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Computers and Chemical Engineering 28 (2004) 1511–1528
Computers & Chemical Engineering
www.elsevier.com/locate/compchemeng

Efficient MILP formulations and valid cuts for multiproduct pipeline scheduling

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Received 5 August 2003; received in revised form 5 December 2003; accepted 8 December 2003

Abstract

Companies are faced with an ever-increasing competitive environment, larger commodity requirements and the need for rapid response to several uncertainties related to distribution and transportation scheduling. The problem addressed in this paper is composed by the short-term scheduling of a real world logistic complex that comprises the distribution of several petroleum derivatives from a single oil refinery to several depots through a single pipeline. The objective of this work is to generalize and to improve the efficiency of the MILP formulation proposed by Rejowski Jr. and Pinto [Comput. Chem. Eng. 27 (2003) 1229]. The model satisfies all operational constraints, such as mass balances, distribution constraints, product demands, sequencing constraints and logical constraints for pipeline operation. Firstly, the original formulation proposed by the authors is stated in a generalized form. Then, special and non-intuitive practical constraints, which minimizes product contamination inside the pipeline segments, are added to the original MILP and the resulting model is analyzed in terms of computational performance and solution quality. Finally, a set of integer cuts that are based on demands and pipeline segment initial inventories is included in the original formulation. All proposed examples are tested in three different demand scenarios. Results show that the formulations with the special constraints find the optimal solution with a higher value when compared to a feasible one of the respective problems without this assumption. When the delivery cuts were considered on the formulation with the special constraints for high demand scenario cases, they improved the CPU time in at least almost 70% when compared to the formulations that did not considered this set of valid cuts.

Keywords: Pipeline; Valid cuts; Logistics; Distribution scheduling; Optimization

1. Introduction

Planning and scheduling activities related to product distribution have received growing attention in the past 20 years. Companies are faced with an ever-increasing competitive environment, larger commodity requirements and the need for rapid response to several uncertainties related to these operations. When applied to logistics complexes, optimization techniques clearly provide tools that help to reduce costs and to improve distribution operations. Nevertheless, heuristic methods are widely utilized to find their operation profiles for a given time horizon. These can only provide feasible solutions and therefore, there is a need for systematic optimization-based approaches in decision-making problems. In the petroleum business, significant improvements can be achieved, since large amounts are always involved in the transportation and storage of oil derivatives.

Several applications of planning and scheduling activities in petroleum complexes have been developed in recent years. Cheng and Duran (2003) developed a decision-making system for the worldwide crude oil transportation problem. The system integrates discrete event dynamic simulation techniques and stochastic optimal control of the inventory and transportation system. The problem is formulated as a Markov decision process that considers travel times and demands as uncertainties of the model. A decomposition scheme based on dynamic programming is used to determine near optimal inventory policies that should minimize the total operational cost. Van den Heever, Grossmann, Vasantharajan, & Edwards, 2000 addressed the design and planning of oilfield infrastructure over a long time horizon. The nonlinear reservoir behavior and complex business characteristics are taken into account. The authors present

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0098-1354/$ – see front matter
doi:10.1016/j.compchemeng.2003.12.001

a disjunctive formulation that is computationally more efficient than a Big-M formulation.

A recent survey of refinery scheduling and planning operations is presented in Pinto, Joly, & Moro, 2000. The authors develop a general planning model and several examples relating real world applications of scheduling operations for refineries are presented. The authors showed a real-world application for diesel production planning that increased the annual profitability of the refinery in several millions dollars. The authors also show real-world scheduling applications, such as crude oil inventory management and problems involving fuel oil and LPG areas of the refinery. Many applications of refinery scheduling deal with tank farm management. These tanks usually store oil or any finished product and they must feed and/or receive unit distillations, a given pipeline, or any other operational installation. Some examples of these applications are the works done by Lee, Pinto, Grossmann, & Park (1996) and Moro and Pinto (in press). The first authors deal with a set of crude tanks that must receive oil from vessels and at the same time supply several unit distillations, whereas the latter show the more detailed operation of a set of tanks that must receive crude oil from a given pipeline and at the same time feed a distillation unit. Moro and Pinto (in press) developed an efficient continuous time representation that considers operations that can last 15 min to several hours.

Although a significant amount of work has been done in the area of refinery operations, the same does not occur in pipeline scheduling. Sasikumar, Prakash, Patil, & Ramani (1997) developed an expert system for scheduling a multi product pipeline, where only interface and pumping costs are minimized. Many models related to pipeline systems are solved by decomposition strategies due to their computational complexity. Besides this fact, this approach can handle models with a higher level of detail. Hane and Ratliff (1995) present the problem of sequencing commodities in a large petroleum products pipeline with extension from New Orleans, Louisiana to Newark, New Jersey. The problem of sequencing inputs products into the pipeline was decomposed into subproblems which were easily handled by a Branch and Bound algorithm. Shah (1996) studied the problem of crude oil transportation from a port to a refinery through a single pipeline. A monthly operation that minimizes volumes stored in the tanks is presented by the author. The system was divided in two models. The first determines operations at the port. Tanks must load the pipeline continuously and at the same time receive crude oil from vessels, while the last one states how tanks at the refinery should be utilized, once the pipeline operation has already been determined. Más and Pinto (2003) use this strategy to solve a short-term crude oil scheduling problem. The pipeline complex comprises ports where vessels must be allocated to piers and unload crude oil to tanks, which must feed pipelines that connect multiple refineries. The approach, which is based on a continuous time formulation, still considers the intermediate pipeline storage infrastructure. The authors firstly address the management of tank farms at the loading of the pipeline. Then its results serve as initial conditions for the models at the pipeline destinations.

The main drawback of decomposition approaches for modeling systems is that only near optimal solutions can be found. This can be critical as the subsystems decomposed share many resources. In pipeline complexes, the common resources to all locations are pipelines and storage tanks. Magalhães and Shah (2003) propose an MILP for the single pipeline crude oil transportation problem. A schedule that minimizes the deviation of the planned feed profile of unit distillations is considered in this work. A continuous time formulation is used for the system that is operated in continuous mode. Simultaneous loading/unloading operations must be avoided in the tanks located at the port and at the refinery. Other constraints such as time settling of tanks and pipeline peak time flow are also taken into account. Rejowski Jr. and Pinto (2003) developed a single MILP model for scheduling a real-world multiproduct pipeline system that contains multiple destinations. The pipeline must distribute gasoline, diesel oil, liquefied petroleum gas (LPG) and Jet Fuel among these depots. Two models based on disjunctive programming and on discrete time representation are presented. The first one divides the pipeline segments into packs of equal size, while the second one relaxes such assumption. Total costs must be minimized, that involve inventory, pumping and interface. Key decisions involve loading and unloading operations of tanks and pipeline. Results include the inventory levels at all locations, the operations of all segments of the pipeline and the best sequencing of products inside the pipeline.

Despite the deficiencies of decomposition strategies, unified scheduling models may also become intractable due to their combinatorial features. Hence, it becomes critical to develop tight formulations in order to find realistic solutions within reasonable computational time. Yee and Shah (1998) improved the efficiency of discrete time scheduling formulations by cut generation and by decision-variable disaggregation. Wolsey (1998) presents a detailed discussion on cut generation techniques for combinatorial optimization problems.

The objective of this work is to improve the efficiency of the MILP formulation proposed by Rejowski Jr. and Pinto (2003). The system reported in this work is composed by an oil refinery, one multiproduct pipeline connected to several depots and to the local consumer markets that must be fed with large amounts of oil products. Firstly, the original formulation proposed by the authors is stated in generalized form. Then, special and non-intuitive practical constraints, which impose that a given segment of the pipeline may be inoperative only if it is filled with a single derivative, are added to the original MILP and the resulting model is analyzed in terms of computational performance and solution quality. Finally, a set of integer cuts that are based on demands and pipeline segment initial inventories is included in the original formulation. Three cases with different demand patterns are presented. The first one is a low demand example, whereas the last two are the most critical cases to be solved, where high demands for some of the products should be met at the depots.

2. Problem definition

Consider a short-term scheduling problem where a refinery must distribute P petroleum products among D depots connected to a single pipeline, which is divided into D segments that may represent decreasing diameters. The pipeline system is represented in Fig. 1.

In the refinery and in the distribution depots, several tanks store the same product, although at most one of these is connected to the pipeline at each time.

In the pipeline, each segment can feed its respective depot or the next segment. In addition, product transfer must satisfy constant volume and maximum flow rate constraints in the pipeline. Besides these constraints, it presents a unique feature that is product contamination. There are also forbidden sequences of products inside the pipeline that must be considered in the problem.

The depots have to control their inventory levels and satisfy product demands determined by the local consumer markets.

The objective is to minimize operating costs that include inventory costs in the refinery as well as in the depots, pumping costs and transition costs between different products inside the pipeline.

A possible configuration of generic segments d and $d+1$ is represented in Fig. 2. It assumes that volumetric capacities of packs inside segment d and segment $d+1$ are the same. Segment d contains L_d packs that represent one product at any time. If product p enters d at time k, the content of the first pack is displaced to the following pack. The same occurs to all subsequent packs within the same segment. According to this configuration, the same amount of product must either leave the segment ($\mathrm{XD}_{p,d,k}$) or be transferred to segment $d+1$ ($\mathrm{XT}_{p,d+1,k}$).

Fig. 3 illustrates a configuration for pipeline systems when packs of segments d and $d+1$ have different capacities. This approach may be necessary when segments present a reduction in their diameter. Furthermore, it allows more than one simultaneous unloading operation from the pipeline. The amount comprised in pack L_d can be either sent integrally to depot d ($\mathrm{XD}_{p,d,k}$) or it can be split between this depot ($\mathrm{XW}_{p,d,k}$) and segment $d+1$. If no product enters d at time k ($\mathrm{XT}_{p,d,k} = 0$), then all packs keep their content.

