Exercise 1 Consider the following static games of complete information played by A and B where the normal-form representation of the game is given. For all of them, find the entire set of Nash equilibria.

(a)

A \ B	f	g
F	2, 4	0, 1
G	1, 6	3, 5

(b)

A \ B	f	g
F	0, 4	3, 0
G	6, 0	0, 5

(c)

A \ B	f	g
F	9, 3	2, 2
G	0, 0	3, 9

(d)

A \ B	f	g
F	2, 2	0, 6
G	6, 0	1, 1

(a) Player B has g as a strictly dominated strategy
Applying IESDS leads to finding the only NE (F,f)
which is in pure strategies

(b) is a discoordination game. No NE in pure strategies
There must be a mixed strategy NE. How to find it?

$\alpha =$ probability that A plays F

B must be indifferent when answering with f or g
because the support of B's mixed strategy is {f,g}

$4\alpha = 5(1-\alpha) \qquad \alpha = 5/9$

$\beta =$ probability that B plays f $\qquad 6\beta = 3(1-\beta) \Rightarrow \beta = 1/3$

(c) this is similar to a Battle of Sexes (coordination game)

A \ B	f	g
F	9, 3	2, 2
G	0, 0	3, 9

two pure strategy NE
(F,f) (G,g)

but there must be
a mixed one, too

$\alpha =$ probability that A plays F

Remember: this is found by setting indifference for player B

$u_B(\alpha, f) = u_B(\alpha, g)$ $\quad 3\alpha = 2\alpha + 9(1-\alpha)$

$\alpha = 9/10$ $\quad \beta =$ probability that B plays f $= 1/10$

(d) is like a Prisoners' dilemma. Only pure NE (G,g)

If we used α β to find a "mixed" NE $\Rightarrow$ non si può usare in questi casi

$2\beta = 6\beta + 1 - \beta \quad -3\beta = 1 \quad \beta \notin [0,1]$

Exercise 2 Consider the following static game of complete information played by A and B where the normal-form representation is given below.

		B		
A	J	K	L	M
X	6, 7	5, 5	3, 8	8, 1
Y	4, 9	9, 2	0, 4	2, 3
Z	8, 4	2, 8	4, 2	3, 6

1. Prove that there is no Nash equilibrium in pure strategies
2. Prove that these (m_A, m_B) are Nash equilibria in mixed strategies:
   - $m_A = (2/3, 0, 1/3)$, $m_B = (5/11, 4/11, 2/11, 0)$
   - $m_A = (0, 4/11, 7/11)$, $m_B = (7/11, 4/11, 0, 0)$
3. List all of the joint pure strategies that are Pareto optimal.

2 means that mA has support = {X,Z} so X and Z give the same payoff against mB, and also this payoff is >= that of Y

$$u_A(X, m_B) = \frac{30}{11} + \frac{20}{11} + \frac{6}{11} = 56/11$$

$$u_A(Z, m_B) = \frac{40}{11} + \frac{8}{11} + \frac{8}{11} = 56/11$$

$$u_A(Y, m_B) = \frac{20}{11} + \frac{36}{11} + 0 = 56/11$$

Analogously, mB has support {J,K,L} so they all give the same payoff against mA

$$u_B(m_A, J) = \frac{14}{3} + \frac{4}{3} = 6$$

$$u_B(m_A, K) = \frac{10}{3} + \frac{8}{3} = 6$$

$$u_B(m_A, L) = \frac{16}{3} + \frac{2}{3} = 6$$

$$u_B(m_A, M) = \frac{2}{3} + \frac{6}{3} < 6$$

3 - to find the Pareto efficient outcomes, we start from the maximum payoffs of A and B

(Y, J) giving 4, 9

(Y, K) giving 9, 2

(X, J) giving 6, 7

(Z, J) giving 8, 4

Come trovarli:

- quando uno massimizza, sicuro Pareto ottimale
- applicare "Pareto efficiente se non si può migliorare uno senza peggiorare altri

Exercise 3 Two students, Charlotte (C) and Daniel (D), need to write their MS Thesis. They need to choose (independently and unbeknownst to each other) a supervising professor. Three professors are available for this role: **X**avier, **Y**uan, and **Z**ingberry. The utility of a student is given by the amount of help he/she receives from the supervisor, which is quantified as 40 for Xavier, 60 for Yuan, 50 for Zingberry. However, if the two students select the *same* professor as their supervisor, they only get 70% of the utility that they would get if the professor had only one of them to supervise.

1. Write the game in normal form.

2. Find all the Nash equilibria of the game in pure strategies

3. Find all the Nash equilibria of the game

D

C	~~X~~	Y	Z
~~X~~	28, 28	40, 60	40, 50
Y	60, 40	42, 42	60, 50
Z	50, 40	50, 60	35, 35

(Y, Z)

(Z, Y)

X sempre dominata → posso escluderla dall'analisi

There also is a third NE in mixed strategies. It is easier to find thanks to symmetry. (the mixture is between Y and Z)

p = probability that C plays Y

$$u_D(p, Y) = u_D(p, Z)$$

$$42p + 60(1-p) = 50p + 35(1-p)$$

$$33p = 25 \qquad p = 25/33 \qquad \text{NE}: (p, p)$$

Exercise 4 A strategic interaction takes place between a taxpayer T and the tax inspector I. T is supposed to pay a share S of its income so as to have a net income after paying taxes equal to R. However, T is considering two alternatives: hide part of the taxes (**H**) to get an additional dishonest income of L (so the tax paid is $S-L$ and the net income is $R+L$) or pay all due taxes in full (**P**). Player I also has two choices: check T for tax evasion (**C**) or do not check T (**D**). Performing a check has a cost equal to E. If the tax inspector discovers that T did not pay the tax, then the taxpayer will be fined and will have to pay an additional amount equal to F that goes into the inspector. The probability of being discovered by a tax inspector after a check is p. The purpose of the taxpayer is to get the maximum possible amount of money. The goal of the tax inspector is to maximize the collected amount (minus the cost). Formalize this conflict in the form of a static game of complete information and find its Nash equilibria.

I

T	C	D
H	$\cancel{R}+(1-p)L-pF$; $\cancel{S}-E+pF-(1-p)L$	$\cancel{R}+L$; $\cancel{S}-L$
P	$\cancel{R}$; $\cancel{S}-E$	$\cancel{R}$; $\cancel{S}$

$\gtrless ? \Rightarrow$ così

$\gtrless ? \Downarrow$ così

$$-E+pF-(1-p)L > -L$$
$$\Downarrow$$
$$-E+pF-\cancel{L}+pL > \cancel{-L}$$
$$\Downarrow$$
$$E < p(F+L)$$

To find the NE it is convenient to set R=S=0

↳ R, S non cambiano ordine di utilità

1) $E > p(F+L)$ the only NE is (H,D)

this is because D strictly dominates C

physical interpretation: the effort to check is too high

2) $E < p(F+L)$ but $p < L/L+F$

the only NE is (H,C) this because H strictly dominates P

physical interpretation: the taxpayer is always dishonest, but the cost of checking is low so I does the control

3) $E < p(F+L)$ and $p > L/L+F$

there is no NE in pure strategies, there must be a mixed one

After some math, you can find that T and I play H and C with respective probabilities

$$\alpha = \frac{L}{p(L+F)} \qquad \beta = \frac{E}{p(L+F)}$$

physical interpretation: there is a mixed behavior. Sometimes the taxpayer is honest, some other times is not