ECE 4960 Spring 2018: Computational and Software Engineering
Reading 2: Differentiation in Local Analysis
Deposit a pdf file of the two tables below to your Git directory before 11:59pm of 2/11

Document your programming environment: Language; development platform; operating system
Prob. 1. (Quadratic function to observe the tradeoffs between the truncation error and round-off error): For $f(x) = x^2$, we know the exact $f'(x=1) = 2$.
1.1 Use Eq. (1) below to estimate $f'(x=1)$ varying the value of h from 0.1 to 10^{-18} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
1.2 Repeat your calculation with $f(x) = x^2 + 10^8$. Add your results to the same table.
1.3 Repeat the above two procedures by using Eq. (2). Add your results to the same table.

$$f'(x) = \frac{f(x+h)-f(x)}{h} + O(h) \qquad (1)$$

$$f'(x) = \frac{f(x+h)-f(x-h)}{2h} + O(h^2) \qquad (2)$$

h	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (1) where $f(x) = x^2 + 10^8$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2$	Error in $f'(x=1)$ by Eq. (2) where $f(x) = x^2 + 10^8$
10^{-1}				
10^{-2}				
10^{-3}				
...				
10^{-18}				

```
-0.1           -4.44089e-16    -0.0999908      1.90735e-06
-0.01          -1.33227e-15    -0.00996399     4.95911e-05
-0.001         1.65201e-13     -0.000808716    0.000144958
-0.0001        7.76046e-13     -0.00271606     -0.00271606
-1e-05         -1.99929e-12    0.0926514       0.0926514
-9.99924e-07   -1.99929e-12    0.0926514       0.0926514
-1.01088e-07   -5.75104e-11    2               2
1.21549e-08    6.60383e-09     2               2
-1.65481e-07   -5.44584e-08    2               2
-1.65481e-07   -1.65481e-07    2               2
-1.65481e-07   -1.65481e-07    2               2
-0.000177801   -6.67789e-05    2               2
0.00159856     0.000488333     2               2
0.00159856     0.00159856      2               2
-0.220446      -0.109424       2               2
2              0.889777        2               2
2              2               2               2
2              2               2               2
```

Prob. 2. (Cubic function to observe the Richardson error estimation): For $f(x) = x^3$, we know the exact value of $f'(x=1) = 3$.
2.1 Use Eqs. (3) – (5) below to estimate $f'(x=1)$ varying the value of h from 2^{-4} to 2^{-40} to observe the relative error in calculating $f'(x)$. Tabulate your results with sufficient precision in a table.
2.2 Estimate η from Eqs. (6) and (7) for each choice of h. Add your results to the same table.

$$f'(x) = \frac{f(x+h)-f(x)}{h} + E(h); \quad E(h) = O(h) = \frac{1}{2}hf''(x) + O(h^2) \qquad (3)$$

$$f'(x) = \frac{f(x+2h)-f(x)}{2h} + E(2h); \quad E(2h) = O(h) = \frac{1}{2}2hf''(x) + O(h^2) \qquad (4)$$

$$f'(x) = \frac{-1}{2h}f(x+2h) - \frac{3}{2h}f(x) + \frac{2}{h}f(x+h) + O(h^2) \qquad (5)$$

$$R(h) \equiv \frac{E(2h)}{E(h)} \cong \eta \qquad (6)$$

$$R(h) \cong \frac{\hat{A}(4h) - \hat{A}(2h)}{\hat{A}(2h) - \hat{A}(h)} \cong \eta \qquad (7)$$

h	Error in $f'(x=1)$ by Eq. (3)	Error in $f'(x=1)$ by Eq. (4)	Error in $f'(x=1)$ by Eq. (5)	η by Eq. (6)	η by Eq. (7)
2^{-4}					
2^{-5}					
2^{-6}					
…					
2^{-40}					

```
0.199219        0.410156        0.193359        2.11765      2.11765
0.0966797       0.196289        0.0952148       2.06061      2.06061
0.0476074       0.0959473       0.0472412       2.03077      2.03077
0.0236206       0.0474243       0.0235291       2.0155       2.0155
0.0117645       0.0235748       0.0117416       2.00778      2.00778
0.00587082      0.0117531       0.0058651       2.0039       2.0039
0.00293255      0.00586796      0.00293112      2.00195      2.00195
0.00146556      0.00293183      0.0014652       2.00098      2.00098
0.000732601     0.00146538      0.000732511     2.00049      2.00049
0.000366256     0.000732556     0.000366233     2.00024      2.00024
0.000183117     0.000366244     0.000183111     2.00012      2.00012
9.15555e-05     0.000183114     9.15541e-05     2.00006      2.00006
4.57771e-05     9.15548e-05     4.57767e-05     2.00003      2.00003
2.28884e-05     4.57769e-05     2.28883e-05     2.00002      2.00002
1.14442e-05     2.28883e-05     1.14441e-05     2.00001      2.00001
5.72205e-06     1.14442e-05     5.72205e-06     2.00001      2.00001
2.86102e-06     5.72205e-06     2.86102e-06     2            2
1.43051e-06     2.86102e-06     1.43051e-06     2            2
7.15256e-07     1.43051e-06     7.15256e-07     2            2
3.57628e-07     7.15256e-07     3.57628e-07     2            2
1.78814e-07     3.57628e-07     1.78814e-07     2            2
8.9407e-08      1.78814e-07     8.9407e-08      2            2
4.47035e-08     8.9407e-08      4.47035e-08     2            2
1.49012e-08     4.47035e-08     0               3            3
2.98023e-08     2.98023e-08     5.96046e-08     0.5          0.5
0               0               0               inf          inf
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
0               0               0               nan          nan
```