Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		2105363

Rapid Bushfire Response for Emergency Response

Summary

How to respond and deal with fires in time when they occur is a question worth thinking about. This paper provides a fire response plan for Victoria through the rational deployment of EOC, drones and forward teams.

In Task 1, the paper establishes the area safety evaluation model with fire frequency, size, and recent fire situation as indicators to classify the danger levels of different areas in Victoria. Then, we confirm that drones should provide different services for high-risk and dangerous areas. In order to increase the service capacity, we optimize both capacity and cost. For capability, we make the average response time of SSA drones to high-risk areas as short as possible, and reserve as many SSA drones to dangerous areas as possible. For cost, we quantify the demand for SSA drones in terms of fire acreage, and take into account the rounds and the attrition rate of drones. We also consider the mix between SSA drones and radio repeater drones, and calculate the number of repeater drones using a greedy mix-based maximum number solving algorithm. The total cost is calculated and finally the quantity optimization model based on the maximum mix rate and minimum cost is obtained. **200 SSA drones and 32 radio repeater drones are needed, respectively.**

In Task 2, we consider countermeasures in terms of both predicting extreme fire events and increasing equipment. We first build a ternary commensurable fire prediction model to predict the probability and magnitude of extreme fire events. Then, we use the expectation of the number of extreme fires to estimate the amount of equipment that should be added to the EOC. Also, based on the size of the fire and the number of firefighters' equipment, we obtain a probability of success formula for extreme fires, and the expected value of the additional equipment, given that the success rate of each extreme fire rescue by the "boots on the ground" team is 80%. The unit cost of the equipment is refined with relevant information to obtain the expected value of the increased equipment cost. **So the total cost of additional equipment is A$4,952.25.**

In Task 3, we consider signal range loss and drone safety to optimize the distribution range of repeater drones. First, we establish a signal loss model with improved Standard Propagation Model (SPM) model to quantify the signal-to-noise ratio of the repeater drone's signal around obstacles of different heights, and eliminate the locations where the repeater signal-to-noise ratio is less than 20 dB. Then, we build a dynamics-based fire spread model to calculate the edge length of fire in each direction. The locations where the distance between the repeater drone and the fire edge is less than 50m are excluded. Finally, we simulate three locations at different altitudes, and finds that **the wind direction directly affects the distribution of the optimized repeater drones, which is concentrated at the edge of the ideal range perpendicular to the wind direction.**

We verify the robustness of the model by sensitivity analysis with respect to the parameters affecting the safety factors, and find that the recent fire conditions have a strong influence. We analyze the influence of different fire frequencies and regions on the number of SSA drones, and the results with higher sensitivity are, **fire frequency in the [1600,2000] interval** and **located in the central region of Victoria**, respectively. Finally, an annotated Budget Request is provided by collating all the expenses.

Keywords: safety factor commensurable signal-to-noise ratio

Contents

1 Introduction

1.1 Problem Background

Australia's hot and dry climate, with many hills and few mountains, makes it one of the most forest fire-prone places in the world. Since the first fire in New South Wales in September 2019, New South Wales and eastern Victoria was severly affected.

Firefighters have used drones for surveillance and situational awareness (SSA) to allow the Emergency Operations Center (EOC) best direct active crews for optimal effect and maximal safety. The main task of drones is to monitor places where fires are violent and where it is easy to grow into active fires.

1.2 Restatement of the Problem

After understanding the relevant background information, the team are supposed to do the following:

- Create a model to determine the optimal numbers and mix of SSA drones and Radio Repeater drones.
- Illustrate how your model adapts to the changing likelihood of extreme fire events over the next decade. Project what equipment cost increases will occur assuming the cost of drone systems stays constant.
- Determine a model for optimizing the locations of hovering VHF/UHF radio-repeater drones.

2 Preparation of the Models

2.1 Assumptions and Justifications

1. **The EOC can be deployed and moved to locations as needed for firefighting missions, with negligible cost of deployment and movement. And it is responsible for fire rescue situations in only one area at a time.**

   EOC can be moved and deployed according to the fire situation, and its cost is labor cost and equipment cost. The labor cost is more complicated and is a fixed expenditure for the fire department, so it is not considered.

2. **When calculating the distance we consider that the distance is a straight line distance in a spatial three-dimensional coordinate system.**

   There is a big difference between the altitude of the Victorian mountains and the plains, so the distance obtained by considering the altitude is more accurate.

3. **The repeaters can receive signals from multiple targets and forward them to multiple targets.**

   This is determined by the function of the repeater. A repeater located between the front lines and the EOC can relay radio signals both from the front lines to the EOC and from the EOC to the front lines.

4. **Drones only serve part of the fire.**

   Fires in Australia are common, thus it is less likely to attract widespread attention when an average sized fire occurs. And can be quickly extinguished by local fire forces, without dedicated drone deployment.

5. **The drone needs to adjust its position when working.**

   To avoid damage to the drone caused by the effects of fire.

2.2 Notations

The primary notations used in this paper are listed in **Table 1**.

Table 1: Notations

Symbol	Definition
r	drone scanning radius
(x, y)	a place with longitude x and latitude y
$f(x, y)$	fire frequency at (x, y)
$size(x, y)$	fire size at (x, y)
$situation(x, y)$	recent fire situation at (x, t)
$S(x, y)$	safety factor at (x, y)
x_i	the distance between the EOC and the high-risk area
$Cost(\text{drone})$	total cost of two types of drones
m	number of SSA drones
n	number radio repeater drones
L_2	signal-to-noise ratio

2.3 Drones voyage model

Considering the drone power problem, the flight range of the drones are related to the average velocity (only consider single trip), and the relationship between them is parabolic. For practical considerations, we formulate that the average speed of the drone cannot exceed $10m/s$ (i.e., $\max\{v\} = 10m/s$). When the drone is in the maximum flight time (i.e. 2.5 hours), the drone range is maximum, and the average velocity of the drone is $30\text{km}/2.5\text{h} \approx 3.3m/s$. The corresponding relationship is shown in the figure below.

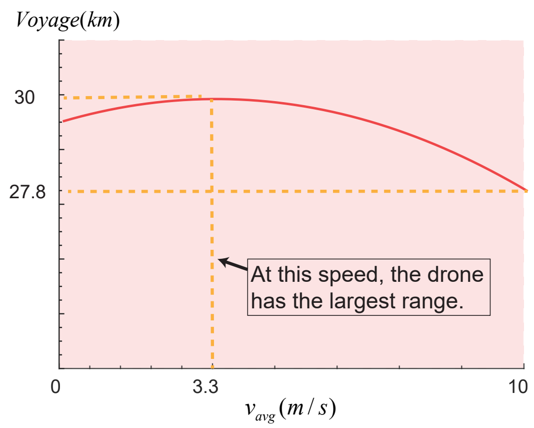


Figure 1: Relationship of $Voyage$ and $v_{\max}$

Figure 1 shows the length of drones' voyage average velocity curve, which is a parabolic. What we obtain is as follows by fitting data.

$$Voyage(v) = -0.2 \times v^2 + 1.32 \times v + 27.8 \quad (\text{km}), \tag{1}$$

where $Voyage(v)$ is the length of vayage when drones travel with the average speed of v.

2.4 Data pre-processing

In order to simplify the problem, the range of the repeater signal in the calculation of the first question is always 20 km. Handheld two-way radios have a range of 2 km in urban, forest and mountainous areas and 5 km elsewhere. This greatly simplifies the calculation and will be further optimized in the third question.

