Algorithm for Optimal Chance Constrained Knapsack with Applications to Multi-robot Teaming

Stony Brook University, New York, USA

Fan Yang, Nilanjan Chakraborty, {fan.yang.3, nilanjan.chakraborty}@stonybrook.edu

Stony Brook University

Introduction

We consider a multi-robot teaming problem.

- A route with known distance (L in Eq.1) need to be traversed for patrolling or environmental sensing (see Fig.1).
- A set of available robots with known operation costs (c_i in Eq.1) and uncertain travel distance capacity (l_i in Eq.1) in one battery charge cycle, which are random variables.
- The goal is to find a team of robots from the given set to cover the route with a pre-specified probability (p in Eq.1) while the total operation cost is minimized.

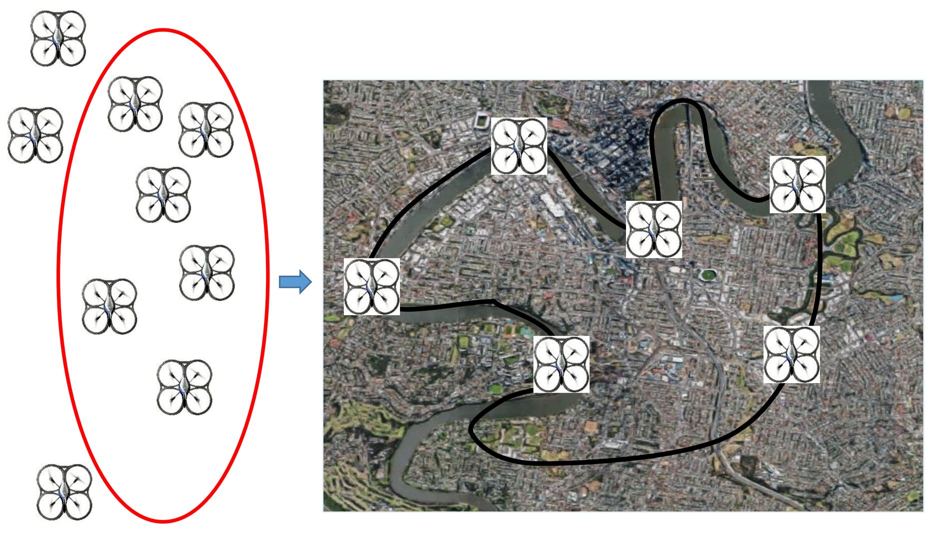

Fig. 1. A route with given length should be traversed by robots with random travel distances. The goal is to select a team of robots from the given set to minimize the total cost such that the route is traversed with probabilistic guarantee.

Chance-Constrained Knapsack

The team selection problem with uncertain travel distance capacity is a chance-constrained knapsack problem (CC-KAP):

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} c_i f_i \\ \text{s.t.} \quad & \mathbb{P}\left(\sum_{i=1}^{n} \ell_i f_i \geq L\right) \geq p \qquad (1) \\ & f_i \in \{0,1\}, \quad \forall i = 1, \ldots, n \end{aligned}$$

- The solution is a vector where each entry f_i is a binary decision variable indicating that a robot is selected when $f_i = 1$.
- The chance constraint guarantees that under any realization of the random travel distance, the selected team covers the route with a pre-specified probability p.
- The chance constraint is equivalent to a deterministic constraint $\sum_i \mu_i f_i - C\sqrt{\sum_i \sigma_i^2 f_i} \geq L$ where C is a constant determined by p.

Risk-Averse Knapsack Problem

It is difficult to solve the CC-KAP directly. Instead we solve a sequence of deterministic problems, called risk-averse knapsack problem (RA-KAP).

