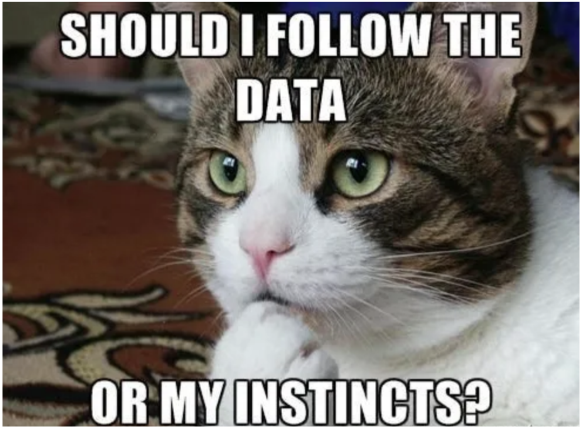
Causal Inference Workshop

Week 1 - Modeling Fundamentals

Causal Inference Workshop

January 19, 2024

Welcome to the Causal Inference Workshop!



Welcome to the Causal Inference Workshop!

- Extremely heavily based on Claire's (2022) and Anna's (2024) awesome courses
- Updated with a section on field experiments
- Slight modification for the DID section
- Slides and codes are uploaded to the Github repository
- Additional reference materials will be added to the `readings` folder.

Workshop outline

A. Causal inference fundamentals
  - Modeling assumptions matter too
  - Conceptual framework (potential outcomes framework)

B. Design stage: common identification strategies
  - IV + RDD [coding]
  - DiD, DiDiD, Event Studies, New TWFE Lit [coding]
  - Synthetic Control / Synthetic DiD [coding]

C. Analysis stage: strengthening inferences
  - Limitations of identification strategies, pre-estimation steps
  - Estimation [controls] and post-estimation steps [supporting assumptions]

D. Other topics in causal inference and sustainable development
  - Inference (randomization inference, bootstrapping)
  - Field experiments
  - Weather data regressions, remote sensing data [coding]

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Workshop Logistics

- 9am - 10am Fridays (before colloquium)
  - Office hours on demand 8am - 9 am, if you want to work through code / discuss anything
- If you’re taking the course for credit (pass/fail) and “research tools” requirement, attendance is expected every week
  - If you’re not taking it for credit, but want to get workshop-related emails, let me know!
- One assignment to turn it at the end (run and modify one of the coding exercises)

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Causal inference

Modeling assumptions of the Classical Linear Regression Model

Departures from usual assumptions

Summary

Mini Coding Exercise

Causal inference, identification, and modeling assumptions

What is causal inference?

- Process by which we use data to make claims about causal relationships
- *Potential outcomes* [framework]
  - Causal effect is difference between two potential outcomes
- *Identification* [application/implementation]
  - Identifying assumptions needed for a statistical estimate to have causal interpretation
  - Removing selection bias
  - E.g., RD, IV, ...
- *Estimation* [application/implementation]

RCT: Identification, Estimand, and Estimator

- **Estimand**
  - Average Treatment Effect (ATE):
  $$\tau = \mathbb{E}[Y(1) - Y(0)]$$
  - A population-level causal effect defined using potential outcomes.
- **Identification Assumptions (assumption about DGP)**
  - Random assignment:
  $$(Y(1), Y(0)) \perp D$$
  - Stable Unit Treatment Value Assumption (SUTVA).
- **Identifiable (under these assumptions)**
  - The estimand can be written using observable quantities:
  $$\mathbb{E}[Y(1)] - \mathbb{E}[Y(0)] = \mathbb{E}[Y \mid D = 1] - \mathbb{E}[Y \mid D = 0]$$
- **Estimator**
  - Difference in sample means:
  $$\hat{\tau} = \bar{Y}_{D=1} - \bar{Y}_{D=0}$$

2 by 2 DiD: Identification, Estimand, and Estimator

- **Estimand**
  - Average Treatment Effect on the Treated (ATT):

$$\tau_{\text{ATT}} = \mathbb{E}[Y(1)_{t=1} - Y(0)_{t=1} \mid D = 1]$$

- **Identification Assumptions (assumption about DGP)**
  - Parallel trends:

$$\mathbb{E}[Y(0)_{t=1} - Y(0)_{t=0} \mid D = 1] = \mathbb{E}[Y(0)_{t=1} - Y(0)_{t=0} \mid D = 0]$$

  - No anticipation assumption
- **Identifiable (under these assumptions)**
  - The counterfactual trend for treated units is recovered from controls:

$$\tau_{\text{ATT}} = (\mathbb{E}[Y_{1,1}] - \mathbb{E}[Y_{1,0}]) - (\mathbb{E}[Y_{0,1}] - \mathbb{E}[Y_{0,0}])$$

- **Estimator**
  - DiD estimator:

$$\hat{\tau}_{\text{DiD}} = (\bar{Y}_{1,1} - \bar{Y}_{1,0}) - (\bar{Y}_{0,1} - \bar{Y}_{0,0})$$

  - Equivalent to the coefficient on *Treatment* $\times$ *Post* in a 2×2 DiD regression.