Fires data processing

Combined with life experience we know that the spread of fire presents a convex polygon, which is approximately estimated as a circle. The circumference of the j^{th} fire with latitude and longitude of (x_j, y_j) is $d_j(x_j, y_j)$, which yields:

- Radius: $r_j(x_j, y_j) = \frac{d_j(x_j,y_j)}{2\pi}$;
- Area: $S_{circle,j}(x_j, y_j) = \pi \times r_j^2(x_j, y_j)$.

We combine fires that occurred on the same day and in close proximity into one fire point.

3 Task 1: Optimal Numbers and Mix of Two Types of Drones

In the model for determining the number and combination of drones, it is necessary to provide an evaluation standard for whether the fire point needs drone rescue to improve rescue efficiency. To do this, we first classify the fires into different safety levels. On the basis of the known safety level of each area, we optimize the number and combination of drones based on maximum mix rate and minimum cost.

3.1 Area safety evaluation model

In order to assess the safety factor for different areas of Victoria, the gridded map is shown in **Figure 2**. The longitude and latitude of each region are denoted by x, y, respectively. For different latitude and longitude (x, y) regions, we establish three evaluation indicators: **fire frequency** $f(x, y)$, **recent fire situation** $situation(x, y), and$**fire size** size(x,y) , which are used to measure the safety factor $S(x, y)$. The safety coefficients of the grid and the adjacent grid are scored to get the safety levels of the different grids on the map.

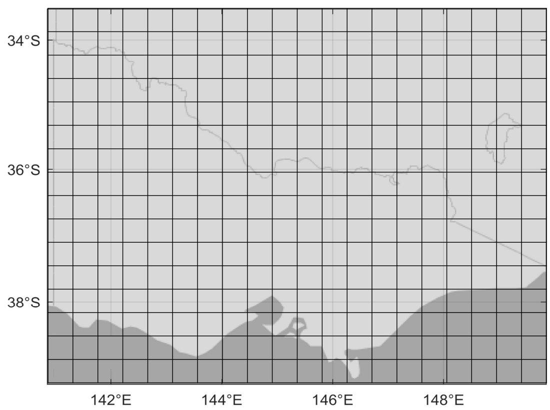


Figure 2: Map gridding schematic

3.1.1 Indicators establishment

(1) Fire frequency $f(x, y)$

In order to fully understand the fire frequency when fires are most severe in different areas, we need the maximum value of the daily fire frequency at that location and explore it on different time scales to estimate that maximum value.

$$f(x, y) = \max\{\frac{u(t_s, i, x, y)}{t_s}\}, \tag{2}$$

where:

- t_s is the time scale in days, $t_s \in [1, 365]$;
- i denotes the number of times, $i \in [1, \lfloor \frac{365}{t_s} \rfloor]$;
- $u(t_s, i, x, y) =$ [number of fires that occurred from day $(1 + (i-1) \times t_s)$ to day $(1 + i \times t_s)$]$/t_s$ (in place **(x,y)**), indicating the number of fires that occurred in the i^{th} time scale.

Figure 3 shows a detailed illustration about the time scale.

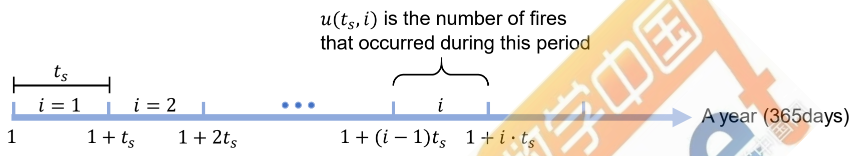


Figure 3: Time scale diagram

(2) Recent fire situation $situation(x, y)$

From the website[2] we know that the "mini-weather zone" formed by smoke and heat absorbing atmospheric moisture can continue to spread the fire through the sparks and form new ignition sites, which is also known as "embers attack". In other words, the area where the fire just broke out is more likely to reignite. during the period t_0 (the time period corresponding to Figure 1 in the title, i.e., October 1, 2019 to January 7, 2020). We use t_0 as the time criterion for determining the presence of recent fires and the period for updating the safety level, making the following divisions.

Table 2: A three-way score table

Recent fire situation	Scores
Burning	3
Happened recently	2
Not happened recently	1

(3) Fire size $size(x, y)$

Based on the data obtained from *Active Fire Data*[1], we extract the circumference $d_j(x, y)$ for the j^{th} fire in the (x, y) region and obtain annual average size:

$$size(x, y) = \sum_{j=1}^{365} \frac{d_j(x, y)}{365}. \tag{3}$$

3.1.2 Safety factors $S(x, y)$ and danger levels

1. Safety factors based on AHP AHP is a systematic and hierarchical multi-objective comprehensive evaluation method. We build the importance matrix of $S(x, y)$:

$$\begin{bmatrix} 1 & 2 & 1/6 \\ 2 & 1 & 1/5 \\ 6 & 5 & 1 \end{bmatrix}.$$

We take the average size of the fire as the most important indicator, and the purpose is to make the area with a high safety factor correspond to a larger average size. The resulting fire is more active or can develop into a more larger size, which is in line with the characteristics of the objects monitored by drones.

The consistency ratio is

$$CR = 0.0739,$$

which is less than 0.1, so the degree of inconsistency of $S(x, y)$ is within the tolerance range. And it passes the consistency test.

In order to eliminate the differences in the different magnitudes, after normalization, we get:

$$S(x, y) = a_1 \times f(x, y) + a_2 \times situation(x, y) + a_3 \times size(x, y). \tag{4}$$

The result of **Formula 4** is $a_1 = 0.1628$, $a_2 = 0.1088$, $a_3 = 0.7285$.

2. Safety Assessment of the area:

When we consider whether a region is safe or not, in addition to the safety factor $S(x, y)$ for that region, for its neighboring regions is also of reference value. We convolve the nine safety factors with a convolution kernel

$$S'(x, y) = \begin{bmatrix} S(x-1, y+1) & S(x, y+1) & S(x+1, y+1) \\ S(x-1, y) & S(x, y) & S(x+1, y) \\ S(x-1, y-1) & S(x, y-1) & S(x+1, y-1) \end{bmatrix} \otimes \begin{bmatrix} 0.05 & 0.05 & 0.05 \\ 0.05 & 0.6 & 0.05 \\ 0.05 & 0.05 & 0.05 \end{bmatrix}.$$

While the safety factor is already a valuable indicator when considering the safety situation, it is clear that we cannot ignore the influence of topography for the Rapid Bushfire Response program. There are significant differences in landscape from region to region. Therefore, fires in forested and vegetated hills and mountains (collectively referred to as ***flammable topography***) are more likely to develop over time into fires that are fierce and difficult to extinguish. And urban, desert, plains and other areas (collectively referred to as ***non-combustible topography***) after the fire is difficult to further develop into a larger scale. Do the following division.

Table 3: Dangerous level classification

Non-flammable topography		Flammable topography	
Safety factor	Danger level	Safety factor	Danger level
0~0.25	very safe	0~0.25	safe
0.25~0.5	safe	0.25~0.5	general
0.5~0.75	general	0.5~0.75	dangerous
0.75~1	dangerous	0.75~1	high-risk

3.1.3 Result analyze of heat map

The map of different security levels is shown in **Figure 4**. The areas in different colors represent areas of different levels.