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} c_i f_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} \mu_i f_i - \lambda \sum_{i=1}^{n} \sigma_i^2 f_i \geq L' \qquad (2) \\ & f_i \in \{0,1\}, \quad \forall i = 1, \ldots, n \end{aligned}$$

In RA-KAP the travel distances of robots are linear combinations of means (μ_i) and variances (σ_i^2), λ is risk-averse parameter and the route length is L'.

Geometric Interpretation

- The problem is analyzed on a 2D variance-mean plane (see Fig.2) where x-axis, y-axis represent the sum of variances and means of the selected robots respectively.
- Any robot team can be represented as a point on variance-mean plane.
- The optimal solution is the point in the non-convex feasible region (above blue parabola in Fig.2a) with the highest objective value.

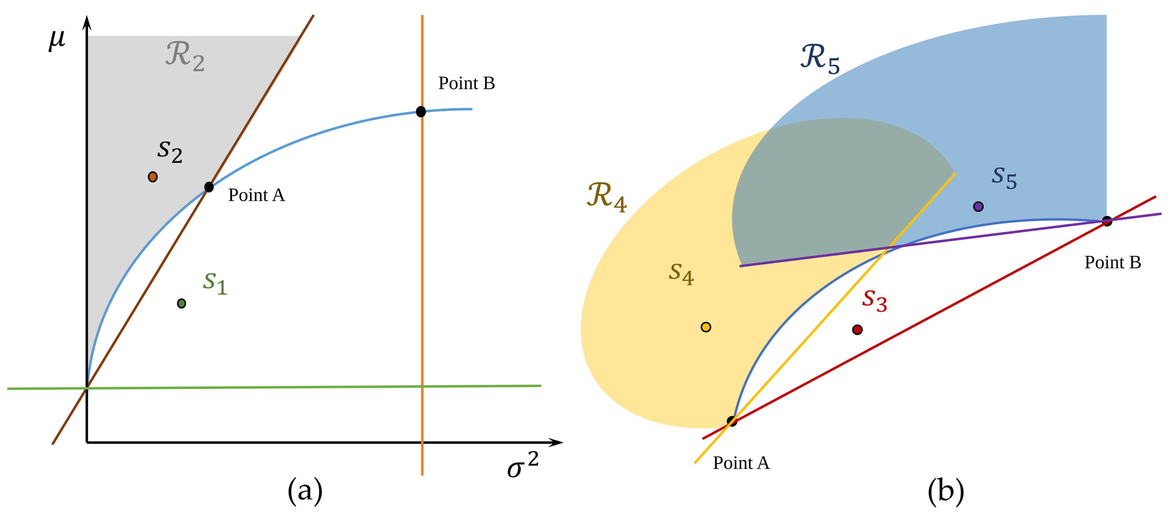

Fig. 2. Illustration of our two-step algorithm. The feasible region of CC-KAP ($\mathcal{R}$) is the non-convex space above the parabola. Fig.(a) illustrates the first step, where we find a feasible solution of CC-KAP (s_2) which is optimal in the intersection region $\mathcal{R}_2$. Fig.(b) zooms in the remaining feasible region $\mathcal{R}\backslash\mathcal{R}_2$. It illustrates the second step, where we find several feasible solutions, (s_4, s_5) that are optimal in the intersection region of CC-KAP and the corresponding RA-KAP, ($\mathcal{R}_4$, $\mathcal{R}_5$) such that the union of those intersection regions and the region in the first step, $\mathcal{R}_2$ covers the feasible region of CC-KAP, e.g. $\mathcal{R} \subseteq \cup_{i\in\{2,4,5\}} \mathcal{R}_i$.

Relationship

Observations on the relationship between CC-KAP and RA-KAP help us to find the optimal solution in non-convex region.

- The optimal solution of CC-KAP is the optimal solution of a RA-KAP with appropriate choice of (λ, L').
- Thus, the problem is converted to a two-dimensional search (λ, L') on variance-mean plane.

