Analysis of Cooperative Games with Matlab and Mathematica

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Bibliography

H. Aarts and T. Driessen. The irreducible core of a minimum cost spanning tree game. *ZOR - Methods and Models of Operations Research*, 38:163–174, 1993. doi: 10.1007/BF01414212. URL https://doi.org/10.1007/BF01414212.

E. Algaba, J. M. Bilbao, P. Borm, and J. J. López. The Position Value for Union Stable Systems. *Mathematical Methods of Operations Research*, 52:221–236, 2000.

E. Algaba, J. M. Bilbao, P. Borm, and J. J. López. The Myerson Value for Union Stable Structures. *Mathematical Methods of Operations Research*, 54:359–371, 2001.

E. Algaba, J. M. Bilbao, and J. J. López. The Position Value in Communication Structures. *Mathematical Methods of Operations Research*, 59:465–477, 2004.

E. Algaba, V. Fragnelli, and J. Sánchez-Soriano, editors. Series in Operations Research. Chapman and Hall/CRC, New York, 1 edition, 2020. doi: 10.1201/9781351241410. URL https://doi.org/10.1201/9781351241410.

J. M. Alonso-Meijide, F. Ferreira, M. Álvarez Mozos, and A. A. Pinto. Two new power indices based on winning coalitions. *Journal of Difference Equations and Applications*, 17(7):1095–1100, 2011. doi: 10.1080/10236190903200677. URL https://doi.org/10.1080/10236190903200677.

J.M. Alonso-Meijide, F. Carreras, M.G. Fiestras-Janeiro, and G. Owen. A comparative axiomatic characterization of the banzhaf–owen coalitional value. *Decision Support Systems*, 43(3):701–712, 2007. ISSN 0167-9236. doi: https://doi.org/10.1016/j.dss.2006.11.008. URL https://www.sciencedirect.com/science/article/pii/S0167923606002028. Integrated Decision Support.

Y. Altman. *Accelerating MATLAB Performance*. CRC Press, Boca Raton, FL. (USA), 1 edition, 2015.

E.D. Andersen, C. Roos, and T. Terlaky. On implementing a primal-dual interior-point method for conic quadratic optimization. *Mathematical Programming*, 95(2):249 – 277, 2002. doi: 10.1007/s10107-002-0349-3. URL https://doi.org/10.1007/s10107-002-0349-3.

J. Arin and V. Feltkamp. The nucleolus and kernel of veto-rich transferable utility games. *International Journal of Game Theory*, 26: 61–73, 1997. URL https://doi.org/10.1007/BF01262513.

J. Arin and E. Iñarra. A Characterization of the Nucleolus for Convex Games. *Games and Economic Behaviour*, 23:12–24, 1998.

J. Arin and I. Katsev. The Coincidence of the Kernel and Nucleolus of a Convex Game: An Alternative Proof. Technical report, University of Basque Country, 2013.

St. Attaway. *MATLAB: A Practical Introduction to Programming and Problem Solving*. Butterworth-Heinemann/Elsevier, Oxford, UK., 5 edition, 2019.

R. J. Aumann. Acceptable Points in General Cooperative n-Person Games. In Tucker and Luce, editors, *Contributions to the theory of games IV, Annals of Mathematics Studies Vol. 40*, pages 287–324, 1959.

R. J. Aumann. A Survey on Cooperative Games without Side Payments. In M. Shubik, editor, *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, pages 3–27, Princeton, 1961. Princeton University Press.

R. J. Aumann and J. H. Drèze. Cooperative Games with Coalition Structures. *International Journal of Game Theory*, 3:217–237, 1974.

R. J. Aumann and M. Maschler. Game Theoretic Analysis of a Bankruptcy Problem from the Talmud. *Journal of Economic Theory*, 36: 195–213, 1985.

J. F. Banzhaf. Weighted voting doesn't work. *Rutgers Law Review*, 19:221–236, 1965.

R.J. Barro and X. Sala-I-Martin. *"Economic Growth"*. McGraw-Hill, New York, 1 edition, 1995.

E.N. Barron. *"Game Theory: An Introduction"*. John Wiley & Sons, Hoboken, 2 edition, 2013.

B. Barry. Is it Better to Be Powerful or Lucky?: Part I. *Political Studies*, 28(2):183–194, 1980a. doi: 10.1111/j.1467-9248.1980.tb01244.x. URL https://doi.org/10.1111/j.1467-9248.1980.tb01244.x.

B. Barry. Is it Better to Be Powerful or Lucky? Part II. *Political Studies*, 28(3):338–352, 1980b. doi: 10.1111/j.1467-9248.1980.tb00473.x. URL https://doi.org/10.1111/j.1467-9248.1980.tb00473.x.

S. Béal, E. Rémila, and Ph. Solal. A Decomposition of the Space of TU-games Using Addition and Transfer Invariance. Technical Report 2013-08, Université de Franche-Comté,, 2013.

G. Bergantiños and J. Vidal-Puga. A review of cooperative rules and their associated algorithms for minimum-cost spanning tree problems. *SERIEs*, 12:73–100, 2021.

C. G. Bird. On cost allocation for a spanning tree: A game theoretic approach. *Networks*, 6(4):335–350, 1976. doi: https://doi.org/10.1002/net.3230060404. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/net.3230060404.

O.J. Blanchard and S. Fischer. *"Lectures on Macroeconomics"*. MIT-Press, Cambridge, Massachusetts, 1 edition, 1989.

O. N. Bondareva. Some Applications of Linear Programming Methods to the Theory of Cooperative Games. *Problemy Kibernet*, 10: 119–139, 1963. in Russian.

P. Borm, G. Owen, and SH. Tijs. On the Position Value for Communication Situations. *SIAM Journal of Discrete Mathematics*, 5: 305–320, 1992.

St. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, Cambridge, eigth reprint edition, 2010.

E. Bozzo, M. Franceschet, and F. Rinaldi. Vulnerability and power on networks. *Network Science*, 3(2):196–226, 2015. doi: 10.1017/nws.2015.8.

Ronald L Breiger and Philippa E Pattison. Cumulated social roles: The duality of persons and their algebras. *Social Networks*, 8(3):215 – 256, 1986. ISSN 0378-8733. doi: https://doi.org/10.1016/0378-8733(86)90006-7. URL http://www.sciencedirect.com/science/article/pii/0378873386900067.

E. Calvo and E. Gutiérrez. A Value for Cooperative Games with a Coalition Structure. Technical report, University of Valencia, Spain, 2012.

E. Calvo and E. Gutiérrez. The Shapley-Solidarity Value for Games with a Coalition Structure. *International Game Theory Review*, 15 (1):1350002–1–1350002–24, 2013.

