

| initialx | | xOpt | | xTrue | | thetaOpt | | | | thetaTrue | | | | objOpt | MLBD | SUBD |
|----------|-------|--------|--------|--------|--------|----------|--------|--------|--------|-----------|--------|--------|--------|--------|-------|------|
| [0] | [1] | [0] | [1] | [0] | [1] | [0][0] | [1][0] | [0][1] | [1][1] | [0][0] | [1][0] | [0][1] | [1][1] | | | |
| 0.314 | 0.039 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.006 | 0.098 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.592 | 0.899 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.896 | 0.988 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.438 | 0.329 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.187 | 0.601 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.883 | 0.082 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.774 | 0.311 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.512 | 0.799 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.82 | 0.825 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.862 | 0.743 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 1 | 0.445 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.143 | 0.039 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.041 | 0.002 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.262 | 0.634 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.294 | 0.486 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.047 | 0.875 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.189 | 0.106 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.734 | 0.014 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.091 | 0.404 | -0.852 | -0.215 | -0.219 | -0.866 | 0.055 | 0.945 | 0.39 | 0.61 | 0.951 | 0.049 | 0.622 | 0.378 | 0 | -0.01 | 0 |
| 0.631 | 0.196 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.846 | 0.944 | 0 | -0.627 | 0.44 | -0.51 | 0.439 | 0.561 | 1 | 0 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.38 | 0.876 | 0 | -0.627 | 0.44 | -0.51 | 0.439 | 0.561 | 1 | 0 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.086 | 0.848 | -0.468 | -0.002 | 0.44 | -0.51 | 0.751 | 0.249 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.912 | 0.028 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.977 | 0.967 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.858 | 0.204 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.808 | 0.506 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.986 | 0.165 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.75 | 0.356 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.521 | 0.683 | 0 | -0.627 | 0.44 | -0.51 | 0.439 | 0.561 | 1 | 0 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.897 | 0.497 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.538 | 0.249 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.838 | 0.616 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.593 | 0.03 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.837 | 0.678 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.468 | 0.279 | -0.627 | 0 | 0.44 | -0.51 | 0.561 | 0.439 | 0 | 1 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.173 | 0.782 | 0 | -0.627 | 0.44 | -0.51 | 0.439 | 0.561 | 1 | 0 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.399 | 0.574 | 0 | -0.627 | 0.44 | -0.51 | 0.439 | 0.561 | 1 | 0 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.096 | 0.396 | 0 | -0.627 | 0.44 | -0.51 | 0.439 | 0.561 | 1 | 0 | 0.167 | 0.833 | 0.553 | 0.447 | 0 | -0.01 | 0 |
| 0.483 | 0.812 | 1.738 | 0.533 | -2.956 | 0.748 | 0.179 | 0.821 | 0.179 | 0.821 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.409 | 0.789 | 3.01 | 0.695 | -2.956 | 0.748 | 0.023 | 0.977 | 0.023 | 0.977 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.188 | 0.905 | 3.527 | 0.178 | -2.956 | 0.748 | 0.17 | 0.83 | 0.17 | 0.83 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.517 | 0.413 | 0.669 | 3.036 | -2.956 | 0.748 | 0.966 | 0.034 | 0.966 | 0.034 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.585 | 0.769 | 3.249 | 0.455 | -2.956 | 0.748 | 0.105 | 0.895 | 0.105 | 0.895 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.256 | 0.107 | 3.22 | 0.485 | -2.956 | 0.748 | 0.096 | 0.904 | 0.096 | 0.904 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.191 | 0.07 | 0.445 | 0.821 | -2.956 | 0.748 | 0.193 | 0.807 | 0.193 | 0.807 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.615 | 0.395 | 0.726 | 0.725 | -2.956 | 0.748 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0.001 | -0.01 | 0 |
| 0.766 | 0.419 | 0.802 | 0.753 | -2.956 | 0.748 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.858 | 0.978 | 1.836 | 0.599 | -2.956 | 0.748 | 0.121 | 0.879 | 0.121 | 0.879 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.667 | 0.338 | 1.242 | 0.747 | -2.956 | 0.748 | 0.002 | 0.998 | 0.002 | 0.998 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.893 | 0.57 | 0.615 | 3.089 | -2.956 | 0.748 | 0.946 | 0.054 | 0.946 | 0.054 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.675 | 0.901 | 0.588 | 2.429 | -2.956 | 0.748 | 0.913 | 0.087 | 0.913 | 0.087 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.868 | 0.887 | 1.836 | 0.599 | -2.956 | 0.748 | 0.12 | 0.88 | 0.12 | 0.88 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.402 | 0.973 | 0.37 | 0.764 | -2.956 | 0.748 | 0.04 | 0.96 | 0.04 | 0.96 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.693 | 0.412 | 1.247 | 0.746 | -2.956 | 0.748 | 0.005 | 0.995 | 0.005 | 0.995 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.275 | 0.927 | 0.164 | 0.69 | -2.956 | 0.748 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0.007 | -0.01 | 0 |
| 0.811 | 0.757 | 0.25 | 3.454 | -2.956 | 0.748 | 0.845 | 0.155 | 0.845 | 0.155 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.379 | 0.938 | 3.561 | 0.143 | -2.956 | 0.748 | 0.177 | 0.823 | 0.177 | 0.823 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |
| 0.257 | 0.748 | 0.75 | 0.758 | -2.956 | 0.748 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | -0.01 | 0 |

Table 1. Test GOP using different seeds on three data sets with scale of M=1, K=2, N=2.

The optimal objective function $||y-xOpt*thetaOpt||^2_2$ is 0.

Constraints for x:

`cons = np.sum(abs(x_star))`

Number of seeds:

`MAXSEED = 20`

Generate different seeds by timing:

`SEED = int(time.time())`

`np.random.seed(SEED)`

Generate initial value for x:

`np.random.random_sample((M,K))`

Tolerance:

`e = 0.01`

Parallel processing in "get unique regions using cell numeration" and "solve master pr

`NUM_CORES = 64`