Market Equilibria

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Market Equillibria

- Walras Model
  - Initial Endowment of divisible goods
  - Utility Function for Consuming goods
  - Every person sells the initial endowment and then buys an optimal bundle of goods with the entire revenue i.e., the market clears
- Walras asked whether a price can be assigned to every good so that this is possible.
- Arrow –Debreu Theorem
- Fisher Model – Two kinds of People , Producers and Consumers
  - Consumers have money and utility function for goods
  - Producers have initial endowment of goods and want to earn money
  - Special Case of Walras‘ Model

Models: Walras vs Fisher

- Walras model - money has no intrinsic value - a scale to measure the value of goods
- Fisher model assumes the value of the money.
- This feedback feature of the Walras model

Mathematical Model

- N people in the system,with initial endowment of divisible goods
- Without loss of generality
  - Each person has a single good
  - Quantity of each good is 1
- Further,
  - Each good has atleast someone interested in it
  - Each person is interested in atleast one good
- With a little loss of generality
  - Each subset of persons has atleast one person outside the set who is interested in a good possessed by a person in the subset.

Non-zero liking graph

- Arrow-Debreu allows for 0-priced goods
- Consider the *non-zero liking graph*
  - If there exists set of goods in whom others are not interested, then it will not be a single SCC
  - Solve the problem for each SCC
  - In the acyclic components of the SCCs, scale the prices appropriately so that people will buy only goods from their own SCC
  - Prices can be set to 0 only for 'initial' SCCs.

Characterization – a non-convex program

- *The feasible region of non-convex program 1 has all and only general market equilibria.*

$$
\begin{aligned}
&\forall j: \ \sum_i x_{ij} = 1 \\
&\forall i,j: \ x_{ij} \geq 0 \\
&\forall i,j: \ \frac{u_{ij}}{p_j} \leq \frac{\sum_k u_{ik} x_{ik}}{p_i} \\
&\forall i: \ p_i > 0
\end{aligned} \tag{1}
$$

Implications

- Prices are market-clearing
- Money is spent optimally

Convex Program 1

- Non-Convex program feasible if and only of No Negative Cycle in the non-zero liking graph

$$\forall j: \sum_i x_{ij} = 1$$

$$\forall i,j: x_{ij} \geq 0$$

$$\text{For every cycle, } C\text{, of } G: \prod_{ij \in C} w(ij) \geq 1$$

$$w(ij) = \frac{\sum_k u_{ik} x_{ik}}{u_{ij}}$$

Convex Program 2

- Non convex program 1 is equivalent to Convex program 2.
- The set of equilibria prices, on a logarithmic scale, is convex.

$$\forall j: \sum_i x_{ij} = 1$$

$$\forall i, j: x_{ij} \geq 0$$

$$\forall i, j \text{ such that } u_{ij} > 0:$$

$$LOGp_i - LOGp_j \leq \log\left(\frac{\sum_k u_{ik} x_{ik}}{u_{ij}}\right)$$

Toward Combinatorial Algorithms

- Combinatorial characterization - No Negative Cycle
- Passive Characterization – doesn't tell us how to fix it
- Active characterization in Fisher's model through Eisenberg-Gale's LP

Possible directions

- Can we discretize our search space?
  - Looking for an objective function whose optimal solutions are market equilibria
- Can we move from price vector to optimal allocation vector and vice-versa easily?

References

K.Jain. A Polynomial Time Algorithm for Computing an Arrow-Debreu Market Equilibrium for Linear Utilities