M. Carter. *Cooperative Games*, pages 167–191. In Varian (1993), 1993.

Ch. Chang and T. S. H. Driessen. (Pre)Kernel Catchers for Cooperative Games. *OR Spectrum*, 17(1):pp 23–26, 1995.

Ch. Chang and Ch. Y. Kan. The Bound of the Kernel. *Mathematical Social Sciences*, 25:87–93, 1992.

Ch. Chang and Ch.-H. Lian. Some Results on (Pre)Kernel Catchers and the Coincidence of the Kernel with Prekernel. *International Game Theory Review*, 4(3):201–211, 2002.

Chih Chang and Ying-Chih Tseng. On the coincidence property. *Games and Economic Behavior*, 71(2):304 – 314, 2011. ISSN 0899-8256. doi: https://doi.org/10.1016/j.geb.2010.04.010. URL http://www.sciencedirect.com/science/article/pii/S0899825610000710.

St. J. Chapman. *MATLAB Programming for Engineers*. Brooks/Cole, Pacific Grove, CA. (USA), 2 edition, 2002.

A. C. Chiang. *Fundamental Methods of Mathematical Economics*. McGraw-Hill, Singapore, third. edition, 1984.

A. Claus and D. J. Kleitman. Cost allocation for a spanning tree. *Networks*, 3(4):289–304, 1973. doi: https://doi.org/10.1002/net.3230030402. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/net.3230030402.

R. H. Coase. The marginal cost controversy. *Economica*, 13(51):169–182, 1946. ISSN 00130427, 14680335. URL http://www.jstor.org/stable/2549764.

J. S. Coleman. *Control of Collectivities and the Power of a Collectivity to Act*. Gordon and Breach, New York, 1 edition, 1971.

I. Curiel. *Cooperative game theory and applications*, volume 16 of *Theory and Decision Library: Series C*. Kluwer Acad. Publ., Boston, 1997.

M. Davis and M. Maschler. The Kernel of a Cooperative Game. *Naval Research Logistic Quarterly*, 12:223–259, 1965.

R. de Roover. *The Rise and Decline of the Medici Bank 1397-1494*. The Norton Library, New York, USA, 1966.

J. Deegan and E.W. Packel. A new index of power for simplen-person games. *International Journal of Game Theory*, 7:113–123, 1978. doi: 10.1007/BF01753239. URL https://doi.org/10.1007/BF01753239.

P. Dehez. On harsanyi dividends and asymmetric values. *International Game Theory Review*, 19(03):1750012–1–36, 2017. doi: 10.1142/S0219198917500128. URL https://doi.org/10.1142/S0219198917500128.

X. Deng and C. H. Papadimitriou. On the complexity of cooperative solution concepts. *Mathematics of Operations Research*, 19:257–266, 1994.

J. Derks and J. Kuipers. On the core and the nucleolus of routing games. Discussion Paper Report M92-07, Department of Mathematics, University of Maastricht, 1992.

J. J. M. Derks and H. Haller. Weighted Nucleoli. *International Journal of Game Theory*, 28:173–187, 1999.

B. Dietzenbacher and P. Sudhölter. Hart–Mas-Colell Consistency and the Core in Convex Games. *International Journal of Game Theory*, pages 1–17, 2021. ISSN 0254-5330. doi: 10.1007/s00182-021-00798-6. URL https://doi.org/10.1007/s00182-021-00798-6.

A. Dixit. A Model of Duopoly suggesting a Theory of Entry Barriers. *Bell Journal of Economics*, 10:20–32, 1979.

A. K. Dixit. *Optimization in Economic Theory*. Oxford University Press, Oxford, 2 edition, 1990.

T. Driessen. *Contributions to the Theory of Cooperative Games: The τ-Value and k-Convex Games*. Ph.d thesis, University of Nijmegen, Nijmegen, 1985.

T. Driessen. *Cooperative Games, Solutions and Applications*. Kluwer Academic Publishers, Dordrecht, 1988.

T. Driessen. The Greedy Bankruptcy Game: An Alternative Game Theoretic Analysis of a Bankruptcy Problem. In L.A. Petrosjan and V.V. Mazalov, editors, *Game Theory and Applications*, volume IV, pages 45–61, Commack, New York, 1998. Nova Science Publishers Inc.

T. Driessen and H. Meinhardt. (Average-) Convexity of Common Pool and Oligopoly TU-games. *International Game Theory Review*, 3: 141–158, 2001.

T. Driessen and H. Meinhardt. Convexity of Oligopoly Games without Transferable Technologies. *Mathematical Social Sciences*, 50(1): 102–126, 2005.

T. S. H. Driessen and H. I. Meinhardt. On the Supermodularity of Homogeneous Oligopoly Games. *International Game Theory Review (IGTR)*, 12(04):309–337, 2010. doi: 10.1142/S0219198910002702. URL https://doi.org/10.1142/S0219198910002702.

P. Dubey and L. S. Shapley. Mathematical properties of the banzhaf power index. *Mathematics of Operations Research*, 4(2):99–131, 1979. ISSN 0364765X, 15265471. URL http://www.jstor.org/stable/3689345.

H. Dym. *Linear Algebra in Action*, volume 78. American Mathematical Society, Providence, Rhode Island, Graduate Studies in Mathematics edition, 2007.

H-D. Ebbinghaus, J. Flum, and W Thomas. *Einführung in die mathematische Logik*. Spektrum Akademischer Verlag, Berlin/Heidelberg, 5 edition, 2007.

L. Elden, L. Wittmeyer-Koch, and Nielsen H. B. *Introduction to Numerical Computation*. Studentlitteratur AB, Lund, Sweden, 2004.

A. Estévez-Fernández, M.G. Fiestras-Janeiro, M.A. Mosquera, and E. Sánchez-Rodríguez. A Bankruptcy Approach to the Core Cover. Technical Report TI 2012-012/1, VU University Amsterdam, 2012.

U. Faigle and M. Grabisch. Linear Transforms, Values and Least Square Approximation for Cooperation Systems. Technical report, Panthéon-Sorbonne, Université Paris 1, 2014.

D.S. Felsenthal. A well-behaved index of a priori p-power for simple n-person games. *Homo Oeconomicus*, 33:367–381, 2016. doi: 10.1007/s41412-016-0031-2. URL https://doi.org/10.1007/s41412-016-0031-2.

V. Feltkamp. Alternative axiomatic characterizations of the Shapley and Banzhaf values. *International Journal of Game Theory*, 24: 179–186, 1995. doi: 10.1007/BF01240041. URL https://doi.org/10.1007/BF01240041.

L.R. Ford and D.R. Fulkerson. *Flows in Networks*. Princeton University Press, Princeton, 1962.

A. Fronzetti Colladon and M. Naldi. Distinctiveness centrality in social networks. *PLoS ONE*, 15(5), 2020. URL https://doi.org/10.1371/journal.pone.0233276.

