

Abstract

Complex networks emerge in diverse areas, ranging from social to biological, economic and technological systems. Among these applications, controlling the complex networks is one of the most important problem, which can guarantee the reliable and efficient operations of the network. Specifically, according to control theory, a dynamic network system is controllable if it can be driven from any initial state to any desired final state with appropriate external inputs. Existing works mainly focus on the problem of finding the minimum number of nodes connected with external input signals under different conditions such that the resulting network system is controllable. However, as we know when the Gramian matrix tend to be singular, the control cost of the network will be prohibitively large, resulting in the network is theoretically controllable but practically uncontrollable. Therefore, controlling the network with minimum cost has been an important problem to be solved.

In this dissertation, we mainly focus on two problems. The first problem is to locate a subset of nodes connected with a given number of external control sources to drive the network at minimum control cost. For this problem, projected gradient method (PGM) and projected trust-region method (PTM) are proposed with established convergence property. We show that PGM is more computationally efficient than PTM. Simulation results demonstrate satisfactory performance of the algorithms. We further extend PGM in Monte-Carlo scenario and revisited projected gradient method (R-PGM) is proposed. It is found that the importance index of a node is strongly correlated to occurrence rate of that node to be selected as a key node in Monte-Carlo realizations and key nodes tends to divide elementary topologies equally when the control cost reaches its minimum.

In addition, we reformulate the minimum-control-cost model in Boolean constraints. By relaxing these constraints into their convex hull, Inexact Alternating Direction Method of Multipliers (ADMM) is applied and a lower bound is obtained based on the global optimal value. We also introduce a simple but efficient algorithm which can greedily identify key nodes one by one and obtain the upper bound control cost. The proposed two algorithms are validated by examples ranging from Erdős-Rényi networks, scale-free networks to some real-life networks.

The second issue we deal with is to design the network topology with given external control sources such that the control cost is minimum. A matrix function optimization model is proposed to study how the network topology evolves when the objective is to achieve an optimal control of the networks. By obtaining the direction of network topology evolution, a normalized gradient descent method (NPGM) based on the obtained gradient of the network topology evolution is developed to solve the proposed optimization model. It is proven that NPGM linearly converges to a local minimum point. We also derive an optimality condition to determine whether a converged solution is global minimum or not, and

such a condition is verified through numerous experimental tests on directed networks. We find that a network adaptively changes its topology in such a way that many sub-networks are gradually evolved towards a pre-established control target. Such a finding enables us to model and explain how real-world complex networks adaptively self-organize themselves to many similar subnetworks during a relatively long evolution process.

We further extend the model to another case, i.e. to investigate how connection strengths between nodes vary in accordance with control cost for complex networks with fixed topological structure. By considering optimal control of the networks, we obtain the optimal connection strength matrix with proposed NPGM. It is discovered that several control-flow subnetworks are self-formed. Moreover, the control cost with optimal weight matrix is smallest when pre-located controller sources distribute evenly. These observations provide a comprehensive understanding of the impact of connection link weight on control cost for complex networks.