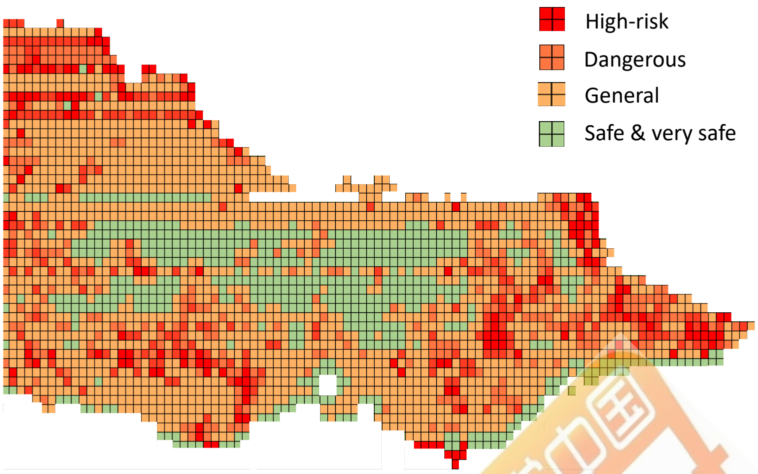


Figure 4: Different danger levels on the Victoria map

As can be seen in **Figure 4**, central Victoria is safer, probably because it is at a low altitude and is not affected much by climate. There are many high-risk areas in the east, west and south, which is because these areas are very vegetated.

For SSA drones, it should maintain timely response capability to the *high-risk area* and have certain prevention capability to the *dangerous area*. Repeater drones, on the other hand, work with SSA drones to maintain communications and surveillance activities in high-risk areas, while reserving a certain number of radio-repeater drones for dangerous areas to support communications that may be established. In order to ensure the safety of the EOC, set the EOC to move freely in the safe area.

3.2 Optimization of the number and mix of drones

After dividing the gridded map into areas with different security levels, we dispatch drones to implement emergency response or preventive deployment based on the security level of the fire, demonstrating the advantages and disadvantages of the solution using capability and cost metrics. We evaluate the mix of the two types of drones and optimize the number of drones based on as much capability and as little cost as possible.

3.2.1 Maximum drone capability

The capability of the drone is reflected in two aspects: the response time of the high-risk area is as short as possible (***response capability***) and the response time of the dangerous area is as reasonable as possible (***prevention capability***). There are M high-risk areas and N precautionary areas in the statistics. These two capabilities are analyzed separately below.

- **Response capability**

We consider the i^{th} EOC to be responsible for communication and surveillance. the role of the EOC is to direct the SSA drone to reach the high risk area from a location closer to the i^{th} high risk area in a safer area, such that the average response time t_{avg} is minimized.

$$t_{avg} = \sum_{i=1}^{M} \frac{t_i}{M},$$
$$t_i = \frac{x_i}{v_{\max}}.$$

To ensure that the drone can travel back to,

$$x_i < 0.5 \times Voyage(v_{\max}). \tag{5}$$

x_i is the distance between the EOC and the high-risk area, and t_i is the time the aircraft flies in the process. In order to maximize the response capability of the drone, t_{avg} should be as small as possible. Therefore, we make the EOC located at the nearest point to the i^{th} high-risk area in the safer region.

- **Prevention capability**

We configure a certain number of SSA drones in the dangerous area, so that when the EOC is movable, we can ensure that there are SSA drones or EOCs in the dangerous area at regular intervals to check whether there are fires. Prepare a certain number of drones, the more the number, the better the ability to prevent, see Section 3.2.3 for quantification.

3.2.2 Minimal drone costs

We calculate the total cost of the two drones and their number separately. Let the number of SSA drones be m and the number of Radio Repeater drones be n. The calculation is as follows.

$$Cost(\text{drone}) = m \times Cost(\text{SSA drone}) + n \times Cost(\text{radio repeater drone}), \tag{6}$$

where $Cost(\text{SSA drone})$ = SSA drones' HD camera $cost+$ remote sensing camera $cost+$ drone $cost$, $Cost(\text{radio repeater drone})$ = repeater $cost+$ Radio Repeater drones' $cost$. So we want

$$\min\{Cost(\text{SSA drone}).\} \tag{7}$$

3.2.3 Number and mix of drones

The m, n are the number of SSA drones and radio-repeater drones, respectively. We determine the number of SSA drones from the demand of the fire scene and the round system, then determine the number of radio repeater drones by the combination relationship between SSA drones and radio repeater drones, and finally prepare some drones to prevent dangerous area.

1. Number of SSA drones

For SSA drones, because SSA drones monitor the fire scene directly, the mixed relationship with the radio-repeater drones does not change their utilization rate in the fire now, so it is sufficient to calculate the number of SSA drones directly according to the fire demand.

(1) For high-risk areas

Let the longitude and latitude of the jth high-risk area be (x'_j, y'_j), and find the number of SSA drones needed to scan the j^{th} high-risk area.

Fires that need to maintain a timely response are very active, large and mostly in terrain that is prone to further expansion.

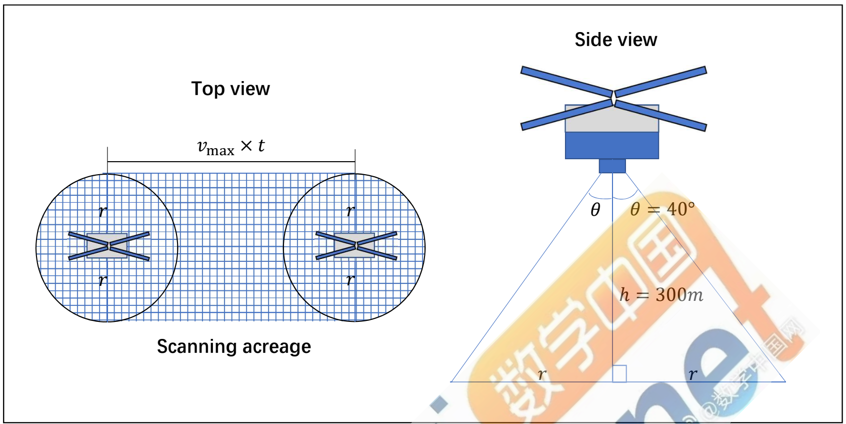


Figure 5: SSA drones surveillance schematic

To ensure firefighting efficiency, SSA drones on scene should assure that they can complete scanning surveillance of the entire scene area within half a minute. The scan radius can be obtained from **Figure 5**.

$$m_{1,j} = \frac{S'_{circle,j}}{\pi r^2 + 2r \cdot v_{max} \times 30 \text{ seconds}}. \tag{8}$$

SSA drones have constant round trips and recharges, and there are handoffs. So the number of SSA required for the j^{th} high-risk area is the product of the number of handoffs and the number of SSA drones required at the same time.

$$m_{2,j} = m_{1,j} \cdot \frac{\frac{2\bar{x}_i}{v} + \text{recharge time}}{\frac{Voyage(v)}{v} - \frac{x_j}{v}}, \tag{9}$$

where the biggest fractional equation in **Formula** (9) represents the rounds of the drone, which is obtained by the quotient of the time returning from the disaster area to recharge and arriving at the disaster area again and the time spent in the disaster area.

