Probability of agreement

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Background

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   Lin, L. , Hedayat, A. S., Sinha, B., & Yang, M. (2002). Statistical methods in assessing agreement: models, issues, and tools. *Journal of the American Statistical Association* 97, 257–270.

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Pandit, V. & Schuller, B. (2020). The many-to-many mapping between the concordance correlation coefficient, and the mean square error. https://arxiv.org/abs/1902.05180:

$MSE_1 < MSE_2$ does not imply $CCC_1 > CCC_2$ (counterintuitive).

Probability of agreement

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Extensions

1. Stevens *et al.* (2018) assume that $M_{ijk} \sim \mathrm{N}(0, \sigma^2_{ij})$, with $\sigma^2_{ij} = \sigma^2_j(s_i) = (\omega_j + \tau_j s_i)^2$.

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The replicates can be balanced (r) or unbalanced (r_{ij}).

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$$Y_{i1k} = S_i + M_{i1k}, \tag{1}$$
$$Y_{ijk} = \alpha_j + \beta_j S_i + M_{ijk}, \tag{2}$$

for $k = 1, \ldots, r$ (replicates), $j = 2, \ldots, m$ (systems), and $i = 1, \ldots, n$ (subjects), noticing that $\alpha_1 = 0$ and $\beta_1 = 1$.
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4 Conditional probability of agreement

$$\theta_j^{(C)} = P(|Y_{ij} - Y_{i1}| \leq c_j \mid Y_{il} = y_l : l \notin \{1, j\}), \quad (4)$$

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Thank you!