

Assessing the Concordance between Two Georeferenced Variables

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Spatial Concordance



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Outline

- Introduction
- Spatial CCC
- A first Extension
- A Nonparametric Perspective

Motivation

- Measurements of agreement are needed to assess the acceptability of new methodology.
- This is important in an assay validation or an instrument validation process.
- Requiring a new measurement to be identical to the truth is often impractical.
- Agreement with continuous measurements (Barnhart: et al., 2007)
 - ▶ Descriptive tools
 - ▶ Unscaled summary indices based on differences
 - ▶ Scaled summary indices attaining values between -1 and 1. (A concordance correlation coefficient was studied by Lin (1989))
- How to include spatial information in a concordance coefficient?

Examples

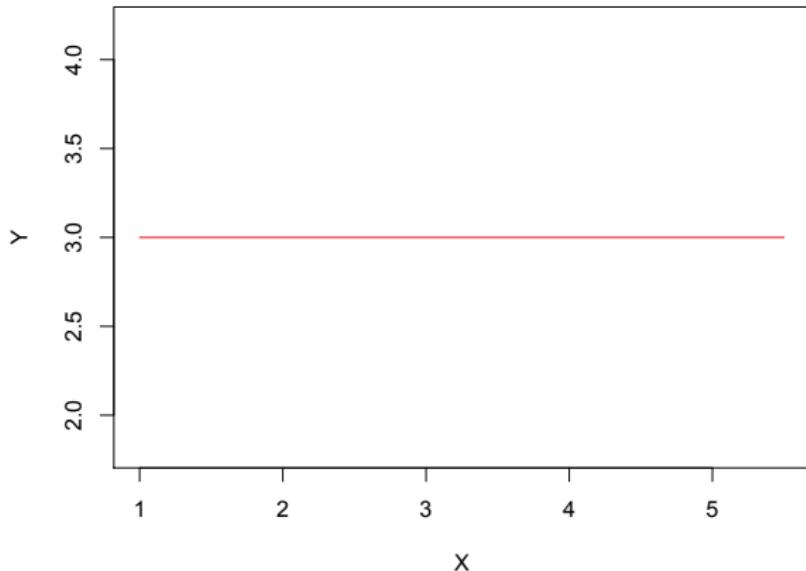
Poor Agreement of the Correlation Coefficient

Values for x

1, 1.5, 2, 2.5, 3,
3.5, 4, 4.5, 5, 5.5

Values for y

3, 3, 3, 3, 3,
3, 3, 3, 3, 3



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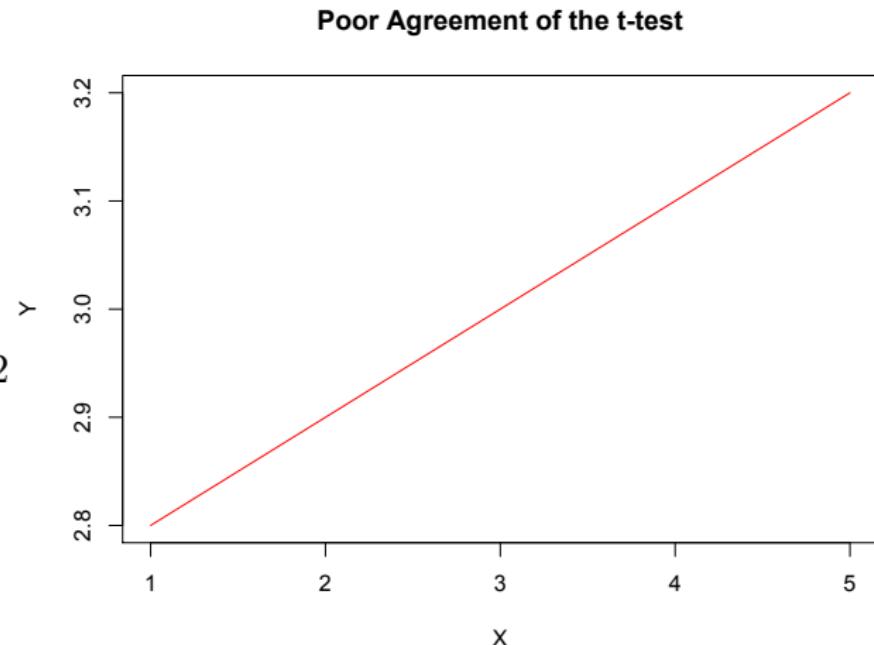