(2) For dangerous areas

For the i^{th} dangerous area, the number of SSA drones should reach a certain threshold in order to check for fires in these areas, such that even in the worst case scenario (i.e., when all dangerous areas are on fire), the number of SSA drones in reserve is sufficient to dispatch one SSA drone for each hazard area for temporary surveillance and situational awareness. Thus there is a total number of SSAs:

$$m = \sum_{i=1}^{M} m_{2,j} + N \times (1 + \varphi), \tag{10}$$

where φ is loss rate. Considering the attrition of reserve drones, we prepare some additional drones to cope with this attrition. φ is the number of extra prepared drones divided by the number of dangerous area drones.

2. Mix of Radio Repeater drones

For the radio-repeater drones, consider its combination with the SSA drones, so that each radio-repeater drone can receive as many signals as possible and forward more signals to determine the best number , optimize utilization.

(1) Greedy and multiplexing based solution algorithms

Using a greedy algorithm, the solution that allows each radio repeater drone to reuse the maximum number of SSA drones is planned, and the minimum number of radio repeater drones, n, is output.

The idea of the algorithm is to find candidate locations of radio repeater drones that can be multiplexed. Starting from the candidate location with the most multiplexing, a radio repeater drone is determined to be placed at that location, while removing the SSA drones in high-risk areas that can use that repeater drone to send and receive signals. Then candidate locations with the second highest reuse rate is identified. Continue this process until all high-risk areas are taken care of by the repeater.

Given the meaning of some symbols:

- n_1, n_2 are the number of prepared radio-repeater drones and the number of radio-repeater drones responding to the high-risk area, respectively. When calculating n_1, the input n is n_1, so does n_2.
- m_i is the number of SSA drones in i^{th}.
- U is the number of corresponding areas when considering high-risk or dangerous areas.
- $R_{radio} = 2$km and $R_{repeater} = 20$km are the ranges of the handheld two-way radios and repeaters, respectively.
- SC_i is the candidate range of radio repeater drone for the i^{th} high-risk area, $SC = \{SC_1, SC_2, \cdots, SC_M\}$.
- SR_i is the number of radio repeater drones multiplexed corresponding to the ith high-risk area (other high-risk areas use this repeater at the same time), equal to 1 means that this radio repeater drone is only responsible for the signal transceiver in this high-risk area. $SR = \{SR_1, SR_2, \cdots, SR_M\}$.
- SN_j is the reuse number of the radio repeater drones corresponding to the j^{th} high-risk region. If another high-risk area multiplexes this repeater, the serial number of this high-risk area is an element of the set SN. $SN = \{SN_1, SN_2, \cdots, SN_M\}$

Algorithm: Radio Repeater drones number solving algorithm

Input: x_i, R_{radio}, $R_{repeater}$, $n = 0$
Output: n

for $i = 1; i \leqslant U; i++$ **do**
  if $x_i > R_{radio}$ **then**
    take the i$^{\text{th}}$ high-risk area as the center and R_{radio} as the radius to make a circle O_1;
    take the i$^{\text{th}}$ EOC as the center and $R_{repeater}$ as the radius to make a circle O_2;
    $SC_i = O_1 \cap O_2$;
    $SR_i = 1$;

for $i = 1; i \leqslant U; i++$ **do**
  for $j = 1; j \leqslant U; j++$ **do**
    if $SC_i \cap SC_j \neq \varnothing$ *and* $i \neq j$ **then**
      $SR_i = SR_i + m_j$;
      $SC_i = SC_i \cap SC_j$;
      $SN_i = SN_i \cup \{j\}$;

while $SR_1 + SR_2 + \cdots + SR_U = 0$ **do**
  $SR_k = \max\{SR\}$, k is the maximum value's serial number in SR;
  $n = n + 1$;
  for $i = 1; i \leqslant U; i++$ **do**
    if $k \in SN_i$ **then**
      $SN_i = \varnothing$;
      $SR_i = 0$;

(2) Maximize the reuse rate $\frac{m}{n}$ Using the reuse rate: $\frac{m}{n}$ indicates the mix relationship of drone configuration, the larger this indicator is the better the mix relationship of two drones. Then we have:

$$\max\{\frac{m}{n}\}. \tag{11}$$

(3) Continuous supply to high-risk areas

Considering the process of repeater drone round trip and charging, there is hand-offs, so it is necessary to get the number of repeater drones that respond to high risk areas:

$$n_1' = n_1 \times \frac{\frac{2\bar{x}_i}{v} + \text{recharge time}}{\frac{Voyage(v)}{v} - \frac{\bar{x}_i}{v}},$$

ditto,

$$3.3 \leqslant v \leqslant 10,$$

$$Voyage(v) > 2 \times \max(x_j).$$

So the total number of repeater drones:

$$n = n_1' + n_2, \tag{12}$$

where n_2 is radio repeater drones that dangerous areas need.

3.3 Optimization model with maximum mix rate and minimum cost

Combining all the above, λ is introduced to reflect the priority of the drone capability indicators. The larger λ is, the higher the priority of the capability indicators in the drone configuration scheme obtained from the planning. The final model is:

$$\min\{(\frac{n}{m})^{\lambda} \times Cost(\text{drone})^{(1-\lambda)}\},$$

$$s.t. \begin{cases} n = n_1' + n_2 \\ 0 \leqslant \varphi \leqslant 1 \\ 0 \leqslant \lambda \leqslant 13.3 \leqslant v \leqslant 10 \\ Voyage(v) > 2 \times max\{x_j\} \end{cases} .$$

3.3.1 Result Analysis

The result of the optimization model with maximum drone capability and minimum cost are shown in **Figure 6**.

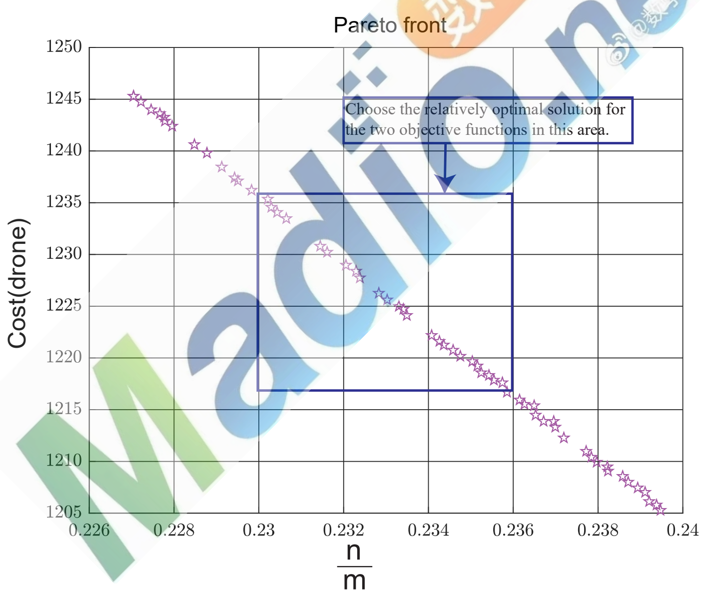


Figure 6: The results of the optimization model

As shown in the **Figure 6**, the inverse of $\frac{m}{n}$ shows an inverse relationship with the cost of the drone, which indicates that our bi-objective optimization is effective. We intercept the middle section of the interval as the solution space, and we can choose a non-inferior solution in the left half of the interval when we prefer the combination rate to be optimal, and in the right half of the interval when we prefer the cost to be minimal.

