Compact SPICE for ODE solution

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Program Organization

In this program, we implemented three ODE solving methods: forward Euler, RK4, and time adaptive RK34. The ODE solvers can take ODEs of any dimensions and any typed equations.

1. forwardEuler.cpp/.h
forwardEuler.cpp contains the forward Euler ODE solver.

2. RK34.cpp/.h
RK34.cpp contains RK3, RK4, and time adaptive RK34 ODE solvers. It also contains a slope function generator which is used by all three methods mentioned.

3. circuit.cpp/.h
circuit.cpp contains the RC circuit ODE and the CS amplifier circuit ODE. It also contains the current source generator and the EKV equation for the circuits.

4. test.cpp/.h
test.cpp contains the single variable ODE and its known solution (Task 3). It also contains a small helper function that calculates percent error.

5. main.cpp
main.cpp provide three example uses of the ODE solver. First is a single variable ODE with known solution. The percent errors compared with ground truth are reported. Second is the AC circuit ODE, results from the three methods are reported and compared side by side. Third is the CS amplifier ODE, results from the three methods are reported and compared side by side.

6. external library

We used the same linear algebra library Eigen as in the last project for easy operations on vectors and matrices. All of our data structure (such as vectors that stores x variables and matrices that store solutions) are stored as Eigen matrices.

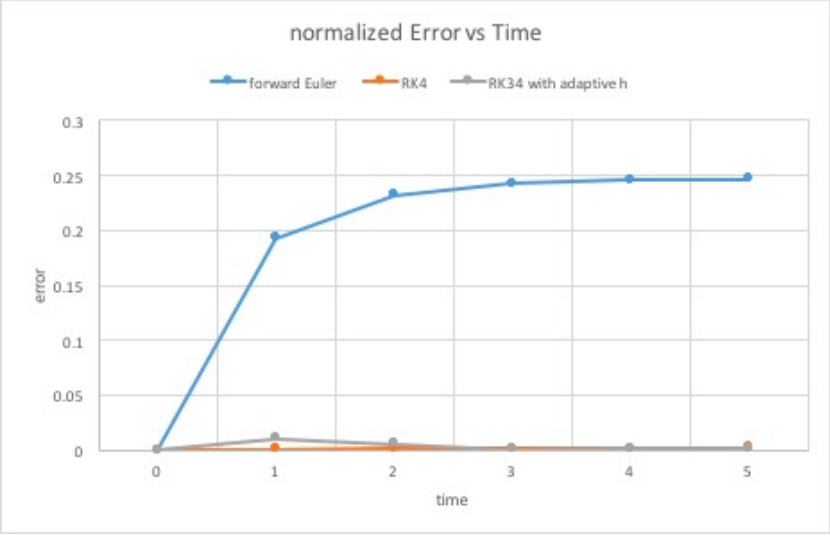
7. inherited code from previous programs (Task 2)

We inherited many codes from our previous hacker practices such as forward Euler and RK34 ODE solvers. The single variable ODE was also solved and tested in previous hacker practices, so we inherited that also. Another part of the program that we inherited from the last project was the EKV model used to solver the CS amplifier.

Test Problem Result (Task 3)

We validate our ODE solvers using a single variable ODE with known solution. Noticed that the forward Euler gives the worst accuracy compared to ground truth while RK4 and RK34 with adaptive h give better solutions and the error is relatively small. As time increases. RK34 with time adaptation gives better performance as time goes on although it performed slightly worse than RK4 at the beginning.

Also noticed that error of forward Euler is an incremental function, which means the error is accumulate with the time. The error accumulates and plateaus as time increases.