Algorithm

We develop a two-step algorithm to solve the CC-KAP optimally:

1. Solve a sequence of RA-KAPs by methodically increasing λ that controls the slope of the straight line until the optimal solution of a RA-KAP (s_2 in Fig. 2a) is feasible to CC-KAP.
2. Solve RA-KAPs by methodically changing parameters (λ, L') that control the slope and y-intercept of the straight line. If the optimal solution of a RA-KAP is feasible to CC-KAP, we obtain a solution that is optimal in the intersection of feasible regions of CC-KAP and RA-KAP, e.g., $\mathcal{R}_4$, $\mathcal{R}_5$ in Fig.2b.

The *procedure terminates* when the intersection regions corresponding to those feasible solutions, e.g., $\mathcal{R}_4$, $\mathcal{R}_5$, cover the remaining feasible region of CC-KAP at the end of the first step, i.e., $\mathcal{R}\backslash\mathcal{R}_2$, in Fig.2a.

Simulation Results

Simulations are performed based on randomly generated means and variances for travel distance capacity of each robot. We count the number of RA-KAPs solved which influence the efficiency and scalability of our algorithm.

- Fig. 3 shows the scalability of our algorithm as a function of the number of robots varying from 10 to 100. The results shown are obtained from 100 randomly generated scenarios.
  - The average numbers of RA-KAP solved is constant (< 3) irrespective of the number of robots while the maximum number is at most 7 (see Fig.3).
- Our algorithm is scalable with the number of robots or variances of travel distances (not shown in poster).

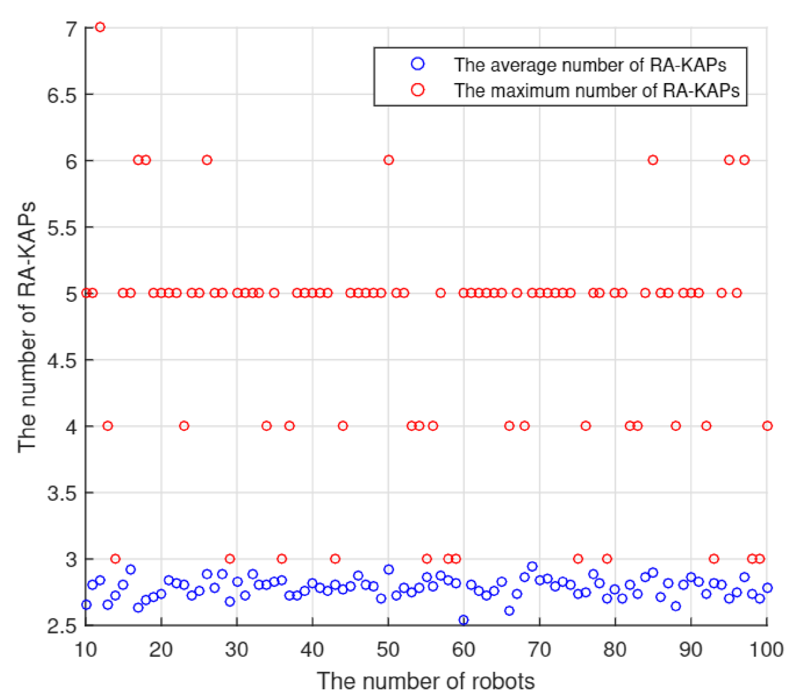

Fig. 3. For a given number of robots (varying from 10 to 100), the average and maximum number of RA-KAPs solved are less than 3 and 7 respectively. Results are based on 100 simulations with randomly generated mean and variance for travel distance of each robot.

Conclusion

- We present a novel approach that uses the solutions of a small number of deterministic RA-KAPs to solve stochastic CC-KAP optimally.
- We analyze the relationship between CC-KAP and RA-KAP on variance-mean plane. The geometric insight is helpful for solving other problems with chance constraint.
- We present simulation results showing that our method is efficient and scalable with the number of robots and the uncertainty inn travel distance capacity.

Acknowledgements

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