4 Task 2: Adaptation of Model and Cost Increases for Equipment

For simplicity, we make the following two assumptions:

- The topography of Australia will not change dramatically in the next ten years (not considering changes in topography caused by unavoidable factors such as earthquakes).
- The probability of fire occurrence in different hazard areas is independent of each other.

4.1 Ternary commensurable fire prediction model

4.1.1 Model building process

Since the terrain does not change dramatically over the course of a decade, the main task of predicting fire is when extreme fire conditions will occur, what the probability of occurrence is, and what the amount of equipment prepared to respond to fires of different sizes is. We obtained data on extreme fire conditions in Victoria over the last few decades by referring to *List of major bushfires in Australia*[5], and used this as the basis for our predictions.

The commensurable prediction model is a method of forecasting special events based on special events, and he bases his forecasting on the universal existence of commensurability of events, which is a generalized periodicity. Commensurability is a law that is common to all elements of a system and reflects the occurrence of special events (small probability events) in nature[6].

1. The general commensurable prediction model is:

$$X_{i+1} = \sum_{j=1}^{l} I_j X_j,$$

where X is entered in chronological order for the past years in which extreme fire events occurred to obtain $X = \{X_1, X_2, ..., X_{17}\}$ in the sequence.. $\{j\} \in \{i\}$, that is, j is an element of the set i. I_j is an integer.

2. In order to get the optimal results, the ternary commensurable prediction model with the best results in general is selected[7]. And the formula is:

$$X_a + X_b - X_c = X_{u(a,b,c)},$$

where $a, b, c, d, e, f, g = 1, 2, \cdots, n$, $u, v, w = 1, 2, \cdots, m$.

3. Enumerate the equations that match the above relationship by enumeration, and process the sequence numbers to obtain a vector

$$V = (b - a, c - b, u - c,' +',' -'),$$

which means the difference between the serial numbers of the first two years, the difference between the serial numbers of the second and third years, the difference between the serial numbers of the third and fourth years, the sign before the second year, and the sign before the third year.

Find among all vectors the vectors that exist identical to itself and obtain the multi-set ternary commensurable forecast formula:

$$X_i V_4 X_{i+V1} V_5 X_{i+V_1+V_2} = X_{i+V_1+V_2+V_3} \quad i = 1, 2, \cdots,$$

let the term with the largest ordinal number in the forecast equation be X_{18}, and the forecast value for multiple years corresponding to X_{18} can be calculated.

4. We sort the past year series and the forecasted year series from left to right by serial number, enumerate four years from all of them, find the ternary commensurable forecast formula they match, and connect them two by two with their differences to get the butterfly diagram.

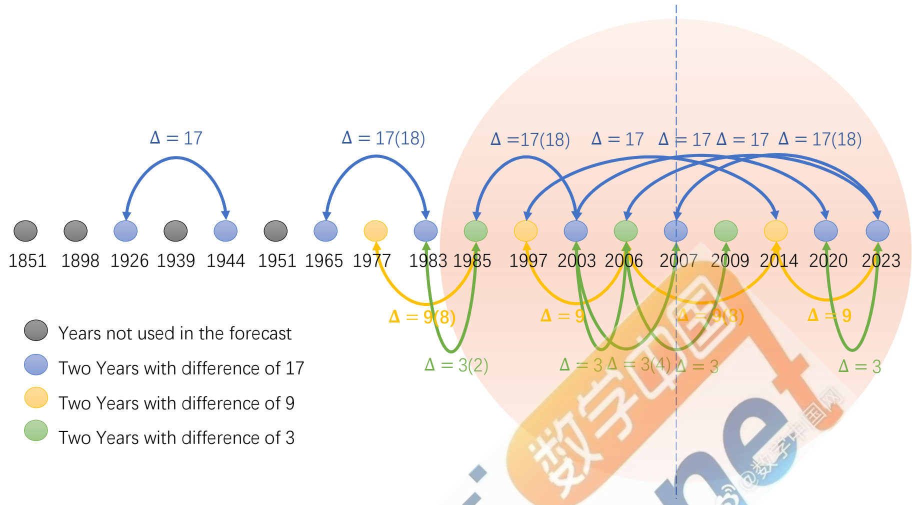


Figure 7: Butterfly structure diagram

According to the K. Pearson-Fisher theorem, **Figure 7** satisfies the butterfly structure probability reference criterion. It reflects the most important quasi-cycles since 1851 as 3, 9, 17a, with 17a being the most typical, occurring seven times.

5. Count statistically the total number of extreme fires used to predict the i^{th} extreme fire situation as n_i, and the number of extreme fires participating in the prediction of the ith extreme fire case is m_i, then the probability of the ith extreme fire event is

$$P(i^{\text{th}}\text{ extreme fire occur}) = \frac{m_i}{n_i},$$

and m_i can be seen by butterfly structure diagram clearly.

For example, our predicted year for the next extreme fire is 2023, which is involved in all three cycles, and the number of years involved in the prediction is 16. The probability of the event prediction is $P = m/n = 16/17 = 94.12\%$, which is the probability of an extreme severe fire in 2023.

6. We use the fire acreage in the data to indicate the severity of extreme fire conditions S_{danger}, and similarly the fire scale can be predicted by the ternary commensurable prediction model.

7. Test the predicted values and calculate the relative residuals:

$$\varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \quad , k = 1, 2, \ldots, n.$$

If for all $\varepsilon|(k)| < 0.1$, the higher requirement is considered to be reached; otherwise, if for all $|\varepsilon(k)| < 0.2$, the general requirement is considered to be reached.

4.1.2 Result of ternary commensurable fire prediction model

1. Table of predicted years and residual test

Table 4: Table of predicted years

Years	Probability	Residual	Residual change
2022	64.71%	53	2.62%
2023	94.12%	54	2.67%
2024	52.94%	55	2.72%
2027	41.18%	58	2.86%

Table 4 shows our predictions for the four possible years of extreme severity fires and the corresponding probabilities of it. The fourth column shows that the residual change in the occurrence of extreme fires for all four years is around 2%, which is less than 5%. This indicates that our predictions have a strong accuracy.

2. Pie map of predicted relative index

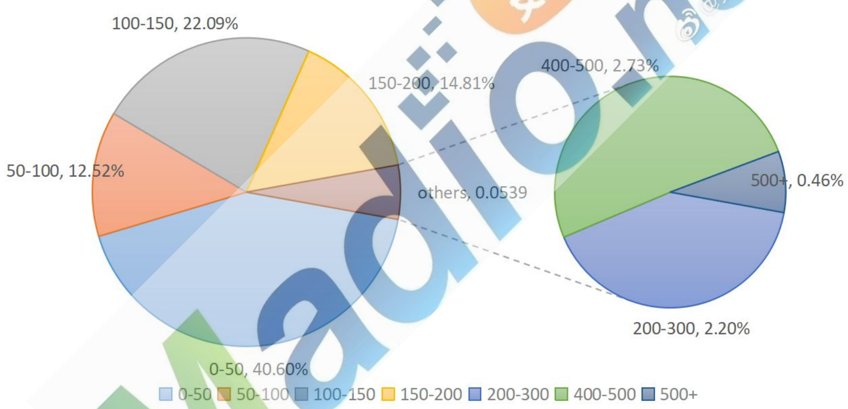


Figure 8: Pie map of predicted relative index

Based on the commensurable model, we predict the relative index of severity that would result from the next extreme severity fire when it occurred. As shown in **Figure 8**, the next extreme severity fire is most likely to produce a fire with a severity index of 0-50. Also, fires with larger severity indices have a high likelihood, accounting for more than half of the likelihood. Therefore, in order to meet future emergency response needs for large fires, we may need to increase the number of drones or "Boots-on-the-ground" Forward Teams.

