

1. Tenemos el conjunto de datos:

$$\mathbf{x} = \left\{ \underbrace{10\,000, 10\,000, \dots, 10\,000}_{500 \text{ observaciones}}, 10\,001, \underbrace{10\,002, 10\,002, \dots, 10\,002}_{500 \text{ observaciones}} \right\}.$$

De este modo, es evidente que

$$\text{me}(\mathbf{x}) = 10\,001,$$

mientras que el promedio muestral es dado por:

$$\begin{aligned} \bar{x} &= \frac{500 \cdot 10\,000 + 10\,001 + 500 \cdot 10\,002}{1001} = \frac{500 \cdot 10\,000 + 10\,000 + 1 + 500(10\,000 + 2)}{1001} \\ &= \frac{500 \cdot 10\,000 + 10\,000 + 500 \cdot 10\,000 + 1 + 500 \cdot 2}{1001} = \frac{10\,000 \cdot 1001 + 1001}{1001} \\ &= \frac{1001(10\,000 + 1)}{1001} = 10\,001. \end{aligned}$$

Sea $u_i = x_i - \bar{x}$, para $i = 1, \dots, 1001$. Es decir, tenemos:

$$\mathbf{u} = \left\{ \underbrace{-1, -1, \dots, -1}_{500 \text{ obs}}, 0, \underbrace{1, 1, \dots, 1}_{500 \text{ obs}} \right\}.$$

Podemos calcular la varianza muestral como:

$$s^2 = \frac{1}{1001 - 1} \sum_{i=1}^{1001} u_i^2 = \frac{1}{1000} (500(-1)^2 + 0 + 500(1)^2) = \frac{1000}{1000} = 1.$$

Como $s = 1$, sigue que $z_i = (x_i - \bar{x})/s = u_i$, para $i = 1, \dots, 1001$. Esto permite calcular

$$b_1 = \frac{1}{1001} \sum_{i=1}^{1001} \left(\frac{x_i - \bar{x}}{s} \right)^3 = \frac{1}{1001} \sum_{i=1}^{1001} z_i^3 = \frac{1}{1001} (500(-1)^3 + 0 + 500(1)^3) = 0.$$

2. Tenemos que

$$\begin{aligned} n &= 6, & \sum_{i=1}^n x_i &= 0, & \sum_{i=1}^n x_i^2 &= 130 \\ \sum_{i=1}^n y_i &= 0.41, & \sum_{i=1}^n y_i^2 &= 10.5009, & \sum_{i=1}^n x_i y_i &= -10.59. \end{aligned}$$

a) Obtenemos $\bar{x} = 0$. Mientras que,

$$\text{me}(\mathbf{x}) = \frac{-2 + (-1)}{2} = -1.5.$$

Como $\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$, sigue que

$$s_x^2 = \frac{1}{6-1} \sum_{i=1}^n x_i^2 = \frac{130}{5} = 26.$$

Además,

$$b_2 = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right)^4 - 3 = \frac{\sum_{i=1}^n (x_i - \bar{x})^4 / (n-1)}{s_x^4} - 3.$$

Tenemos que $\bar{x} = 0$, de este modo $\sum_{i=1}^n (x_i - \bar{x})^4 = \sum_{i=1}^n x_i^4 = 10354$. Ahora,

$$b_2 = \frac{10354/5}{26^2} - 3 = \frac{2070.8}{676} - 3 = 0.0633.$$

Por otro lado, $\bar{y} = 0.41/6 = 0.0683$. Para el cálculo de la mediana, considere el conjunto de datos ordenados,

$$\{-1.44, -1.32, -0.04, 0.00, 0.73, 2.48\}.$$

De este modo, $\text{me}(\mathbf{y}) = (-0.04 + 0.00)/2 = -0.02$. Ahora,

$$\sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n y_i^2 - n\bar{y}^2 = 10.5009 - 6 \cdot 0.41^2 = 10.4729.$$

Luego, $s_y^2 = 10.4729/5 = 2.0946$. Finalmente,

$$b_2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^4 / (n-1)}{s^4} - 3 = \frac{42.9104/5}{2.0946^2} - 3 = -1.0439$$

Para el cálculo de la correlación, tenemos

$$\text{cor}(\mathbf{x}, \mathbf{y}) = \frac{\text{cov}(\mathbf{x}, \mathbf{y})}{\sqrt{\text{var}(\mathbf{x}) \text{var}(\mathbf{y})}}.$$

Ahora,

$$\text{cov}(\mathbf{x}, \mathbf{y}) = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{n-1} \left(\sum_{i=1}^n x_i y_i - n\bar{x}\bar{y} \right).$$

De ahí que, $\text{cov}(\mathbf{x}, \mathbf{y}) = -10.59/5 = -2.118$. De este modo,

$$\text{cor}(\mathbf{x}, \mathbf{y}) = \frac{-2.1180}{\sqrt{26 \cdot 2.0946}} = -0.2870.$$

3. Tenemos que

$$\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2,$$

luego

$$\begin{aligned} \kappa &= \frac{\sum_{i=1}^n x_i^2}{(n-1)s^2} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2 + n\bar{x}^2}{(n-1)s^2} = \frac{(n-1)s^2 + n\bar{x}^2}{(n-1)s^2} \\ &= 1 + \left(\frac{n}{n-1} \right) \frac{\bar{x}^2}{s^2} = 1 + \left(\frac{n}{n-1} \right) \left(\frac{s}{\bar{x}} \right)^{-2} \\ &= 1 + \left(\frac{n}{n-1} \right) CV^{-2}. \end{aligned}$$