Y. Funaki. Upper and Lower Bounds of the Kernel and Nucleolus. *International Journal of Game Theory*, 15:121–129, 1986.

Y. Funaki and H. I. Meinhardt. A Note on the Pre-Kernel and Pre-Nucleolus for Bankruptcy Games. *The Waseda Journal of Political Science and Economics*, 363:126–136, 2006.

R. J. Gaylord, S. N. Kamin, and P. R. Wellin. *An Introduction to Programming with Mathematica*. Telos/Springer, Santa Clara, CA, sec. edition, 1996.

J. Getán, J. Izquierdo, J. Montes, and C. Rafels. The Bargaining Set and the Kernel of Almost-Convex Games. Technical report, University of Barcelona, Spain, 2012.

H.-G. Gräbe and M. Kofler. *Mathematica 6: Einführung, Grundlagen und Beispiele*. Scientific Tools. Pearson Studium, München, 5 edition, 2007.

D. Granot and G. Huberman. Minimum cost spanning tree games. *Mathematical Programming*, 21:1–18, 1981. URL https://doi.org/10.1007/BF01584227.

D. Granot and G. Huberman. On the core and nucleolus of minimum cost spanning tree games. *Mathematical Programming*, 29:323–347, 1984. URL https://doi.org/10.1007/BF02592000.

D. Granot, F. Granot, and W. R. Zhu. Circular Network Games. Discussion paper, Faculty of Economics and Business Admistration, UBC Vancouver, 1994.

D. Granot, F. Granot, and W. R. Zhu. Characterization sets for the nucleolus. *International Journal of Game Theory*, 27:359–374, 1998.

Daniel Granot and Gur Huberman. The relationship between convex games and minimum cost spanning tree games: A case for permutationally convex games. *SIAM Journal on Algebraic Discrete Methods*, 3(3):288–292, 1982. doi: 10.1137/0603029. URL https://doi.org/10.1137/0603029.

Z. Gu, E. Rothberg, and R. Bixby. Gurobi Software documentation. Technical report, Gurobi Optimization, Inc., Houston, Texas, 2017a. Gurobi Optimizer Reference Manual Version 7.0.

Zonghao Gu, Edward Rothberg, and Robert Bixby. Gurobi Software program. Technical report, Gurobi Optimization, Inc., Houston, Texas, 2017b. Gurobi Optimizer Version 7.0.

B. D. Hahn and D. T. Valentine. *Essential MATLAB for Engineers and Scientists*. Academic Press, Cambridge, MA. (USA), 7 edition, 2019.

S. Hart and M. Kurz. Endogenous Formation of Coalitions. *Econometrica*, 51(4):1047–1064, 1983.

S. Hart and A. Mas-Colell. Potential, Value and Consistency. *Econometrica*, 57:589–614, 1989.

D.A. Harville. *Matrix Algebra From A Statistican's Perspective*. Springer Publisher, New York, Heidelberg, 1997.

M. Herceg, M. Kvasnica, C. N. Jones, and M. Morari. Multi-Parametric Toolbox 3.0. In *Proc. of the European Control Conference*, pages 502–510, Zurich, Switzerland, July 17–19 2013. European Control Conference. http://control.ee.ethz.ch/~mpt.

L. Hernández-Lamoneda, R. Juárez, and F. Sánchez-Sánchez. Dissection of solutions in cooperative game theory using representation techniques. *International Journal of Game Theory*, 35(3):395–426, Feb 2007. ISSN 1432-1270. doi: 10.1007/s00182-006-0036-3. URL https://doi.org/10.1007/s00182-006-0036-3.

Ch. Hibbert. *The Rise and Fall of the House of Medici*. Harper Perennial, New York, USA, reprint 2003 edition, 1974.

D. J. Higham and N. J. Higham. *MATLAB Guide*. SIAM, Philadelphia, PA. (USA), 1 edition, 2000.

M. J. Holler. Forming Coalitions and Measuring Voting Power. *Political Studies*, 30:262–271, 1982.

M. J. Holler. The Public Good Index: A Brief Introduction. *Finnish-German Yearbook of Political Economy*, 1:31–39, 2018.

M. J. Holler and E. W. Packel. Power, Luck and the Right Index. *Journal of Economics*, 43:21–29, 1983.

M. J. Holler and F. Rupp. Power in Networks: The Medici. Technical report, CCR-Munich, Munich, Germany, 2020. Forthcoming in HOMO OECONOMICUS.

Harold Hotelling. The general welfare in relation to problems of taxation and of railway and utility rates. *Econometrica*, 6(3):242–269, 1938. ISSN 00129682, 14680262. URL http://www.jstor.org/stable/1907054.

J. Hougaard, B. Peleg, and L. Thorlund-Petersen. On the set of lorenz-maximal imputations in the core of a balanced game. *IJGT*, 30: 147–165, 2001. ISSN 0167-6687. doi: https://doi.org/10.1007/s001820100070. URL https://link.springer.com/article/10.1007%2Fs001820100070.

Jens Leth Hougaard and Aleksandrs Smilgins. Risk capital allocation with autonomous subunits: The lorenz set. *Insurance: Mathematics and Economics*, 67:151–157, 2016. ISSN 0167-6687. doi: https://doi.org/10.1016/j.insmatheco.2015.12.002. URL https://www.sciencedirect.com/science/article/pii/S0167668715301712.

Cl. J. Huang and Ph. S. Crooke. *Mathematics and Mathematica for Economists*. Blackwell, Malden, Massachusetts, 1 edition, 1997.

G. Huberman. *The nucleolus and essential coalitions*, volume 28 of *Lecture Notes in Control and Information Sciences*. Elsevier, Amsterdam, 1980.

Y.-A. Hwang and P. Sudhölter. Axiomatizations of the core on the universal domain and other natural domains. *International Journal of Game Theory*, 29:597–623, 2001.

T. Ichiishi. Super-modularity: Applications to convex games and to the greedy algorithm for lp. *Journal of Economic Theory*, 25(2): 283–286, 1981. ISSN 0022-0531. doi: https://doi.org/10.1016/0022-0531(81)90007-7. URL https://www.sciencedirect.com/science/article/pii/0022053181900077.

T. Ichiishi. *Microeconomic Theory*. Cambridge, MA., 1997.

Publisher IEEE. IEEE Standard for Floating-Point Arithmetic. *IEEE Std 754-2019 (Revision of IEEE 754-2008)*, pages 1–84, 2019. doi: 10.1109/IEEESTD.2019.8766229.

E. Iñarra and J.M. Usategui. The Shapley value and average convex games. *International Journal of Game Theory*, 22:13–29, 1993.