3. Optimization model

The main assumptions of the optimization model are as follows:

(A1) all products have constant densities;
(A2) production rates and demands are known during the time horizon;
(A3) all tanks are treated as aggregated capacities;
(A4) at most one tank at the refinery and at all depots is connected to the pipeline at any time;
(A5) the pipeline segments are always completely filled;
(A6) set-up times for switching from one tank to another in the refinery are embedded in the formulation.

The nomenclature for the proposed model is listed below.

Indices and sets

$d = 1, \ldots, D$	depots or pipeline segments
D_c	set of segments that present a constant diameter with respect to the next segment
$\mathrm{DC}_{p,d,r}$	set of products and depots that belong to a given operational situation

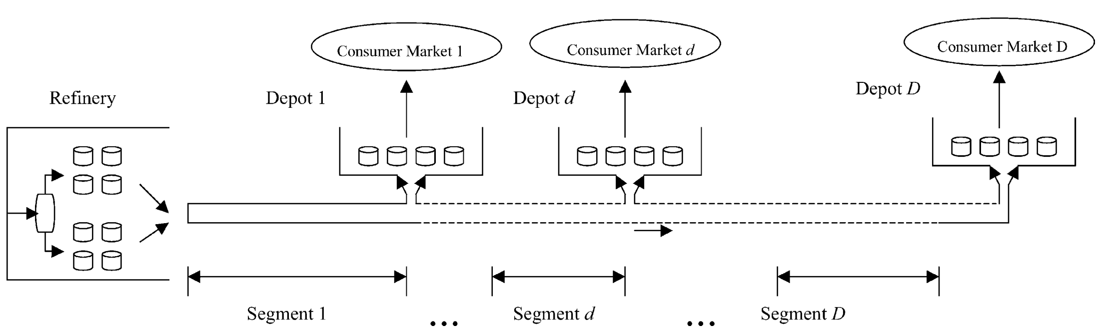

Fig. 1. Distribution pipeline system.

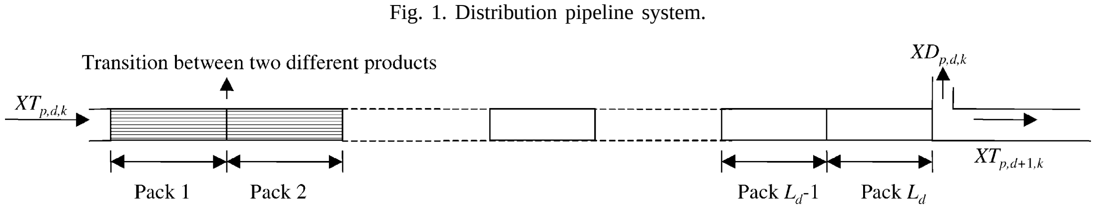

Fig. 2. Segments d and $d+1$ with packs of equal diameter capacities.

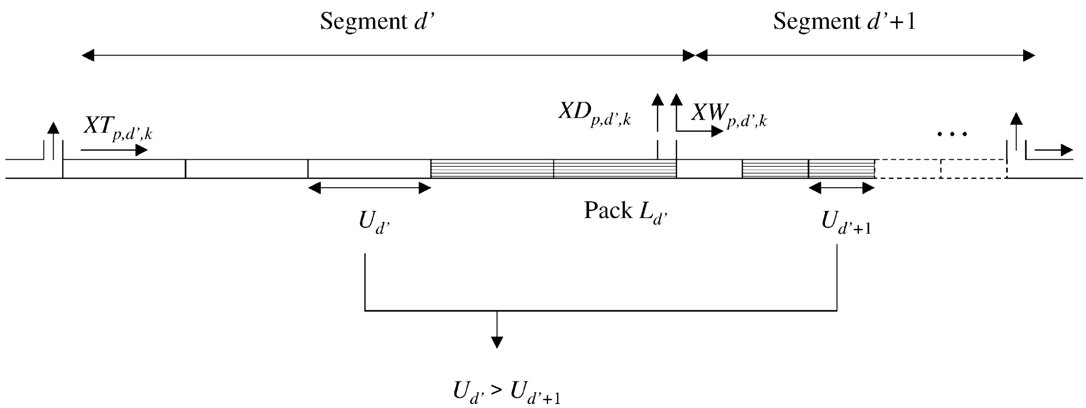

Fig. 3. Pipeline system with packs of different sizes.

$FS_{p,p'}$ set of forbidden sequences between products p and p'
$k = 1, \ldots, K$ time intervals
$l = 1, \ldots, L_d$ packs
$p = 1, \ldots, P$ products
$r = 1, \ldots, R$ operational situations
$TS_{p,p'}$ set of all possible sequences between products p and p'

Parameters

$CED_{p,d}$ inventory unit cost of product p at depot d
CER_p inventory unit cost of product p at the refinery
$CONTACT_{p,p'}$ transition cost from products p to p'
$CP_{p,d,k}$ unit cost for pumping product p to depot d at time interval k
$DEM_{p,d}$ demand of product p at consumer market supplied at depot d
L_d number of packs comprised by segment d
$LMAX_{p,d}$ nearest pack of segment d located from depot d that contains product p at the beginning of the operation
$LMIN_{p,d}$ farthest pack of segment d located from depot d that contains product p at the beginning of the operation
$RP_{p,k}$ production rate of product p at the refinery at time interval k
U_d volume of packs of segment d
$UM_{p,d,k}$ upper bound on the volume of product p sent by depot d at time interval k
$VDMAX_{p,d,k}$ maximum volumetric capacity of product p at depot d at time interval k
$VDMIN_{p,d,k}$ minimum volumetric capacity of product p at depot d at time interval k
$VDZERO_{p,d}$ initial inventory level of product p at depot d
$VRMAX_{p,k}$ maximum volumetric capacity of product p at the refinery at time k
$VRMIN_{p,k}$ minimum volumetric capacity of product p at the refinery at time k
$VRZERO_p$ initial inventory level of product p at the refinery
$XDC_{d,r}$ 1 if depot d must take from the pipeline any product comprised by set $DC_{p,d,r}$ within the operation, 0 otherwise
$XSMIN_{d,r}$ minimum number of time that segment d must be operated according to operational situation r
$XVMIN_{p,d}$ number of minimum time intervals for the arrival to depot d of product p from the farthest/nearest pack
$XVZERO_{p,d,1}$ 1 if pack 1 of segment d stores product p at the beginning of the operation, 0 otherwise

Binary variables

$XD_{p,d,k}$ 1 if depot d receives product p from the pipeline at time k
$XR_{p,k}$ 1 if the refinery discharges p in the pipeline at time k
$XT_{p,d,k}$ 1 if product p is sent to segment d at time k
$XW_{p,d,k}$ denotes if pack L_d from segment d is sent to depot d and to segment $d+1$

Continuous variables

C total cost to be minimized
$TY^d_{p,p',k}$ denotes if product p succeeds p' in segment d at time k
$VD_{p,d,k}$ volumetric inventory level of product p at depot d at time k
$VOD_{p,d,k}$ volume of product p received from segment d and/or volume of product p sent to $d+1$ at time k by depot d at time k

$\mathrm{VOM}_{p,d,k}$	volume of product p sent by depot d to the local consumer market at time k
$\mathrm{VOR}_{p,k}$	volume of product p sent by the refinery to the pipeline at time k
$\mathrm{VR}_{p,k}$	volumetric inventory level of product p at the refinery at time k
$\mathrm{XS}_{d,k}$	denotes if segment d is under operation at time k
$\mathrm{XV}^{d}_{p,l,k}$	denotes if pack l from segment d contains product p at time k

The MILP proposed originally in Rejowski Jr. and Pinto (2003) is generalized in this paper as follows. The objective function of the model is presented by Eq. (1).

$$\min C = \left[\sum_{p=1}^{P}\sum_{k=1}^{K}\mathrm{CER}_p\mathrm{VR}_{p,k} + \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{k=1}^{K}\mathrm{CED}_{p,d}\mathrm{VD}_{p,d,k}\right]\delta + \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{k=1}^{K}\mathrm{CP}_{p,d,k}[\mathrm{XD}_{p,d,k}U_d + \mathrm{XW}_{p,d,k}(U_d - U_{d+1})] + \sum_{p=1}^{P}\sum_{p'=1}^{P}\sum_{d=1}^{D}\sum_{k=1}^{K}\mathrm{CONTACT}_{p,p'}\mathrm{TY}^{d}_{p,p',k} \quad (1)$$

The terms account for inventory costs in the refinery and in the depots (in brackets), pumping costs and product transitions. The model constraints are detailed below.

