Game theory

A course for the
MSc in ICT for Internet and multimedia

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Game theory

Introduction

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What we will not see

- The term "game" is usually associated with

videogames

sports

gambling

- This is not what game theory is about ☹

What is a game?

- It's a <u>multi-person</u> <u>multi-objective</u> problem

problemi di ottimizzazione con più agenti

- And we talk about mathematical models
  - the key is really "theory" not just "games"
- You want to keep your models simple
  - this is why many real-life scenarios are not considered, they are just too complex
  - our games normally have easy math

vogliamo problemi trattabili

Engineers often use math!

- What is the maximum of this function?
- What is a good approximation of reality?
- How can this program/device/system achieve overall efficiency > X (or cost < X)

- We will not consider these problems either
  - we just take some very simplified versions… as other subjects teach how to solve them!
  - we rather give an added twist to them

Problem & rewards

- In most cases, engineers give mathematical models of single-person problems

goal to meet

reward

- Success solely depends on your ability
- Uncertainties: data availability, noise randomness, approximations

Many people: it gets messy

- When multiple agents are involved…

- Goals and rewards are not so clear-cut
- They depend on the choices of others, too

Characteristics of a game

- Involvement of multiple **players**, all with their individual objectives
- Common assumptions:
  - The rules of the game are known in advance
  - Players are rational hanno obiettivo da massimizzare, e sanno come fare
- The outcome of the game depends on the joint choices of all involved participants

Applications of game theory

- Game theory needs attitude towards mathematical modeling
- It was born as a branch of micro-economics and it is usually studied within this subject
- Many applications of game theory arise in the field of economics:
  - Micro level: trading, auctions, bargaining
  - Intermediate level: markets, firms
  - Macro level: countries, monetary authorities

Applications of game theory

- Game theory is nowadays widely used in

Economics (classic application)

players = enterprises

game = market

Engineering, biology (more recent)

prediction instrument

for distributed systems

Predictions

- Game theory introduces a further uncertainty that players know their moves but can only conjecture about the others'
- Still, some predictions can be obtained
  - we will discuss this in detail
  - predictions are actually harder for human beings and easier for digital information entities (algorithms, computers…)

Application examples

- Resource/channel access
  - users want service despite congestion
  - the more they ask, the more congestion
- Network behaviors
  - how to promote cooperation in a network
  - viral spreading (also pandemics, fake news)
- The Internet of Things
  - myriads of heterogeneous devices
  - their only driver: their own interest

Application examples

- Information security (adversarial)
  - attackers are not fully predictable
  - the defender just hopes to counteract
- Truthfulness of information
  - issues with reliability, manipulability, trust
  - online reputation, collusions
- Massive online platforms
  - electronic voting, auctions, social networks

Some background

A review of some useful concepts

Monopoly – oligopoly

- Monopolies or trusts damage the consumer
- Interestingly, some notable examples of monopoly or trust can be found today in the area of information engineering:
  - Power supply provisioning and similar utilities
  - Operating systems and related software
  - Wireless telephony and related services
  - Last mile in broadband Internet access
- Anti-trust offices punish monopolists. Why?

Market

- It is thought that the best social outcome is obtained when the market takes decisions
- Demand / offer laws, "invisible hand"
  - The market clearing price obtained in a competition is lower than monopoly price

spiegato fino a qui ^

- A (rational) monopolist with enough freedom pursues their own profit, not social welfare
- State monopoly admitted for public services or special goods (tobacco, lotto, alcohol)

Substitute goods

- Demand of a good is generally a decreasing function of its price.
  - Notable exceptions: Giffen goods, Veblen goods
- Two goods are substitute for each other if one can replace the other
  - e.g., margarine/butter, coffee/tea
- Formally, x and y are substitute goods if y's demand increases with x's price

Discount factor

- Any object does not have the same value today and tomorrow: today's value is <u>higher</u>
- Intuitions behind this: greed of the users, uncertainty, deterioration, price inflation…
- To keep this into account we may use a discount factor $\delta < 1$ describing this loss
- A euro at time 0 is valued 1
- At time t, it is δ^t

Discount factor

- Another common interpretation of the discount factor is that players may not be willing to play forever
  - some act like "there's no tomorrow"

- Very strong interpretation for network problems: some users may be joining the service only for a short while

saltata

Decision problems

How to select our preferred choice?

