Abstract

Fractional differential equations have received much attention in recent decades likely due to its powerful ability in modeling 'memory' processes, which are mostly observed in real world. Fractional models have been widely investigated and applied in many fields such as physics, biochemistry, electrical engineering, continuum and statistical mechanics. It is shown by many researchers that fractional derivatives can provide more accurate models than integer order derivatives do. However, for most models involving fractional operators, it is very challenging if not impossible to get analytical solutions. This motivates us to propose numerical techniques, especially those that can obtain numerical approximations both efficiently and effectively.

In this thesis, we aim to construct high order accuracy numerical scheme for two categories: (i) four classes of typical fractional problems, and (ii) some classes of generalized fractional problems. In tackling the first category, we focus on the improvement of the theoretical spatial convergence order mainly using *parametric quintic spline*. The parametric spline method has been used in previous work and has been numerically shown to get satisfying performance. Unfortunately, no strict theoretical result is established and we note that the analysis is not trivial. Due to this reason, we are greatly motivated to further investigate this method and provide strict analysis which is extremely important for a numerical method. Our contributions for topics in the first category mainly lies in two aspects: 1) we first apply the *parametric quintic spline* method to the fractional problems; 2) we succeed to establish the solvability, stability and convergence results of the proposed methods that are not given in previous work relating to parametric spline method.

The other category is about numerical treatment on some classes of generalized fractional problems. As the name suggests, the generalized fractional problems include typical fractional problems as special cases and may represent even more: both existing and non existing cases. In fact, it is shown that many integral equations can be solved in a much elegant way using generalized fractional operators and it can somehow blur the distinction between the differential and integral equations. For this topic, the numerical investigations are very scarce and the temporal convergence order of the existing work is not so satisfying and may not meet the demand in reality. Motivated by these two observations, we shall construct some new approximations of generalized fractional derivatives through generalizations of some typical techniques, e.g. $L2-1_\sigma$ method and weighted shifted Grünwald-Letnikov method to improve the temporal convergence order. This will help to solve this kind of problems more efficiently and effectively.