The tanks at the refinery are modeled by constraints (2) to (4). The volumes that leave tanks and feed the pipeline are related to binary variables $\mathrm{XR}_{p,k}$ in Eq. (4) and constant parameter U_d for each segment d of the pipeline. Note that the refinery may supply the pipeline in discontinuous mode of operation, according to (5) and (6). Constraint (5) activates the 0–1 variable $\mathrm{XS}_{d,k}$ that denotes operation of segment d. An important feature is that when constraint (6) is imposed, this variable can be represented as continuous.

$$\mathrm{VR}_{p,k} = \mathrm{VRZERO}_p + \mathrm{RP}_{p,k}\delta - \mathrm{VOR}_{p,k}, \quad \forall p,\ k=1 \quad (2a)$$

$$\mathrm{VR}_{p,k} = \mathrm{VR}_{p,k-1} + \mathrm{RP}_{p,k}\delta - \mathrm{VOR}_{p,k}, \quad \forall p,\ k=2,\ldots,K \quad (2b)$$

$$\mathrm{VRMIN}_{p,k} \le \mathrm{VR}_{p,k} \le \mathrm{VRMAX}_{p,k}, \quad \forall p,\ k \quad (3)$$

$$\mathrm{VOR}_{p,k} = \mathrm{XR}_{p,k}U_d, \quad \forall p,\ k,\ d=1 \quad (4)$$

$$\sum_{p=1}^{P}\mathrm{XR}_{p,k} = \mathrm{XS}_{d,k}, \quad \forall d=1,\ k \quad (5)$$

$$\mathrm{XS}_{d,k} \le 1, \quad \forall d,\ k \quad (6)$$

Pipeline operation is controlled by the assignment of logical variables $\mathrm{XV}^{d}_{p,l,k}$ to each set (*product*, *depot*, *pack*, *time*), according to constraints (7) to (21).

$$\sum_{p=1}^{P}\mathrm{XV}^{d}_{p,l,k} = 1, \quad \forall d,\ l,\ k \quad (7)$$

Due to assumption (A6), there is always one product comprised in each pack of all segments of the pipeline at each time interval k, as enforced by constraint (7). For example, consider the operation of the first segment of the pipeline. If there is transfer of product p at time k ($\mathrm{XR}_{p,k} = 1$ and from (5) $\mathrm{XS}_{1,k} = 1$) constraint (8) activates $\mathrm{XV}^{1}_{p,1,k}$. The other movements between packs ($l = 2,\ldots,L_d$) are activated through (9a) and (10a). On the other hand, when a segment of the pipeline is not operational at time k ($\sum_p \mathrm{XR}_{p,k} = \mathrm{XS}_{d,k} = 0$), all products comprised in every pack l at $k-1$ remains in it, as imposed by (9b) and (10b). Note that constraints (9) consider initial amounts inside the segments of the pipeline, denoted by parameters $\mathrm{XVZERO}_{p,d,l}$. For this reason, they are assigned to the first time interval ($k = 1$) of the operation.

$$\mathrm{XV}^{d}_{p,l,k} \ge \mathrm{XR}_{p,k}, \quad \forall p,\ d=1,\ l=1,\ k \quad (8)$$

$$\mathrm{XV}^{d}_{p,l,k} \ge \mathrm{XVZERO}_{p,d,l-1} - [1-\mathrm{XS}_{d,k}], \quad \forall p,\ d,\ l=2,\ldots,L_d,\ k=1 \quad (9a)$$

$$\mathrm{XV}^{d}_{p,l,k} \ge \mathrm{XVZERO}_{p,d,l} - \mathrm{XS}_{d,k}, \quad \forall p,\ d,\ l \le L_d,\ k=1 \quad (9b)$$

$$\mathrm{XV}^{d}_{p,l,k} \ge \mathrm{XV}^{d}_{p,l-1,k-1} - [1-\mathrm{XS}_{d,k}], \quad \forall p,\ d,\ l=2,\ldots,L_d,\ k=2,\ldots,K \quad (10a)$$

$$\mathrm{XV}^{d}_{p,l,k} \ge \mathrm{XV}^{d}_{p,l,k-1} - \mathrm{XS}_{d,k}, \quad \forall p,\ d,\ l \le L_d,\ k=2,\ldots,K \quad (10b)$$

For any intermediate segment of the pipeline, constraint (8) is replaced by (11). This is necessary since these segments are no longer supplied by the refinery, but by their preceding segment.

$$\mathrm{XV}^{d}_{p,l,k} \ge \mathrm{XT}_{p,d,k} + \mathrm{XW}_{p,d-1,k}, \quad \forall p,\ d>1,\ l=1,\ \forall k \quad (11)$$

For segments with no reduction in diameter (given by the set D_c), two options are available for the product comprised in the last pack of any given segment at any given time interval k. It can be either sent to the respective depot d, denoted by variable $\mathrm{XD}_{p,d,k}$, or it can feed the next segment of the pipeline, a condition that is stated by variable $\mathrm{XT}_{p,d,k}$, according to constraints (12), (13) and (14). Another operational alternative is represented by variable $\mathrm{XW}_{p,d,k}$, which indicates whether the product inside the last pack of segment d is split between its respective depot and the consecutive segment at time interval k. For intermediate segments with no reduction in diameter (segments that belong to set

D_c) and for the last one, all variables $XW_{p,d,k}$ must be zero, according to Eq. (16).

$$\sum_{p=1}^{P}[XT_{p,d,k} + XW_{p,d-1,k}] = XS_{d,k}, \quad \forall k,\ d > 1, \tag{12}$$

$$\sum_{p=1}^{P}[XD_{p,d,k} + XT_{p,d+1,k} + XW_{p,d,k}] = XS_{d,k}, \quad \forall k,\ d < D \tag{13}$$

$$XD_{p,d,k} + XT_{p,d+1,k} + XW_{p,d,k} \geq XVZERO_{p,d,L_d} - [1 - XS_{d,k}], \quad \forall p,\ d < D,\ k = 1 \tag{14a}$$

$$XD_{p,d,k} + XT_{p,d,+1,k} + XW_{p,d,k} \geq XV^{d}_{p,L_d,k} - [1 - XS_{d,k}], \quad \forall p,\ d < D,\ k = 2, \ldots, K \tag{14b}$$

$$VOD_{p,d,k} = [XD_{p,d,k} + XT_{p,d+1,k}]U_d + XW_{p,d,k}[U_d - U_{d+1}], \quad \forall p,\ d < D,\ k \tag{15}$$

$$XW_{p,d,k} = 0, \quad \forall p,\ k,\ d \in D_c \tag{16}$$

For the last segment of the pipeline, constraints (17), (18), (19) replace (13), (14) and (15), respectively. In this segment ($d = D$), the product inside the last pack can only be sent for the last depot connected to the pipeline.

$$\sum_{p=1}^{P}XD_{p,d,k} = XS_{d,k}, \quad \forall k,\ d = D \tag{17}$$

$$XD_{p,d,k} \geq XVZERO_{p,d,L_d} - [1 - XS_{d,k}], \quad \forall p,\ d = D,\ k = 1 \tag{18a}$$

$$XD_{p,d,k} \geq XV^{d}_{p,L_d,k-1} - [1 - XS_{d,k}], \quad \forall p,\ d = D,\ k = 2, \ldots, K \tag{18b}$$

$$VOD_{p,d,k} = XD_{p,d,k}U_d, \quad \forall p,\ d = D,\ k \tag{19}$$

A major and unique characteristic of this problem is the formation of an interface between two different products, which is detected by constraints (20) and (21) at the beginning of each segment of the pipeline, by variable $TY^{d}_{p,p',k}$. A complicating factor of this specific operation is that some of the product pairs involved in this operation cannot form an interface. This is stated by Eq. (21), where $TY^{d}_{p,p',k}$ always assume values equal to zero for two products p and p' that belong to the subset of forbidden sequences ($FS_{p,p'}$).

Table 1
Summary of model proposed by Rejowski and Pinto (2003)

Operational system	Constraints
Objective function	(1)
Refinery	(2), (3), (4), (5)
First segment of the pipeline	(6), (7), (8), (9), (10), (13), (14), (15), (16)
Intermediate segment of the pipeline	(6), (7), (9), (10), (11), (12), (13), (14), (15), (16)
Terminal segment of the pipeline	(6), (7), (9), (10), (11), (12), (17), (18), (19)
Sequencing constraints	(20), (21)
Depots	(22), (23), (24), (25)

$$TY^{d}_{p,p',k} \geq XV^{d}_{p,1,k} + XV^{d}_{p,2,k} - 1 \quad \{\forall p, p', d| \in TS_{p,p'}\},\ \forall k \tag{20}$$

$$TY^{d}_{p,p',k} = 0 \quad \{\forall p, p', d| \in FS_{p,p'}\},\ \forall k \tag{21}$$

The constraints related to depots connected to the pipeline are listed below. Constraints (22) state mass balances, whereas lower and upper bounds of storage capacity are given by (23). Local markets are fed according to the flow rate upper bound $UM_{p,d,k}$, stated by constraint (24), whereas demand satisfaction for a given product at the end of the operational horizon time obeys Eq. (25).

$$VD_{p,d,k} = VDZERO_{p,d} + XD_{p,d,k}U_d + XW_{p,d,k}[U_d - U_{d+1}] - VOM_{p,d,k}, \quad \forall p,\ d,\ k = 1 \tag{22a}$$

$$VD_{p,d,k} = VD_{p,d,k-1} + XD_{p,d,k}U_d + XW_{p,d,k}[U_d - U_{d+1}] - VOM_{p,d,k}, \quad \forall p,\ d,\ k = 2, \ldots, K \tag{22b}$$

$$VDMIN_{p,d,k} \leq VD_{p,d,k} \leq VDMAX_{p,d,k}, \quad \forall p,\ d,\ k \tag{23}$$

$$VOM_{p,d,k} \leq UM_{p,d,k} \quad \forall p,\ d,\ k \tag{24}$$

$$\sum_{k=1}^{K}VOM_{p,d,k} = DEM_{p,d}, \quad \forall p,\ d \tag{25}$$

Table 1 summarizes the resulting MILP model. Note that the only binary variables of the model are $XR_{p,k}$ (1 if the refinery supplies the pipeline with product p at time k, 0 otherwise), $XD_{p,d,k}$ (1 if pipeline feeds depot d with product p at time k, 0 otherwise) and $XT_{p,d,k}$ (1 if segment d is fed with product p at time k, 0 otherwise).

4. Special constraints and delivery cuts

Rejowski Jr. and Pinto (2003) added one set of integer cuts to the MILP summarized in Table 1 to improve solution performance by the determination of the minimum number of times that depot d must receive product p along the horizon. The authors related demand of product p at a local consumer market with the difference between the initial amount stored at the respective depot and the inventory lower bound of this derivative. These cuts are presented in (27) that makes use of parameter $\mathrm{XDMIN}_{p,d}$ that is shown in (26). The authors assumed that this lower bound remains unchanged during the time horizon. This cut provided improvements in the original formulation, although for real-world cases the optimal solution was not achieved.

$$\mathrm{XDMIN}_{p,d} = \max\left\{0, \left\lceil \frac{\mathrm{DEM}_{p,d} - (\mathrm{VDZERO}_{p,d} - \mathrm{VDMIN}_{p,d})}{U_d} \right\rceil\right\}, \quad \forall p,\ d \quad (26)$$

$$\sum_{k=1}^{K} \mathrm{XD}_{p,d,k} \geq \mathrm{XDMIN}_{p,d}, \quad \forall p,\ d \quad (27)$$

where $\lceil\cdot\rceil$ denotes the ceil operator.