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Examples

Values for x
1, 2, 3, 4, 5

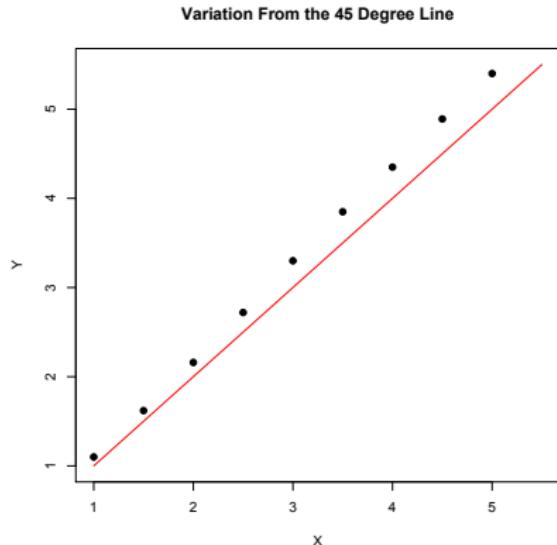
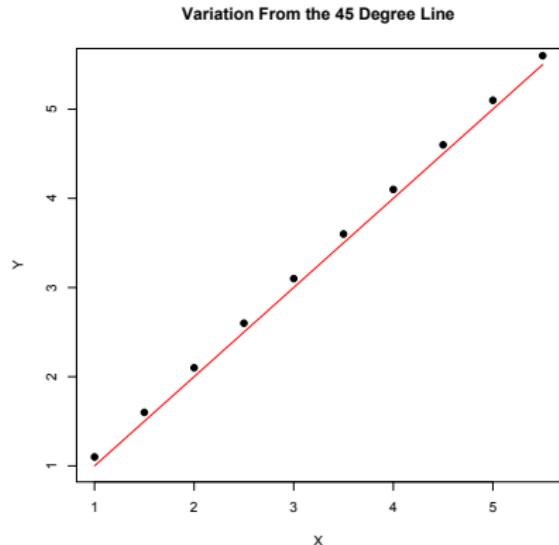
Values for y
2.8, 2.9, 3, 3.1, 3.2



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Examples

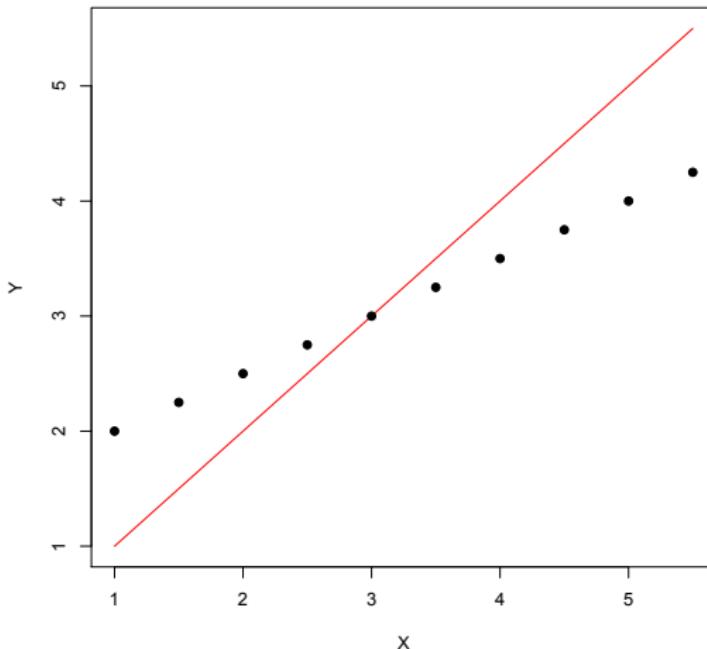


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Examples

Variation From the 45 Degree Line



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The Concordance Correlation Coefficient

Definition

Assume that the joint distribution of X and Y has finite second moment with means μ_1 and μ_2 , variances σ_1^2 and σ_2^2 , and covariance σ_{21} .