A. Isbell. A Class of Majority Games. *Quarterly Journal of Mathematics*, 7:183–187, 1956.

A. Isbell. On the Enumeration of Majority Games. *Mathematical Tables and other Aids to Computation*, 13:21–28, 1959.

Vocabulary ISO/IEC/IEEE. ISO/IEC/IEEE International Standard - Systems and software engineering–Vocabulary. *ISO/IEC/IEEE 24765:2017(E)*, pages 1–541, Aug 2017. doi: 10.1109/IEEESTD.2017.8016712.

G. Jentzsch. Some Thoughts on the Theory of Cooperative Games. In M. Dreshler, L.S. Shapley, and A.W. Tucker, editors, *Advances in Game Theory, Annals of Mathematical Studies*, number 52, pages 407–442. Princeton University Press, Princeton, 1964.

M. Jerison. Russell on gorman's engel curves: A correction. *Economics Letters*, 43(2):171 – 175, 1993. ISSN 0165-1765. doi: https://doi.org/10.1016/0165-1765(93)90032-8. URL http://www.sciencedirect.com/science/article/pii/0165176593900328.

M. Justman. Iterative Processes with 'Nucleolar' Restrictions. *International Journal of Game Theory,*, 6:189–212, 1977.

Ehud Kalai and Eitan Zemel. Generalized network problems yielding totally balanced games. *Operations Research*, 30(5):998–1008, 1982a. doi: 10.1287/opre.30.5.998. URL https://doi.org/10.1287/opre.30.5.998.

Ehud Kalai and Eitan Zemel. Totally balanced games and games of flow. *Mathematics of Operations Research*, 7(3):476–478, 1982b. doi: 10.1287/moor.7.3.476. URL https://doi.org/10.1287/moor.7.3.476.

Anirban Kar, Manipushpak Mitra, and Suresh Mutuswami. On the coincidence of the prenucleolus and the shapley value. *Mathematical Social Sciences*, 57(1):16 – 25, 2009. ISSN 0165-4896. doi: https://doi.org/10.1016/j.mathsocsci.2008.08.004. URL http://www.sciencedirect.com/science/article/pii/S0165489608000851.

K. Kido. A nonlinear Approximation of the Nucleolus. In W. Takahashi and T. Tanaka, editors, *Proceedings of the International Conference on Nonlinear Analysis and Convex Analysis*, pages 307–317, Tokyo, 2004. Yokohama Publishers, Yokohama, Japan.

K. Kido. Convergence Theorems for l_p-Norm Minimizers with respect to p. *Journal of Optimization Theory and Applications*, 125: 577–589, 2005.

K. Kido. A Modified Kohlberg Criterion and a Nonlinear Method to compute the Nucleolus of a Cooperative Game. *Taiwanese Journal of Mathematics*, 12:1581–1590, 2008.

N. L. Kleinberg and J. W. Weiss. Equivalent N-Person Games and the Null Space of the Shapley Value. *Mathematics of Operations Research*, 10(2):233–243, May 1985.

J. Kleppe, H. Reijnierse, and P. Sudhölter. Axiomatizations of Symmetrically Weighted Solutions. Technical Report No. 2013-007, Tilburg University, February 2013.

E. Kohlberg. On the Nucleolus of a Characteristic Function Game. *SIAM Journal of Applied Mathematics*, 20:62–66, 1971.

E. Kohlberg. The Nucleolus as a Solution of a Minimization Problem. *SIAM, Journal of Applied Mathematics*, 23:34–39, 1972.

A Kopelowitz. Computation of the Kernels of Simple Games and the Nucleolus of N-Person Games. Technical report, RM 31, Research Program in Game Theory and Mathematical Economics, The Hebrew University of Jerusalem, 1967. mimeo.

R. W. Krause and A. Caimo. Missing Data Augmentation for Bayesian Exponential Random Multi-Graph Models. volume 221, pages 63–72. International Workshop on Complex Networks, 2019. URL doi:10.21427/PME8-MT48.

Joseph B. Kruskal. On the shortest spanning subtree of a graph and the traveling salesman problem. *Proceedings of the American Mathematical Society*, 7(1):48–50, 1956. ISSN 00029939, 10886826. URL http://www.jstor.org/stable/2033241.

J-J. Laffont and J. Tirole, editors. *A Theory of Incentives in Procurement and Regulation*. The MIT Press, Cambridge, MA, 1993.

A. Lardon. The γ-core in Cournot oligopoly TU-games with capacity constraints. *Theory and Decision*, 72(3):387–411, 2012. ISSN 0040-5833. doi: 10.1007/s11238-011-9256-5. URL http://dx.doi.org/10.1007/s11238-011-9256-5.

P. V. Lekeas. Coalitional beliefs in cournot oligopoly tu games. *International Game Theory Review*, pages 1–21, 2013. doi: 10.1142/S0219198913500047. URL http://dx.doi.org/10.1142/S0219198913500047.

Mingming Leng and Mahmut Parlar. Analytic solution for the nucleolus of a three-player cooperative game. *Naval Research Logistics (NRL)*, 57(7):667–672, 2010. doi: 10.1002/nav.20429. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/nav.20429.

N. Lindfield and J. Penny. *Numerical Methods Using MATLAB*. Prentice Hall, Upper Saddle River, NJ. (USA), 2 edition, 2000.

S. Littlechild. A Simple Expression for the Nucleolus in a Simple Case. *International Journal of Game Theory*, 3:21–29, 1974a.

S. C. Littlechild. A simple expression for the nucleolus in a special case. *International Journal of Game Theory*, 3:21–29, 1974b. URL https://doi.org/10.1007/BF01766216.

S. C. Littlechild and G. Owen. A simple expression for the shapely value in a special case. *Management Science*, 20(3):370–372, 1973. ISSN 00251909, 15265501. URL http://www.jstor.org/stable/2629727.

S. C. Littlechild and G. Owen. A further note on the nucleolus of the airport game. *International Journal of Game Theory*, 5(2):91–95, 1976. URL https://doi.org/10.1007/BF01753311.

S. C. Littlechild and G. F. Thompson. Aircraft landing fees: A game theory approach. *The Bell Journal of Economics*, 8(1):186–204, 1977. ISSN 0361915X. URL http://www.jstor.org/stable/3003493.

S. Lorenzo-Freire. New characterizations of the owen and banzhaf–owen values using the intracoalitional balanced contributions property. *TOP*, 25(3):579–600, 2017. doi: 10.1007/s11750-017-0446-3. URL https://doi.org/10.1007/s11750-017-0446-3.

S. Lorenzo-Freire, J.M. Alonso-Meijide, B. Casas-Méndez, and M.G. Fiestras-Janeiro. Characterizations of the deegan–packel and johnston power indices. *European Journal of Operational Research*, 177(1):431 – 444, 2007. ISSN 0377-2217. doi: https://doi.org/10.1016/j.ejor.2005.08.025. URL http://www.sciencedirect.com/science/article/pii/S0377221705009057.