The resulting MILP achieved good results for real-world examples using the formulation presented in the previous section added to cut (27). Nonetheless, two major aspects have to be considered. The first one is the contamination of products inside the pipeline that can be critical if segments remain inoperative during a time interval and store more than one derivative, which is addressed in Section 4.1. Section 4.2 presents the second one, which concerns generation of valid cuts that can drastically improve CPU performance of the proposed model by Rejowski Jr. and Pinto (2003).

4.1. Special constraints

Therefore, in order to minimize product contamination inside the pipeline, a set of special constraints must impose that a given segment of the pipeline is inoperative only if it contains a single derivative. This new operational condition is a crucial characteristic, especially when low viscosity products must be transported through a single pipeline. The cost of an interface formed inside a pipeline between two different products in adjacent position is measured by its volume, which tends to increase as the products remain inside the pipeline while it is inoperative. The interface volume formed also depends on the physical properties of the products. These costs are critical for pipelines that operate in intermittent mode. Interface detection can be carried out by on-line density analysis, volumetric calculation (Moro & Pinto, in press), and on-line ultra-sonic analysis. Most refineries separate a given volume before and after the interface arrival to assure that they do not contaminate any product tank. In multiproduct pipeline systems, the interfaces are usually stored in separate tanks and many operational strategies with different costs are available to recover the products. Jones and Paddock (1982) suggest that a laminar flow regime would lead to complete product contamination inside the pipeline. The authors adopt critical Reynolds values according to several pipeline diameters, which minimize product contamination, although is well known that this phenomenon is rigorously explained by liquid dispersions and mixtures. Immediately after a pipeline shutdown, the volume of interfaces expands inside the pipeline due to diffusive mass transfer.

Hence, because of the lack of diffusive mass transfer equations that represent this phenomenon there is a strong necessity to include constraints in the scheduling models that impose stopping conditions ($\mathrm{XS}_{d,k} = 0$) for a segment d of the pipeline only when it does not contain any interface of any products within all its length. Therefore, it is necessary to relate this parameter (L_d) with the interface variables of the original model ($\mathrm{TY}^d_{p,p',k}$). Fig. 4 illustrates a typical situation where this relationship takes place.

According to Fig. 4, at time interval $k-1$, the pipeline segment does not contain any interface. At this time interval, the segment can either move the only product inside ($\mathrm{XS}_{d,k} = 1$) or stop product transfer ($\mathrm{XS}_{d,k} = 0$). Assume that it is under operation at time $k-1$ and that in the next time interval (k) another product enters the pipeline and consequently an interface is formed ($\mathrm{TY}^d_{p,p',k} = 1$). Under this new condition, the pipeline segment must operate in the next time intervals. Operation must continue until any interface detected at time interval k is completely removed from the segment. This removal occurs at time interval $k + L_d - 1$. This new operational condition gives rise to a set of new constraints that must be included to the original model proposed by Rejowski Jr. and Pinto (2003), which is shown by constraints (28). Note that these should be written until the time interval that the interface detected at time k is removed ($k + L_d - 1$) from the segment.

$$\begin{aligned}
&\textstyle\sum_{p=1}^{P}\sum_{p'=1}^{P} \mathrm{TY}^d_{p,p',k} \leq \mathrm{XS}_{d,k+1}, && \forall d \\
&\textstyle\sum_{p=1}^{P}\sum_{p'=1}^{P} \mathrm{TY}^d_{p,p',k} \leq \mathrm{XS}_{d,k+2}, && \forall d \\
&\vdots && \vdots \\
&\textstyle\sum_{p=1}^{P}\sum_{p'=1}^{P} \mathrm{TY}^d_{p,p',k} \leq \mathrm{XS}_{d,k+L_d-1}, && \forall d
\end{aligned} \quad (28)$$

Note that the set of special constraints described by (28) is composed by $L_d - 1$ constraints. Finally, these constraints are summarized by constraint (29). Note that this situation is only valid until time interval $K + L_d - 1$.

$$\sum_{p=1}^{P}\sum_{p'=1}^{P} \mathrm{TY}^d_{p,p',k}[L_d - 1] \leq \sum_{k'=k+1}^{k+L_d-1} \mathrm{XS}_{d,k'}, \quad \forall d,\ k \leq K - L_d + 1 \quad (29)$$

Although constraint (29) is necessary, it is not sufficient to cover the time horizon of the operation. Thus, another set of constraints must be developed that follows the same idea of constraints (28). These are imposed between time

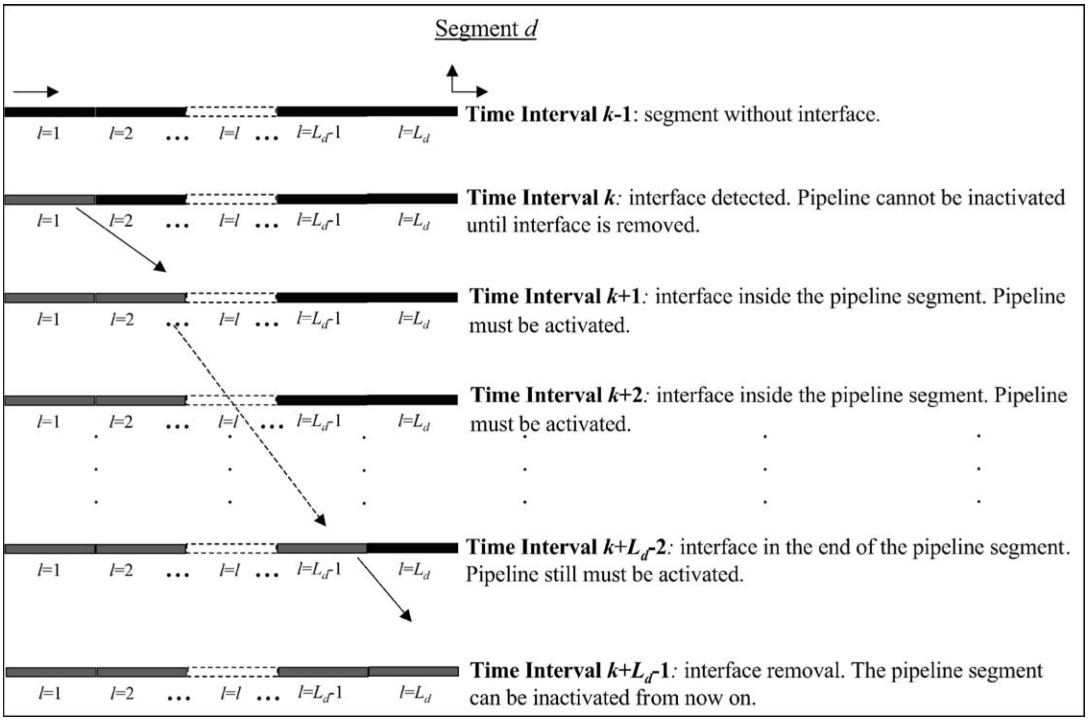

Fig. 4. Pipeline segment operation under new conditions.

interval $K - L_d$ and the horizon time (K), as shown by constraint (30).

$$\sum_{p=1}^{P}\sum_{p'=1}^{P} \mathrm{TY}^{d}_{p,p',k}[K-k] \le \sum_{k'=k+1}^{K} \mathrm{XS}_{d,k'}, \quad \forall d,\ k > K - L_d + 1 \quad (30)$$

Nevertheless, constraints (29) and (30) do not take into account the initial storage condition of the pipeline segment. They only consider interfaces that are formed within this time horizon. For this reason, a third constraint that considers the position of initial interfaces inside each pipeline segment is given by parameter $\mathrm{TINI}^{d}_{p,p',k}$, in Eq. (31).

$$\mathrm{TINI}^{d}_{p,p',l} = \max\{0, (\mathrm{XVZERO}_{p,d,l} + \mathrm{XVZERO}_{p',d,l+1} - 1)\} \quad \{\forall p, p'| \in \mathrm{TS}_{p,p'}, d\},\ \forall l < L_d \quad (31)$$

This equation states whether there exists an interface between products p and p'. Moreover, it determines its position inside segment d at the beginning of operation. Any initial interface placed at pack l and $l+1$ implies that the respective segment must continuously transfer products in the first $L_d - l$ time intervals of the operational time horizon. This idea is stated by constraint (32). Note also that if there is no initial interface in the pipeline, constraints (32) are trivially satisfied.

$$\sum_{p=1}^{P}\sum_{p'=1}^{P} \mathrm{TINI}^{d}_{p,p',l}[L_d - l] \le \sum_{k \le L_d - l} \mathrm{XS}_{d,k}, \quad \forall d, l \le L_d \quad (32)$$

Finally, constraints (29), (30), (32) and Eq. (31) guarantee that all segments of the pipeline will only be out of operation if they do not comprise any interface within all their extension for the whole scheduling horizon.

4.2. Delivery cuts

Delivery cuts can be generated by relating parameter $\mathrm{XDMIN}_{p,d}$ with initial amounts inside the pipeline ($\mathrm{XVZERO}_{p,d,l}$). These determine lower bounds on the number of time intervals that all segments of the pipeline must operate within the time horizon. Overall, six operational situations (r) arise for this case, each of them described as follows. Furthermore, sets that assign products and depots to each operational situation must be considered ($\mathrm{DC}_{p,d,r}$). Parameter $\mathrm{XDC}_{d,r}$ is a binary parameter that assumes the unitary value if at least one product for a given depot d belongs to set $\mathrm{DC}_{p,d,r}$.