- The mean squared deviation of $D = Y - X$ is

$$\begin{aligned} \text{MSD} &= \epsilon^2 = \mathbb{E}[D^2] = \mathbb{E}[(Y - X)^2] \\ &= (\mu_1 - \mu_2)^2 + \sigma_2^2 + \sigma_1^2 - 2\sigma_{21}. \end{aligned}$$

- The Concordance Correlation Coefficient (Lin, 1989) is

$$\rho_c = 1 - \frac{\epsilon^2}{\epsilon^2 | \rho = 0} = \frac{2\sigma_{21}}{\sigma_2^2 + \sigma_1^2 + (\mu_2^2 - \mu_1^2)^2}.$$

The Concordance Correlation Coefficient

Properties

- $\rho_c = \alpha \cdot \rho$, where $\alpha = \frac{2}{w + 1/w + v^2}$, $w = \frac{\sigma_2}{\sigma_1}$, $v = \frac{\mu_2 - \mu_1}{\sqrt{\sigma_2 \sigma_1}}$, and $\rho = \text{corr}(X, Y)$.
- $|\rho_c| \leq 1$.
- $\rho_c = 0$ if and only if $\rho = 0$.
- $\rho_c = \rho$ if and only if $\sigma_2 = \sigma_1$ and $\mu_2 = \mu_1$.

Sample Concordance

The sample counterpart of ρ_c is given as

$$\hat{\rho}_c = \frac{2s_{21}}{s_2^2 + s_1^2 + (\bar{y} - \bar{x})^2}.$$

Spatial Concordance



Our Proposal

- The goal is to construct a concordance coefficient that takes into account the spatial lag \mathbf{h} , similarly to the variogram and cross-variogram.
- $C_{11}(\mathbf{h}) = \text{cov}[X(s), X(s + \mathbf{h})]$,
 $C_{22}(\mathbf{h}) = \text{cov}[Y(s), Y(s + \mathbf{h})]$.
 $C_{12}(\mathbf{h}) = \text{cov}[X(s), Y(s + \mathbf{h})]$.
- The idea is to define a new coefficient of the form

$$\rho_c = 1 - \frac{\epsilon^2}{\epsilon^2 | \rho = 0 },$$

but using the above ingredients.

Our Definition

Definition

Let $(X(\mathbf{s}), Y(\mathbf{s}))^\top$ be a bivariate second order stationary random field with $\mathbf{s} \in \mathbb{R}^2$, mean $(\mu_1, \mu_2)^\top$ and covariance function

$$\mathbf{C}(\mathbf{h}) = \begin{pmatrix} C_{11}(\mathbf{h}) & C_{12}(\mathbf{h}) \\ C_{21}(\mathbf{h}) & C_{22}(\mathbf{h}) \end{pmatrix}.$$

Then the spatial concordance coefficient is defined as

$$\rho_c(\mathbf{h}) = \frac{\mathbb{E}[(Y(\mathbf{s} + \mathbf{h}) - X(\mathbf{s}))^2]}{\mathbb{E}[(Y(\mathbf{s} + \mathbf{h}) - X(\mathbf{s}))^2 | C_{12}(\mathbf{0}) = 0]} = \frac{2C_{21}(\mathbf{h})}{C_{11}(\mathbf{0}) + C_{22}(\mathbf{0}) + (\mu_1 - \mu_2)^2}.$$

Some Features

- $\rho_c(\mathbf{h}) = \eta \cdot \rho_{21}(\mathbf{h})$, where $\eta = \frac{2\sqrt{C_{11}(\mathbf{0})C_{22}(\mathbf{0})}}{C_{11}(\mathbf{0}) + C_{22}(\mathbf{0}) + (\mu_1 - \mu_2)^2}$.
- $|\rho_c(\mathbf{h})| \leq 1$.
- $\rho_c(\mathbf{h}) = 0$ iff $\rho_{21}(\mathbf{h}) = 0$.
- $\rho_c(\mathbf{h}) = \rho_{21}(\mathbf{h})$ iff $\mu_1 = \mu_2$ and $C_{11}(\mathbf{0}) = C_{22}(\mathbf{0})$.
- For the nonseparable covariance function

