1. Tenemos el conjunto de datos:

$$\boldsymbol{x} = \{\underbrace{10\,000, 10\,000, \cdots, 10\,000}_{500 \text{ observaciones}}, 10\,001, \underbrace{10\,002, 10\,002, \cdots, 10\,002}_{500 \text{ observaciones}}\}.$$

De este modo, es evidente que

$$\operatorname{me}(\boldsymbol{x}) = 10\,001,$$

mientras que el promedio muestral es dado por:

$$\begin{aligned}
\overline{x} &= \frac{500 \cdot 10\,000 + 10\,001 + 500 \cdot 10\,002}{1001} = \frac{500 \cdot 10\,000 + 10\,000 + 1 + 500(10\,000 + 2)}{1001} \\
&= \frac{500 \cdot 10\,000 + 10\,000 + 500 \cdot 10\,000 + 1 + 500 \cdot 2}{1001} = \frac{10\,000 \cdot 1001 + 1001}{1001} \\
&= \frac{1001(10\,000 + 1)}{1001} = 10\,001.
\end{aligned}$$

Sea $u_i = x_i - \overline{x}$, para $i = 1, \ldots, 1001$. Es decir, tenemos:

$$\boldsymbol{u} = \{\underbrace{-1, -1, \cdots, -1}_{500 \text{ obs}}, 0, \underbrace{1, 1, \cdots, 1}_{500 \text{ obs}}\}.$$

Podemos calcular la varianza muestral como:

$$s^2 = \frac{1}{1001 - 1} \sum_{i=1}^{1001} u_i^2 = \frac{1}{1000}\big(500(-1)^2 + 0 + 500(1)^2\big) = \frac{1000}{1000} = 1.$$

Como $s = 1$, sigue que $z_i = (x_i - \overline{x})/s = u_i$, para $i = 1, \ldots, 1001$. Esto permite calcular

$$b_1 = \frac{1}{1001} \sum_{i=1}^{1001} \Big(\frac{x_i - \overline{x}}{s}\Big)^3 = \frac{1}{1001} \sum_{i=1}^{1001} z_i^3 = \frac{1}{1001}\big(500(-1)^3 + 0 + 500(1)^3\big) = 0.$$

2. Tenemos que

$$\begin{aligned}
&n = 6, && \sum_{i=1}^{n} x_i = 0, && \sum_{i=1}^{n} x_i^2 = 130 \\
&\sum_{i=1}^{n} y_i = 0.41, && \sum_{i=1}^{n} y_i^2 = 10.5009, && \sum_{i=1}^{n} x_i y_i = -10.59.
\end{aligned}$$

a) Obtenemos $\overline{x} = 0$. Mientras que,

$$\operatorname{me}(\boldsymbol{x}) = \frac{-2 + (-1)}{2} = -1.5.$$

Como $\sum_{i=1}^n (x_i - \overline{x})^2 = \sum_{i=1}^n x_i^2 - n\overline{x}^2$, sigue que

$$s_x^2 = \frac{1}{6-1}\sum_{i=1}^n x_i^2 = \frac{130}{5} = 26.$$

Además,

$$b_2 = \frac{1}{n-1}\sum_{i=1}^n \Big(\frac{x_i - \overline{x}}{s_x}\Big)^4 - 3 = \frac{\sum_{i=1}^n (x_i - \overline{x})^4/(n-1)}{s_x^4} - 3.$$

Tenemos que $\overline{x} = 0$, de este modo $\sum_{i=1}^n (x_i - \overline{x})^4 = \sum_{i=1}^n x_i^4 = 10354$. Ahora,

$$b_2 = \frac{10354/5}{26^2} - 3 = \frac{2070.8}{676} - 3 = 0.0633.$$

Por otro lado, $\overline{y} = 0.41/6 = 0.0683$. Para el cálculo de la mediana, considere el conjunto de datos ordenados,

$$\{-1.44, -1.32, -0.04, 0.00, 0.73, 2.48\}.$$

De este modo, $\mathrm{me}(\boldsymbol{y}) = (-0.04 + 0.00)/2 = -0.02$. Ahora,

$$\sum_{i=1}^n (y_i - \overline{y})^2 = \sum_{i=1}^n y_i^2 - n\overline{y}^2 = 10.5009 - 6 \cdot 0.41^2 = 10.4729.$$

Luego, $s_y^2 = 10.4729/5 = 2.0946$. Finalmente,

$$b_2 = \frac{\sum_{i=1}^n (y_i - \overline{y})^4/(n-1)}{s^4} - 3 = \frac{42.9104/5}{2.0946^2} - 3 = -1.0439$$

b) Para el cálculo de la correlación, tenemos

$$\mathsf{cor}(\boldsymbol{x}, \boldsymbol{y}) = \frac{\mathsf{cov}(\boldsymbol{x}, \boldsymbol{y})}{\sqrt{\mathsf{var}(\boldsymbol{x})\,\mathsf{var}(\boldsymbol{y})}}.$$

Ahora,

$$\mathsf{cov}(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{n-1}\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y}) = \frac{1}{n-1}\Big(\sum_{i=1}^n x_i y_i - n\overline{x}\,\overline{y}\Big).$$

De ahí que, $\mathsf{cov}(\boldsymbol{x}, \boldsymbol{y}) = -10.59/5 = -2.118$. De este modo,

$$\mathsf{cor}(\boldsymbol{x}, \boldsymbol{y}) = \frac{-2.1180}{\sqrt{26 \cdot 2.0946}} = -0.2870.$$

3. Tenemos que

$$\sum_{i=1}^n (x_i - \overline{x})^2 = \sum_{i=1}^n x_i^2 - n\overline{x}^2,$$

luego

$$\begin{aligned}
\kappa = \frac{\sum_{i=1}^n x_i^2}{(n-1)s^2} &= \frac{\sum_{i=1}^n (x_i - \overline{x})^2 + n\overline{x}^2}{(n-1)s^2} = \frac{(n-1)s^2 + n\overline{x}^2}{(n-1)s^2} \\
&= 1 + \Big(\frac{n}{n-1}\Big)\frac{\overline{x}^2}{s^2} = 1 + \Big(\frac{n}{n-1}\Big)\Big(\frac{s}{\overline{x}}\Big)^{-2} \\
&= 1 + \Big(\frac{n}{n-1}\Big)CV^{-2}.
\end{aligned}$$