Firstly, consider an operational situation ($r = 1$) in which a depot d must take product p from the pipeline ($\mathrm{XDMIN}_{p,d} > 0$) and p is not initially present in its respective segment ($\sum_l \mathrm{XVZERO}_{p,d,l} = 0$). A set of ordered pairs of product-depot is defined for this operational circumstance, according to (33). Let $\mathrm{XSMIN}_{d,1}$ be the minimum number of time intervals that segment d of the pipeline must be operative during the time horizon. In this situation, this lower bound depends on the total extension of the pipeline segment and on the sum of the minimum number of times that the pipeline must deliver all products to the respective depot, according to Eq. (34). Note that for the correct application of this equation, the total extension of the segment d (L_d) must be multiplied by binary parameter $\mathrm{XDC}_{d,1}$, otherwise parameter $\mathrm{XSMIN}_{d,1}$ would assume values greater than or equal to L_d for any circumstance.

$$\mathrm{DC}_{p,d,1} = \left\{ (p,d) \middle| (\mathrm{XDMIN}_{p,d} > 0) \wedge \left(\sum_l \mathrm{XVZERO}_{p,d,l} = 0 \right) \right\}, \quad \forall p,\ d \tag{33}$$

$$\mathrm{XSMIN}_{d,1} = L_d \mathrm{XDC}_{d,1} + \sum_{p \in \mathrm{DC}_{p,d,1}} \mathrm{XDMIN}_{p,d}, \quad \forall d \tag{34}$$

Another possible situation ($r = 2$) is characterized by the insufficient initial amounts of products inside the pipeline segment ($\sum_l \mathrm{XVZERO}_{p,d,l} < \mathrm{XDMIN}_{p,d}$). The set defined for this situation ($\mathrm{DC}_{p,d,2}$) relates products that are initially inside of segment d but in insufficient amounts to fulfill demands.

$$\mathrm{DC}_{p,d,2} = \left\{ (p,d) \middle| \left(\sum_l \mathrm{XVZERO}_{p,d,l} > 0 \right) \wedge \left(\sum_l \mathrm{XVZERO}_{p,d,l} < \mathrm{XDMIN}_{p,d} \right) \right\}, \quad \forall p,\ d \tag{35}$$

In this case, the minimum number of times that a segment d must operate is given by (36). For any segment d of the pipeline under this circumstance, lower bound $\mathrm{XSMIN}_{d,2}$ depends on two terms. The first one is the total segment extension, given by parameter L_d multiplied by binary parameter $\mathrm{XDC}_{d,2}$, whereas the second term in brackets represents the total difference between demands and initial amounts of all products comprised by set $\mathrm{DC}_{p,d,2}$ that fill initially this segment.

$$\mathrm{XSMIN}_{d,2} = L_d \mathrm{XDC}_{d,2} + \left[\sum_{p \in \mathrm{DC}_{p,d,2}} \left(\mathrm{XDMIN}_{p,d} - \sum_l \mathrm{XVZERO}_{p,d,l} \right) \right], \quad \forall d \tag{36}$$

Eqs. (37)–(40) are applied when product amounts inside the pipeline at $k = 0$ are exactly sufficient to satisfy demands ($r = 3$, $\sum_l \mathrm{XVZERO}_{p,d,l} = \mathrm{XDMIN}_{p,d}$). For this operational situation, the set $\mathrm{DC}_{p,d,3}$ relates initial amounts of products inside segment d that meet the demand required by the local consumer market. Let $\mathrm{LMIN}_{p,d}$ be the farthest pack of segment d from its respective depot occupied by products that belong to set $\mathrm{DC}_{p,d,3}$, according to Eq. (38). Parameter $\mathrm{XVMIN}_{p,d}$ defines the minimum number of time intervals for the products to reach depot d. It is determined by the extension of segment d and the farthest pack from depot d filled by product p, described by Eq. (39). Then, the lower bound $\mathrm{XSMIN}_{d,3}$ ($\mathrm{XSMIN}_{d,3} \geq 0$) is finally given by Eq. (40), where it is stated as the maximum necessary number of time intervals among all products within this operational situation.

$$\mathrm{DC}_{p,d,3} = \left\{ (p,d) \middle| \left(\sum_l \mathrm{XVZERO}_{p,d,l} > 0 \right) \wedge \left(\sum_l \mathrm{XVZERO}_{p,d,l} = \mathrm{XDMIN}_{p,d} \right) \right\}, \quad \forall p,\ d \tag{37}$$

$$\mathrm{LMIN}_{p,d} = \min_{l | \mathrm{XVZERO}_{p,d,l} > 0} \{ l\, \mathrm{XVZERO}_{p,d,l} \}, \quad \forall p,\ d \in \mathrm{DC}_{p,d,3} \tag{38}$$

$$\mathrm{XVMIN}_{p,d} = L_d - \mathrm{LMIN}_{p,d} + 1, \quad \forall p,\ d \in \mathrm{DC}_{p,d,3} \tag{39}$$

$$\mathrm{XSMIN}_{d,3} = \left[\max_{p \in \mathrm{DC}_{p,d,3}} (\mathrm{XVMIN}_{p,d}) \right], \quad \forall d \tag{40}$$

Set $\mathrm{DC}_{p,d,4}$ relates products p which initial inventory amounts inside segment d are higher than demands established ($r = 4$, $\sum_l \mathrm{XVZERO}_{p,d,l} > \mathrm{XDMIN}_{p,d}$). For this set, parameter $\mathrm{XVMIN}_{p,d}$ depends on the segment extension L_d, on parameter $\mathrm{XDMIN}_{p,d}$, and on the nearest pack from depot d that comprises a product in this situation. The lower bound $\mathrm{XSMIN}_{d,4}$ ($\mathrm{XSMIN}_{d,4} \geq 0$) is given by Eq. (44).

$$\mathrm{DC}_{p,d,4} = \left\{ (p,d) \middle| (\mathrm{XDMIN}_{p,d} > 0) \wedge \left(\sum_l \mathrm{XVZERO}_{p,d,l} > 0 \right) \wedge \left(\sum_l \mathrm{XVZERO}_{p,d,l} > \mathrm{XDMIN}_{p,d} \right) \right\}, \quad \forall p,\ d \tag{41}$$

$$\mathrm{LMAX}_{p,d} = \max_l \{ l\, \mathrm{XVZERO}_{p,d,l} \}, \quad \forall p,\ d \in \mathrm{DC}_{p,d,4} \tag{42}$$

$$\mathrm{XVMIN}_{p,d} = L_d - \mathrm{LMAX}_{p,d} + \mathrm{XDMIN}_{p,d}, \quad \forall p,\ d \in \mathrm{DC}_{p,d,4} \tag{43}$$

$$\mathrm{XSMIN}_{d,4} = \left[\max_{p \in \mathrm{DC}_{p,d,4}} (\mathrm{XVMIN}_{p,d}) \right], \quad \forall d \tag{44}$$

The fifth operational situation ($r = 5$) presented for a given segment d of the pipeline relates the first and the second operational situations. It takes into account the pairs of products p and depot d that belong to set $DC_{p,d,1}$ and another pair formed by product p' ($p' \neq p$) and the same depot d, which are related to set $DC_{p',d,2}$. The lower bound $XSMIN_{d,5}$ is given by Eq. (45). The first term is composed by the overall segment extension (L_d) and binary parameters $XDC_{d,1}$ and $XDC_{d,2}$. The second term takes into account the minimum number of times that depot d must take all products related by set $DC_{p,d,1}$. The third one relates the difference between parameter $XDMIN_{p,d}$ and all initial amount comprised in segment d at the beginning of the operation ($\sum_l XVZERO_{p,d,l}$) considered by set $DC_{p,d,2}$ multiplied by binary parameter $XDC_{d,2}$. Binary parameters $XDC_{d,r}$ are considered in all their respective terms in Eq. (45). This is necessary for tighter lower bounds that relate simultaneously the first and the second operational situations.

$$\begin{aligned} XSMIN_{d,5} &= \left\{ L_d + \left[\sum_{p \in DC_{p,d,1}} XDMIN_{p,d} \right] \right\} (XDC_{d,1} XDC_{d,2}) \\ &\quad + \left[\sum_{p' \in DC_{p',d,2}} \left(XDMIN_{p',d} - \sum_l XVZERO_{p',d,l} \right) \right] \\ &\quad \times (XDC_{d,1} XDC_{d,2}), \quad \forall d \end{aligned} \tag{45}$$

Finally, when no demand is required ($r = 6$), all parameters are set to zero. Note that all equations presented in all operational situations are linear.

The delivery cut proposed is then described by constraint (46). It is formulated as an integer cut and it imposes a lower bound on the number of times during the time horizon that a pipeline segment must be operative. Note that parameter $XSMIN_{r,d}$ must be set as the maximum among all the operational situations presented.

$$\sum_{k=1}^{K} XS_{d,k} \geq \max_r \{XSMIN_{r,d}\}, \quad \forall d \tag{46}$$

In the following section, several examples are presented to test both the effect of the special constraints proposed and of the set of delivery cuts on the original formulation proposed by Rejowski Jr. and Pinto (2003).

5. Case studies

The example studied in this paper deals with the Petrobras system, which is shown in Fig. 5. The system is composed by the REPLAN refinery, the pipeline OSBRA and five depots (and consequently segments). This pipeline transports diesel oil, gasoline, liquified petroleum gas (LPG) and jet fuel. As the only diameter reduction is comprised from the fourth to the fifth segments of the pipeline, the set D_c is composed by segments ($d = 1, 2, 3, 5$). According to Eq. (16), logical variables $XW_{p,d,k}$ are set to zero. Variables $XW_{p,4,k}$ can be stated as continuous. Furthermore, there are 15 time intervals with 5 h each.