Decision problem

- It consists of three elements:
  - **actions** belonging to a set A, which are what can be decided upon
  - **outcomes**, i.e. the results of said actions
  - **preferences**: a way to describe what outcomes are preferable for the decider

Preferences

- We have a set A of alternatives (at least two).
- A **preference** is a binary relationship $\succcurlyeq$ on A
  - If a, b $\in A$ and a $\succcurlyeq$ b then a is ranked above b
  - formally, it is reflexive and antisymmetric
- A preference $\succcurlyeq$ is said to be
  - **complete**, if $\forall$a,b$\in A$, then a $\succcurlyeq$ b or b $\succcurlyeq$ a (or both)
  - **transitive** if $\forall$a,b,c$\in A$, a $\succcurlyeq$ b and b $\succcurlyeq$ c $\Rightarrow$ a $\succcurlyeq$ c
- If $\succcurlyeq$ is complete+transitive, we call it **rational**
  - mathematicians would call it a total order relation

Utility functions

- Utilities (also called payoff functions) are an arbitrary quantification $u(q)$ of the goodness coming from some input q di solito raggiungono asintoto
- If q is a countable good, $u(q)$ is generally an increasing function of q
  - and economists would say $u'(q) \geq 0, \quad u''(q) \leq 0$
- The exact formulation of u does not matter, it just maps the order via ≥ on numbers
- Preference relationship:
  - if $u(q) > u(q')$ a rational user prefers q over q'

Rationality

- The definition of utility hinders an important concept: players are **rational**

se definisco utilità e sono razionale, voglio massimizzare utilità -> egoismo dipende da definizione utilità

- This means that:
  - They act for their own good (selfishness)
  - They are aware of all consequences of their acts

- The first point may seem arguable
  - also does it imply that rational = selfish?
- As we will see, the **second** is indeed critical

Rationality

- Many economists argue that human beings are far from being rational (they are often crazy or simply generous, or make mistakes)
  - These criticisms are irrelevant when dealing with computers, algorithms, autonomous agents
- The actual problem is the accuracy of the model, not rationality
  - We can simply modify the utility by accounting for generosity, illogical preferences, and so on

Preferences and utilities

- A preference $\succcurlyeq$ can be put in relationship with a **utility function** $u : A \rightarrow \mathbb{R}$.
- We say that u **represents** $\succcurlyeq$ if for all a, b $\in A$

$$a \succcurlyeq b \Leftrightarrow u(a) \geq u(b)$$

non ci interessano numeri, ma piuttosto ordinamento

- **Theorem**: On a finite set A, $\succcurlyeq$ can be represented by u iff it is rational.

$\Rightarrow$ Immediate (due to properties of $\geq$ over $\mathbb{R}$)

$\Leftarrow$ A suitable utility function can be

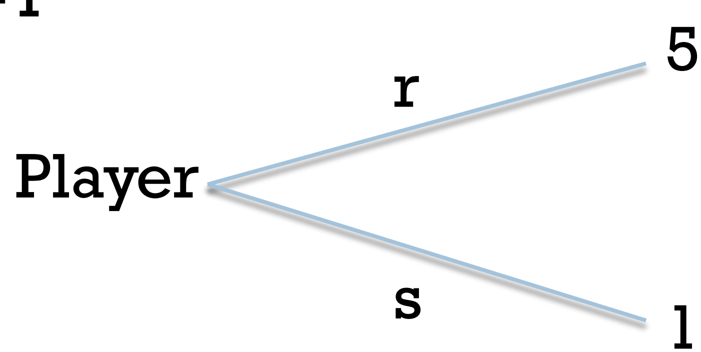
$$u(a) = |\{b \in A : a \succcurlyeq b\}|$$

Decision trees

- To represent the decision process, from now on we refer to the following example of a student going to the canteen to eat lunch
  - the goal is to get the preferred food available
- The canteen has several choices of food, and also subsequent choices to make
- For starters, let's consider that two options are available: ravioli (Italian pasta) or soup