$$C_{ij}(\mathbf{h}) = \rho_{ij}\sigma_i\sigma_j R(\mathbf{h}, \boldsymbol{\psi}_{ij}), \quad \rho_{ii} = 1, i, j = 1, 2.$$

where $R(\mathbf{h}, \boldsymbol{\psi})$ is a univariate correlation function, we have

$$\rho_c(\mathbf{h}) = \eta \cdot \rho_{12},$$

$$\text{where } \eta = \frac{2\sigma_1\sigma_2R(\mathbf{h}, \boldsymbol{\psi}_{12})}{\sigma_1^2R(\mathbf{0}, \boldsymbol{\psi}_{11}) + \sigma_2^2R(\mathbf{0}, \boldsymbol{\psi}_{22})}.$$

Some Features

- In particular, If $C_{11}(\mathbf{h}) = \sigma_1^2 M(\mathbf{h}, \nu_1, a_1)$,
 $C_{22}(\mathbf{h}) = \sigma_2^2 M(\mathbf{h}, \nu_2, a_2)$, and
 $C_{21}(\mathbf{h}, \nu_{12}, a_{12}) = \rho_{12} \sigma_1 \sigma_2 M(\mathbf{h}, \nu_{12}, a_{12})$, where
 $M(\mathbf{h}, \nu, a) = \frac{2^{1-\nu}}{\Gamma(\nu)} (a\|\mathbf{h}\|)^\nu K_\nu(a\|\mathbf{h}\|)$, and $K_\nu(\cdot)$ is a modified Bessel function of the second type and $\rho_{12} = \text{cor}[X(s_i), Y(s_j)]$,
 Then

$$\rho_c(\mathbf{h}) = \frac{2\sigma_1 \sigma_2 \rho_{12} M(\mathbf{h}, \nu_{12}, a_{12})}{\sigma_1^2 + \sigma_2^2} = \eta \cdot \rho_{12},$$

where $\eta = \frac{2\sigma_1 \sigma_2 M(\mathbf{h}, \nu_{12}, a_{12})}{\sigma_1^2 + \sigma_2^2}$.

- In the previous scheme, if $\nu_{12} = 1/2$, then

$$\rho_c(\mathbf{h}) = \eta \cdot \rho_{12},$$

where $\eta = \frac{2\sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2} e^{-a_{12}\|\mathbf{h}\|}$.

A Local Approach

- Let $(X(\mathbf{s}), Y(\mathbf{s}))^\top, \mathbf{s} \in D \subset \mathbb{Z}^2$ be a bivariate random field.
 - Suppose now that D is a finite rectangular grid of \mathbb{Z}^2 and that we split D into p subgrids, say $D_i, i = 1, \dots, p$. Then the process $(X_i(\mathbf{s}), Y_i(\mathbf{s}))^\top, \mathbf{s} \in D_i$, represents two subimages defined on D_i .
 - Assume that each process $(X_i(\mathbf{s}), Y_i(\mathbf{s}))^\top$ has a covariance function of the form
- $$C_{jk}^i(\mathbf{h}) = [\rho_{jk}^i \sigma_j^i \sigma_k^i R(\mathbf{h}, \boldsymbol{\psi}_i)]_{j,k=1}^2, \quad \rho_{jk}^i = 1, i = 1, \dots, p, \quad j, k = 1, 2.$$
- Let $\rho_c^i(\mathbf{h})$ be the spatial concordance correlation coefficient of each $(X_i(\mathbf{s}), Y_i(\mathbf{s}))^\top$. Then

$$\rho_c^i(\mathbf{h}) = \frac{2\sigma_1^i \sigma_2^i}{(\sigma_1^i)^2 + (\sigma_2^i)^2} \rho_{12}^i R(\mathbf{h}, \boldsymbol{\psi}_i).$$

A Local Approach

- In order to summarize the local concordance coefficients defined for each window, we propose two global concordance coefficients

$$\square \quad \rho_1(\mathbf{h}) = \frac{1}{p} \sum_{i=1}^p \rho_c^i(\mathbf{h}).$$

$$\square \quad \rho_2(\mathbf{h}) = \frac{2\bar{\sigma}_1\bar{\sigma}_2}{\bar{\sigma}_1^2 + \bar{\sigma}_2^2} \bar{\rho}_{12} R(\mathbf{h}, \bar{\psi}),$$

where $\bar{\sigma}_1, \bar{\sigma}_2, \bar{\rho}_{12}, \bar{\psi}$ are the average of the values computed for each sub-image.