W. F. Lucas. A Game with no Solution. *Bull. Amer. Math. Soc.*, 74:237–239, 1968.

W. F. Lucas. A Proof that a Game may have not a Solution. *Bull. Amer. Math. Soc.*, 137:219–229, 1969.

W. F. Lucas and M. Rabie. Games with no solutions and empty cores. *Mathematics of Operations Research*, 7(4):491–500, 1982. ISSN 0364765X, 15265471. URL http://www.jstor.org/stable/3689474.

D. Léonard and N. Van Long. *Optimal Control Theory and Static Optimization in Economics*. Cambridge University Press, Cambridge, second reprint edition, 1994.

R. Maeder. *Programming in Mathematica*. Addison Wesley, Reading Massachusetts, 3 edition, 1996.

R. Maeder. *Computer Science with Mathematica*. Cambridge University Press, Cambridge, UK, 1 edition, 2000.

N. G. Mankiw. *"Macroeconomics"*. Worth, New York, 1 edition, 1992.

J-E. Martinez-Legaz. Dual Representation of Cooperative Games based on Fenchel-Moreau Conjugation. *Optimization*, 36:291–319, 1996.

A. Mas-Colell, M. Whinston, and J. Green. *Microeconomic Analysis*. Oxford University Press, Oxford, 1 edition, 1995.

M. Maschler. The Bargaining Set, Kernel and Nucleolus. In R. J. Aumann and S. Hart, editors, *Handbook of Game Theory*, volume 1, chapter 18, pages 591–668. Elsevier Science Publishers, Amsterdam, 1992.

M. Maschler and B. Peleg. A Characterization, Existence Proof and Dimension Bounds for the Kernel of a Game . *Pacific Journal of Mathematics*, 18(2):289–328, 1966.

M. Maschler, B. Peleg, and L.S. Shapley. The Kernel and Bargaining Set for Convex Games. *International Journal of Game Theory*, 1: 73–93, 1972.

M. Maschler, B. Peleg, and L. S. Shapley. Geometric Properties of the Kernel, Nucleolus, and Related Solution Concepts. *Mathematics of Operations Research*, 4:303–338, 1979.

M. Maschler, E. Solan, and Zamir Sh. *Game Theory*. Cambridge University Press, Cambridge, 1 edition, 2013.

MathWorks. *Second-Order Cone Programming Algorithm - MATLAB & Simulink*. MathWorks Inc., Natick, Massachusetts, 2021.

MATLAB. *version 9.13.0 (R2022b)*. The MathWorks Inc., Natick, Massachusetts, 2022.

R. Meessen. Communication Games. Master's thesis, University of Nijmegen, the Netherlands, 1988. in Dutch.

H. Meinhardt. Common Pool Games are Convex Games. *Journal of Public Economic Theory*, 2:247–270, 1999a.

H. Meinhardt. An LP Approach to Compute the Pre-Kernel for Cooperative Games. *Computers and Operations Research*, 33/2:pp 535–557, 2006.

H. I. Meinhardt. Convexity and k-Convexity in Cooperative Common Pool Games. Discussion Paper 11, Institute for Statistics and Economic Theory, University Karlsruhe, Karlsruhe, 1999b.

H. I. Meinhardt. *Cooperative Decision Making in Common Pool Situations*, volume 517 of *Lecture Notes in Economics and Mathematical Systems*. Springer, Heidelberg, 2002.

H. I. Meinhardt. *The Pre-Kernel as a Tractable Solution for Cooperative Games: An Exercise in Algorithmic Game Theory*, volume 45 of *Theory and Decision Library: Series C*. Springer Publisher, Heidelberg/Berlin, 2013. ISBN 978-3-642-39548-2. doi: 10.1007/978-3-642-39549-9. URL https://doi.org/10.1007/978-3-642-39549-9.

H. I. Meinhardt. A Note on the Computation of the Pre-Kernel for Permutation Games. Technical Report MPRA-59365, Karlsruhe Institute of Technology (KIT), May 2014. URL http://mpra.ub.uni-muenchen.de/59365/.

H. I. Meinhardt. The Incorrect Usage of Propositional Logic in Game Theory: The Case of Disproving Oneself. *ArXiv e-prints*, abs/1509.05883, 2015. URL http://arxiv.org/abs/1509.05883.

H. I. Meinhardt. Finding the nucleoli of large cooperative games: A disproof with counter-example. *CoRR*, abs/1603.00226, 2016a. URL http://arxiv.org/abs/1603.00226.

H. I. Meinhardt. The Incorrect Usage of Propositional Logic in Game Theory: The Case of Disproving Oneself. *MPRA*, 75876, 2016b. URL https://mpra.ub.uni-muenchen.de/75876/. Revised Version.

H. I. Meinhardt. Simplifying the Kohlberg Criterion on the Nucleolus: A Disproof by Oneself. Technical Report MPRA-77143, Karlsruhe Institute of Technology (KIT), March 2017a. URL http://mpra.ub.uni-muenchen.de/77143/.

H. I. Meinhardt. Simplifying the Kohlberg Criterion on the Nucleolus: A Correct Approach. *ArXiv e-prints*, 2017b. URL http://arxiv.org/abs/1706.08076.

H. I. Meinhardt. MatRep: A Matlab Toolbox for Representation Theory (Symmetric Groups), 2017c. URL https://de.mathworks.com/matlabcentral/fileexchange/62142-matrep--a-matlab-representation-theory-toolbox--symmetric-groups-?s_tid=prof_contriblnk.

H. I. Meinhardt. A Manual of the Matlab Representation Theory Toolbox MatRep Version 0.1: A Quick Reference. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2017d. URL https://www.researchgate.net/publication/315661547_A_Manual_of_the_Matlab_Representation_Theory_Toolbox_MatRep_Version_01_A_Quick_Reference. Version 0.1.

H. I. Meinhardt. *The Pre-Kernel as a Fair Division Rule for Some Cooperative Game Models*, pages 235–266. Springer International Publishing, Cham, 2018a. ISBN 978-3-319-61603-2. doi: 10.1007/978-3-319-61603-2_11. URL https://doi.org/10.1007/978-3-319-61603-2_11.

H. I. Meinhardt. The Modiclus Reconsidered. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2018b. URL http://dx.doi.org/10.13140/RG.2.2.32651.75043.

H. I. Meinhardt. Reconsidering Related Solutions of the Modiclus. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2018c. URL http://dx.doi.org/10.13140/RG.2.2.27739.82729.

H. I. Meinhardt. On the Replication of the Pre-Kernel and Related Solutions. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2018d. URL http://dx.doi.org/10.13140/RG.2.2.21053.10725.

