probability p, called **persistence**, goes to the next document or with probability $1 - p$ stops
  - typical value for p are: 0.5 for impatient users, 0.8 for patient users, and 0.95 for extremely patient users

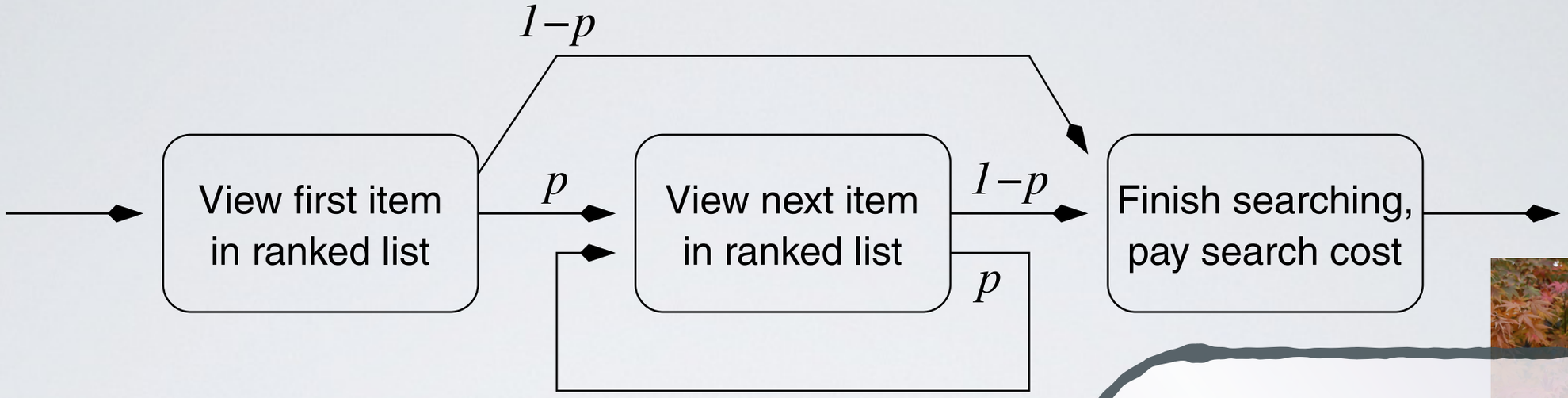
$$RBP = (1-p)\sum_{n=1}^{N} p^{n-1} r_n = (1-p)\sum_{k \in \mathcal{R}} p^{k-1}$$

Alistair Moffat

Justin Zobel

Moffat, A. and Zobel, J. (2008). Rank-biased Precision for Measurement of Retrieval Effectiveness. *ACM Transactions on Information Systems (TOIS)*, 27(1):2:1–2:27.

Rank-based Measures: Rank-Biased Precision



- The user starts from the top ranked document and with probability p, called **persistence**, goes to the next document or with probability $1 - p$ stops
  - typical value for p are: 0.5 for impatient users, 0.8 for patient users, and 0.95 for extremely patient users

$$RBP = (1-p)\sum_{n=1}^{N} p^{n-1} r_n = (1-p)\sum_{k \in \mathcal{R}} p^{k-1}$$

RBP is independent from the recall base

Alistair Moffat

Justin Zobel

Moffat, A. and Zobel, J. (2008). Rank-biased Precision for Measurement of Retrieval Effectiveness. *ACM Transactions on Information Systems (TOIS)*, 27(1):2:1–2:27.

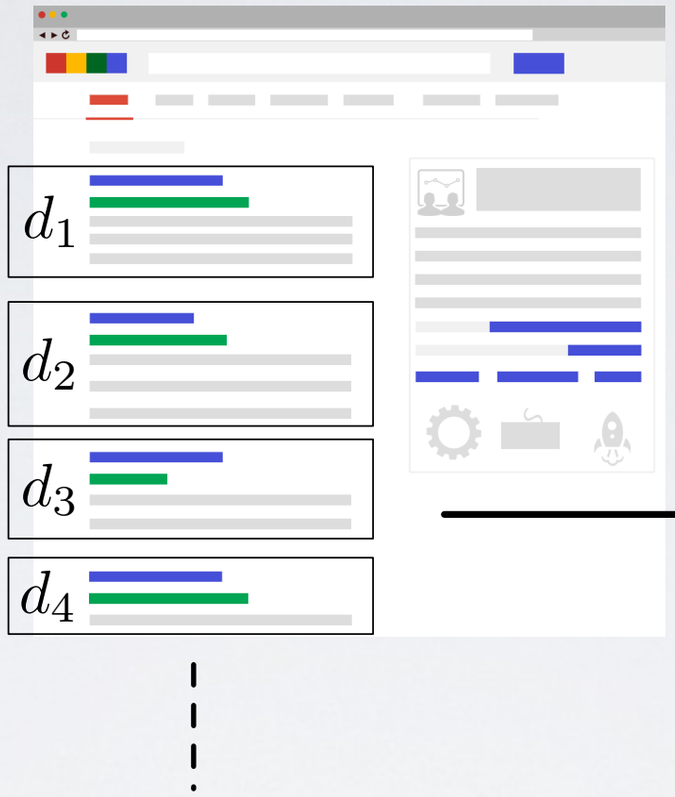
Rank-based Measures: Example of Rank-biased Precision

Assume

- $p = 0.8$ a patient user
- Lenient mapping to binary relevance degrees

Topic

Run



Assessed Run

1	Highly Relevant
2	Not Relevant
3	Partially Relevant
4	Fairly Relevant
5	Not Relevant
6	Not Relevant
7	Not Relevant
8	Fairly Relevant
9	Not Relevant
10	Not Relevant

Binary Weighted Assessed Run

1	1
2	0
3	1
4	1
5	0
6	0
7	0
8	1
9	0
10	0

$$RBP = (1-0.8)\left(0.8^{1-1} + 0.8^{3-1} + 0.8^{4-1} + 0.8^{8-1}\right) = 0.4723$$

Compute it Yourself!

```
% A topic of a run.
% Each position is a retrieved document; first element of the vector is
% the top of the rank.
% Assume binary relevance: 0 for not relevant; 1 for relevant.
run = [0 1 1 0 1 0 0 1 1 0];

% the persistence
p = 0.8;
```

```
% Find the rank positions of the relevant retrieved documents
ranks = find(run);

% compute rank-biased precision, if any relevant document has been retrieved
if ~isempty(ranks)
    rbp = (1 - p) .* sum(repmat(p, 1, length(ranks)) .^ (ranks - 1) );
else
    rbp = 0;
end
```

Questions?


It works...It doesn't....It works.....
Just a hunch....maybe we do need a better way to measure results.....