- The sample counterparts of $\rho_1(\mathbf{h})$ and $\rho_2(\mathbf{h})$ are

$$\hat{\rho}_1(\mathbf{h}) = \frac{1}{p} \sum_{i=1}^p \hat{\rho}_c^i(\mathbf{h}), \quad \hat{\rho}_2(\mathbf{h}) = \frac{2\hat{\bar{\sigma}}_1\hat{\bar{\sigma}}_2}{\hat{\bar{\sigma}}_1^2 + \hat{\bar{\sigma}}_2^2} \hat{\bar{\rho}}_{12} R(\mathbf{h}, \hat{\bar{\psi}}).$$

Estimation and Asymptotics

- Let $\{Y(s) : s \in D \subset \mathbb{R}^d\}$ be a Gaussian random field such that $Y(\cdot)$ is observed on $D_n = \{s_1, s_2, \dots, s_n\} \subset D$.
- Denote $\mathbf{Y} = (Y(s_1), \dots, Y(s_n))^\top$ and assume that $\mathbb{E}[\mathbf{Y}] = \mathbf{X}\boldsymbol{\beta}$, $\text{cov}(Y(t), Y(s)) = \sigma(t, s; \boldsymbol{\theta})$, \mathbf{X} is $n \times p$ with $\text{rank}(\mathbf{X}) = p$, $\boldsymbol{\beta} \in \mathbb{R}^p$ and $\boldsymbol{\theta} \in \mathbb{R}^q$.
- Let $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}(\boldsymbol{\theta})$ be the covariance matrix of \mathbf{Y} such that the ij -th element of $\boldsymbol{\Sigma}$ is $\sigma_{ij} = \sigma(s_i, s_j; \boldsymbol{\theta})$.
- The estimation of $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$ can be made by ML estimation, maximizing

$$L = L(\boldsymbol{\beta}, \boldsymbol{\theta}) = \text{Conts} - \frac{1}{2} \ln |\boldsymbol{\Sigma}^{-1}| - \frac{1}{2} (\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})^\top \boldsymbol{\Sigma}^{-1} (\mathbf{Y} - \mathbf{X}\boldsymbol{\beta})$$

Estimation and Asymptotics

Theorem

(Mardia and Marshall, 1984) Let $\lambda_1 \leq \dots \leq \lambda_n$ be the eigenvalues of Σ , and let those of $\Sigma_i = \frac{\partial \Sigma}{\partial \theta_i}$ and $\Sigma_{ij} = \frac{\partial^2 \Sigma}{\partial \theta_i \partial \theta_j}$ be λ_k^i and λ_k^{ij} , $k = 1, \dots, n$, such that $|\lambda_1^i| \leq \dots \leq |\lambda_n^i|$ and $|\lambda_1^{ij}| \leq \dots \leq |\lambda_n^{ij}|$ for $i, j = 1, \dots, q$. Suppose that as $n \rightarrow \infty$

- (i) $\lim \lambda_n = C < \infty$, $\lim |\lambda_n^i| = C_i < \infty$ y $\lim |\lambda_n^{ij}| = C_{ij} < \infty$ for all $i, j = 1, \dots, q$.
- (ii) $\|\Sigma_i\|^{-2} = \mathcal{O}(n^{-\frac{1}{2}-\delta})$ for some $\delta > 0$, for $i = 1, \dots, q$.
- (iii) For all $i, j = 1, \dots, q$, $a_{ij} = \lim \left[t_{ij}/(t_{ii}t_{jj})^{\frac{1}{2}} \right]$ exists, where $t_{ij} = \text{tr}(\Sigma^{-1}\Sigma_i\Sigma^{-1}\Sigma_j)$ and $A = (a_{ij})$ is nonsingular.
- (iv) $\lim(\mathbf{X}^\top \mathbf{X})^{-1} = 0$.

