EXERCISE 1 RESOURCE SELECTION

$R = \{r_1, r_2, \dots, r_n\}$ universe of resource types

T is a set of m tasks.

In order to be executed each task t may use either $Y_t^1 \subseteq R$ or $Y_t^2 \subseteq R$ (subsets of resource types)

We wish to execute all tasks using the minimum number of resource types.

For $1 \leq t \leq m$ select one between Y_t^1 and Y_t^2 – say $Y_t^{s_t}$, $s_t \in \{1, 2\}$ – so to minimize

$$\left| \bigcup_{t=1}^{m} Y_t^{s_t} \right|$$

1. Prove that the decision version of the problem RESOURCE SELECTION (RS)

$$\langle R, \{Y_t^j : 1 \leq t \leq m, 1 \leq j \leq 2\}, k \rangle \in NPH$$

2. Provide an ILP formulation of the problem

3. Use LP rounding to get a 2-approximation algorithm

Possible scenario: R is a set of drugs $\{r_1, r_2, ---, r_n\}$
Each task is a therapy for a disease j, $1 \le j \le m$ which may use either drug combination Y_t^1 or Y_t^2. We want to minimize the number of drug types used for all the therapies

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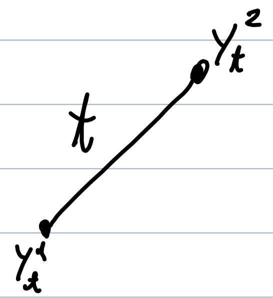
1.



$\Rightarrow VC <_p RS$

$$\langle G=(V,E), K\rangle \overset{f}{\rightsquigarrow} f(\langle G=(V,E),K\rangle) = \langle R_G, \{Y^1_{G_i}, Y^2_{G_i}\}, K_G\rangle$$

$$R_G = V,\ e_i \in E = \{u,v\} \to Y^1_{G_i} = \{u\},\ Y^2_{G_i} = \{v\},\ K_G = K$$

f é ptc (lineare)

$$\langle G=(V,E),K\rangle \in VC \Rightarrow \exists V' \subseteq V, |V'| = K: \forall \{u,v\} \in E\ (u \in V') \wedge (v \in V') \Rightarrow$$
$$\Rightarrow (\text{select } Y^1_{G_i} \text{ se per } e_i = \{u,v\}\ u \in V', \text{ altrimenti } Y^2_{G_i}) \Rightarrow$$
$$\Rightarrow \exists \{Y^{x_i}_{G_i} : 1 \le i \le m,\ x_i \in \{1,2\}\} : \left|\bigcup_{i=1}^{m} Y^{x_i}_{G_i}\right| \le K \Rightarrow$$
$$\Rightarrow f(\langle G,K\rangle) \in RS$$

2. $x_1, \dots, x_n \in \{0,1\} \Rightarrow$ selezione di risorsa i
$y_j^1, y_j^2 : 1 \leq j \leq m \Rightarrow y_j^K = 1$ se seleziono insieme K di risorse per T_j

$$\begin{cases} \min \sum_{i=1}^{m} x_i \\ y_j^1 + y_j^2 \geq 1 & \forall j \in [1,m] \\ x_i \geq y_j^K & \forall r_i \in Y_j^K,\ i \in [1,n],\ K \in \{1,2\},\ j \in [1,m] \\ x_i, y_j^K \in \{0,1\} & \forall i \in [1,n],\ K \in \{1,2\},\ j \in [1,m] \end{cases}$$

3. LP: $$\begin{cases} \min \sum_{i=1}^{m} x_i \\ y_j^1 + y_j^2 \geq 1 & \forall j \in [1,m] \\ x_i \geq y_j^K & \forall r_i \in Y_j^K,\ i \in [1,n],\ K \in \{1,2\},\ j \in [1,m] \end{cases}$$

soluzione: $(\vec{x}^*, \vec{y}^*)$

$$\hat{x}_i = \text{rounding}(x_i^*) = \begin{cases} 1 & \text{se } x_i^* \geq 1/2 \\ 0 & \text{altrimenti} \end{cases}$$

$$\hat{y}_j^K = \text{rounding}(y_j^{K*}) = \begin{cases} 1 & \text{se } y_j^{*K} \geq 1/2 \\ 0 & \text{altrimenti} \end{cases}$$

ammissibilità di $(\hat{x}, \hat{y})$?

$y_j^1 + y_j^2 \geq 1$ in LP $\Rightarrow \exists K : y_j^{*K} \geq 1/2 \Rightarrow \hat{y}_j^K = 1$ ∎

$x_i^* \geq y_j^{*K} \geq 1/2 \Rightarrow x_i^* \geq 1/2\ \forall r_i \in Y_j^K \Rightarrow \hat{x}_i = 1$ ∎

$\hat{x}_i \leq 2 x_i^* ?$

$$\text{cost}(\hat{x}, \hat{y}) = \sum_{i=1}^{m} \hat{x}_i \leq 2 \sum_{i=1}^{m} x_i^* = 2 \,\text{cost}(x^*, y^*) \leq 2 \,\text{cost}(\text{opt})$$

$$\frac{\text{cost}(\hat{x}, \hat{y})}{\text{cost}(\text{opt})} \leq 2$$

EXERCISE Extend to F subsets for each task. Obtain an F-approximation algorithm

EXERCISE 2 MAX NODE-CUT

Given $G=(V,E)$ a node cut is any bipartition of V: $(W, V-W)$, $W \subseteq V$.
The size of a node cut is
$s(W, V-W) = |\{e=\{u,v\} \in E : u \in W, v \in V-W\}|$
We wish to find the node cut of maximum size.
The decision problem associated to MAX-CUT is NP-Complete (observe that the minimization problem is polynomial)

Provide a deterministic 2-approximation algorithm for MAX-CUT

Idea: applico CONTINUOUS REFINEMENT (local search):
da semplice soluzione ammissibile, cerco nodi da spostare fuori da W che alzano dim.

EXERCISE 3 Develop a randomized 2-approximation algorithm for MAX-CUT