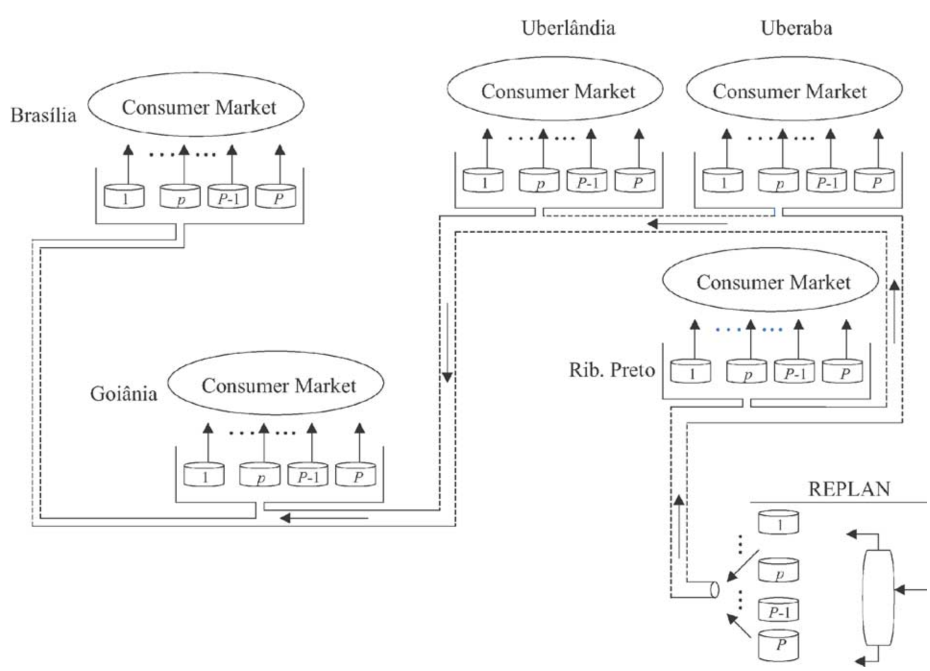

Fig. 5. Flowsheet of the distribution system.

Table 2
Volumetric capacities and lots of the OSBRA pipeline

Segment	Actual capacity (m^3)	Discrete capacity (m^3)	Difference (%)	U_d ($\times 10^{-2}$ m^3)	L_d lots
REPLAN-Rib. Preto	39,759	40,000	0.61	50	8
Rib. Preto-Uberaba	25,879	25,000	3.52	50	5
Uberaba-Uberlândia	25,321	25,000	1.27	50	5
Uberlândia-Goiânia	59,676	60,000	0.54	50	12
Goiânia-Brasília	13,739	13,500	1.74	27	5

Table 3
Common data for all scenarios

Product	Inventory costs		Initial condition in pipeline				
	CER_p (US\$/m^3 h)	$CED_{p,d}$ (US\$/m^3 h)	$XVZERO_{p,1,l}$	$XVZERO_{p,2,l}$	$XVZERO_{p,3,l}$		
Gasoline (1)	0.020	0.100	1 ($l = 1, 2, 3, 4, 5, 6, 7, 8$)	1 ($l = 4, 5$)	1 ($l = 1, 2, 3, 4, 5$)		
Diesel oil (2)	0.023	0.155	1 ($l = 4$)	1 ($l = 1, 2, 3$)			
LPG (3)	0.070	0.200	0	0	0		
Jet fuel (4)	0.025	0.170	0	0	0		
	Initial condition in pipeline		Pumping costs $CP_{p,d}$ (US\$/m^3)	$CONTACT_{p,p'}$ ($\times 10^{-2}$ US\$)			
	$XVZERO_{p,4,l}$	$XVZERO_{p,5,l}$		$p' = 1$	$p' = 2$	$p' = 3$	$p' = 4$
Gasoline (1)	1 ($l = 1, \ldots, 10$)	0	3.5/4.5/5.5/6.0/6.9	0	30	37	35
Diesel oil (2)	0	1 ($l = 1, 2, \ldots, 5$)	3.6/4.6/5.6/6.2/7.3	30	0	X	X
LPG (3)	1 ($l = 11, 12$)	0	4.8/X/6.8/7.9/8.9	37	X	0	X
Jet fuel (4)	0/0	0	X/X/X/6.1/7.0	35	X	X	0

Common data for this example are listed from Tables 2–5. Table 2 compares the actual capacity of each segment of the pipeline with its discrete capacity adopted for all case studies. It also illustrates parameters U_d and L_d. It can be seen that the longest segment of the pipeline system is the fourth one, which connects depots located at Uberlândia and Goiânia, whereas the last segment is the shortest one.

Fig. 6 shows the production profile given by the refinery for all derivatives involved in the transfer operation. In the first 10 time intervals, the tank farm at the refinery receives gasoline and diesel oil, while in the last five time intervals, only LPG and Jet Fuel are generated.

Table 3 provides parameters for the model and initial conditions for the pipeline. Tables 4 and 5 show upper and lower bounds for inventory levels as well as initial inventory conditions at the refinery and at all depots, respectively.

Another important common feature to all scenarios presented in this work is that not all depots store all products. An example is the depot located at Uberaba, which only stores gasoline and diesel oil. Note that LPG can only form an interface with gasoline. The same occurs with jet fuel.

Table 4
Refinery conditions

Product	Refinery conditions ($\times 10^{-2}$ m^3)	
	$VRZERO_p$	$VRMAX_{p,k}$/$VRMIN_{p,k}$
Gasoline (1)	1000	2500/300
Diesel oil (2)	1050	2250/300
LPG (3)	100	300/30
Jet fuel (4)	315	600/120

Three different demand scenarios are addressed in this section:

(S1) scenario of low product demands;
(S2) scenario of medium product demands;
(S3) scenario of high product demands.

The following models are considered:

(M1) the original formulation proposed by Rejowski Jr. and Pinto (2003);
(M2) the original formulation and the special constraints;
(M3) the original formulation proposed by the authors, the special constraints and the delivery cuts.

All cases were implemented in GAMS modeling language (Brooke, Kendrick, & Meeraus, 2000) in a PC platform with *Pentium II* 400 MHz processor. The MILP solver chosen was CPLEX (ILOG, 2000).

5.1. Scenario 1—low demand patterns

Scenario 1 (S1) is characterized by low demand patterns. Demands for this scenario are listed in Table 6.

For this scenario, most of the initial inventories at the depots are sufficient to satisfy the product demands. Table 7 provides computational results for formulations S1M1, S1M2 and S1M3. The first one corresponds to the original formulation proposed by Rejowski Jr. and Pinto (2003), whereas S1M2 considers the original formulation and the special constraints. Finally, case S1M3 comprises the original formulation, the special constraints and the delivery cuts.

Table 5
Capacity bounds and initial condition at depots

Depot	Product	Max/min capacities ($\times 10^{-2}$ m^3) VDMAX$_{p,d,k}$/VDMIN$_{p,d,k}$	Initial condition ($\times 10^{-2}$ m^3) VDZERO$_{p,d}$
Rib. Preto (1)	Gasoline	190/50	100
	Diesel	270/90	180
	LPG	120/20	90
	Jet fuel	0/0	0
Uberaba (2)	Gasoline	90/30	40
	Diesel	190/50	150
	LPG	0/0	0
	Jet fuel	0/0	0
Uberlândia (3)	Gasoline	90/20	50
	Diesel	270/90	180
	LPG	120/20	60
	Jet fuel	0/0	0
Goiânia (4)	Gasoline	190/50	110
	Diesel	720/150	350
	LPG	180/20	40
	Jet fuel	140/30	90
Brasília (5)	Gasoline	180/50	90
	Diesel	720/150	330
	LPG	92/20	40
	Jet fuel	136/25	110

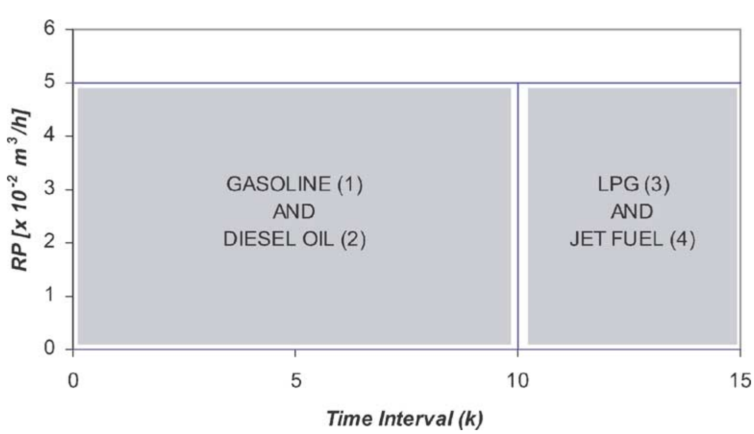

Fig. 6. Production profile in the refinery for all examples.

In all cases the optimal schedule is achieved. Although formulation S1M1 provides a good schedule, it would lead the products inside the pipeline to a very high contamination degree. In this case, products inside the pipeline must be transferred only to fulfill demands at the end of the time horizon.

Formulation S1M2 shows the effect of special constraints, which clearly reduces the feasible region. This is due to the fact that the pipeline must operate whenever an interface is detected. Thus, its optimal objective value is larger than the solution found in case S1M1. It is also desired that many shut-down and start up procedures be avoided along

Table 6
Demands for the proposed examples of scenario 1 (S1)

Product	DEM$_{p,1}$ ($\times 10^{-2}$ m^3)	DEM$_{p,2}$ ($\times 10^{-2}$ m^3)	DEM$_{p,3}$ ($\times 10^{-2}$ m^3)	DEM$_{p,4}$ ($\times 10^{-2}$ m^3)	DEM$_{p,5}$ ($\times 10^{-2}$ m^3)
Gasoline (1)	0	0	0	60	10
Diesel oil (2)	50	70	0	0	120
LPG (3)	0	0	0	50	0
Jet fuel (4)	0	0	0	0	0

Table 7
Results for scenario 1 (S1)

	Formulation		
	S1M1	S1M2	S1M3
Relaxed solution (C (US$ × 10^{-2}))	24,740.85	27,495.44	27,495.44
Solution found (C (US$ × 10^{-2}))	24,888.62	31,042.50	31,042.50
Gap (%)	0.000	0.000	0.000
CPU time (s)	124	250	250
Visited nodes	101	278	278

the horizon time. Interestingly, although the present formulation does not explicitly impose such condition, the optimal solution shows that the pipeline is operative in the first ten time intervals, whereas it remains deactivated in the last five time intervals. This occurs because of the differences in inventory costs at the refinery and at the depots. The optimal solution is encountered in 250 s. In this case, the interfaces formed by products inside the pipeline at the beginning of the operation ($k = 0$) must be removed in each of the segments of the pipeline before or at the end of the time horizon. Furthermore, in such cases, the special constraints cause an increase in the inventory levels for some of the products stored at the depots and sometimes these reach levels that are very close to the maximum capacity of the depot.