Then $(\widehat{\beta}^\top, \widehat{\theta}_n^\top)^\top \xrightarrow{d} \mathcal{N}((\beta^\top, \theta^\top)^\top, \mathbf{F}_n(\theta)^{-1})$, as $n \rightarrow \infty$, in an increasing domain sense, where $\mathbf{F}_n(\theta)$ is the Fisher information matrix of β and θ .



Related Work

- Mardia y Marshall (1984) proved that for a Gaussian process with exponential covariance function these conditions are satisfied.
- Acosta and Vallejos (2018) showed that for the covariance (Matérn, $\nu_{12} = n + 1/2$)

$$C_{ij}(\mathbf{h}) = \rho_{ij}\sigma_i\sigma_j \exp(-a_{12}\|\mathbf{h}\|) \sum_{k=0}^n c_k (2a_{12}\|\mathbf{h}\|)^{n-k}, \quad i = 1, 2,$$

where $c_k = \frac{(n+k)!}{(2n)!} \binom{n}{k}$, the conditions of the Theorem are satisfied.

Here $\boldsymbol{\theta} = (\sigma_1^2, \sigma_2^2, \rho_{12}, a_{12})$.

- Bevilacqua et al. (2015) proved that for a bivariate Gaussian process with the covariance (Matérn, $\nu_{12} = 1/2$)

$$C_{ij}(\mathbf{h}) = \rho_{ij}\sigma_i\sigma_j \exp(a_{12}\|\mathbf{h}\|), \quad i = 1, 2,$$

the conditions of the Theorem hold. Here $\boldsymbol{\theta} = (\sigma_1^2, \sigma_2^2, \rho_{12}, a_{12})$.

Asymptotics for the Spatial Concordance

- ◻ $\widehat{\rho}^c(\mathbf{h}) = \widehat{\eta} \cdot \widehat{\rho}_{12}$
- ◻ If $\boldsymbol{\theta} = (\sigma_1^2, \sigma_2^2, \boldsymbol{\psi}_{11}^\top, \boldsymbol{\psi}_{22}^\top, \boldsymbol{\psi}_{12}^\top)^\top$. Then $\widehat{\rho}^c(\mathbf{h}) = g(\widehat{\boldsymbol{\theta}}_n)$.
- ◻ The Theorem of Mardia and Marshall (1984) works here for $\widehat{\boldsymbol{\theta}}_n$. The asymptotic normality of $g(\widehat{\boldsymbol{\theta}}_n)$ can be handled via the Delta Method for $g(\cdot)$ differentiable. Indeed,

$$\left(\nabla g(\boldsymbol{\theta})^\top \mathbf{F}_n(\boldsymbol{\theta})^{-1} \nabla g(\boldsymbol{\theta}) \right)^{-1/2} (g(\widehat{\boldsymbol{\theta}}_n) - g(\boldsymbol{\theta})) \xrightarrow{d} \mathcal{N}(0, 1), \text{ as } n \rightarrow \infty.$$

Computation of the Asymptotic Variance

- For the covariance Matérn, $\nu_{12} = 1/2$,

$$\begin{aligned}\nabla g(\boldsymbol{\theta})^\top \mathbf{F}_n(\boldsymbol{\theta})^{-1} \nabla g(\boldsymbol{\theta}) &= \frac{2\sigma_1^2\sigma_2^2}{(\sigma_1^2 + \sigma_2^2)^2} \left[\frac{\rho_{12}^2(\sigma_1^2 - \sigma_2^2)^2}{(\sigma_1^2 + \sigma_2^2)^2 n \mathbf{C}} \right. \\ &\quad + \frac{2n}{\mathbf{C}} + \frac{2\|\mathbf{h}\|^2\rho_{12}^2(\rho_{12}^2 - 1)^2}{n \mathbf{C}} \\ &\quad \left. + \frac{\rho_{12}^2(\sigma_1^2 - \sigma_2^2)^2([\text{tr}(\mathbf{B})]^2 - 2\rho_{12}^2 \mathbf{C})}{(\sigma_1^2 + \sigma_2^2)^2} \right] \\ &\quad \times \exp(-2a_{12}\|\mathbf{h}\|),\end{aligned}$$