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Today: Modeling assumptions

- In econometrics, focus of causal inference is usually on *identification*
- But we are *never* in a perfect identification setup in observational studies
  - Model specification has influence
  - Modeling assumptions must hold (*especially* because we make causal claims based on statistical significant (e.g., p-values))
  - May want more than just unbiasedness (e.g., precision, external validity)

→ Need to think about modeling assumptions and estimator properties

The Classical Linear Regression Model

- The Classical Linear Regression Model (CLRM):

$$y|X \sim \mathcal{F}(X\beta, \sigma^2 I)$$

- Represent specific distribution induced by the data generating process (DGP)
- Regression models focus on *conditional distribution* $y|X$
- We can therefore write them as *conditional mean* $+/\times$ *error*

$$y = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k + e, \qquad e \overset{\text{iid}}{\sim} \mathcal{F}(0, \sigma^2 I)$$

$$y = \mathbb{E}[y|X] + e, \qquad \mathbb{E}[y|X] = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k, \qquad e \overset{\text{iid}}{\sim} \mathcal{F}(0, \sigma^2 I)$$

RCT: Regression Estimator for ATE

- Write observed outcome in terms of potential Outcomes:

$$Y = XY_1 + (1 - X)Y_0$$

- Decompose potential outcomes into mean and deviation:

$$\begin{aligned} Y_1 &= \mathbb{E}[Y_1] + \epsilon_1, \qquad \mathbb{E}[\epsilon_1] = 0 \\ Y_0 &= \mathbb{E}[Y_0] + \epsilon_0, \qquad \mathbb{E}[\epsilon_0] = 0 \end{aligned}$$

- Substituting these back into the observed outcome equation:

$$\begin{aligned} Y &= X(\mathbb{E}[Y_1] + \epsilon_1) + (1 - X)(\mathbb{E}[Y_0] + \epsilon_0) \\ Y &= \mathbb{E}[Y_0] + X(\mathbb{E}[Y_1] - \mathbb{E}[Y_0]) + \underbrace{\epsilon_0 + X(\epsilon_1 - \epsilon_0)}_{e} \end{aligned}$$

- Identification assumption gives externality:

$$Y = \alpha + \beta X + e, \qquad \text{where } \beta = \mathbb{E}[Y_1 - Y_0] = \text{ATE}$$

$$\mathbb{E}[e|X = 0] = \mathbb{E}[\epsilon_0] = 0 \text{ and } \mathbb{E}[e|X = 1] = \mathbb{E}[\epsilon_1] = 0; \text{ thus } \mathbb{E}[e|X] = 0.$$

Assumptions of the CLRM

- A set of assumptions that describe a DGP (CLRM)
- The assumptions (aka Gauss-Markov assumptions), by decreasing order of importance:

Notation:	System of n equations	Matrix
Model:	$y_i = x_i'\beta + e_i \;\; (i = 1, ..., n)$	$y = X\beta + e$
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A4. spherical errors	$e_i \| X \overset{\text{iid}}{\sim} (0, \sigma^2)$	$\mathbb{V}[e \| X] = \sigma^2 I_N$
-independent errors	$cov[e_i, e_j \| X] = 0$	
-homoskedastic errors	$\mathbb{V}[e_i \| X] = \sigma^2 \quad \cancel{\sigma_i^2}$	

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-homoskedastic errors	$\mathbb{V}[e_i \vert X] = \sigma^2 \quad \cancel{\sigma_i^2}$	
A5. normal errors	$e_i \vert X \sim \mathcal{N}(0, \sigma^2)$	$e_i \vert X \sim \mathcal{N}(0_{N\times 1}, \sigma^2 I_N)$

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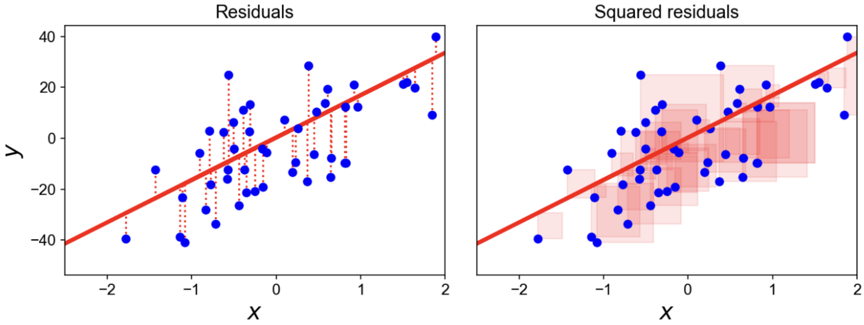
$$\hat{\beta}_{\text{OLS}} = \underset{\beta}{\text{argmin}} \sum_{i=1}^{n} r_i^2 = \sum_{j=1}^{n} (y_i - X_i'\beta)^2 = \ldots = (X'X)^{-1}X'y = \beta_0 + (X'X)^{-1}X'e$$