Key decisions for formulation S1M3 are the same as the ones considered in case S1M2. However, the delivery cuts proposed in Section 4.2 do not have any impact on the CPU performance. This can be seen in Table 7, where it can be observed that the value of the relaxed solution of case M2 and case M3 are the same, as well as the number of visited nodes.

5.2. Scenario 2—medium demand patterns

Table 8 lists values for a scenario characterized by medium level demand requirements. Note that one difference between scenarios S1 and S2 is that in the latter, depot 5 must take gasoline from the pipeline. Note also that the last segment of the pipeline is entirely fulfilled by diesel oil at the beginning of the operation ($k = 0$). Three cases are proposed for this scenario. Case S2M1 considers the original formulation proposed by Rejowski Jr. and Pinto (2003). Case S2M2 takes into account the original formulation plus the special constraints, whereas case S2M3 contemplates the original formulation, the special constraints and the delivery cuts proposed in Section 4.2. Computational results are shown in Table 9.

Table 9
Results for scenario 2 (S2)

	Formulation		
	S2M1	S2M2	S2M3
Relaxed solution (C (US$ × 10^{-2}))	25,669.02	27,125.49	28,072.47
Solution found (C (US$ × 10^{-2}))	30,199.75	31,401.62	31,401.62
Gap (%)	8.4	0.000	0.000
CPU time (s)	10,000	2,807	5,274
Visited nodes	938	5695	6507

Formulation S2M2 found the optimal solution that has a higher value than the solution found by case S2M1, which is simply a feasible one. The pumping costs in case S2M2 increase the value of the optimal solution, when compared to the solution obtained in case S2M1. It is important to note that the original formulation (S2M1) did not find the optimal solution for this scenario.

In case S2M2 the pipeline must be operated to supply demands at the last depot and to take interfaces out of all segments of the pipeline. Although this scenario is characterized by medium demand pattern, the special constraints impose that the pipeline must be operative in all time intervals. In this case the optimal solution is found in more than 2800 s. Note also from the comparison of cases S2M2 and S1M2 that the value of the objective function increases as demands for products at the depots become higher.

The optimal solution was also found for case S2M3. Surprisingly, the delivery cuts proposed in Section 4.2 provide a negative impact on the CPU performance. They improve the value of the relaxed solution, but a larger number of nodes is examined. The CPU time in case S2M3 is almost twice as large as that of case S2M2.

5.3. Scenario 3—high demand patterns

Table 10 provides data for scenario 3 in which high demand patterns are imposed for the example. Two series are proposed in this example.

There is only one difference between data for series 1 and 2 of this scenario. Data for series 2 are the same as the ones imposed for series 1, with exception of demand established for diesel oil at depot located at Ribeirão Preto ($d = 1$), which is shown in parenthesis in the current table.

Table 8
Demands for the proposed examples of scenario 2 (S2)

Product	$DEM_{p,1}$ ($\times 10^{-2}$ m^3)	$DEM_{p,2}$ ($\times 10^{-2}$ m^3)	$DEM_{p,3}$ ($\times 10^{-2}$ m^3)	$DEM_{p,4}$ ($\times 10^{-2}$ m^3)	$DEM_{p,5}$ ($\times 10^{-2}$ m^3)
Gasoline (1)	0	0	0	300	80
Diesel oil (2)	50	70	0	0	120
LPG (3)	0	0	0	50	0
Jet fuel (4)	0	0	0	0	0

Table 10
Demands for the proposed examples of scenario 3 (S3)

Product	$DEM_{p,1}$ ($\times 10^{-2}$ m^3)	$DEM_{p,2}$ ($\times 10^{-2}$ m^3)	$DEM_{p,3}$ ($\times 10^{-2}$ m^3)	$DEM_{p,4}$ ($\times 10^{-2}$ m^3)	$DEM_{p,5}$ ($\times 10^{-2}$ m^3)
Gasoline (1)	0	0	0	300	80
Diesel oil (2)	90 (150)	150	0	0	120
LPG (3)	0	0	0	50	0
Jet fuel (4)	0	0	0	0	0

From Tables 3, 5 and 10 it can be noted that there are clearly high transfer requirements for some of the products and depots. An example of this situation is composed by gasoline and the depots located at Ribeirão Preto ($d = 1$) and Brasília ($d = 5$). These depots must take this product at least once from the pipeline, but initially their respective segments do not contain any gasoline. Results for all cases of both series are shown in Table 11. Formulations S3.1M1 and S3.2M1 are as the same as the original formulation presented by Rejowski Jr. and Pinto (2003). Formulations S3.1M2 and S3.2M2 show the effect of the special constraints on the original formulation, whereas cases S3.1M3 and S3.2M3 show the effect the special constraints and the delivery cuts shown in Section 4.2.

In examples S3.1M1 and S3.2M1, a solution with 5.6% gap is found in 10,000 s. Although the original formulation provides a good schedule for both cases, products inside the pipeline would be completely contaminated due to the lack of the special constraints.

Examples S3.1M2 and S3.2M2 show the effect of special constraints in this scenario. Note that the value of the optimal solution is higher when compared to scenario 1 but it is lower than the value obtained for scenario 2. For this scenario, the pipeline is operative at all time intervals. However, as demands become higher, there is a decrease in the inventory costs at the depots. For series 1, the optimal solution is found in more than 7700 s and in more than 5300 s for series 2.

In cases S3.1M3 and S3.2M3, the value of the relaxed solution is improved when compared to the one obtained by cases S3.1M2 and S3.2M2, respectively. The addition of the delivery cuts drastically reduces the CPU time to approximately 1200 s (84%) for case S3.1M3. For case S3.2M3, the CPU time is decreased to nearly 1800 s (67%).

Figs. 7 and 8 show operational results for formulations S3.2M2 and S3.2M3.

Fig. 8 shows that a given segment and its previous one are activated whenever an interface is inside of it. In the first two time intervals of the operation, for both cases, the last two packs of the fourth segment of the pipeline, which initially comprises LPG are integrally sent to its respective depot. This is expected because if one of them were split between its respective depot and the last segment of the pipeline, it would form a forbidden sequence of derivatives inside the fifth segment. After this, diesel oil amounts are sent to the depot at Uberaba ($d = 3$). Subsequently, large amounts of gasoline are integrally delivered to the depot located at Goiânia ($d = 4$) and then split between this depot and the last segment of the pipeline. Note that in the last depot, diesel oil demand is satisfied before the end of the operation. Finally, the first depot is supplied with diesel oil at the last time interval.

For all cases in scenarios of medium and high demand patterns, the delivery cuts improve the relaxed solution for the problem, when compared to cases that do not consider them. This occurs because both the special constraints and the delivery cuts impose valid lower limits for the number of time intervals that the segments of the pipeline must be activated. The first set imposes these lower limits due to operational characteristics of the transportation problem, which is motivated by the contamination of the products inside the pipeline segments.

On the other hand, delivery cuts provide lower bounds by means of relations between demand patterns and initial inventory condition of the pipeline segments. Nevertheless, for medium to low demand scenarios, the inclusion of the delivery cuts may not be beneficial. For instance, in scenario 2 the improvement on the value of the relaxed solution of the problems provided by the delivery cuts is not sufficient to guarantee a better CPU performance. However, when they are considered in high demand examples the delivery cuts

Table 11
Results for scenario 3 (S3)

	Series 1			Series 2		
Formulation	S3.1M1	S3.1M2	S3.1M3	S3.2M1	S3.2M2	S3.2M3
Relaxed solution*	25,170.36	26,485.34	27,423.25	24,999.60	26,082.80	27,277.89
Solution found*	29,267.33	30,196.22	30,196.22	29,333.92	30,245.97	30,245.97
Gap (%)	5.6	0.000	0.000	5.6	0.000	0.000
CPU time (s)	10,000	7,775	1,209	10,000	5,386	1,775
Visited nodes	6,482	19,716	2,698	5,258	13,758	5,007

* Unit for the values of the solution found: (C ($\times 10^{-2}$ US$)).

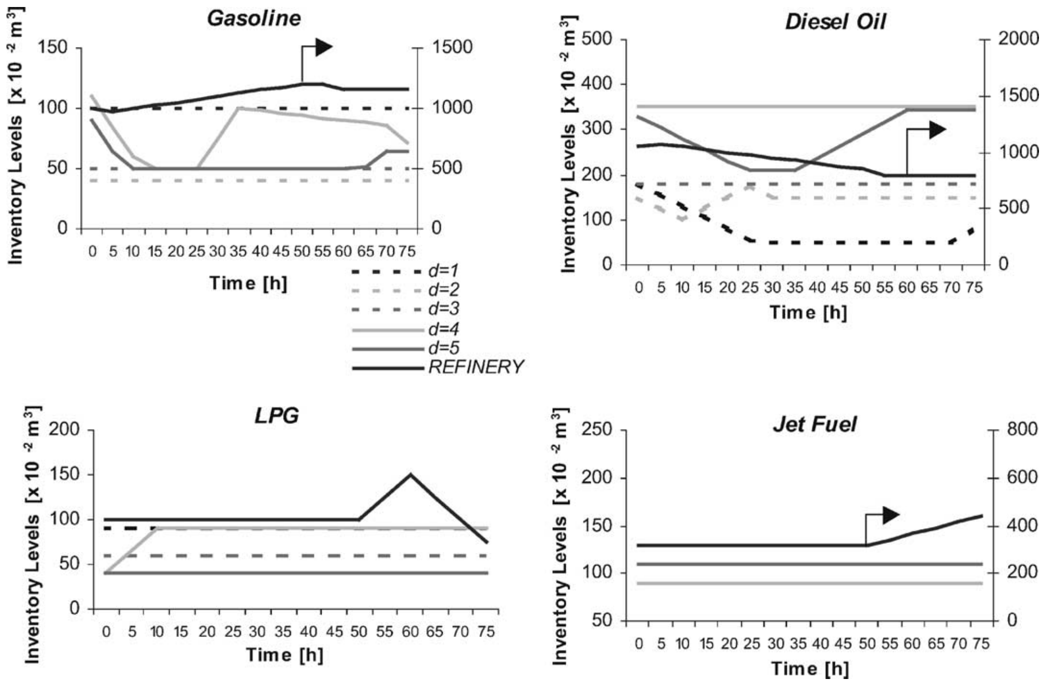

Fig. 7. Inventory levels at all locations for examples S3.2M2 and S3.2M3.

not only improve the value of the relaxed solution, but they drastically reduce the CPU time for the problems.