- $\mathbf{B} = \left(\mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial a_{12}} \right),$
- $\mathbf{C} = n\text{tr}(\mathbf{B}^2) - [\text{tr}(\mathbf{B})]^2.$

Hypothesis Testing

- As a consequence of the asymptotic normality, an approximate hypothesis testing problem of the form

$$H_0 : \rho_c(\mathbf{h}) = \rho_0 \text{ versus } H_1 : \rho_c(\mathbf{h}) \neq \rho_0,$$

can be implemented, for a fixed \mathbf{h} .

- An approximate confidence interval of the form

$$CI(\rho_c(\mathbf{h})) = \hat{\rho}_c(\mathbf{h}) \pm z_{\alpha/2} \sqrt{v},$$

can be constructed.

- The computation of the variance is a challenging problem for other correlation structures.
- Resampling techniques could be one alternative.

Main Result

Theorem

Let $(X(s), Y(s))^\top$ be a zero mean Gaussian random field with a Wendland-Gneiting bivariate covariance function of the form

$$C_{ij}(\mathbf{h}) = \left[\rho_{ij} \sigma_i \sigma_j \left(1 + (\nu + 1) \frac{\|\mathbf{h}\|}{b_{12}} \right) \left(1 - \frac{\|\mathbf{h}\|}{b_{12}} \right)_+^{\nu+1} \right]^2, \quad i, j = 1, 2.$$

where $\nu > 0$ is fixed. Then

- $\rho_c(\mathbf{h}) = g(\sigma_1^2, \sigma_2^2, \rho_{12}, b_{12}) = \frac{2\rho_{12}\sigma_1\sigma_2}{\sigma_1^2\sigma_2^2} \left(1 + (\nu + 1) \frac{\|\mathbf{h}\|}{b_{12}} \right) \left(1 - \frac{\|\mathbf{h}\|}{b_{12}} \right)_+^{\nu+1}$
- The ML estimator of $\boldsymbol{\theta}$, $\hat{\boldsymbol{\theta}}_n$, is asymptotically normal. i.e., $(\nabla g(\boldsymbol{\theta})^\top \mathbf{F}_n(\boldsymbol{\theta})^{-1} \nabla g(\boldsymbol{\theta}))^{-1/2} (g(\hat{\boldsymbol{\theta}}_n) - g(\boldsymbol{\theta})) \xrightarrow{d} \mathcal{N}(0, 1)$, as $n \rightarrow \infty$.

An Application

- Two images of size 1600×1200 from Harvard Forest have been considered.



Figure 1: Two images taken from the same site in Harvard Forest, mainly red oak cups. Left: Image taken with an outdoor StarDot NetCam XL 3MP camera. Right: Image taken with an outdoor Axis 223M camera.

- Both images have been transformed to a grey scale and preprocessed.

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Global Model Fitting

- Lin's concordance: $\hat{\rho}_c = 0.3177$.
- We consider a Gaussian the random field $(X(\mathbf{s}), Y(\mathbf{s}))^\top$, $\mathbf{s} \in \mathbb{R}^2$.
- We fit a bivariate Matérn covariance model of the form

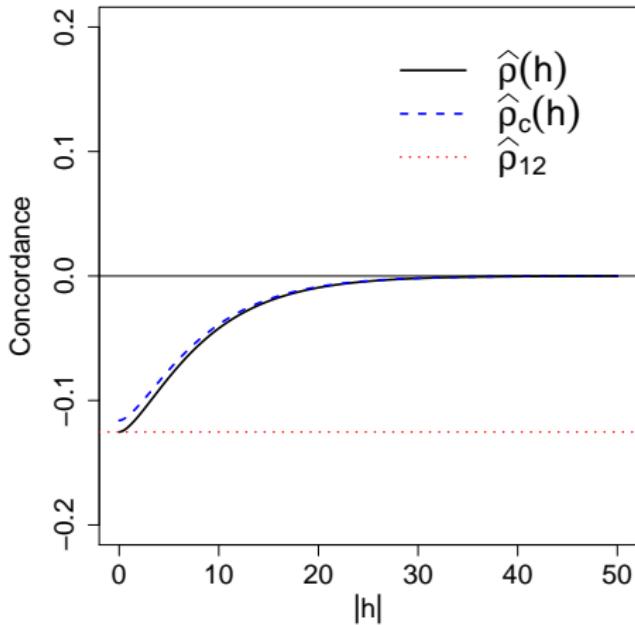
$$C_{11}(\mathbf{h}) = \sigma_1^2 M(\mathbf{h} | \nu_1, a_1),$$