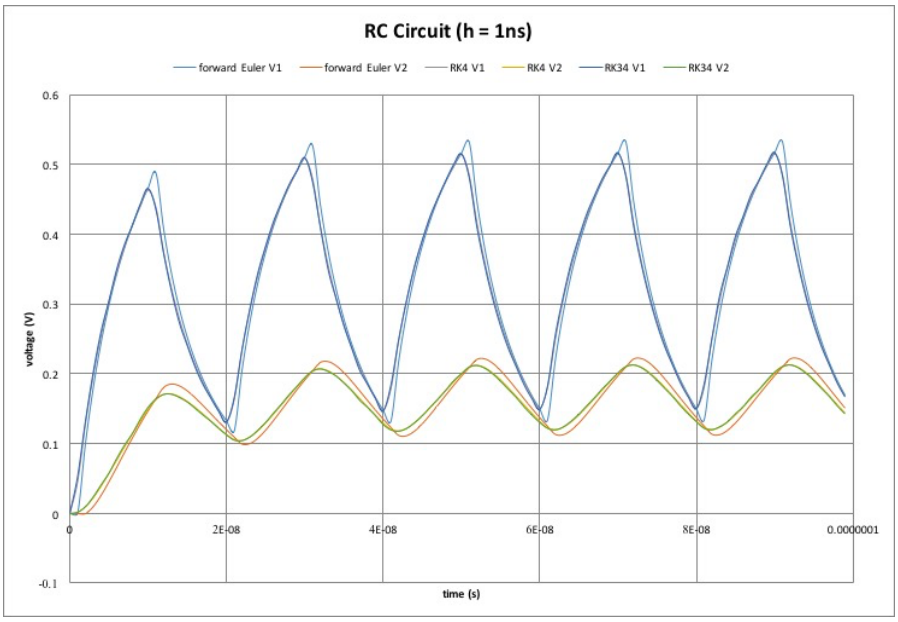
RC Circuit Result (Task 4)

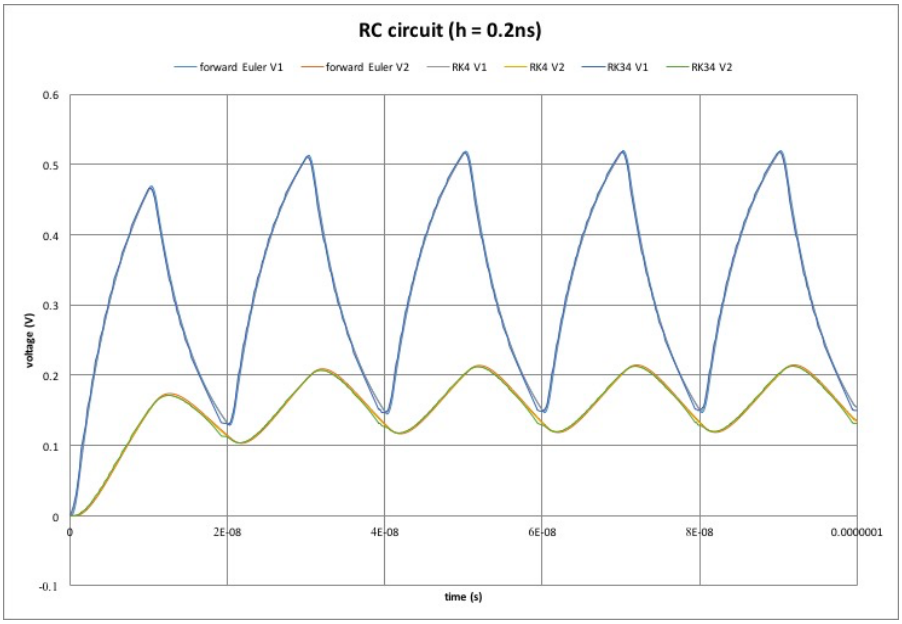
In this task, we solved a RC circuits with a given current source which has a period of 20ns. The resulted voltage 1 and voltage 2 also show periodic changes, as triangle waves. Result of forward Euler is slightly different from result of RK4 and RK34 with adaptive h. That is because forward Euler is equivalent to RK1, which is only first order accurate. RK4 and RK34 are fourth order accurate.

When we make the matching steps smaller, we can observe the result from different methods are closer to each other, especially forward Euler method which improved accuracy a lot. We thus conclude that the smaller time step the higher accuracy, but also for the cost of larger computation time and space.

The shape of the figure shows the charge and discharge of the circuit. Register and capacitor can temporarily store energy from the current source.