H. I. Meinhardt. Deduction Theorem: The Problematic Nature of Common Practice in Game Theory. *arXiv e-prints*, art. arXiv:1908.00409, Aug 2021a. URL https://arxiv.org/abs/1908.00409v2.

H. I. Meinhardt. Disentangle the Florentine Families Network by the Pre-Kernel. Technical report, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany, 2021b.

H. I. Meinhardt. TuGames: A Mathematica Package for Cooperative Game Theory, 2022a. URL https://github.com/himeinhardt/TuGames.

H. I. Meinhardt. MatTuGames: A Matlab Toolbox for Cooperative Game Theory, 2022b. URL http://www.mathworks.com/matlabcentral/fileexchange/74092-mattugames.

A. Meseguer-Artola. Using the Indirect Function to characterize the Kernel of a TU-Game. Technical report, Departament d'Economia i d'Història Econòmica, Universitat Autònoma de Barcelona, Nov. 1997. mimeo.

MOSEK. The MOSEK optimization toolbox for MATLAB manual. Version 8.0. Technical report, MOSEK ApS, Fruebjergvej 3, Boks 16, 2100 Copenhagen O, 2017. URL http://docs.mosek.com/8.0/toolbox/index.html.

H. Moulin. Deterrence and cooperation: A classification of two person games. *European Economic Review*, 15:179–193, 1981.

H. Moulin. *Game Theory for the Social Sciences*. New York University Press, New York, 2 edition, 1986.

H. Moulin. *Axioms of Cooperative Decision Making*. Econometric Society Monographs No. 15. Cambridge University Press, Cambridge, 1988.

H. Moulin. *Fair Division and Collective Welfare*. MIT Press, Cambridge, Massachusetts, 2003.

D. Mueller. *Investitionsrechnung und Investitionscontrolling*. Springer-Lehrbuch. Springer Publisher, Heidelberg, 2 edition, 2018.

D. Mueller. *Investitionscontrolling: Entscheidungsfindung bei Investitionen II*. Springer-Lehrbuch. Springer Publisher, Heidelberg, 3 edition, 2022.

R. B. Myerson. Graphs and Cooperation in Games. *Mathematics of Operations Research*, 2(3):225–229, August 1977.

R. B. Myerson. Conference Structures and Fair Allocation Rules. *International Journal of Game Theory*, 9(3):169–182, 1980.

R. B. Myerson. *Game Theory: Analysis of Conflict*. Harvard University Press, Cambridge, Massachusetts, London, England, 1991.

J. F. Nash. Noncooperative Games. *Annals of Mathematics*, 54:289–295, 1951.

H. Norde, K. H. Pham Do, and S. Tijs. Oligopoly Games with and without Transferable Technologies. *Mathematical Social Sciences*, 43:187–207, 2002.

A. S. Nowak and T. Radzik. A Solidarity Value for n-Person Transferable Utility Games. *International Journal of Game Theory*, 23: 43–48, 1994.

B. O'Neill. A Problem of Rights Arbitration from the Talmud. *Mathematical Social Sciences*, 2:345–371, 1982.

G. Orshan. The Prenucleolus and the Reduced Game Property: Equal Treatment Replaces Anonymity. *International Journal of Game Theory*, 22:241–248, 1993.

G. Orshan and P. Sudhölter. Reconfirming the Prenucleolus. *Mathematics of Operations Research*, 28:283–293, 2003.

A. Ostmann. Classifying Three-Person Games. Technical Report 140, University of Bielefeld, 1984.

A. Ostmann. On the Minimal Representation of Homogeneous Games. *International Journal of Game Theory*, 16(1):69–81, 1987.

A. Ostmann. Limits of Rational Behavior in Cooperatively Played Normalform Games. In R. Tietz, W. Albers, and R. Selten, editors, *Bounded Rational Behavior in Experimental Games and Markets, Lecture Notes in Economics and Mathematical Systems 314*, pages 317–332, Berlin, 1988. Springer-Verlag.

A. Ostmann. A Note on Cooperation in Symmetric Common Dilemmas. mimeo, 1994.

A. Ostmann and H. I. Meinhardt. Non-Binding Agreements and Fairness in Commons Dilemma Games. *Central European Journal of Operations Research*, 15(1):63–96, 2007.

A. Ostmann and H. I. Meinhardt. Toward an Analysis of Cooperation and Fairness That Includes Concepts of Cooperative Game Theory. In A. Biel, D. Eek, T. Garling, and M. Gustafson, editors, *New Issues and Paradigms in Research on Social Dilemmas*, pages 230–251. Springer-Verlag, 2008.

J. Antonio Rivero Ostoic. Compositional equivalence with actor attributes: Positional analysis of the the florentine families network. *CoRR*, abs/1804.09427, 2018. URL http://arxiv.org/abs/1804.09427.

G. Owen. A note on the nucleolus. *International Journal of Game Theory*, 3(2):101–103, 1974. URL https://doi.org/10.1007/BF01766395.

G. Owen. Values of Games with a priori Unions. In R. Henn and O. Moeschlin, editors, *Essays in Mathematical Economics and Game Theory*, pages 76–88. Springer-Verlag, New York, 1977.

G. Owen. *Game Theory*. Academic Press, San Diego, 3 edition, 1995.

J.F. Padgett and C.K. Ansell. Robust Action and the Rise of the Medici, 1400-1434. *American Journal of Sociology*, 98:1259–1319, 1993.

John F Padgett. Open elite? social mobility, marriage, and family in florence, 1282–1494. *Renaissance Quarterly*, 63(2):357–411, 2010. ISSN 00344338, 19350236. URL http://www.jstor.org/stable/10.1086/655230.

B. Peleg. On the kernel of constant-sum simple games with homogeneous weights. *Illinois J. Math.*, 10(1):39–48, 03 1966. doi: 10.1215/ijm/1256055199. URL https://doi.org/10.1215/ijm/1256055199.

B. Peleg. On the reduced Game Property and its converse. *International Journal of Game Theory*, 15:187–200, 1986.

B. Peleg. On the reduced Game Property and its converse. A correction. *International Journal of Game Theory*, 16:290, 1987.

B. Peleg and P. Sudhölter. *Introduction to the Theory of Cooperative Games*, volume 34 of *Theory and Decision Library: Series C*. Springer-Verlag, Heidelberg, 2 edition, 2007.

B. Peleg, J. Rosenmüller, and P. Sudhölter. The kernel of homogeneous games with steps. In N. Megiddo, editor, *Essays in Game Theory in Honor of Michael Maschler*, pages 163–192, Heidelberg, 1994. Springer Publisher.

Bezalel Peleg and Joachim Rosenmüller. The least core, nucleolus, and kernel of homogeneous weighted majority games. *Games and Economic Behavior*, 4(4):588–605, 1992. ISSN 0899-8256. doi: https://doi.org/10.1016/0899-8256(92)90039-U. URL https://www.sciencedirect.com/science/article/pii/089982569290039U.