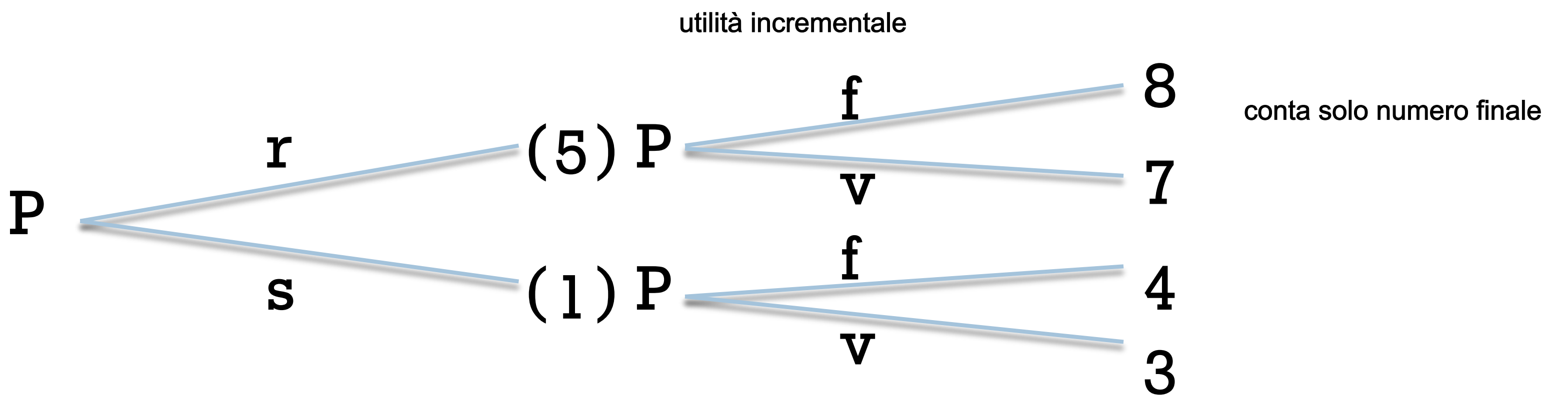
Decision trees

- Setting a preference among alternatives is like choosing a path at a road fork
- To represent A = {ravioli(r), soup(s)} and to explain that we prefer ravioli to soup, we can
  - either set a relation order for this preference
  - or to set u(r)=5, u(s)=1
- The latter can be seen graphically with a **decision tree**



Decision trees

- On the leaves, we have the utilities
- On the branches, we have the alternatives
- The root is the person to move



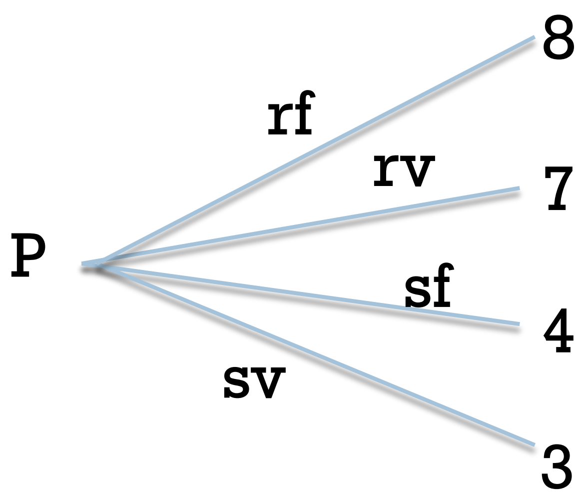
- If Player (P) has a further choice of fish(f) or vegetables(v), then we add levels to the tree

Decisions

- Setting a preference among alternatives is "easy" → just a maximization problem
- There can be subproblems:
  - Example 1: take the best route. It is the combination of multiple best routes (additive?)
  - Example 2: have the best meal. You can choose entree, main course, side, wine
- Strong assumption: one single smart agent can always solve such problems

Decision trees

- We assume we can always collapse every single-player tree into one layer



- We assume to be always able to solve such single-player games via classic optimization
- Things get interesting for multiple players