The Congestion Option

July 14, 2012

Abstract

Network Congestion is a negative externality which affects each player involved with the modern multi-party ecosystem of the Internet. In this paper, we develop an option-like financial instrument, called the 'Congestion Option' for the Internet Economy consisting of Content Providers (CPs) and the Internet Service Providers (ISPs). Conventional Option Pricing literature involves the price of protection against risk involved with the economic value of the asset over a period of time T.In our model, the CPs compete for a last-mile bottleneck capactity so as to service their users, and thus run the risk of losing economic utility due to congestion uncertainty.

The Model:

We consider a model where n CPs are competing for a last-mile bottleneck capacity, or network bandwidth, denoted by μ.For simplicty, assume a single ISP for now. Each CP i expresses the amount of bandwidth it wants to receive by submitting a 'bid' b_i.

Assumption 1:The congestion level depend on time in so far as the CP's bid values depend on time. The same bid values at different points of time should correspond to the same level of congestion.

Assumption 2: The bid value for CP i at time t, $b_i(t)$ is a random variable following a probability distribution. I suggest a $N(\mu_i(t), \sigma_i(t))$ probability distribution, where $\mu_i(t)$ and $\sigma_i(t)$ are known beforehand, possibly through statistical techniques.Intuitively, the bid value $b_i(t)$ corresponds to CP i's throughput requirements (perceived need/demand) at time t.

Let the congestion level be measured by the function $\phi(\mathbf{b})$, where $\mathbf{b} = (b_1, b_2, ..., b_n)$. Thus, $\mathbf{b}$ is a random vector, called the bid vector. ϕ can be any function such that it is nondecreasing in $\mathbf{b}$, for example we can take $\phi = \begin{cases} 0 & \sum b_i < \mu \\ (\sum b_i/\mu) - 1 & \sum b_i \geqslant \mu \end{cases}$, where μ is the bandwidth

This gives us the stochastic process for congestion, the congestion process, $\phi(t) = \phi(b_1(t), b_2(t), ..., b_n(t)) = \phi(\mathbf{b(t)})$

When the bid vector is $\mathbf{b} = (b_1, b_2, ..., b_n)$, let the congestion be$\varphi =$$\phi(b_1, b_2, ..., b_n)$, and the achievable/allocated throughput rate under congestion equillibrium be

$x_i(\varphi, b_i)$. The function $x_i(\varphi, b_i)$ should be non-increasing in φ for a fixed demand, and non-decreasing in b_i for a fixed congestion level. For example, the allocation might be $x_i = \frac{b_i}{\sum b_j} * \mu$.

We assume the Utility function for CP i to be $u_i = v_i x_i$, and thus $u_i(\varphi, b_i) = v_i x_i(\varphi, b_i)$ and $u_i(t) = v_i x_i(\phi(t), b_i(t))$. (Later we can generalise our results to a larger class of utility functions)

The 'risk' in our model is that the all the bid values might take large values simultaneously i.e the throughput need of all the CPs being large at the same time. In this case the allocated throughput rate x_i, and consequently the obtained utility for CP i i.e. u_i, will be very less.Through the congestion option, we seek to transfer this risk from the buyer of the option, CP i , to the ISP.

The option:

If the congestion is high, the CP i runs the risk of suffering an economic loss. Under the contract, the ISP would like to ensure the CP of a threshold value of congestion at time t, say $\hat{\varphi}_t$. If the congestion at time t is higher than this value, $\phi(t) > \hat{\varphi}_t$, then the CP will 'excercise' the contract, and the ISP will pay some 'compensation'.

Note that the utility for CP i , u_i, depends on:

1. the congestion level - φ, and
2. the throughput demand - b_i.

Assumption 3: The ISP should compensate for the CP i's loss in utility due to high congestion, and not due to loss in utility due to low throughput demand for CP i's content.

Under the contract, the payoff for CP i will be:

$P_i(t) = \begin{cases} u_i(\phi(t), b_i(t)) & \phi(t) < \hat{\varphi}_t \\ u_i(\hat{\varphi}_t, b_i(t)) & \phi(t) \geqslant \hat{\varphi}_t \end{cases}$ (Note that the threshold congestion $\hat{\varphi}_t$ is similar in flavour to the Strike Price K in Option Pricing literature)

Thus, the Option payoff, or value of the Congestion Option at time t, $V_i(t)$, is given by:

$V_i(t) = \begin{cases} 0 & \phi(t) < \hat{\varphi}_t \\ u_i(\hat{\varphi}_t, b_i(t)) - u_i(\phi(t), b_i(t)) & \phi(t) \geqslant \hat{\varphi}_t \end{cases}$, or in this case:

$V_i(t) = \begin{cases} 0 & \phi(t) < \hat{\varphi}_t \\ v_i[x_i(\hat{\varphi}_t, b_i(t)) - x_i(\phi(t), b_i(t))] & \phi(t) \geqslant \hat{\varphi}_t \end{cases}$ (Note that since the utility function is non-increasing in the congestion level at the same demand, the value of the Congestion Option is always non-negative)

The Pricing:

For pricing should be fair, the Congestion Option Price, p_i should be equal to the expected future value of the option contract:

$p_i(t) = E[V_i(t)]$, where the expectation is taken over all possible values of congestion $\varphi = \phi(t) = \phi(\mathbf{b(t)})$, and over all possible values of $b_i(t)$. Since the distribution of the bids, and thus of the random vector $\mathbf{b(t)}$ is known, we can calculate this expectation.

Note that:

1. The Value and Price of the Congestion Option for CP i at time t depends on the threshold congestion value $\hat{\varphi_t}$ too. If $\hat{\varphi}_{t,1} > \hat{\varphi}_{t,2}$, $V_i(t, \hat{\varphi_{t,1}}) \leqslant V_i(t, \hat{\varphi_{t,2}})$ for every value of $b_i(t)$ and φ, and thus the $p_i(t, \hat{\varphi}_{t,1}) \leqslant p_i(t, \hat{\varphi}_{t,2})$.In particular, if $\hat{\varphi_t} \geqslant max\{\phi(t)\}$, then $p_i = 0$, as should be the case.

2. The compensation that the ISP pays to the CP i at a particular congestion level depends on the demand for CP i's content as well. If the demand is low, the compensation is low. If the demand is high, the compensation is high.

3. Though the compensation that the ISP pays to the CP i depends on the throughput demand at that time, the price of the option is independent of it, and depend only on its distribution. This is because the expectation is taken over all values of $b_i(t)$.

4. The Price of the Congestion Option is different for different Content Providers.

5. The risk in the model does not arise due to uncertainty associated with time. This means our uncertainty or 'risk' doesnt grow with time.This is a fundamental difference from traditional Option Pricing . Thus, increasing the value of time will not necessary increase the price of the option.

Possible shortcomings :

1. We have not assumed any relation between the bid values at successive time intervals. In particular, our stochastic process $\phi(t)$ is not continuous, and might have arbitrary jumps. This might or might not be realistic.

2. The Model requires the ISPs to know or have an estimation of the Content Provider's revenue model.In practice, this might not be obtainable, or might be difficult to ascertain.

3. We have not considered the time value of money, or have not discounted the future expected payoff back to present time, as is done in conventional Option Pricing. I'm not sure how this affects the scenario.