H. Peters. *Game Theory: A Multi-Leveled Approach*. Springer-Verlag, Heidelberg, 1 edition, 2008.

H. Peters. *Game Theory: A Multi-Leveled Approach*. Springer-Verlag, Heidelberg, 2 edition, 2015.

N. Ploskas and N. Samaras. *GPU Programming in MATLAB*. Morgan Kaufmann/Elsevier, Cambridge, MA. (USA), 1 edition, 2016.

N. Ploskas and N. Samaras. *Linear Programming Using MATLAB*, volume 127 of *Springer Optimization and its Applications*. Springer, Heidelberg, Germany, 1 edition, 2018.

J. A. M. Potters, H. Reijnierse, and M. Ansing. Computing the Nucleolus by Solving a Prolonged Simplex Algorithm. *Mathematics of Operations Research*, 21(3):757–768, 1996.

R. C. Prim. Shortest connection networks and some generalizations. *The Bell System Technical Journal*, 36(6):1389–1401, 1957. doi: 10.1002/j.1538-7305.1957.tb01515.x.

T.E.S. Raghavan and Sudhölter. The Modiclus and Core Stability. *International Journal of Game Theory*, 33:467–478, 2005. URL https://doi.org/10.1007/s00182-005-0207-7.

H. Reijnierse and J. Potters. The $\mathscr{B}$-Nucleolus of TU-Games. *Games and Economic Behaviour*, 24:77–96, 1998.

M. Richardson. On finite projective games. *Proceedings of the American Mathematical Society*, 7(3):458–465, 1956. doi: 10.1090/S0002-9939-1956-0079543-2. URL https://doi.org/10.1090/S0002-9939-1956-0079543-2.

R. Rockafellar. *Convex Analysis*. Princeton University Press, Princeton, 1970.

J. Rosenmüller. *The Theory of Games and Markets*. North-Holland, Amsterdam, 1981.

J. Rosenmüller. Weighted majority games and the matrix of homogeneity. *Zeitschrift für Operations Research*, 28(5):123 – 141, 1984. doi: 10.1007/BF01920914. URL https://doi.org/10.1007/BF01920914.

J. Rosenmüller. Homogeneous games: Recursive structure and computation. *Mathematics of Operations Research*, 12(2):309–330, 1987. URL https://www.jstor.org/stable/3689693.

J. Rosenmüller and P. Sudhölter. Cartels via the modiclus. *Discrete Applied Mathematics*, 134(1):263 – 302, 2004. ISSN 0166-218X. doi: https://doi.org/10.1016/S0166-218X(03)00227-0. URL http://www.sciencedirect.com/science/article/pii/S0166218X03002270.

L. M. Ruiz, F. Valenciano, and J. M. Zarzuelo. The Least Square Pre-Nucleolus and the Least Square Nucleolus. Two Values for TU Games based on the Excess Vector. *International Journal of Game Theory*, 25:113–134, 1996.

T. Russell. On a theorem of gorman. *Economics Letters*, 11(3):223 – 224, 1983. ISSN 0165-1765. doi: https://doi.org/10.1016/0165-1765(83)90139-8. URL http://www.sciencedirect.com/science/article/pii/0165176583901398.

T. Russell. Gorman demand systems and Lie transformation groups: A reply. *Economics Letters*, 51(2):201 – 204, 1996. ISSN 0165-1765. doi: https://doi.org/10.1016/0165-1765(96)00806-3. URL http://www.sciencedirect.com/science/article/pii/0165176596008063.

T. Russell and F. Farris. The geometric structure of some systems of demand equations. *Journal of Mathematical Economics*, 22(4):309 – 325, 1993. ISSN 0304-4068. doi: https://doi.org/10.1016/0304-4068(93)90020-L. URL http://www.sciencedirect.com/science/article/pii/030440689390020L.

T. Russell and F. Farris. Integrability, Gorman systems, and the Lie bracket structure of the real line. *Journal of Mathematical Economics*, 29(2):183 – 209, 1998. ISSN 0304-4068. doi: https://doi.org/10.1016/S0304-4068(97)00806-9. URL http://www.sciencedirect.com/science/article/pii/S0304406897008069.

B. E. Sagan. *"The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions"*, volume 203 of *Graduate Texts in Mathematics*. Springer Publisher, Heidelberg, 2 edition, 2000.

N. Schlömer. Matlab2Tikz. Technical report, GitHub, 2022. URL https://github.com/matlab2tikz/matlab2tikz. Version 2022.

D. Schmeidler. The Nucleolus of a Characteristic Function Game. *SIAM, Journal of Applied Mathematics*, 17:1163–1170, 1969.

D. Schoch. netrankr: Analyzing Partial Rankings in Networks. Technical report, Manchester, UK, 2020a. URL https://cran.r-project.org/web/packages/netrankr/index.html. Version 0.3.

D. Schoch. networkdata: Package of Different Network Datasets. Technical report, Manchester, UK, 2020b. URL https://github.com/schochastics/networkdata. Version 0.1.

R. W. Sebesta. *Concepts of Programming Languages*. Pearson, 11 edition, 2016.

L. S. Shapley. A Value of N-Person Games. In H. W. Kuhn and A. W. Tucker, editors, *Contributions to the Theory of Games II*, volume 28, pages 307–317, Princeton, NJ., 1953. Annals of Mathematical Studies, Princeton University Press.

L. S. Shapley. On Balanced Sets and Cores. *Naval Research Logistics Quarterly*, 14:453–460, 1967.

L. S. Shapley. Cores of Convex Games,. *International Journal of Game Theory*, 1:11–26, 1971.

L. S. Shapley and M. Shubik. A method for evaluating the distribution of power in a committee system. *American Political Science Review*, 48:787–792, 1954.

L. S. Shapley and M. Shubik. Quasi-cores in a monetary economy with nonconvex preferences. *Econometrica*, 34(4):805–827, 1966. ISSN 00129682, 14680262. URL http://www.jstor.org/stable/1910101.

L. S. Shapley and M Shubik. On Market Games. *Journal of Economic Theory*, 1:9–25, 1969.

L. S. Shapley and M Shubik. An Assignment Game I: The Core. *International Journal of Game Theory*, 2:111–130, 1972.

M. Shubik. *"Market structure and behavior"*. Harvard University Press, Cambridge, 1980.

A. J. Sobolev. The characterization of optimality principles in cooperative games by functional equations. In N. N. Vorobjev, editor, *Matematicheskie Metody v Sotsial'nykh Naukakh, Proceedings of the Seminar*, pages 94–151, Vilnius, 1975. Institute of Physics and Mathematics, Academy of Sciences of the Lithuanian SSR. (in Russian, English summary).