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Based on the assumptions, $\hat{\beta}_{OLS}$ has the following:

- finite sample properties:

  (A1-A3) $\rightarrow$ $\hat{\beta}_{OLS}$ **unbiased**
  (A4) $\rightarrow$ $\hat{\beta}_{OLS}$ **efficient** (Best **Linear** Unbiased Estimator (BLUE))
  (A5) $\rightarrow$ $\hat{\beta}_{OLS}$ **efficient** ($\hat{\beta}_{MLE}$; Best Unbiased Estimator; normal)

- asymptotic properties:
  - $\hat{\beta}_{OLS}$ is consistent and is asymptomatically unbiased, normally distributed, and efficient

(Assumption 5) Normal errors

- Normal error assumption required for making inferences (computing confidence intervals or p-values)

- Without (A5), t and F tests are invalid
  - One-sample t-test for β ($\beta = 0$) assumes sampling distribution of $\hat{\beta}$ is normal (which means errors have to be normal)

- When (A5) is violated, appeal to asymptotics:
  - When n is large enough, Laws of Large Numbers (LLNs) and Central Limit Theorem (CLT) say that asymptotic sampling distribution of $\hat{\beta}$ is normal
  - If n is large, t and F tests are robust to departures from normality
  - In the case of highly non-normal error, may want to consider alternative (e.g., bootstrap)

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(A4) Non-spherical errors

Assuming (A1)-(A3), the asymptotic distribution of $\hat{\beta}_{OLS}$ is:

$$\hat{\beta}_{OLS} \overset{a}{\sim} \mathcal{N}\left(\beta_0, (X'X)^{-1} X' \Sigma X (X'X)^{-1'}\right)$$

$\rightarrow$ need a consistent estimate of the asymptotic vcov matrix in order to do sampling-based statistical inference $\rightarrow$ need Σ (the vcov matrix of the error term)

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- Spherical e: $\Sigma = \sigma^2 I$, so we can simply consistently estimate the population variance σ^2 by the unbiased sample variance
- Heteroskedastic e: compute White SEs
- Autocorrelated e: compute Newey-West/Conley SEs if correlated in time/space...
- Clustered e: compute block-diagonal matrix using residuals

Notes on sandwich estimators

- If errors are autocorrelated in any way, it means your model is not capturing some feature of the DGP
  - Adjust for it after fitting the model (e.g., cluster SEs if autocorrelated by group)
  - Or incorporate into the structure of the model (e.g., multilevel structure)
- Sandwich estimators rely on asymptotics
  - Don't cluster SEs if you have too few clusters!
- These SEs only make sense for the linear regression model

Limited outcome models

A linear regression isn't appropriate when there is a *limited* y

- Binary: $y \in 0, 1$
- Count: $y \in 0, 1, 2, 3, \ldots$
- Censored

Limited outcome models

A linear regression isn't appropriate when there is a *limited y*

- Binary: $y \in 0, 1 \rightarrow$ probit, logit, ...
- Count: $y \in 0, 1, 2, 3, \ldots \rightarrow$ Poisson, negative binomial,...
- Censored $\rightarrow$ censored regression models

Use **generalized** linear models (GLMs) - flexible generalization of OLS:

$$g(\mathbb{E}[y|X]) = \beta_0 + \beta_1 X_1 + \ldots + \beta_k X_k, \quad e \overset{\text{iid}}{\sim} \mathcal{F}(0, \ldots)$$

- Invertible link function $g()$, which relates:
- $\mathbb{E}[y|X]$ to linear predictor vector $X\beta$
- Assume some data distribution $\mathcal{F}()$

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The Bottom-Line

In practice, causal inference in observational studies means, *both*

- A good design (identifying assumptions) $\rightarrow$ unbiasedness. Make causal effects recoverable from observed data.
- A model (modeling assumptions). Determines how we estimate and do inference for identified effects (functional form, SEs, efficiency)

You shouldn't overlook the modeling assumptions!

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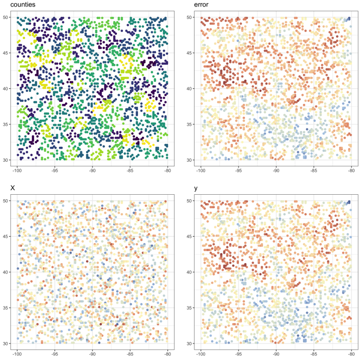
Mini coding exercise

- bit.ly/causal2026
- Create fake data with spatial autocorrelation
- Test clustered and Conley SEs
- You can play around with:
  - Parameters of DGP
  - Strength and type of spatial correlation
  - Conley SEs distance parameter
  - Clustered SEs (number of clusters, etc.)
  - Omitted variable bias
  - R vs. Stata functions



Questions? Comments?

Thank you!

References

Heavily based on Claire Palandri's 2022 version and Anna Papp's 2024 version of the Causal Inference Workshop.