4.2 Increased Equipment Costs

After predicting the fire, to accommodate changes in the likelihood of extreme fire events over the next decade, we increase the equipment configuration of the EOC and "boots-on-the-ground" firefighters for extreme fire events only, getting the increased equipment costs.

Equipment includes handheld radios for the "boots-on-the-ground" teams, wearable devices, and two-way radios for the EOC. From a safety perspective, we assume that each "boots on the ground" squad firefighter has one handheld radio and one set of wearables, and that each EOC has one two-way radio for surveillance and communications and one two-way radio for backup.

4.2.1 Additional equipment costs for EOCs $Cost(\textbf{EOC})$

When extreme fires occur, the drone needs the appropriate EOC to support communication and surveillance. Find the expected value $E(number, extreme)$ of the number of extreme fires that have a probability of occurring in the next ten years, which is equal to the sum of the probabilities of occurrence in the fire prediction model.

$$E(number, extreme) = \sum_{i=1}^{N} P(\text{fires in } i \text{ areas}) \cdot P(\text{no fires in } N - i \text{ areas}) \cdot i,$$

where P denotes the probability.

From **Task 1**, we know that the number of EOCs responsible for extreme fires is equal to the number of extreme fires. So the cost of the additional equipment for the EOC is calculated as

$$Cost(\text{EOC}) = 2 \times E(number, extreme) \cdot Cost(\text{radio}). \tag{13}$$

4.2.2 Additional equipment costs for firefighters $Cost(\textbf{firefighter})$

The number of "boots on the ground" teams on each attendance is determined by the EOC after considering the severity of the fire. The probability model for the fire department to handle an extreme fire event is established, and the probability of successful response to the fire is estimated based on the number of team members Num_i and the severity $S_{danger,i}$ in i^{th} extreme fire event.

Combined with the actual situation, the more serious the fire, the smaller the possibility of "boots-on-the-ground" squad to put out the fire, while the more the number of squad, the greater the ability to deal with the fire. This change in the relationship should be the first slow then fast and finally slow trend, so the sigmoid function is selected to fit.

$$P(success, i) = \frac{1}{1 + \exp(-\frac{Num_i}{S_{danger,i}} + C)}, \tag{14}$$

where $i \in [1, E(number, extreme)]$ and C is a constant, from the article *Brief introduction of Polish volunteer fire brigade*[3], it is known that the average number of Polish fire department is 24, we can use $F_{number,i} = 24$, $S_{danger,i} =$ average of S_{danger} when the success rate is 0.8 to estimate C.

The EOC should ensure that the success rate of each team's rescue is above 80% when making decisions. Substitute Sdanger,i, calculate the number of team members Pnumber,i for the ith extreme fire with a success rate of exactly 0.8. And utilize the expected value of the increase in the number of "boots-on-the-ground" firefighters to estimate the number of increases needed.

$$E(Num_i) = \sum_{i=1}^{M} P(Num_i) \cdot P(i^{\text{th}} \text{ extreme fire occur}).$$

The cost of additional equipment for firefighters of "boots on the ground" is:

$$Cost(\text{firefighter}) = E(Num_i) \cdot (Cost(\text{radio}) + Cost(\text{wearable device})). \tag{15}$$

4.2.3 Results of additional equipment

Equipment	Number	Single price	Total price
EOC radios	5	92.725	463.625
"Boots-on-the-ground" handheld radios	19	94.741	1,800
"Boots-on-the-ground" team wearable equipment	19	141.507	2,688.625

Figure 9: Number of additional equipment

5 Task 3: Location Optimization

In the greedy and multiplexing based solution algorithms of Task 1, we have obtained the ideal distribution range of each repeater drone. Based on this, we consider the signal loss and safety of the repeater drone to optimize its optimal position in the ideal distribution range.

5.1 Signal loss model

SPM model is suitable for propagation loss prediction for distances from 1 to 20 km and frequencies from 150 to 3500 MHz, running GSM and other communication systems. This model takes the feature classification map and effective antenna height into account to calculate the path loss. Its basic equation is shown in **Formula 16**[4].

$$\begin{aligned} L_1 = {} & K_1 + K_2 \lg d + K_3 \lg H_{Txeff} + K_4 Diff\_loss \\ & + K_5 \lg H_{Txeff} \lg d + K_6 H_{Rxeff} + K_{chitter} f(clutter), \end{aligned} \tag{16}$$

where K_4 is the multiplicative factor of the bypassing amount; $Diff\_loss$(unit: dB) is the loss caused by bypassing the obstacle; K_6 is the multiplicative factor of H_{Rxeff}, $Rxeff$ is the mobile station receiving antenna height. K_1, K_2 and other variables are related to the height and distance of the signal base station as well as environment of drones.

For the repeater drone, when the siting is varied within the ideal distribution, K_1, K_2, K_3, H_{Txeff}, K_4, K_5, K_6, and Kchitter are unchanged, and the small change of d is negligible, only H_{Rxeff} and $Diff\_loss$ will change. Therefore, the signal loss model of the repeater drone is established as follows.

$$L2 = K4 \cdot Diff\_loss + K6 \cdot H_{Rxeff} + L_0, \tag{17}$$

$$K_4 = 44.9 - 6.55 \log H_{Rxeff}, \text{[8]}$$

$$K_6 = 1.1 \log f_{repeater} - 0.7, \text{[8]}$$

$$L_0 = 20 \log_{10}\left(\frac{4\pi}{\lambda_{\text{transmitted wave}}}\right), \text{[8]}$$

where f_{radio} is frequency of repeater. $\lambda_{\text{transmitted wave}}$ is wavelength of transmitted wave in meters.

For $Rxeff$, drone altitude = altitude of location + height above ground.

For $Diff\_loss$, we assume that the height of the obstacle is linearly related to the loss of the repeater range, with the most severe loss for obstacle heights greater than $1.5 \cdot Rxeff$ and considered almost no loss effect for obstacle heights less than $0.5 \cdot Rxeff$. In the ideal case the loss ratio δ is 1.0, the worst case is 0.8. The loss ratio of the i^{th} obstacle :

$$\delta_i = (H_i/Rxeff) \cdot (1 - 0.7)/(1.5 - 0.5) + 0.7.$$

where H_i is the height of the ith obstacle Counting K obstacles on the linear path between the repeater drone and the fire area in 1000m. Calculate $Diff\_loss$:

$$Diff\_loss = \prod_{i=1,2,\ldots,K} \delta_i. \tag{18}$$

From the website[9] we can see that we should ensure that the signal-to-noise ratio is not less than 20dB, that is,

$$L_2 \geqslant 20\text{dB}. \tag{19}$$

We use this formula to filter out the areas that do not meet the requirements.

5.2 Drone safety

Depending on the scale of the fire, repeater drones need to avoid nearby fires to ensure safe operations. Because of the limited signal range of handheld radio, repeater drones are often located 2 to 5 km from the center of the fire, and when the fire is large and the terrain is flammable, it is easy to spread to the repeater drone is located, causing damage to the drone in serious cases. We perform a quantitative analysis of safety based on information related to fires.