5.4. Model M4—original formulation plus delivery cuts

Finally, another interesting feature of the delivery cuts is that they can be in principle applied to the model regardless of the special constraints (model M4). They can be added to the original formulation proposed by Rejowski Jr. and Pinto (2003) to verify how the non-optimal solutions obtained in the previous sections can be improved. Table 12 lists all previous examples for which only feasible solutions were encountered and their maximum relative gap obtained from the optimal solution. They comprise all original formulations tested in the previous sections with exception of the low demand scenario. Furthermore, Table 12 also illustrates the solutions obtained by their respective formulations M4 and their maximum gap from the optimal solution. A CPU time of 10,000 s is fixed for all examples listed in this table.

Table 12 shows that the computational performance from models M4 are much better than the original formulation proposed by Rejowski Jr. and Pinto (2003). Nevertheless, the optimal solution cannot be found within the CPU time stipulated for all cases related in this table. For these examples, the delivery cuts improved the value of the relaxed solution, the quality of the solution obtained and consequently the maximum relative gap from the best integer solution. When the special constraints are dropped from the formulation the problems present an enormous number of feasible solutions, even for high demand patterns (scenario 3), for that the pipeline should be activated in almost all time intervals. Moreover, the absence of the special constraints in the formulation implies that the problems of medium demand patterns (scenario 2) become the most difficult to solve. Once the pipeline must be operated discontinuously, the number of alternatives to find the optimal solution is even larger when compared to the problems of high demand patterns. The transportation of the products by means of the pipeline in this scenario can be made in the beginning of the operation,

Table 12
Results for the original formulation and model M4

	Scenario 2			Scenario 3		
Formulation	S2M1	S2M4	S3.1M1	S3.1M4	S3.1M1	S3.2M4
Relaxed solution*	25,669.02	26,789.09	25,170.36	26,286.80	24,999.60	26,143.89
Solution found*	30,199.75	29,838,40	29,267.33	29,176.65	29,333.92	28,932.56
Gap (%)	8.4	5.9	5.6	2.7	5.6	2.3

* Unit for the values of the solution found: (C ($\times 10^{-2}$ US$)).

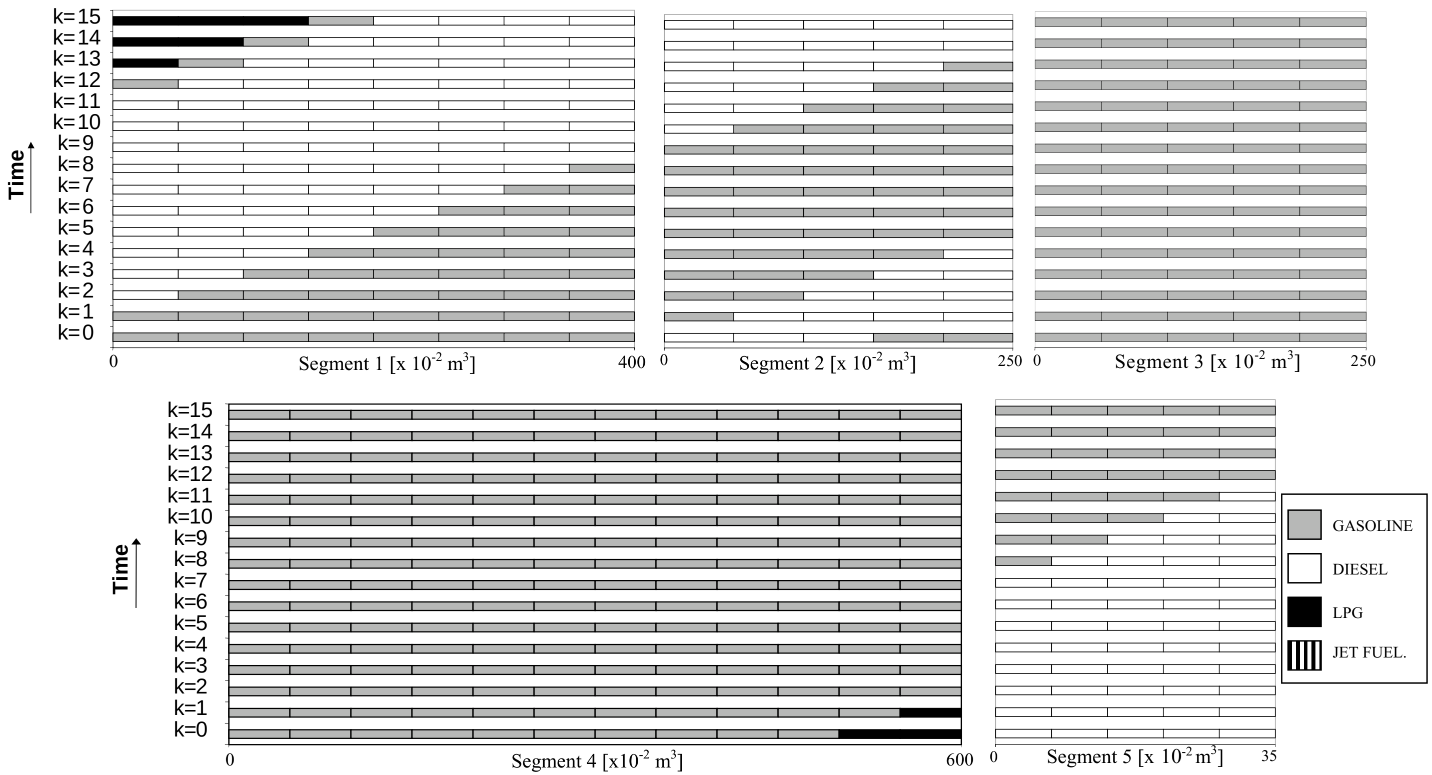

Fig. 8. Pipeline transportation profile for examples S3.2M2 and S3.2M3.

or near at the end, or it can still be uniformly distributed along the whole time horizon. That is the main cause for a larger maximum relative gap for problem of scenario 2 when compared to problems of scenario 3.

6. Conclusions

Several improvements were achieved on the formulation of the problem of scheduling of multiproduct pipeline systems. The problem addressed relates a real-world system from Petrobras that comprises an oil refinery, one pipeline and several depots connected to it. One of the major concerns of this problem is the contamination of the products inside the pipeline segments. Another important issue is the poor computational performance for the majority of the cases that were presented.

The contamination of the products becomes critical when the segments of the pipeline store two or more products and at the same time they are inactivated. To overcome this obstacle, a set of non-intuitive and practical constraints was added to the original MILP formulation proposed by Rejowski Jr. and Pinto (2003). The constraints, which impose that a segment of the pipeline can be stopped only if it stores exactly one product, consider initial interfaces that are present at the beginning of the operation and the ones that are formed during the time horizon. For all examples presented, they find the optimal solution with a higher value when compared to a feasible one of the respective formulations without this assumption. The constraints increase the pumping costs because they force the segments of the pipeline to operate whenever an interface is detected. Furthermore, inventory costs at the depots are also increased, especially for low demand scenario cases. The special constraints also give a better computational performance for medium and high demand scenario cases.

Another improvement on the formulation was the generation of delivery cuts that relate the product demands established at the depots and the initial amounts stored in the respective segment of the pipeline. These cuts were imposed as integer cuts and they determined lower bounds on the number of times that a segment of the pipeline must operate within the time horizon. They were added to the formulation with the special constraints and to the original formulation proposed by Rejowski Jr. and Pinto (2003). When considered on the formulation with the special constraints, they had no effect on the computational performance for low demand scenario cases. For medium demand cases, although they improved the value of the relaxed solution, the computational performance was poor. For high demand scenario cases, which were the most difficult problems of solving, they generated better values for the relaxed solution. Moreover, they reduced drastically the CPU time of all examples of this scenario. These improvements reached at least almost 70% when compared to the formulations that did not considered this set of valid cuts. When they were included on the original formulation, they generated better values for the relaxed solution for medium and high demand scenario cases and better feasible solutions were found within the CPU time stipulated. Despite the efficiency of the proposed cuts, other valid constraints may generate tight bounds for the operation and deactivation of the pipeline in such operational systems. However, the main objective is to reduce computational effort, and there is a trade-off between the improvement that cuts generate on the relaxed problem and the number of constraints that they consequently add to the model.

Although significant improvements were achieved in the present formulation, there is a necessity to investigate formulations that take into account hydraulic considerations such as friction and potential losses along the pipeline extension and the enormous amounts of energy consumed by the booster stations. Nonetheless, special decomposition techniques for solving large mixed integer problems must be considered due to the significant increase in the number of the decision variables of this practically relevant scheduling problem.

Acknowledgements

The authors would like to acknowledge the support received from FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) under grant 01/10944-9.

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