$$C_{22}(\mathbf{h}) = \sigma_2^2 M(\mathbf{h} | \nu_2, a_2),$$

$$C_{12}(\mathbf{h}) = C_{21}(\mathbf{h}) = \rho_{12} \sigma_1 \sigma_2 M(\mathbf{h} | \nu_{12}, a_{12}).$$

σ_1^2	σ_2^2	ρ_{12}	ν_1	ν_2	ν_{12}	$1/a_1$	$1/a_2$	$1/a_{12}$
0.04 (0.006)	0.09 (0.037)	-0.12 (0.214)	0.1 (0.015)	0.29 (0.026)	0.99 (2.002)	162.24 (65.973)	162.24 (95.791)	5.64 (2.9589)

Results



Spatial Concordance



Local Model Fitting

- We splitted the original images into 110 rectangular images of size 19×26 .
- We fitted bivariate Matérn covariance functions of the form

$$C_{11}(\mathbf{h}) = \sigma_1^2 M(\mathbf{h} | \nu_1, a_1),$$

$$C_{22}(\mathbf{h}) = \sigma_2^2 M(\mathbf{h} | \nu_2, a_2),$$

$$C_{12}(\mathbf{h}) = C_{21}(\mathbf{h}) = \rho_{12} \sigma_1 \sigma_2 M(\mathbf{h} | \nu_{12}, a_{12}).$$

for the 110 subimages.

Local Model Fitting

- We compute the global spatial concordance coefficients

$$\hat{\rho}_1(\mathbf{h}) = \frac{1}{p} \sum_{i=1}^p \hat{\rho}_c^i(\mathbf{h}),$$

and

$$\hat{\rho}_2(\mathbf{h}) = \frac{2\hat{\sigma}_1\hat{\sigma}_2}{\hat{\sigma}_1^2 + \hat{\sigma}_2^2} \hat{\rho}_{12} M(\mathbf{h} | \hat{\nu}_{12}, \hat{a}_{12}),$$

where $\bar{\sigma}_1$, $\hat{\sigma}_2$, $\hat{\rho}_{12}$, $\hat{\nu}_{12}$, and \hat{a}_{12} are the average estimations computed using the 110 subimages.

Results

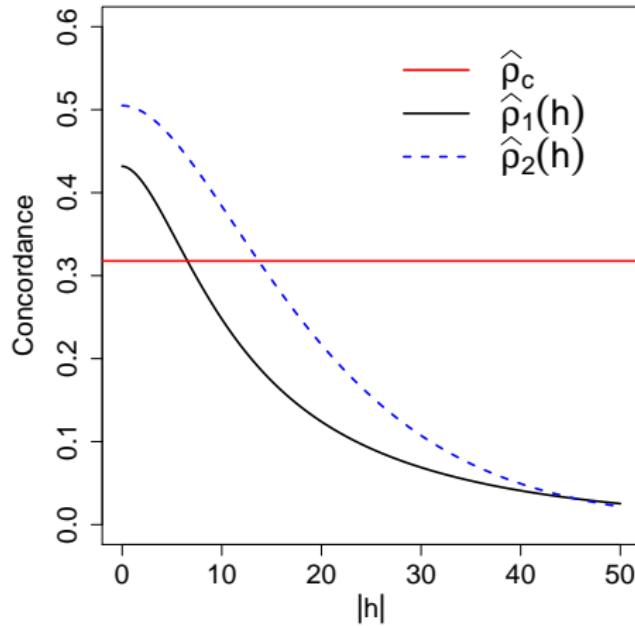


Figure 2: Global concordance coefficients.

Spatial Concordance



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Probability of Agreement

References

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