Kinetics-based fire spread model:

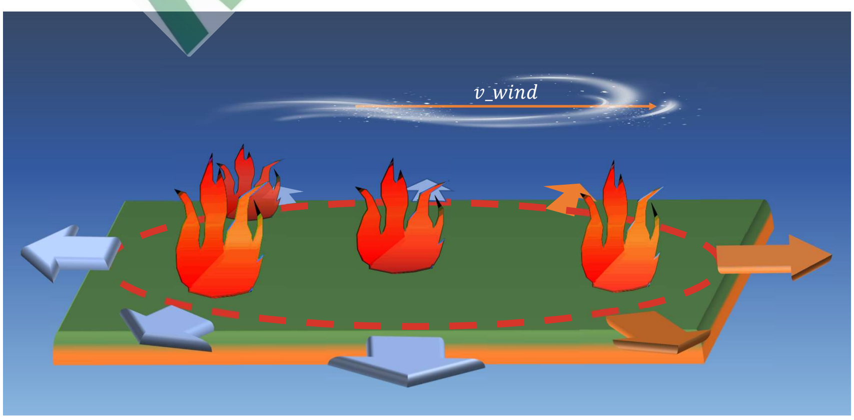


Figure 10: Kinetic-based fire spread model schematic

As shown in the **Figure 10**, the fire spread into eight directions, of which the direction of the angle with the wind speed less than 45 degrees will spread outside (such as the direction of the yellow arrow), the other directions remain unchanged (such as the direction of the blue arrow).

$$\vec{V}_{spread} = \vec{V}_{wind} \times \exp((D + \text{topography scoring}) \cdot \text{fire size} + E),$$

where $\vec{S}_r$ denotes the rate of spread in the vector direction, and D and E both denote constants.

In the fire spread range, the amount of change in the length of the fire $\vec{k}$ direction is

$$\Delta\vec{L}_k = \begin{cases} V_{spread} \times \vec{k} \times \text{unit time}, V_{spread} \times \vec{k} > 0 \\ 0, others \end{cases},$$

and length of fire $\vec{k}$ direction is

$$\vec{L}_k = \vec{L}_k(\text{last time}) + \Delta\vec{L}_k.$$

Wind speeds were obtained from SSA drones collecting data from "boots-on-the-ground" teams, and terrain was scored to create a scoring table as follows.

Table 5: Terrain scoring table

Terrains	Scores
Forests, grassy hills, grasslands	3
Plains, other terrain	2
Not happened recently	1

To ensure safety, the distance between the ith repeater drone position $(x_{drone,i}, y_{drone,i})$, which is in a certain direction of the fire, and the edge of the fire should be made no less than the safety threshold, which is set here to 50 m. Therefore,

$$|(x_{drone,i}, y_{drone,i}) - (x, y) \text{ of center}| \geqslant 50 + \text{the range length of the fire}. \quad (20)$$

where "(x, y) of the center" denotes the latitude and longitude of the fire center. "The range length of the fire" means the range length in a certain direction of the fire at a certain moment.

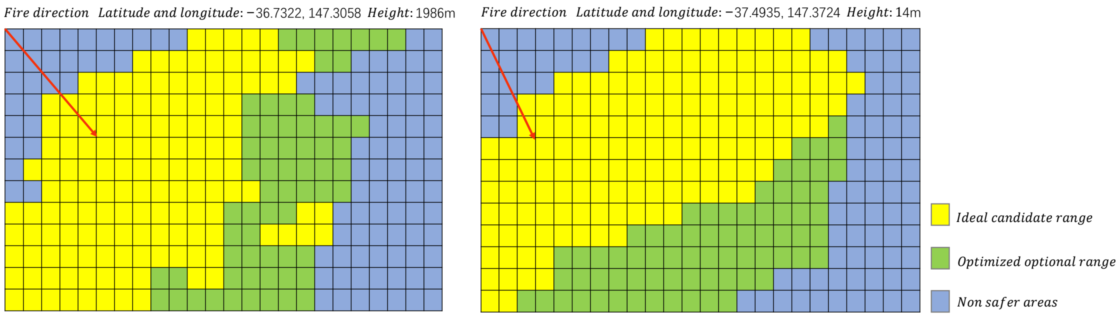
Location optimization:

The signal-to-noise ratio and distance from the edge of the fire are calculated for each location in the ideal distribution range, and reasonable locations are selected:

$$L_2 \geqslant 20\text{dB}.$$ [9]

We use this constraint to optimize to get the drone range from the point of view of preventing fire spread hazards by drones.

5.3 Result analysis



(a) The highest altitude (b) The lowest altitude

Figure 11: Optimization of the location of the highest and lowest points of altitude

Obviously, the wind direction directly affects the distribution of the optimized repeater drones, which is concentrated at the edge of the ideal range perpendicular to the wind direction. High altitude areas are more likely to be affected by the altitude of different locations, which manifests itself in the form of a discontinuous and interrupted range of the optimized distribution.

6 Sensitivity analysis

6.1 Sensitivity Analysis of safety factor $S(x, y)$

In **Task 1**, we classify the areas of Victoria as different danger levels by calculating the safety factor. Therefore, the safety factor $S(x, y)$ is a crucial parameter, which is calculated by **fire frequency** $f(x, y)$, **fize size** $size(x, y)$, **recent fire situation** $situation(x, y)$ (see **Formula 4**). As a result, we do sensitivity analysis on the coefficients a_1, a_2 and a_3 of these three parameters and the results are shown in **Figure 12**.

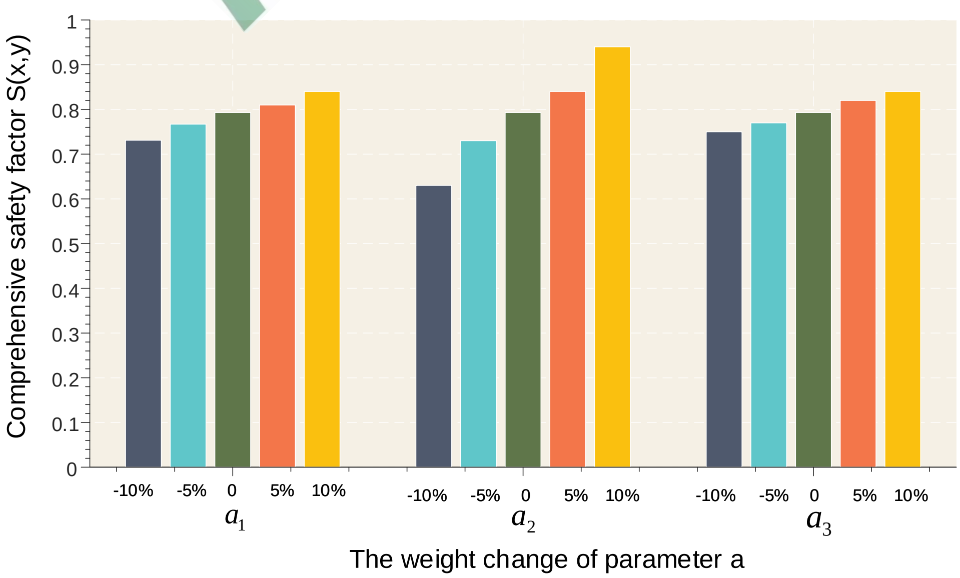


Figure 12: Sensitivity Analysis of $S(x, y)$

In **Figure 12**, we select a high-risk area as a place to test sensitivity, and the coordinate is $(x, y) = (-36.0634, 148.2911)$

From the figure we can conclude that When the variation of a_1, a_2 and a_3 change from -10% to 10%, the changes in the values of $S(x, y)$ corresponding to a_1 and a_3 are small and the overall trend is flat. Therefore, we believe that **the frequency and size of fires are more robust** and do not affect S too much. Meanwhile, a_2 corresponds to a relatively large amount of change in $S(x, y)$, so we think **the sensitivity of the recent fire situation to be strong**. This result verifies the reasonableness of our consideration of $situation(x, y)$ in Section 3.1.1.