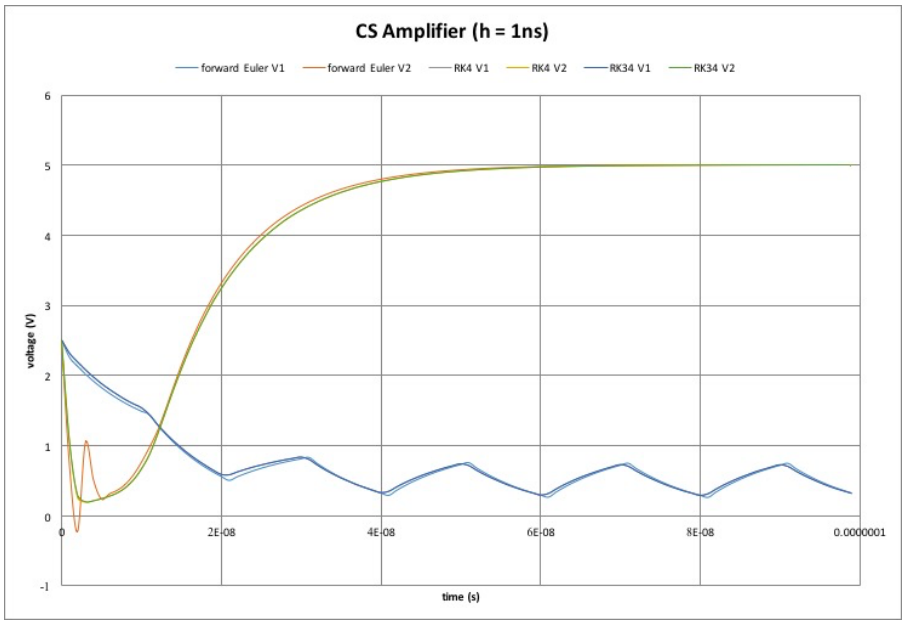
Interesting debug issue: originally, we plot the figure with periodic increasing voltage, which reaches around 50 volts in the end. As this is a real circuit, the voltage cannot be that large. Turns out we give the circuit an increasing current with time. It gives the reasonable result after we modifies the current.

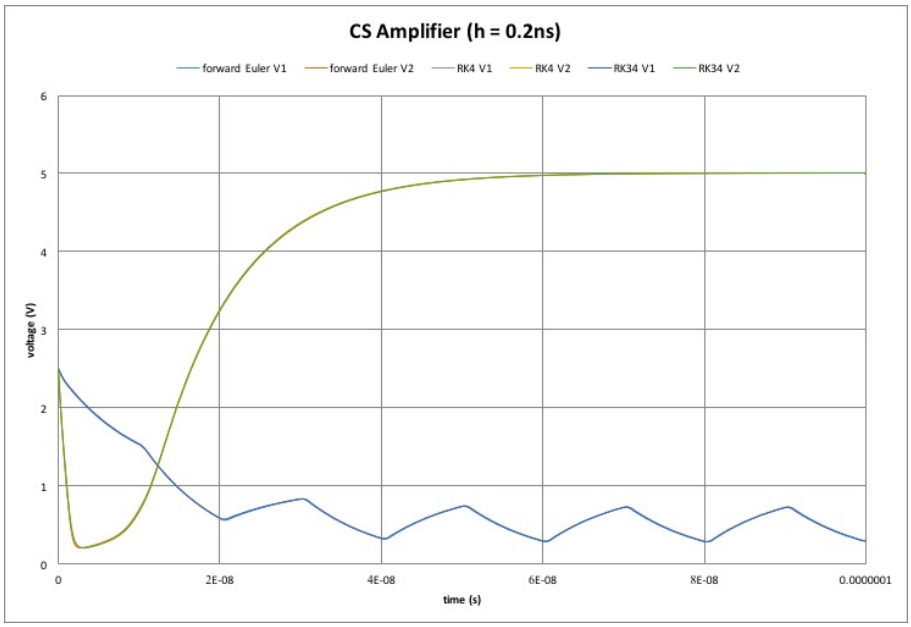




CS Amplifier Result (Task 5)

In this task, we solved a common source amplifier with small signal model. The amplifier will amplify v1 to v2, the gain is around 10. V1 is the input signal, v2 is the amplified signal. As time goes on, the amplified signal becomes stable and finally reaches around 5 volts. Also notice that forward Euler is less accurate than RK4 and RK34 with adaptive h. And forward Euler will oscillate at the beginning with a large h. Smaller time step give more accurate result for forward Euler and RK4, so the plots overlap more.


CS Amplifier (h = 1ns)
forward Euler V1
forward Euler V2
RK4 V1
RK4 V2
RK34 V1
RK34 V2
voltage (V)
time (s)
6
5
4
3
2
1
0
-1
0
2E-08
4E-08
6E-08
8E-08
0.0000001


CS Amplifier (h = 0.2ns)
forward Euler V1
forward Euler V2
RK4 V1
RK4 V2
RK34 V1
RK34 V2
voltage (V)
time (s)
6
5
4
3
2
1
0
0
2E-08
4E-08
6E-08
8E-08
0.0000001