T. Solymosi. The kernel is in the least core for permutation games. *Central European Journal of Operations Research*, pages 1–15, March 2014. ISSN 1435-246X. doi: 10.1007/s10100-014-0342-y. URL http://dx.doi.org/10.1007/s10100-014-0342-y.

T. Solymosi and TES Raghavan. An algorithm for finding the nucleolus of assignment games. *International Journal of Game Theory*, 6:94–151, 1994.

T. Solymosi and B. Sziklai. Universal Characterization Sets for the Nucleolus in Balanced Games. Technical Report MT-DP – 2015/12, Corvinus University of Budapest, 2015.

Tamás Solymosi and Balázs Sziklai. Characterization sets for the nucleolus in balanced games. *Operations Research Letters*, 44(4): 520 – 524, 2016. ISSN 0167-6377. doi: http://dx.doi.org/10.1016/j.orl.2016.05.014. URL http://www.sciencedirect.com/science/article/pii/S0167637716300414.

R.E. Stearns. Convergent Transfer Schemes for N-Person Games. *Transaction of the American Mathematical Society*, 134:449–459, 1968.

P. Sudhölter. Star-Shapedness of the Kernel of Homogeneous Games. *Mathematical Social Sciences*, 32:179–214, 1996.

P. Sudhölter. The modified nucleolus: Properties and axiomatizations. *International Journal of Game Theory*, 26(2):147–182, Jun 1997a. ISSN 1432-1270. doi: 10.1007/BF01295846. URL https://doi.org/10.1007/BF01295846.

P. Sudhölter. Nonlinear Self Dual Solutions for TU-Games. In T. Parthasarathy, B. Dutta, J.A.M. Potters, T.E.S. Raghavan, D. Ray, and Sen A., editors, *Game Theoretical Applications to Economics and Operations Research*, volume 18 of *Theory and Decision Library: Series C*, pages 33–50, Boston, MA, 1997b. Springer.

P. Sudhölter. *The Modified Nucleolus of a Cooperative Game*. Habilitation thesis, University of Bielefeld, Bielefeld, 1993.

P. Sudhölter. The modified nucleolus as canonical representation of weighted majority games. *Mathematics of Operations Research*, 21 (3):734–756, 1996. ISSN 0166-218X. URL http://www.jstor.org/stable/3690307.

J.G. Sánchez Léon. *Mathematica beyond Mathematics: The Wolfram Language in the Real World*. CRC Press, Boca Raton, US., 2017.

R. M. Thrall and W. F. Lucas. N-person games in partition function form. *Naval Research Logistics*, pages 281–298, 1963. doi: 10.1002/nav.3800100126.

S. H. Tijs. Bounds on the Core and the τ-value. In O. Moeschlin and D. Pallaschke, editors, *Game Theory and Mathematical Economics*, pages 123–132. North-Holland, Amsterdam, 1981.

J. Tirole, editor. *The Theory of Industrial Organization*. The MIT Press, Cambridge, MA, 1988.

D. M. Topkis. *Supermodularity and Complementarity*. Frontiers of Economic Research. Princeton University Press, Princeton, New Jersey, 1998.

E. van Damme. *Stability and Perfection of Nash Equilibria*. Springer-Verlag, Heidelberg/Berlin, 2 edition, 1991.

A. van den Nouveland, P. Borm, and SH. Tijs. Allocation Rules for Hypergraph Communication Situations. *International Journal of Game Theory*, 20:255–268, 1992.

H. R. Varian. *Microeconomic Theory*. Norton, New York, 3 edition, 1992.

H. R. Varian, editor. *Economic and Financial Modeling with Mathematica*. Telos/Springer, Santa Clara, CA, 1993.

V.A. Vasil'ev. The Shapley value for cooperative games of bounded polynomial variation. *Optimazacija Vyp*, 17:5–27, 1975. in Russian.

V.A. Vasil'ev. On a class of imputations in cooperative games. *Soviet Mathematics Dokladi*, 23:53–57, 1981. in Russian.

X. Vives. *Oligopoly Pricing: Old Ideas and New Tools*. MIT Press, Cambridge, Massachusetts, 1999.

J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, 1944.

Ing. Walter. *Die Strozzi: Eine Familie im Florenz der Renaissance*. Verlag C.H.Beck, München, Germany, 2011.

R. J. Weber. *Probabilistic Values for Games*, pages 101–119. Cambridge University Press, Cambridge, 1988.

R. Webster. *Convexity*. Oxford University Press, Oxford, 1994.

P. Wellin. *Programming with Mathematica: An Introduction*. Cambridge University Press, Cambridge, UK., 2013.

P. Wellin. *Essentials of Programming in Mathematica*. Cambridge University Press, Cambridge, UK., 2016.

Inc. Wolfram Research. Mathematica Edition: Version 13.1. Technical report, Wolfram Research, Inc., Champaign, Illinois, 2022.

H. Wolsey. The Nucleolus and Kernel for Simple Games or Special Valid Inequalities for 0-1 Linear Integer Programs. *International Journal of Game Theory*, 5(4):227–238, 1976.

D. Xue and Y. Chen. *Solving Applied Mathematical Problems with MATLAB*. CRC Press, 2010.

W. Y. Yang, W. Cao, T-S. Chung, and J. Morris. *"Applied Numerical Methods Using Matlab"*. John Wiley & Sons, Inc, 2005.

K. Yokote, Y. Funaki, and Y. Kamijo. Linear Basis Approach to the Shapley Value. Working Paper 1303, Waseda University, 2013.

E. Zeidler, W. Hackbusch, and H. R. Schwarz. *Teubner-Taschenbuch der Mathematik*. Vieweg & Teubner„ Stuttgart/Leipzig, 1996.

J. Zhao. β-Core Existence Result and its Application to Oligopoly Markets. *Games and Economic Behavior*, 27:153–168, 1999a.

J. Zhao. A Necessary and Sufficient Condition for the Convexity in Oligopoly Games. *Mathematical Social Sciences*, 37:189–204, 1999b.

J. Zhao. The existence of TU α-core in normal form games. *International Journal of Game Theory*, 28:25–34, 1999c.

J. Zhao. The relative interior of Base Polyhedron and the Core. *Economic Theory*, 18:635–648, 2001.

J. Zhao. Estimating Merging Costs by Merger Preconditions. *Theory and Decision*, 66:373–399, 2009.

J. Zhao. Merger Profitability in All Linear Oligopolies with Four Firms. Discussion paper, Department of Economics, University of Saskatchewan, Saskatoon, 2016. mimeo.

J. Zhao. TU Oligopoly Games and Industrial Cooperation. In Corchon and Marini, editors, *Handbook of Game Theory and Industrial Organization*, volume I, page 392–422. Edward Elgar Publisher, Cheltenham, 2018.

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