NOTES ON STATISTICAL PHYSICS OF SOFT AND BIOLOGICAL MATTER

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ABSTRACT

These are personal notes on *Statistical Physics of Soft and Biological Matter*, for personal review only. This is NOT an origianl work, all the ideas and texts may be from some other sources, mainly lecture notes by Prof. Dr. Gerhard Gompper.

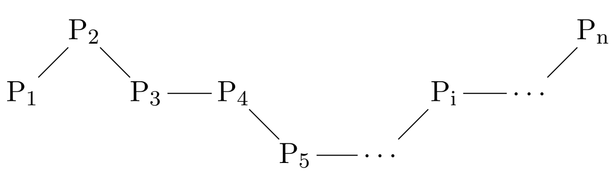
1. Introduction

2. Physics of Polymers

Polymers are chains of repetitive units (monomers), homopolymers is defined as $(\cdots)_n$ where n is degree of the polymer.

2.1. Conformation of a Polymer Chain

This is a model for fully flexible chain without self avoidance.



If the bond length $\mathrm{P_i} - \mathrm{P_{i+1}}$ is l, $L = Nl_0$ is then the contour length. While the end-to-end length is:

$$\vec{R}_l = \vec{R}_N - \vec{R}_0 = \sum_{i=1}^{N} \vec{r}_i \tag{2.1}$$

Where $\vec{R}_N$ and $\vec{R}_0$ are the vector from original point to the ends.

$$\langle \vec{R}_l \rangle = \sum_i \langle \vec{r}_i \rangle = 0 \tag{2.2}$$

$$\langle (\vec{R}_l)^2 \rangle = \sum_{ij} \langle \vec{r}_i \vec{r}_j \rangle = \sum_i \langle (\vec{r}_i)^2 \rangle + \sum_{i \neq j} \langle \vec{r}_i \vec{r}_j \rangle \tag{2.3}$$

$$= N l_0^2 \tag{2.4}$$

because that $\sum_{i \neq j} \langle \vec{r}_i \vec{r}_j \rangle = 0$.

2.2. Radius of Gyration

$$R_g^2 = \frac{1}{2N^2} \sum_i \sum_j (\vec{R}_i - \vec{R}_j)^2 \tag{2.5}$$

$$\langle (\vec{R}_i - \vec{R}_j) \rangle = \sum_{n,m=i+1}^{j} \langle \vec{R}_n \cdot \vec{R}_m \rangle = \sum_{n=i+1}^{j} \langle (\vec{R}_n)^2 \rangle \tag{2.6}$$

$$= |j - i| l_0^2 \tag{2.7}$$

$$\langle R_g^2 \rangle = \frac{1}{2N^2} \sum_i \sum_j |j - i| l_0^2 \tag{2.8}$$

$$= \frac{1}{2N^2} \int_0^N \mathrm{d}x \int_0^N \mathrm{d}y |x - y| l_0^2 \tag{2.9}$$

$$= \frac{l_0^2}{2N^2} \int_0^N \mathrm{d}x \left(x^2 - Nx + \frac{1}{2} N^2 \right) \tag{2.10}$$

$$= \frac{l_0^2}{2N^2} \left(\frac{1}{3} N^3 - \frac{1}{2} N^2 + \frac{1}{2} N^2 \right) \tag{2.11}$$

$$= \frac{1}{6} N l_0^2 \tag{2.12}$$

$$\langle R_g^2 \rangle = \frac{1}{6} \langle R_l^2 \rangle \tag{2.13}$$

2.3. Free Rotating Chain

The angle is defined as:

$$\langle \vec{r}_i \cdot \vec{r}_{i-1} \rangle = l_0^2 \cos\theta \tag{2.14}$$

The average projection of $\vec{r}_m$ can be denoted as a vector:

$$\langle \vec{r}_m \rangle_m = \vec{r}_{m-1} \cos\theta \tag{2.15}$$

$$\begin{aligned}
\langle \vec{r}_n \cdot \vec{r}_m \rangle &= \vec{r}_n \cdot \vec{r}_{m-1} \cos\theta && (2.16)\\
&= \vec{r}_n \cdot \vec{r}_{m-2} \cos^2\theta && (2.17)\\
&= (\vec{r}_n)^2 \cos^{|m-n|}\theta && (2.18)\\
\langle \vec{r}_n \cdot \vec{r}_m \rangle &= l_0^2 \cos^{|m-n|}\theta && (2.19)\\
& && (2.20)
\end{aligned}$$

$$\begin{aligned}
\langle (\vec{R}_l)^2 \rangle &= \sum_{m,n=1}^{N} \langle \vec{r}_n \cdot \vec{r}_m \rangle && (2.21)\\
&= \sum_{n=1}^{N} \sum_{k=1-n}^{N-n} \langle \vec{r}_n \cdot \vec{r}_{n+k} \rangle && (2.22)\\
&\approx l_0^2 \sum_{n=1}^{N} \sum_{k=-\infty}^{\infty} \cos^k\theta \ (N \gg 1) && (2.23)\\
&= l_0^2 \sum_{n=1}^{N} \left(1 + 2 \sum_{k=1}^{\infty} \cos^k\theta \right) && (2.24)\\
&= l_0^2 \sum_{n=1}^{N} \frac{1+\cos\theta}{1-\cos\theta} && (2.25)\\
&= N l_0^2 \frac{1+\cos\theta}{1-\cos\theta} && (2.26)
\end{aligned}$$

Therefore:

$$\sqrt{\langle (\vec{R}_l)^2 \rangle} \sim N^{1/2} \tag{2.27}$$

When $\theta \to 0$ then $\langle (\vec{R}_l)^2 \rangle \to \infty$ while $\langle (\vec{R}_l)^2 \rangle = N l_0^2$.

For small θ:

$$\langle \vec{r}_1 \cdot \vec{R}_i \rangle = \sum_{j=1}^{N} \langle \vec{r}_1 \cdot \vec{r}_j \rangle = l_0^2 + \sum_{j=2}^{N} \langle \vec{r}_1 \cdot \vec{r}_j \rangle \tag{2.28}$$

$$= l_0^2 \sum_{j=1}^{N} \cos^{i-1}\theta \overset{N \to \infty}{\equiv} l_0^2 \frac{1}{1-\cos\theta} \tag{2.29}$$

$$\equiv \xi_p - l_0 \tag{2.30}$$

The persistence length, ξ_p, is:

$$\xi_p \simeq 2 l_0 \theta^{-2}, \ \theta \ll 1 \tag{2.31}$$