EXERCISE Design a decision algorithm A_L for a generic $L \in NP$ and analyze its running time

$L \in NP \Rightarrow$ esiste V_L poly-time verifier

$\forall x \in \{0,1\}^*: x \in L \rightarrow \exists y: |y| \le C \cdot |x|^{k_1}, k \ge 1: V_L(x,y) = 1$

$T_{V_L}(|x|+|y|) \le C_2 \cdot (|x|+|y|)^{k_2}, k_2 \ge 1$

$\Downarrow$

non serve provare ogni certificato, bastano questi

prova ogni stringa binaria di dim. $\le C_1 \cdot |x|^{k_1}$ come candidati

$A_L(x)$
foreach $y \in \{0,1\}^*$, $|y| \le C_1 \cdot |x|$ do:
  if $V_L(x,y) = 1$ then return 1;
return 0;

Correttezza: dimostrare $x \in L_{A_L} \Leftrightarrow x \in L$

$x \in L_{A_L} \Leftrightarrow A_L(x) = 1 \Leftrightarrow \exists y \in \{0,1\}^*, |y| \le C_1 \cdot |x|^{k_1}: V_L(x,y) = 1 \Leftrightarrow x \in L$

$\Downarrow$

$L_{A_L} = L$

Running time:

- # iterazioni: $\sum_{i=0}^{C_1 \cdot |x|^{k_1}} 2^i = \Theta(2^{C_1 \cdot |x|^{k_1}})$
- costo / iterazione: $O((|x|+|y|)^{k_2}) = O((|x| + |x|^{k_1})^{k_2}) = O(|x|^{k_1 \cdot k_2})$

$T_{A_L}(|x|) = O(|x|^{k_1 k_2} \cdot 2^{C_1 \cdot |x|^{k_1}})$

EXERCISE Consider

HALTING-PROBLEM (RAM version) (HP-RAM)

I : $\langle C, x \rangle$ $C(x)$ RAM pseudocode with parameter $x \in \{0,1\}^*$

Q : Does $C(x)$ terminate ?

Prove that HP-RAM $\in$ NPH

$\Downarrow$

$\forall L \in NP$, $L <_p$ HP-RAM $\Rightarrow$ facciamo tutte riduzioni:

$\forall L \in NP$, scriviamo pseudocodice che usa V_L e termina iff $x \in L$ (non ci preoccupiamo di tempo)

$f(x) \rightsquigarrow \langle H_L, x \rangle$ $\Rightarrow$ pseudocodice

$H_L(x')$:

foreach $y \in \{0,1\}^*, |y| \leq C_1 \cdot |x'|^{K_1}$ do:

if $V_L(x', y) = 1$ then return 1;

while true do. $y \leftarrow y$ // qualunque istruzione

$x \in L \Leftrightarrow \exists y \in \{0,1\}^*, |y| \leq C_1 \cdot |x|^{K_1} : V_L(x,y) = 1 \Leftrightarrow H_L(x)$ terminates $\Leftrightarrow$

$\Leftrightarrow \langle H_L, x \rangle = f(x) \in$ HP-RAM

Dimensione H_L: costante $\Rightarrow$ running time: $\Theta(|x|) \Rightarrow f$ è ptc

EXERCISE

Prove the following proposition:

$$\overset{Hp1}{(L' <_p L)} \wedge \overset{Hp2}{(L \in NP)} \Rightarrow L' \in NP$$

Hp1: $L' <_p L$: $\exists f(x)$ ptc: $x \in L' \Leftrightarrow f(x) \in L$

$T_{A_f}(|x|) = O(|x|^{k_1})$, $k_1 \geq 1$

Hp2: $\exists V_L(x,y)$: $x \in L \Leftrightarrow \exists y \in \{0,1\}^*$, $|y| \in O(|x|^{k_2})$, $k_2 \geq 1$,

$V_L(x,y) = 1$, $T_{V_L}(|x|, |y|) = O((|x| + |y|)^{k_3})$, $k_3 \geq 1$

Th: $L' \in NP$

$V_{L'}(x,y)$: return $V_L(A_f(x), y)$;

$$T_{V_{L'}}(|x| + |y|) = T_{A_f}(|x|) + T_{V_L}(|f(x)| + |y|) = O(|x|^{k_1}) + O((\overbrace{|x|^{k_1}}^{f(x)} + |y|)^{k_3}) =$$

$$= O((|x| + |y|)^{k_1 k_3})$$

Provare correttezza

EXERCISE

Consider this decision problem:

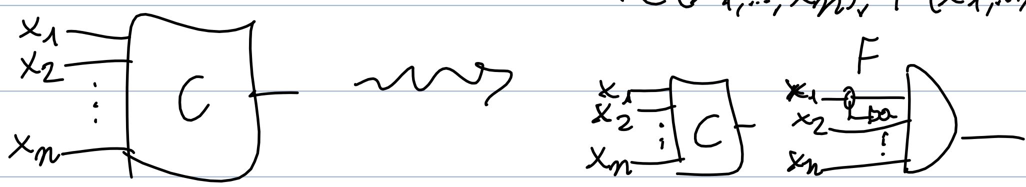
TESTING

$$\begin{cases} I: \langle C_1(x_1,\dots,x_n),\ C_2(x_1,\dots,x_n)\rangle \\ Q: \exists \vec{b} \in \{0,1\}^n : C_1(\vec{b}) \neq C_2(\vec{b})\ ? \end{cases}$$

Prove that TESTING $\in$ NPH

BC-SAT $<_p$ TESTING:

$\langle C(x_1,\dots,x_n)\rangle$ $\qquad\qquad$ $\langle C(x_1,\dots,x_n),\ F(x_1,\dots,x_n)\rangle$



$C \in$ BC-SAT: $\exists \vec{b} \in \{0,1\}^\ell : C(\vec{b}) = 1 \Rightarrow \exists \vec{b} : C(\vec{b}) = 1,\ F(\vec{b}) = 0 \Rightarrow$

$\Rightarrow f(\langle C\rangle) = \langle C, F\rangle \in$ TESTING

$C \notin$ BC-SAT: $\forall \vec{b} \in \{0,1\}^\ell : C(\vec{b}) = 0 \Rightarrow F(\vec{b}) = 0 \Rightarrow f(\langle C\rangle) \notin$ TESTING

EXERCISE Consider the following problem

KNAPSACK

I : $\langle a_1, a_2, \ldots, a_n, b; c_1, c_2, \ldots, c_n, K \rangle$,

a_i : weights of n items

b : total weight bound (capacity)

c_i : profit / value of the n items

K : lower bound on total profit

$\in \mathbb{N}$

Q : Is there a subset of items of total weight $\leq b$ and total profit $\geq K$?

Prove that KNAPSACK $\in$ NPH