6.2 Sensitivity analysis of fire frequency in different regions per year

The annual frequency of fires in different regions has a significant impact on the number of SSA drones. Therefore, we divid Victoria into central, eastern, western, and southern regions, and do sensitivity analysis on the fire frequency in each of these four geographic regions.

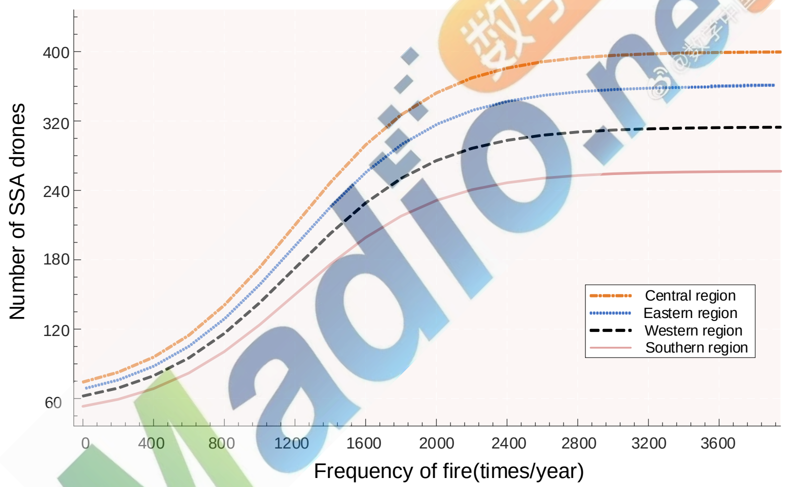


Figure 13: Sensitivity analysis of fire frequency in different regions per year

We can analyze **Figure 13** in two ways. **For fire frequency**, the number of SSA drones vary the most when the number of fires in Victoria is in the range of 1600~2000 per year. That is, the fire frequency is sensitive in the interval [1600,2000]. Therefore, when CFA uses our model, if the measured number of fires in the year is within this interval, they should calculate the number of SSA drones very carefully.

For different regions, the number of SSA drones in the four regions change significantly when the fire frequency is greater than 1600. The central region of Victoria has the highest number, while the southern region has the lowest. Therefore, regions are also sensitive to the number of SSA drones, and CFA's drone dispatch should be different in different regions.

7 Strengths and Weaknesses

7.1 Strengths

- We innovatively use safety factors to classify classes for areas with different fire conditions, which improves the efficiency of drone response for emergency situations and provides a clear idea for drone dispatching and distribution.
- The combination and reuse relationship between drones is used to optimize the utilization rate of repeater drones, which both fits the actual situation and has a universal solution to other configuration problems.
- The ternary commensurable prediction model is a model that predicts the probability of future special situations for data of special situations, and the commensurabiliby it considers is very accurate in the face of special situations with the superiority that other forecasting models do not have.
- We have reviewed a large amount of literature, not only with sufficient data, but also with estimation and calculation methods for the parameters of each model from specialized literature, and the results are scientific and rigorous.

7.2 Weaknesses

- In the problem of optimizing the position of the repeater drone, we are limited by the data for the simulation solution, and the results may be somewhat different from the real situation.
- The index estimated by using AHP model has some subjective influence, and sensitivity analysis is needed to calculate the reliability of the model.

8 Annotated Budget Request

Budget Justification for Cost Proposal (Unit: A$)

We are a numerical modeling competition team from MCM and will be responsible for the overall management and direction of the Victorian National Fire Service's drone acquisition program for the Rapid Bushfire Response Division and equipment acquisition program for extreme fires over the next decade, based on the soundness of our models through testing and analysis of results.

1. "Rapid Jungle Response" Project

Direct costs: 2,401,030

Drones	Number	Single price	Total price
SSA drones	200	10,390	2,077,200
Radio repeater drones	32	10,094.741	323,030

The number of SSA drones procured to respond to emergency fire situations in Victoria includes 134 shift SSA drones for emergency response to high-risk areas, 62 SSA drones for fire preparedness in prepared areas, and 4 standby drones for consideration of wear and tear. The cost of each SSA drone includes the cost of a bare drone at

A$10,000, an HD camera at A$47.370, and a thermal imaging camera at A$338.648; the repeater drones include 21 repeater drones in combination with SSA drones in high-risk areas and 21 repeater drones in combination with SSA drones in preparedness areas 11, the cost of each repeater drone includes a bare drone of A$10,000, the cost of a repeater of A$94.741.

With this drone configuration, the drones can detect all 373 locations (including 154 high-risk areas and 219 precautionary areas) throughout the state with high frequency and large fire sizes, and have an average response time to emergency fires of only 5 minutes and 17 seconds. In addition, each repeater drone achieved a combined rate of 6.25 with SSA drones. the overall system had excellent responsiveness and excellent combined relationships.

Other costs: None.

Electricity cost for drone charging: A$0.

Our analysis of the infrastructure within the Victorian State Fire Service concluded that the infrastructure within the Fire Service can accommodate the normal conduct of activities for other costs and therefore the cost is 0.

2. Equipment procurement items to respond to extreme fires over the next ten years

Direct Costs: 148,950

Additional equipment (abbreviated): $A 4952.25

Equipment	Number	Single price	Total price
EOC radios	5	92.725	463.625
"Boots-on-the-ground" handheld radios	19	94.741	1,800
"Boots-on-the-ground" team wearable equipment	19	141.507	2,688.625

From a safety perspective, we consider a one-to-one EOC for extreme fires and adjust the number of firefighters in the Ground Boots team based on the predicted size of the extreme fire. We provide a handheld radio and a set of wearables for each firefighter in the "boots on the ground" team, with one two-way radio and a backup two-way radio per EOC. The unit cost of the wearables included a fire axe for $11.288, a heat suit for $120.442, and a fire extinguisher for $9.877. According to our modeling, this scenario has a success rate of more than 80% for "boots on the ground" teams for extreme fires.

Staff cost: 144,000

Staff	Number	Single price	Total price
Command staff within EOC	5	6,000	30,000
Firefighters in "Boots-on-the-ground"	19	6,000	114,000

We consider subsidizing a bonus for firefighters to encourage their performance in fire rescue, and similarly, for each EOC, the hard work of the command staff is worth a bonus. We refer to the federal government's subsidy for volunteer firefighters in New South Wales for the bonus assessment.

Other costs: None

Cost of apparatus consumption: None

Wear and tear of firefighting equipment: A$0 Loss of drones: $0

The primary job of the "boots on the ground" team is to detect first-hand information about the fire, make judgments about the rapidly changing situation, and pass additional information to the EOC. Therefore, fire fighting is not their main job, the consumption of firefighting equipment is negligible. The wear and tear of drones is already considered in the cost of drones.

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