EXERCISE (Complementary languages)

Let $L^c = \{0,1\}^* - L$. Prove that

$(L^c)^c = L$

$\forall L \subseteq \{0,1\}^*$:

$(L \in NPC) \wedge (L^c \in NP) \Rightarrow L^c \in NPC$

HP1 HP2 TH

HP1 $\forall \hat{L} \in NP : \hat{L} <_p L$ $(L \in NP)$

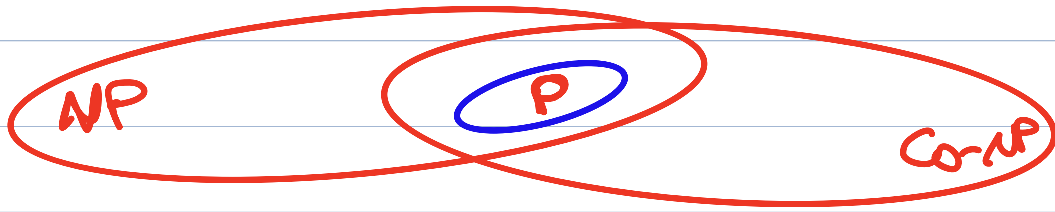
HP2 $L^c \in NP$

EXERCISE Define

$$co\text{-}NP = \{L \subseteq \{0,1\}^* : L^c \in NP\}$$

$$\equiv L' = L^c \longrightarrow \left(= \{L'^c \subseteq \{0,1\}^* : L' \in NP\}\right)$$

1) Prove that $P \subseteq NP \cap co\text{-}NP$



2) Prove that

if $NPC \cap co\text{-}NP \neq \emptyset \Rightarrow NP = co\text{-}NP$

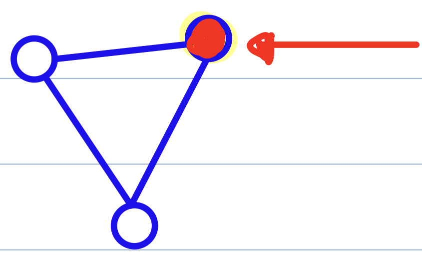
EXERCISE

DEF Given an undirected graph $G=(V,E)$ a dominating set $V' \subseteq V$ is such that:

$$\forall v \in V: (v \in V') \vee (\exists \{u,v\} \in E : u \in V')$$

In other words, a node is either in the dominating set or adjacent to a node in the dominating set

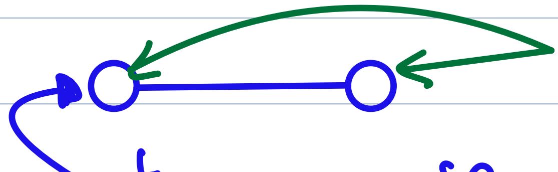
EXAMPLE:



dominating set for G

Observe that a dominating set is not a vertex-cover

VICEVERSA: In a graph with no isolated nodes a vertex cover is also a dominating set:



one or the other in the VC

(each vertex is the endpoint of an edge: either it is in the vertex cover or the other endpoint must be for the edge to be covered)

DOMINATING SET (DS)

I : $\langle G = (V, E), k \rangle$

Q : Does G have a DS of size k ?

Prove that DS $\in$ NPH

EXERCISE Consider the following problem:

L-PATH

I: $\langle G=(V,E), u, v, k \rangle$,
$G=(V,E)$ undirected graph, $u, v \in V$, $1 \leq k \leq |V|$

Q: Is there a simple path from u to v of length $\geq k$?

(decision problem for LONGEST-PATH)

Prove that HAMILTON $<_p$ L-PATH

(thus L-PATH $\in$ NPH)

EXERCISE Assume that there exists a polynomial algorithm $A_{SS}(\langle S,t\rangle)$ for SS:

$$\begin{cases} I: \langle S,t\rangle : S \subseteq \mathbb{N} \text{ finite}, t \in \mathbb{N} \\ Q: \exists S' \subseteq S : \sum_{s \in S'} s = t \,? \end{cases}$$

Design a polynomial algorithm SUBSET$(\langle S,t\rangle)$ which uses A_{SS} as a subroutine and, in case $\langle S,t\rangle \in SS$, returns $S' : \sum_{s \in S'} s = t$.

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EXERCISE Consider the following problem:

PARTITION

$$\begin{cases} I: \langle S \rangle,\ S \subset \mathbb{N} \text{ finite} \\ Q: \exists S_1, S_2 \subset S, (S_1 \cup S_2 = S) \wedge (S_1 \cap S_2 = \phi): \\ \qquad \sum_{s \in S_1} s = \sum_{s \in S_2} s \ ? \end{cases}$$

Show that PARTITION $\in$ NPH